Treatment of the semiclassical Boltzmann equation for magnetic multilayers

L. G. Pereira, J. L. Duvail, and D. K. Lottis

Citation: Journal of Applied Physics **88**, 4772 (2000); doi: 10.1063/1.1289782
View online: http://dx.doi.org/10.1063/1.1289782
View Table of Contents: http://scitation.aip.org/content/aip/journal/jap/88/8?ver=pdfcov
Published by the AIP Publishing

Treatment of the semiclassical Boltzmann equation for magnetic multilayers

L. G. Pereira[a)]
Instituto de Fisica—UFRGS, Av. Bento Gonçalves 9500, Caixa Postal 15051, 91501-970—Porto Alegre, RS, Brazil

J. L. Duvail
Laboratoire de Physique des Solides, Bât 510, Université Paris-Sud, 91405 Orsay, France

D. K. Lottis
School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455

(Received 30 March 2000; accepted for publication 28 June 2000)

We present an analytical treatment of the Camley–Barnaś theory of the giant magnetoresistance (GMR) in magnetic layered structures and obtain an exact and general expression for the resistivity. We used this expression to evaluate the resistivity and GMR numerically, comparing the results with experimental observations. © *2000 American Institute of Physics.* [S0021-8979(00)08619-9]

I. INTRODUCTION

This report describes an extension of the semiclassical approach originally introduced by Camley and Barnas[1] to modeling magnetoresistance in thin-film multilayers. The basis for the model is an evaluation of the electrical resistivity in a multilayer by means of the Boltzmann transport equation, with recognition given to the asymmetry between the conduction by carriers in the various channels. Another important consideration is that similarly asymmetric (or *spin-dependent*) scattering also takes place at the interface between magnetic and nonmagnetic metals. This approach has been used and improved by various authors in recent years.[1–6]

The Camley–Barnas approach has proven to be very effective in providing theoretical predictions of the magnetoresistive behavior in magnetic multilayers. However, up until now the treatment given has usually been numerical since the large number of interfaces that exist in a multilayer complicate the introduction of boundary conditions for the system as a whole. A common approach is to use an approximation known as the ''infinite multilayer,'' in which the difficulty with boundary conditions at outer interfaces is avoided by determining the conductivity of an idealized multilayer with an infinite number of periods.

In this article we present a detailed development of a calculation in which we obtain a compact analytical form for the conductivity in a thin-film multilayer composed of N layers, where N is an arbitrary integer. The contribution from each layer can be examined independently, but of course the electronic conduction in such a system is characterized by the influence that physical processes within one layer have on the conduction in other layers. We also consider, in establishing the Boltzmann equation for the system, the possibility of spin-mixing effects (and therefore thermal effects) are included in the calculations that we will present in what follows. This article is divided in the following manner: in the next section, we formulate the problem to be solved. Next, we present the central aspects of the calculation of the conductivity in a thin-film metallic multilayer. In the fourth section we incorporate the effect of thermal fluctuations on free mean path and interface coefficients. We finish with a comparison between results obtained using our calculation and experimental data.

II. GENERAL CONSIDERATIONS

We define the magnetoresistance (MR) by

$$\mathrm{MR}(\%)=100\frac{\sigma_P-\sigma_{\mathrm{AP}}}{\sigma_{\mathrm{AP}}}, \tag{1}$$

where σ_P and σ_{AP} correspond, respectively, to conductivity in parallel and antiparallel configurations of the magnetization of the magnetic layers. This definition, written in terms of the conductivities for the two magnetic configurations, is easily shown to be equivalent to the more commonly used expression containing the corresponding resistivities. We can find σ by the relation

$$\mathbf{J}=\boldsymbol{\sigma}\cdot\mathbf{E}, \tag{2}$$

where $\mathbf{E}$ is the applied electrical field and $\mathbf{J}_\eta$ is the current density and is given by

$$\mathbf{J}_\eta=-e\left(\frac{m}{\hbar}\right)^3\int \mathbf{v}f_\eta(\mathbf{r},\mathbf{v})dv_x dv_y dv_z dz, \tag{3}$$

where the $f_\eta(\mathbf{r},\mathbf{v})$ is the distribution function of the conduction electrons, the index η corresponds to spin direction, e and m are the charge and the mass of an electron, respectively, and $\hbar$ is Planck's constant. We have taken the layers parallel to the (x,y) plane, as shown in Fig. 1.

