UNIVERSIDADE FEDERAL DO RIO GRANDE DO SUL
INSTITUTO DE INFORMÁTICA
PROGRAMA DE PÓS-GRADUAÇÃO EM COMPUTAÇÃO

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Read-polarity-once functions

Thesis presented in partial fulfillment
of the requirements for the degree of
Master of Science in Computer Science

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Porto Alegre, July 2012

CIP – CATALOGAÇÃO NA PUBLICAÇÃO

Callegaro, Vinicius

Read-polarity-once functions / Vinicius Callegaro. – Porto Alegre: PPGC da UFRGS, 2012.

65 f.:il.

Thesis (Master) –Universidade Federal do Rio Grande do Sul.

Programa de Pós-Graduação em Computação, Porto Alegre, BR–RS, 2010. Advisor: André Inácio Reis; Co-advisor: Renato Perez Ribas.

1. Boolean functions. 2. Factoring. 3. Logic synthesis. 4. Incompletely specified functions 5. Read-once functions. 6. Read-polarity-once functions. I. Reis, André Inácio. II. Ribas, Renato Perez. III. Read-polarity-once functions.

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ACKNOWLEDGEMENTS

I would like to dedicate this thesis to my parents Aldo and Marta, to my sister Thalita and to my fiancée Monica, who were essential to me during the master years.

I am grateful to my advisor Andre Inácio Reis and my co-advisor Renato Perez Ribas for their knowledge and effort to make this thesis possible.

Special thanks go to my colleagues in the Logic Circuit Synthesis (LogiCS) lab who helped me with discussions, ideas, and fun.

I would like to thank the members of my thesis committee, professors Raul Fernando Weber, Sérgio Bampi and Marcelo de Oliveira Johann who helped me to improve this work in several ways. Special acknowledgments go to Raul Fernando Weber by his pioneering classes in the study of logic synthesis in our university.

The research presented in the thesis was supported by Nangate under a Nangate/UFRGS research agreement, by CAPES and CNPq Brazilian funding agencies, by FAPERGS under grant 11/2053-9 (Pronem), and by the European Community's Seventh Framework Programme under grant 248538 – Synaptic.

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LIST OF ABBREVIATIONS AND ACRONYMS

BDD	Binary Decision Diagram
CMOS	Complementary Metal Oxide Semiconductor
CSF	Completely Specified Boolean Function
EDA	Electronic Design Automation
GF	Good Factor
HDL	Hardware Description Language
IPOS	Irredundant Sum-Of-Products
IROF	Is Read-Once Function
ISF	Incompletely Specified Boolean Functions
ISF2RO	Incompletely Specified Boolean Functions to *read-once*
ISF2RPO	Incompletely Specified Boolean Functions to *read-polarity-once*
ISOP	Irredundant Sum-Of-Products
JPHI	John P. Hayes Improved
LA	Logic Arrangement
NPN	Negation-Permutation-Negation
POS	Product-Of-Sums
QF	Quick Factor
RO	Read-Once
ROBDD	Reduced and Ordered Binary Decision Diagram
RPO	Read-Polarity-Once
RTL	Register Transfer Level
SOP	Sum-Of-Products
SPFD	Sets of Pairs of Functions to be Distinguished
VHDL	VHSIC Hardware Description Language
VHSIC	Very-High-Speed Integrated Circuits

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ABSTRACT

Efficient exact factoring algorithms are limited to *read-once* functions, in which each variable appears once in the final Boolean equation. However, those algorithms present two main constraints: (1) they do not consider incompletely specified Boolean functions; and (2) they are not suitable for binate functions. To overcome the first drawback, it is proposed an algorithm that finds *read-once* formulas for incompletely specified Boolean functions, whenever possible. With respect to the second limitation, a domain transformation that splits existing binate variables into two independent unate variables is presented. Such domain transformation leads to incompletely specified Boolean functions, which can be efficiently factored by applying the proposed algorithm. The combination of both contributions gives optimal results for a novel broader class of Boolean functions named as *read-polarity-once* functions, where each polarity (positive or negative) of a variable appears at most once in the factored form. Experimental results over ISCAS'85 benchmark circuits have shown that *read-polarity-once* functions are significantly more frequent than *read-once* functions, for which many works have already been devoted in the literature.