The electron distribution function for ηth spin is written in the form

$$f_\eta(z,\mathbf{v})=f_\eta^0(\mathbf{v})+g_\eta(z,\mathbf{v}), \tag{4}$$

where $f_\eta^0(\mathbf{v})$ is the equilibrium distribution in the absence of an electrical field, and $g_\eta(z,\mathbf{v})$ is a correction to the distri-

[a)]Author to whom correspondence should be addressed; electronic mail: lgp@if.ufrgs.bv

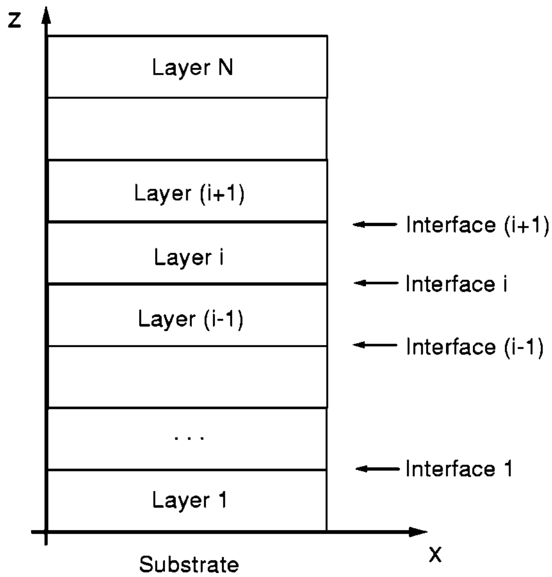

FIG. 1. Schematic diagram of arbitrary multilayers.

bution function due to scattering. Here $g_\eta(\mathbf{r},\mathbf{v})$ was substituted by $g_\eta(z,\mathbf{v})$, because the system is isotropic in respect to x and y coordinates. The linear-response Boltzmann transport equation, for an electric field in the x direction, is given by

$$\frac{\mathbf{F}}{m}\,\partial_{\mathbf{v}} f^0_\eta(\mathbf{v})+\mathbf{v}\partial_{\mathbf{r}} g_\eta(z,\mathbf{v})=\partial_t g_\eta(z,\mathbf{v})_{\mathrm{col}}. \tag{5}$$

Now, it will be necessary to determine the boundary conditions at surfaces and interfaces

$$\begin{aligned} &g^-_\eta(z=0,\mathbf{v})=0,\\ &g^+_\eta(z=\mathrm{total},\mathbf{v})=0,\\ &g^+_\eta(z=z_i,\mathbf{v})=Q^\eta_i g^+_\eta(z=z_{i-1},\mathbf{v}),\\ &g^-_\eta(z=z_i,\mathbf{v})=Q^\eta_i g^-_\eta(z=z_{i+1},\mathbf{v}),\end{aligned} \tag{6}$$

where the Q^η_i are the transmission coefficients and the sign $+$ and $-$ stands for the direction of the electron velocities with respect to the z axis. We suppress this symbol because we consider the multilayer symmetric in velocity with respect to the z axis.

III. CALCULATING THE CONDUCTIVITY

The collision term for the system with spin mixing can be written as in Ref. 3

$$\partial_t g_\eta(z,\mathbf{v})_{\mathrm{col}}=-\frac{g_\eta(z,\mathbf{v})}{\tau^\eta}-\frac{g_\eta(z,\mathbf{v})-g_{-\eta}(z,\mathbf{v})}{\tau^{\uparrow\downarrow}}. \tag{7}$$

Here τ_η is the relaxation time for electrons of spin η and $\tau^{\uparrow\downarrow}$ describes the contribution of the spin-flip processes. Combining expression (7) with Eq. (5) yields

$$\frac{E}{m}e\,\partial_v f^0(\mathbf{v})+v\,\partial_z g_\eta(z,\mathbf{v})=-\frac{g_\eta(z,\mathbf{v})}{\tau_\eta}-\frac{g_\eta(z,\mathbf{v})-g_{-\eta}(z,\mathbf{v})}{\tau^{\uparrow\downarrow}}, \tag{8}$$

which is actually a system of two coupled equations, one for spin up, and the other for spin down.

The general solution of this system is

$$g_{\eta i}(z,\mathbf{v})=\left(\frac{E}{m}e\,\partial_v f^0(\mathbf{v})\right)B_{\eta i}(1-A_{\eta i}e^{-q_{\eta i}z}), \tag{9}$$

where

$$\lambda^\eta_i=v_x\tau_{i\eta},$$

$$q_{\eta i}=-\frac{1}{v_x}\left[\frac{1}{\lambda^\eta_i}+\frac{1}{\lambda^{-\eta}_i}+\frac{2}{\lambda^{\uparrow\downarrow}_i}-\sqrt{\left(\frac{1}{\lambda^\eta_i}-\frac{1}{\lambda^{-\eta}_i}\right)^2+\left(\frac{2}{\lambda^{\uparrow\downarrow}_i}\right)^2}\,\right],$$

$$B_{\eta i}=\frac{\dfrac{1}{\lambda^\eta_i}+\dfrac{2}{\lambda^{\uparrow\downarrow}_i}}{\dfrac{1}{\lambda^\eta_i\lambda^{-\eta}_i}+\dfrac{1}{\lambda^\eta_i\lambda^{\uparrow\downarrow}_i}+\dfrac{1}{\lambda^{-\eta}_i\lambda^{\uparrow\downarrow}_i}},$$

where $A_\eta(v)$ is a constant resulting from the integration and is a characteristic of any layer i. (See the Appendix for a more detailed discussion of this result.)

From Eqs. (4) and (9) we obtain

$$\sigma_\eta=e^2\frac{m^2}{\hbar^3}\sum_i B_{\eta,i}\int v_x\partial_v f^0(\mathbf{v})\times[1-A_{\eta i}(\mathbf{v})e^{-q_{\eta i}z}]dv_x dv_y dv_z dz. \tag{10}$$

By changing from Cartesian velocity components (v_x,v_y,v_z) to spherical ''coordinates'' for the velocities, (v,φ,θ) and then integrating over φ, θ, z we obtain

$$\sigma_\eta=K\sum_i B_{\eta,i}\left[\frac{4}{3}\Delta z_i-q_{\eta,i}\int d\mu F(\mu)\times A_{\eta,i}(\mu)(e^{-q_{\eta i}z_i\mu}-e^{-q_{\eta i}z_j\mu})\right], \tag{11}$$

where v_F is the Fermi velocity and

$$\begin{aligned} &K=\pi e^2 v_F^2\frac{m^2}{\hbar^3},\\ &F(\mu)=(1-\mu^{-2})\mu^{-3},\\ &\Delta z_i=z_j-z_i,\\ &j=i+1,\end{aligned} \tag{12}$$

$$\mu=\frac{1}{\cos\theta}. \tag{13}$$

The expression (11) cannot be integrated immediately, because the dependence of $A_{\eta i}$ in v is not explicit. To find a general form of $A_{\eta i}$ as a function of v, we will use the boundary conditions given in (6)

$$g_{\eta 1}(z=0,v_z)=0 \Rightarrow B_{\eta 1}[1-A_{\eta 1}(v_z)]=0,$$

$$A_{\eta 1}=c^1_{\eta 1}e^{a^1_{\eta 1}/v_z},$$

where

$$c^1_{\eta 1}=1, \quad a^1_{\eta 1}=0, \tag{14}$$

and

$$g_{\eta j}(z=z_i,v_z)=Q^{\eta}_i g_{\eta i}(z=z_i,v_z),$$

$$B_{\eta j}(1-A_{\eta j}e^{-q_{\eta j}z_i/v_z})=Q^{\eta}_i B_{\eta i}(1-A_{\eta i}e^{-q_{\eta i}z_i/v_z}), \tag{15}$$

$$A_{\eta j}=\alpha_{\eta ij}e^{q_{\eta j}z_i/v_z}+\beta_{\eta ij}A_{\eta i}e^{(q_{\eta j}-q_{\eta i})(z_i/v_z)},$$

where

$$\alpha_{\eta ij}=1-\beta_{\eta ij}, \quad \beta_{\eta ij}=Q_{\eta i}\frac{B_{\eta i}}{B_{\eta j}}. \tag{16}$$

Beginning with Eq. (15) it is possible to write expressions for $A_{\eta 2}, A_{\eta 3}, A_{\eta 4} \ldots$.

For $A_{\eta 2}$:

$$A_{\eta 2}=\alpha_{\eta 12}e^{q_{\eta 2}z_1/v_z}+\beta_{\eta 12}A_{\eta 1}e^{(q_{\eta 2}-q_{\eta 1})(z_1/v_z)},$$

$$A_{\eta 2}=c^1_{\eta 2}e^{a^1_{\eta 2}/v_z}+c^2_{\eta 2}e^{a^2_{\eta 2}/v_z}, \tag{17}$$

$$A_{\eta 2}=\sum_{n=1}^{2} c^n_{\eta 2}e^{a^n_{\eta 2}/v_z},$$

where

$$\begin{aligned} c^1_{\eta 2}&=\alpha_{\eta 12},\\ c^2_{\eta 2}&=\beta_{\eta 12}c^1_{\eta 1},\\ a^1_{\eta 2}&=q_{\eta 2}z_1,\\ a^2_{\eta 2}&=a^1_{\eta 1}+(q_{\eta 2}-q_{\eta 1})z_1. \end{aligned} \tag{18}$$