Keywords: Boolean function, factoring, logic synthesis, incompletely specified functions, read-once function, read-polarity-once function.

Funções *read-polarity-once*

RESUMO

Algoritmos exatos para fatoração estão limitados a funções Booleanas *read-once*, onde cada variável aparece uma vez na equação final. No entanto, estes algoritmos apresentam duas restrições principais: (1) eles não consideram funções Booleanas incompletamente especificadas, e (2) eles não são adequados para as funções *binate*. Para superar o primeiro inconveniente, é proposto um algoritmo que encontra equações *read-once* para funções Booleanas incompletamente especificadas, sempre que possível, é proposto. Com respeito à segunda limitação, é apresentada uma transformação de domínio que divide variáveis *binate* existentes em duas variáveis unate independentes. Tal transformação de domínio conduz a funções Booleanas incompletamente especificadas, que podem ser eficientemente fatoradas mediante a aplicação do algoritmo proposto. A combinação das duas contribuições dá resultados ótimos para uma nova classe de funções Booleanas chamada *read-polarity-once*, onde cada polaridade (positiva ou negativa) de uma variável aparece no máximo uma vez na forma fatorada da expressão Booleana. Resultados experimentais sobre circuitos ISCAS'85 mostrou que funções *read-polarity-once* são significativamente mais frequentes em circuitos reais quando comparado com a classe de funções *read-once*, a qual muitos trabalhos já foram dedicados na literatura.

Palavras-chave: Função Booleana, fatoração, síntese lógica, funções Booleanas incompletamente especificadas, funções *read-once*, funções *read-polarity-once.*

1 INTRODUCTION

The circuit synthesis design flow is usually divided into three major steps: architectural synthesis, logic synthesis and physical synthesis (MICHELI, 1994). The architectural synthesis, often called high-level synthesis, consists of transforming an algorithmic description of a desired behavior into a hardware format that implements that behavior, as in RTL (Register Transfer Level) format. Usually, those algorithmic descriptions are represented in a C like format (e.g. System C) or a behavioral HDL (Hardware Description Language) (e.g. VHDL and Verilog) format.

The logic synthesis process has been one of the most commercially successful areas of electronic design automation (*EDA*). This commercial success indicates that all the digital devices that we use in our day-to-day life have been designed with a set of logic synthesis tools. The logic synthesis task consists of several steps. These steps may be different according to the nature of the circuit, e.g. sequential or combinational. The goal of logic synthesis is to determine the primitive structure of a circuit, i.e., its gate level representation. It is typically divided in three phases: technology independent optimizations, technology mapping and technology dependent optimizations. The first one applies transformations that do not depend on the technology, but on the functional behavior of a Boolean network, e.g. factorization algorithms. Then, the technology mapping phase match portions of the circuit to a cell with technology information. The technology dependent phase applies optimizations in the mapped circuit, such as cell resizing and logic duplication.

The physical synthesis, or geometrical level synthesis, consists mainly of two major tasks: placement of blocks and routing of wires. The former distributes physically the cells whereas the later performs the signal interconnections.

This work addresses the synthesis of Boolean functions in the scope of a digital circuit design flow, more precisely in the logic synthesis phase. However, the focus can be considered broader since this work proposes a new class of Boolean functions that may have application in different areas other than circuit synthesis.

1.1 Motivation

The process of factoring Boolean functions is a fundamental operation in algorithmic logic synthesis (BRAYTON, 1987; HACHTEL; SOMENZI, 2006). Factoring is the process of deriving a parenthesized algebraic equation, or factored form, representing a given logic function, usually provided initially in sum-of-products (SOP) or product-of-sums (POS) form. For instance, $f=a*b+a*c*d+a*c*e$ can be factored into the logically equivalent equation $f=a*(b+c*(d+e))$.