For $A_{\eta 3}$:

$$A_{\eta 3}=\alpha_{\eta 23}e^{q_{\eta 3}z_2/v_z}+\beta_{\eta 23}A_{\eta 2}e^{(q_{\eta 3}-q_{\eta 2})(z_2/v_z)},$$

$$\begin{aligned} A_{\eta 3}&=\alpha_{\eta 23}e^{q_{\eta 3}z_2/v_z}+\beta_{\eta 23}(c^1_{\eta 2}e^{b(a^1_{\eta 2}/v_z)}+c^2_{\eta 2}e^{a^2_{\eta 2}/v_z})\\ &\quad\times e^{(q_{\eta 3}-q_{\eta 2})(z_2/v_z)}, \end{aligned} \tag{19}$$

$$A_{\eta 3}=c^1_{\eta 3}e^{b(a^1_{\eta 3}/v_z)}+c^2_{\eta 3}e^{a^2_{\eta 3}/v_z}+c^3_{\eta 3}e^{a^3_{\eta 3}/v_z},$$

$$A_{\eta 3}=\sum_{n=1}^{3} c^n_{\eta 3}e^{a^n_{\eta 3}/v_z},$$

where

$$\begin{aligned} c^1_{\eta 3}&=\alpha_{\eta 23},\\ c^2_{\eta 3}&=\beta_{\eta 23}c^1_{\eta 2},\\ c^3_{\eta 3}&=\beta_{\eta 23}c^2_{\eta 2},\\ a^1_{\eta 3}&=q_{\eta 3}z_2,\\ a^2_{\eta 3}&=a^1_{\eta 2}+(q_{\eta 3}-q_{\eta 2})z_2,\\ a^3_{\eta 3}&=a^2_{\eta 2}+(q_{\eta 3}-q_{\eta 2})z_2. \end{aligned} \tag{20}$$

And finally for $A_{\eta 4}$:

$$A_{\eta 4}=\alpha_{\eta 34}e^{q_{\eta 4}z_3/v_z}+\beta_{\eta 34}A_{\eta 3}e^{(q_{\eta 4}-q_{\eta 3})(z_2/v_z)},$$

$$\begin{aligned} A_{\eta 4}&=\alpha_{\eta 34}e^{q_{\eta 4}z_3/v_z}\beta_{\eta 34}(c^1_{\eta 3}e^{b(a^1_{\eta 3}/v_z)}+c^2_{\eta 3}e^{a^2_{\eta 3}/v_z}\\ &\quad+c^3_{\eta 3}e^{a^3_{\eta 3}/v_z})e^{(q_{\eta 4}-q_{\eta 4})(z_3/v_z)}, \end{aligned} \tag{21}$$

$$A_{\eta 4}=c^1_{\eta 4}e^{b(a^1_{\eta 4}/v_z)}+c^2_{\eta 4}e^{a^2_{\eta 4}/v_z}+c^3_{\eta 4}e^{a^3_{\eta 4}/v_z}+c^4_{\eta 4}e^{a^4_{\eta 4}/v_z},$$

$$A_{\eta 4}=\sum_{n=1}^{4} c^n_{\eta 4}e^{a^n_{\eta 4}/v_z},$$

where

$$\begin{aligned} c^1_{\eta 4}&=\alpha_{\eta 34},\\ c^2_{\eta 4}&=\beta_{\eta 34}c^1_{\eta 3},\\ c^3_{\eta 4}&=\beta_{\eta 34}c^2_{\eta 3},\\ c^4_{\eta 4}&=\beta_{\eta 34}c^3_{\eta 3},\\ a^1_{\eta 4}&=q_{\eta 4}z_3,\\ a^2_{\eta 4}&=a^1_{\eta 3}+(q_{\eta 4}-q_{\eta 3})z_3,\\ a^3_{\eta 4}&=a^2_{\eta 3}+(q_{\eta 4}-q_{\eta 2})z_3,\\ a^4_{\eta 4}&=a^3_{\eta 3}+(q_{\eta 4}-q_{\eta 2})z_3. \end{aligned} \tag{22}$$

From the results for $A_{\eta 1}$, $A_{\eta 2}$, $A_{\eta 3}$, and $A_{\eta 4}$, one may deduce a general expression for $A_{\eta j}$:

$$A_{\eta j}(v_z)=\sum_{n=1}^{j} c^n_{\eta j}e^{a^n_{\eta j}/v_z}, \tag{23}$$

where

$$\begin{aligned} a^1_{\eta 1}&=0,\\ a^1_{\eta j}&=z_i q_{\eta j},\\ a^n_{\eta j}&=a^{n-1}_{\eta i}+z_i(q_{\eta j}-q_{\eta i}),\\ c^1_{\eta 1}&=1,\\ c^1_{\eta j}&=\alpha_{ij\eta},\\ c^n_{\eta j}&=\beta_{ij\eta}c^{n-1}_{i\eta}. \end{aligned} \tag{24}$$

Expression (23) for the $A_{\eta i}$ are now in the desired form, that is, with an explicit dependence on v_z. This result will allow the analytical integration of (4).

Using the earlier mentioned relation for the constants A, Eq. (11) can be rewritten as

$$\sigma_\eta=\sum_{\eta,i} B_{\eta i}\left[\frac{4}{3}\Delta z_i-\sum_{n=1}^{i} q_{\eta i}c^n_{\eta j}\int d\mu F(\mu) \times(e^{\Gamma^{1,n}_{i\eta}\mu}-e^{\Gamma^{2,n}_{i\eta}\mu})\right], \tag{25}$$

where $\Gamma^{l,n}_{i\eta}$ is given by

$$\Gamma^{1,n}_{i\eta}=a^n_{\eta i}-q_{\eta i}z_j, \tag{26}$$

$$\Gamma^{2,n}_{i\eta}=a^n_{\eta i}-q_{\eta i}z_i. \tag{27}$$

We can integrate this equation

$$\sigma_\eta=\sum_i^N B_{\eta i}\left[\frac{4}{3}\Delta z_i-q_{\eta i}\sum_n^i c^n_{\eta i}(L^{1,n}_{i\eta}-L^{2,n}_{x\eta}+M^{1,n}_{i\eta}-M^{2,n}_{x\eta})\right], \tag{28}$$

$$L^{m,n}_{i\eta}=[\tfrac{1}{4}-\tfrac{5}{12}\Gamma^{m,n}_{i\eta}-\tfrac{1}{24}(\Gamma^{m,n}_{i\eta})^2+\tfrac{1}{24}(\Gamma^{m,n}_{i\eta})^3]e^{-\Gamma^{m,n}_{i\eta}},$$

$$M^{m,n}_{i\eta}=\frac{(\Gamma^{mn}_{i\eta})^2}{2}\left(1-\frac{1}{12}(\Gamma^{mn}_{i\eta})^2\right)\int\frac{e^{-\Gamma^{mn}_{i\eta}\mu}}{\mu}d\mu,$$

$$x=i-1. \tag{29}$$

Equation (28) is an analytic expression that can be used along with Eq. (1) to determine the magnetoresistance. In this manner we can simultaneously obtain values for the resistivity, conductance, and magnetoresistance of the system.

IV. TEMPERATURE DEPENDENCE OF THE PARAMETERS *Q* AND λ

In order to apply the formalism developed in the preceding section to temperature-dependent systems, it is necessary to examine the thermal evolution of the parameters that appear in the calculation. For the nonmagnetic layer, the resistivity is written as

$$\rho(T)=\rho(4.2\ \text{K})+\delta\rho(T), \tag{30}$$

where $\delta\rho(T)$ is related to the phonon scattering and is a characteristic of each metallic element.

For the magnetic layers, the resistivity is also written in the form given by Eq. (30) and the dependence of the free-mean path on temperature is given by[9,10]

$$\rho=\frac{\rho^\uparrow\rho^\downarrow+\rho^{\uparrow\downarrow}(\rho^\uparrow+\rho^\downarrow)}{\rho^\uparrow+\rho^\downarrow+4\rho^{\uparrow\downarrow}},\qquad \rho^\downarrow=\alpha\rho^\uparrow, \tag{31}$$

where ρ and α are experimental data, see Refs. 12 and 13.

We have related the resistivity and mean-free-path using

$$\rho_\eta(\mu\Omega\ \text{cm})\lambda_\eta(\text{Å})=1940,$$

see Ref. 1.

We suppose the mechanism of the dependence on temperature for transmission coefficients through the interfaces has the form

$$Q(T)=Q(0)+\omega T^\gamma, \tag{32}$$

where $Q(0)$ is the diffusion term at 0 K and ω and γ are deduced from the fits.[11]

The use of these two functional forms for $\lambda(T)$ and $Q(T)$ in the temperature will preserve the thermal evolution of the simulations and allow to work with experimental values, once the resistivity of the materials as a function of the temperature is known. In other words, once the fundamental parameters of the simulation have been defined at 0 K, the system will evolve on its own, without the need of external interference in the program.

V. EXAMPLES

In order to illustrate the results found in the previous sections, we apply the results obtained via Eqs. (1) and (28) to reproduce some experimental results already known in the literature. The parameters for λ and Q used in this example are in perfect agreement with those reported in the literature (for an example, see Ref. 7).