Any given logic function can be represented by different factored equations. The task of factoring Boolean functions into shorter, more compact logically equivalent formulae is one of the basic operations at the early stages of algorithmic logic synthesis (HACHTEL; SOMENZI, 2006). In most design styles, like conventional CMOS gates, the electrical implementation of a Boolean function corresponds directly to its factored equation in terms of literals and device count. Generating an optimum factored form, i.e. the shortest length equation, is an NP-hard problem (GOLUMBIC; MINTZ, 1999). Hence, heuristic algorithms have been developed in order to obtain good factored solutions (BRAYTON, 1987; STANION; SECHEN, 1994; MINTZ; GOLUMBIC, 2005; HACHTEL; SOMENZI, 2006; YOSHIDA; FUJITA, 2011). Some well-known heuristic algorithms include XFactor (MINTZ; GOLUMBIC, 2005), which provides good results but does not guarantee the minimal equations. In (LAWLER, 1964), the author claims to provide the exact factoring. However, Lawler's method is not scalable and becomes impractical even for some functions with only 4 variables. Recently, new approaches have improved the factoring process for exact solutions, but the scalability and runtime still remain the main bottlenecks (YOSHIDA; IKEDA; ASADA, 2006; YOSHIDA; FUJITA, 2011; MARTINS ET AL., 2012).

Efficient exact algorithms exist for a sub-class of functions known as *read-once* (RO) functions (LEE; WANG, 2007; GOLUMBIC; MINTZ; ROTICS, 2008). A Boolean function is considered *read-once* if it can be represented in a factored form where each variable appears only once (GOLUMBIC; MINTZ; ROTICS, 2001). Reviewing the example above, the function $f=a*(b+c*(d+e))$ is RO. In Table 1.1 it is possible to see a sample of Boolean function gathered from the ISCAS benchmarks. The class of RO functions is of special interest in logic synthesis because they are quite frequent in circuit applications.

Table 1.1: The abundance of RO functions in a sample of Boolean function gathered from the ISCAS benchmarks.

	Logic level 2			**Logic level 3**			**Logic level 4**			**Logic level 5**		
Circuit name	No. Eqns	RO eqns No.	RO eqns %	No. Eqns	RO eqns No.	RO eqns %	No. Eqns	RO eqns No.	RO eqns %	No. Eqns	RO eqns No.	RO eqns %
C432	142	79	55	142	91	64	114	19	17	111	18	16
C499	162	0	0	146	0	0	138	0	0	52	0	0
C880	430	312	73	278	157	57	257	103	40	247	52	21
C1355	496	326	66	409	168	41	377	96	25	350	50	14
C1908	822	810	98	734	569	77	648	283	43	361	0	0
C2670	1087	979	90	994	644	64	873	282	32	837	168	20
C3540	1588	1404	88	1550	1119	72	1464	764	52	1334	449	34
C5315	2108	1789	84	1844	1174	64	1805	396	21	1700	204	12
C6288	2159	1696	79	2147	603	28	2114	92	4	2143	109	5
C7552	3404	3290	97	3290	2410	73	3121	328	11	2919	367	14
Total	**12398**	**10685**	**86**	**11534**	**6935**	**60**	**10911**	**2363**	**22**	**10054**	**1417**	**14**

Source: PEER; PINTER, 1995.

However, exact algorithms for RO functions present two important limitations: (1) they do not consider incompletely specified Boolean functions; and (2) they are not suitable for functions with binate variables (see Subsection 2.2.5). In this context, a

question can be raised: "*Since RO functions are quite frequent in real circuit applications, suppose we extend the boundaries of this class, creating a new class of functions, in order to overcome the limitation (1) and (2). How broader is this new class with regard to the RO class? How much more frequent are the functions of this new class in real circuit applications?*" These questions summarize our motivation.

1.2 Objective

The objective of this thesis is to transcend the boundaries of the RO class of functions. One way of doing that is to create a new broad class of Boolean functions, which can deal with binate functions and where factorized forms can be guaranteed minimal. An approach is to split the positive and negative phases of binate variables into two independent variables. This way, it is possible to represent the positive phase in one variable and the negative phase into another variable, being these two variables unate and independent each other. This domain transformation leads to ISBF. Therefore, it is interesting to develop a method that is able to factorize ISBF into RO forms, which are known by resulting minimal forms. The combination of both strategies can lead to a novel broader class of functions called *read-polarity-once* (RPO) functions, where each polarity (positive or negative) of a variable appears at maximum once in the minimal factored form, e.g. *f=(!a*d+c)*(a+b)*. In this sense, this work aims to provide an efficient algorithm that guarantees minimal factored forms for the class of RPO Boolean functions.