The first example to be presented shows the variation of the magnetoresistance with the number of layers. The experimental data are from work published by Parkin,[8–13] where the author presents a set of experimental data for the variation of the magnetoresistance in the Si/Cr (9 Å) [Fe (18 Å)/Cr (9 Å)]$_N$/Cr (9 Å) system with the number of layers. Here we shall take as basis for comparison the empirical equation for the magnetoresistance obtained by the author

$$\text{MR}=168\,\frac{2N-2}{T_N-15N}, \tag{33}$$

where N is the number of bilayers and T_N is the total thickness of the multilayer. To reproduce Parkin's experimental results, we use the following set of parameters:

$$\lambda_{\text{Fe}}=7.6\ \text{nm},$$

$$\lambda_{\text{Cr}}=4.5\ \text{nm},$$

$$Q^\uparrow=1,$$

$$Q^\downarrow=0.1.$$

Figure 2 shows the experimental curve [solid line ob-

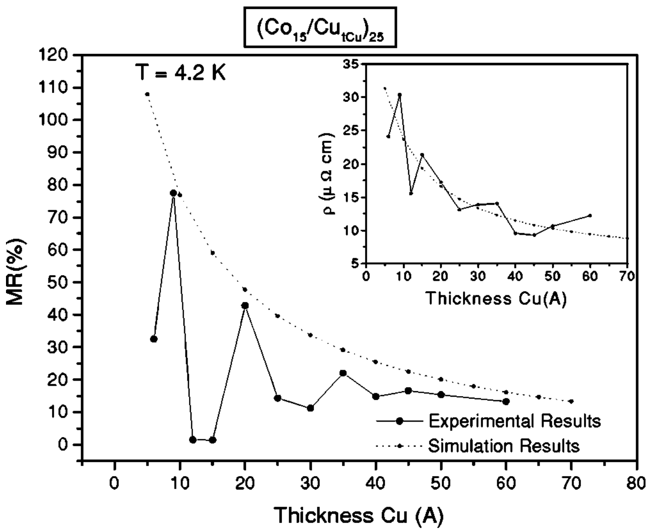

FIG. 3. Magnetoresistance vs Cu spacer layer thickness at 4.2 K for several series of multilayers of the form [Co (15 Å)/Cu (t_{Cu} Å)]$_{25}$. Inset: the dependence of resistivity vs t_{Cu}.

tained from Parkin's phenomenological expression (33)], along with the results obtained using Eq. (28). Note that there is good agreement between the results.

In what follows, we present a comparative set of experimental and computational results for [Co (15 Å)/Cu (9 Å)]$_{25}$ system (see Ref. 14). We start by showing the variation of the resistivity and magnetoresistance for the copper layer thickness evolution (t_{Cu}=6–60 Å) for two different temperatures, 4.2 and 300 K: Figs. 3 and 4, respectively.

Finally, in Fig. 5, the dependence of the magnetoresistance in a large range of temperatures ($T=4.2-300$ K), was shown.

The parameters used in these three last cases are

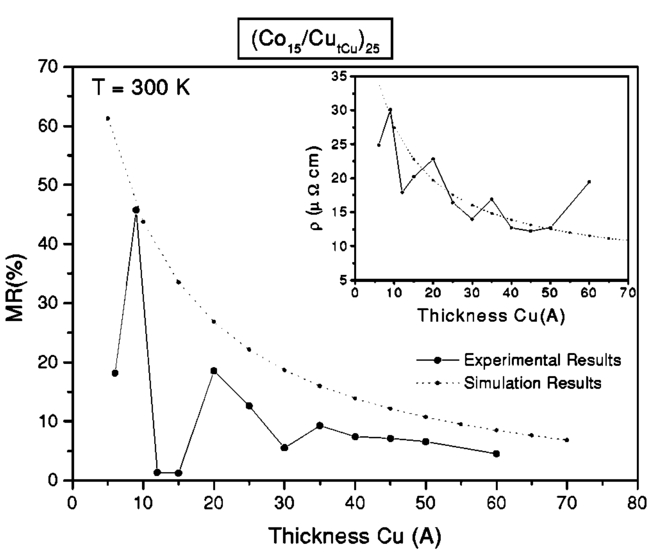

FIG. 4. Magnetoresistance vs Cu spacer layer thickness at room temperature for several series of multilayers of the form [Co (15 Å)/Cu(t_{Cu} Å)]$_{25}$. Inset: the dependence of resistivity vs t_{Cu}.

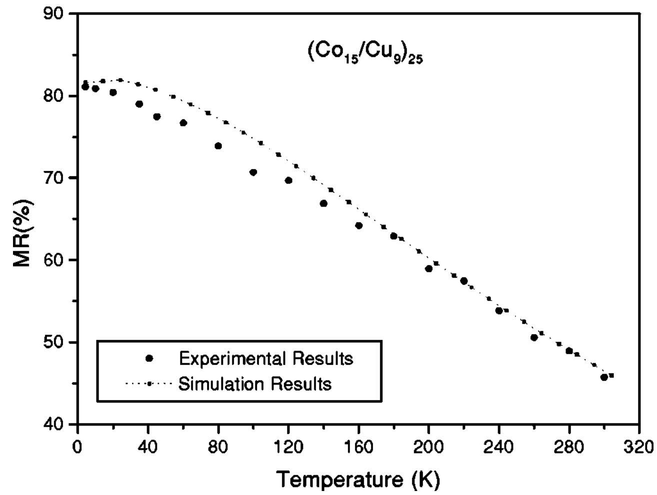

FIG. 5. Calculated thermal variation of GMR in a [Co (15 Å)/Cu (9 Å)]$_{25}$ (dashed line). Dots correspond to the experimental results.