1.3 Thesis Organization

The remaining of this thesis is organized as follows.

Chapter 2: *BOOLEAN LOGIC CONCEPTS* — Provides to the reader basic and consolidated knowledge that is needed to understand the concepts presented in this work.

Chapter 3: *READ-ONCE BOOLEAN FUNCTIONS* — Describes the evolutionary line of RO factoring algorithms, and presents the two most recent methods that are related to the methods proposed herein.

Chapter 4: *FACTORING INCOMPLETELY SPECIFIED BOOLEAN FUNCTIONS INTO READ-ONCE EQUATIONS* — Presents a complete method for factoring ISBF into RO forms, when it is possible. This method, called *ISF2RO* could also be used in other applications, as discussed in Chapter 5.

Chapter 5: *READ-POLARITY-ONCE FUNCTIONS* — Defines a new class of Boolean functions, where each polarity (positive or negative) of a variable appears at maximum once in the minimum factored equation. Results of several experiments are also presented and discussed.

Chapter 6: *CONCLUSIONS AND FUTURE WORK* — Summarizes the major contributions of this work, and discusses some promising future work.

2 BOOLEAN LOGIC CONCEPTS

This chapter introduces notation and preliminaries necessary to the understanding of this work. It gives to the reader a brief description of the basis of Boolean algebra.

2.1 Sets, membership and basic operations

This section presents basic concepts of set theory. This includes the concepts of membership, sets, subsets and associated operation.

Set: a collection of distinct elements. The usual way of describing a set is by defining the characteristics of the elements belonging to the set. For instance, if the set A is defined as the set of all positive even elements, the set A is completely defined. However, it is not possible to explicitly list all the elements in this set, as the number of elements is infinite. This way, the set A could be described as: $A = \{2, 4, 6, 8, 10, \ldots\}$.

Membership: If an element a is member of the set A, this is denoted as $a \in A$.

One way of describing a set is by listing all elements of the set. One example of it is the definition of the binary set B such that $B=\{0, 1\}$; meaning that $0 \in B$ and $1 \in B$.

Subsets: Let A and B be two sets. We say $B \subseteq A$ if all elements in B are also into the set A. In this case, we say that B is a subset of A.

With this very basic definition set, we can now provide several fundamental operations between sets, in order to provide new sets. Table 2.1 shows possible operations that can be performed over sets.

Table 2.1: Possible operations over sets.

Operation	Venn Diagram
Complement: If a universe **U** is defined, A^c creates a new set that contains all elements of a universe **U** that are not in A: $\mathbf{U} \setminus A$	
Union: $A \cup B$ creates a new set that contains all elements that are member of either A or B.	
Intersection: $A \cap B$ creates a new set that contains all elements that are members of both A and B.	
Symmetric difference: A Δ B creates a new set that contains all elements which are in either of the sets and not in their intersection, or more formally: $(A \cup B) \setminus (A \cap B)$	

The goal of this review is just to present some very basic definitions and operations. For more information or formalism, (HALMOS, 1960) is suggested.

2.2 Boolean functions

Let $B = \{0,1\}$, $Y = \{0,1,\mathbf{X}\}$. A Boolean function f in n-input variables, $x_1,\ldots,x_n$, is a function:

$$f : B^n \rightarrow Y$$

where $x = [x_1,\ldots,x_n] \in B^n$ is the input of f. A **completely specified Boolean function** (*CSF*) f is a logic function that allow only values $\{0,1\}$ in Y. Notice that **incompletely specified Boolean functions** (*ISF*) differs from completely specified functions in the fact that the former may also assume *don't-care* (***X***) values, besides the binary values *0* and *1* (BRAYTON ET AL., 1984).

An element $m \in B^n$ is called *term*. The number of terms in B^n is 2^n. We can define the **on-set** f^{ON} as the terms $m \in B^n$ such that $f(m) = 1$, the **off-set** f^{OFF} such that $f(m) = 0$ and the ***don't-care*** **set** f^{DC}, such that $f(m) = \mathbf{X}$. The on-set and the off-set terms are also called *minterms* and *maxterms*, respectively. A CSF that contains no elements in the off-set ($f^{OFF} = \emptyset$) is defined as constant **ONE**. In analog way, a CSF that contains no elements in the on-set ($f^{ON} = \emptyset$) is defined as constant **ZERO**. In this work, Boolean function and function are used interchangeably.