$$\lambda_{Co}^{\uparrow}=120 \text{ Å},$$

$$\lambda_{Co}^{\downarrow}=10 \text{ Å},$$

$$\lambda_{Cu}=210 \text{ Å},$$

$$Q^{\uparrow}=0.98,$$

$$Q^{\downarrow}=0.2.$$

VI. CONCLUSIONS

Using the semiclassical formalism for the electrical conductivity σ in multilayered thin films, we have developed a calculation that resulted in an analytical expression for σ. The greatest difficulty in obtaining this expression was finding a form for $A(v)$ that could be integrated, that is, that explicitly showed the velocity dependence. The quantity $A(v)$ is obtained from the boundary conditions and depends on the interface under consideration. Since an N-layer multilayer has $N-1$ interfaces, the function $A(v)$ will have $N-1$ different forms. By means of a mathematical manipulation of the boundary conditions we were able to find a compact expression that represents $A(v)$ at any interface in the multilayer and that explicitly shows its velocity dependence, thus allowing a general expression for the conductivity to be obtained by integration.

In order to encompass the greatest number of situations possible, we have included the temperature dependence. This was done directly at the initial formulation, that is, the spin-mixing term was included in the Boltzmann equation. The system's thermal evolution is incorporated in the simulations by using experimental results obtained from resistivity studies of bulk metals and alloys.

The simulations were developed using parameters that are well known in the literature and showed good agreement with experimental data. In addition to allowing for numerical studies, the general expression for the electrical conductivity can be explored in its analytical form.

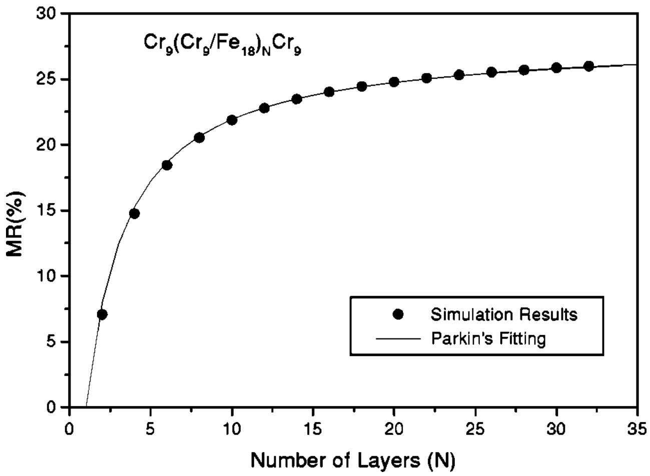

FIG. 2. Dependence of magnetoresistance on number of bilayers for Si/Cr (9 Å) [Fe (18 Å)/Cr (9 Å)]$_N$/Cr (9 Å). Full line corresponds to Parkin's results and dots are the calculated bilayers variation.

The model that has been developed here is limited but can be improved by taking certain physical processes into account, such as reflections at the interfaces.

ACKNOWLEDGMENTS

The authors thank A. Fert and B. Dieny for helpful discussion. This work has been supported by the Brazilian agencies CAPES and CNPq.

APPENDIX

The Boltzmann system is

$$\frac{E}{m} e\,\partial_v f^0(\mathbf{v}) + v\,\partial_z g_\eta(z,\mathbf{v}) = -\frac{g_\eta(z,\mathbf{v})}{\tau_\eta} - \frac{g_\eta(z,\mathbf{v}) - g_{-\eta}(z,\mathbf{v})}{\tau^{\uparrow\downarrow}}, \tag{A1}$$

or

$$g'_\eta + B g_\eta + C g_{-\eta} + A = 0, \tag{A2}$$

$$g'_{-\eta} + D g_{-\eta} + E g_\eta + A = 0, \tag{A3}$$

where $' \equiv \partial_z$. To find the solution of the system we derived one of the equations in z:

$$g''_\eta + B g'_\eta + C g'_{-\eta} = 0, \tag{A4}$$

and it uses (A2) and (A3) to eliminate $g'_{-\eta}$. The new equation presents the following form:

$$g''_\eta + \alpha g'_\eta + \beta g_\eta + \gamma = 0. \tag{A5}$$

The general solution of the new equation is

$$g_\eta = D^1_\eta(1 - A^1_\eta e^{p^1_\eta z/v_z}) + D^2_\eta(1 - A^2_\eta e^{p^2_\eta z/v_z}). \tag{A6}$$

To determine which expression corresponds to $g_\uparrow$ and $g_\downarrow$, we examine the limit $\tau^{\uparrow\downarrow} \to \infty$, which is just the nonspin-flip problem

$$\lim_{\tau^{\uparrow\downarrow}\to\infty} = g_{\eta i}(z,\mathbf{v}) \Rightarrow \frac{E}{m} e\,\partial_{v_x} f^0(\mathbf{v})\, \tau_{\eta i}(1 - A_{\eta i}(\mathbf{v}) e^{-z/\tau_{\eta i} v_z}). \tag{A7}$$

In this limit we have

$$D_0 = \frac{E}{m} e\,\partial_{v_x} f^0(\mathbf{v})\,\tau,$$

$$D^\uparrow = \frac{E}{m} e\,\partial_{v_x} f^0(\mathbf{v}) \frac{\dfrac{1}{\tau^\downarrow} + \dfrac{2}{\tau^{\uparrow\downarrow}}}{\dfrac{1}{\tau^\uparrow \tau^\downarrow} + \dfrac{1}{\tau^\uparrow \tau^{\uparrow\downarrow}} + \dfrac{1}{\tau^\downarrow \tau^{\uparrow\downarrow}}},$$

$$D^\downarrow = \frac{E}{m} e\,\partial_{v_x} f^0(\mathbf{v}) \frac{\dfrac{1}{\tau^\uparrow} + \dfrac{2}{\tau^{\uparrow\downarrow}}}{\dfrac{1}{\tau^\uparrow \tau^\downarrow} + \dfrac{1}{\tau^\uparrow \tau^{\uparrow\downarrow}} + \dfrac{1}{\tau^\downarrow \tau^{\uparrow\downarrow}}}, \tag{A8}$$

$$p_0 = -\frac{1}{\tau},$$

$$p^\uparrow = -\left[\frac{1}{\tau^\uparrow} + \frac{1}{\tau^\downarrow} + \frac{2}{\tau^{\uparrow\downarrow}} - \sqrt{\left(\frac{1}{\tau^\uparrow} - \frac{1}{\tau^\downarrow}\right)^2 + \left(\frac{2}{\tau^{\uparrow\downarrow}}\right)^2}\right],$$

$$p^\downarrow = -\left[\frac{1}{\tau^\uparrow} + \frac{1}{\tau^\downarrow} + \frac{2}{\tau^{\uparrow\downarrow}} + \sqrt{\left(\frac{1}{\tau^\uparrow} - \frac{1}{\tau^\downarrow}\right)^2 + \left(\frac{2}{\tau^{\uparrow\downarrow}}\right)^2}\right],$$

where D_0 and q_0 correspond to the nonmagnetic layers.

We can write (A6) in the general form

$$g_{\eta i}(z,\mathbf{v}) = D_\eta(1 - A_\eta e^{p_\eta z/v_z}). \tag{A9}$$

[1] R. E. Camley and J. Barnas, Phys. Rev. Lett. **63**, 664 (1989).
[2] J. Barnas, A. Fuss, R. E. Camley, P. Grünberg, and W. Zinn, Phys. Rev. B **42**, 8110 (1990).
[3] J. L. Duvail, A. Fert, L. G. Pereira, and D. K. Lottis, J. Appl. Phys. **75**, 7070 (1994).
[4] A. Barthelemy and A. Fert, Phys. Rev. B **43**, 13124 (1991).
[5] B. L. Johnson and R. E. Camley, Phys. Rev. B **44**, 9997 (1991).
[6] R. Q. Hood and L. M. Falicov, Phys. Rev. B **46**, 8287 (1992).
[7] B. Dieny, J. Phys.: Condens. Matter **4**, 8009 (1992).
[8] S. S. P. Parkin, in Proceedings of the Workshop on Magnetic Thin Films, Brasilia, September 1993 (unpublished).
[9] A. Fert and I. A. Campbell, J. Phys. F: Met. Phys. **6**, 849 (1976).
[10] I. A. Campbell and A. Fert, in *Ferromagnetic Materials*, edited by E. P. Wohlfarth (North-Holland, Amsterdam, 1982), Vol. 3, p. 769.
[11] B. Dieny, A. Granovsky, A. Vedyaev, N. Ryzhanova, C. Cowache, and L. G. Pereira, J. Magn. Magn. Mater. **151**, 378 (1995).
[12] G. K. Write and S. B. Woods, Philos. Trans. R. Soc. London, Ser. A **251**, 273 (1959).
[13] B. Loegel and F. Gautier, J. Phys. Chem. Solids **32**, 2723 (1971).
[14] D. H. Mosca, F. Petroff, A. Fert, P. Schroeder, W. P. Pratt, Jr., R. Loloee, and S. Lequien, J. Magn. Magn. Mater. **94**, L1 (1991).