2.2.1 Representing Boolean functions

There are several ways to represent a Boolean function. The most straightforward one is the ***truth table***. In this representation, the value of the output is specified for all possible input vectors $[x_1,\ldots,x_n]$. For example, let f: $B^3 \rightarrow Y$ be specified by the *truth table* in Table 2.2. It is also possible to represent the same *truth table* in a **bit string**, that have the most significant bit on the left and the less significant bit on the right: $f(x1,x2,x3)=\mathtt{11001001}_2$. A well-established way of representing Boolean functions is the **Karnaugh map** (KARNAUGH, 1953), and an example demonstrating its use is depicted in Figure 2.1.

Table 2.2: A *truth table* representation of $f(x1,x2,x3)=\mathtt{11001001}_2$.

x1	***x2***	***x3***	***f***
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

	00	01	11	10
0	1 (0)	0 (2)	1 (6)	0 (4)
1	0 (1)	1 (3)	1 (7)	0 (5)

Figure 2.1: A Karnaugh map representation of *f(x1, x2, x3)* = 11001001_2.

Although being a very simple way of representing Boolean functions, the *truth table* data structure is not scalable in practice, since it uses 2^n bits to store the information. For functions with more than 20 input variables, representing Boolean functions as *truth table* data structure starts to be infeasible. In order to overcome this limitation, a graph-based approach has been proposed by (AKERS, 1978): **Binary Decision Diagram** (*BDD*).

A BDD is a rooted, directed graph with vertex set V containing two types of vertices. A *nonterminal* vertex v has as attributes an input variable and two children $low(v), high(v) \in V$. A terminal vertex v has as attribute a value $v \in \{0,1\}$. The same function represented by the truth Table 2.2 above could be represented by a BDD as it is possible to see in Figure 2.2. The dashed edges represent the low child nodes, while the non-dashed lines are the high child nodes.

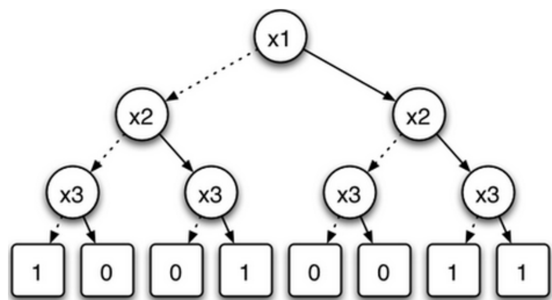


Figure 2.2: Binary decision diagram (BDD) representation of a Boolean function.

In order to make the BDD a canonical data structure, Bryant (BRYANT, 1986) proposed a **reduced and ordered binary decision diagram** (*ROBDD*). ROBDD is similar to the representation introduced by (AKERS, 1978), but with further restrictions on the ordering of decision variables in the graph. A ROBDD representing the same function of the BDD of Figure 2.2 could be seen in the Figure 2.3. Notice that ROBDD is also a more compact way of representing BDDs.

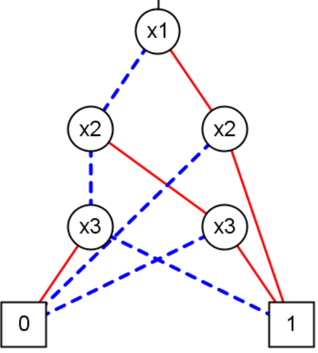


Figure 2.3: A reduced and ordered binary decision diagram (ROBDD) of a Boolean function.

In general, BDDs represent only completely specified Boolean functions. In order to represent incompletely specified Boolean functions, two BDDs are necessary: one BDD to represent both on- and off-set, and another BDD for representing the *don't-care* set of the function.

2.2.2 Operations on completely specified Boolean functions

In this subsection we define some basic Boolean operators for completely specified Boolean functions (*CSF*). After the basic operations definition, the next subsection defines how these operators work on incompletely specified Boolean functions (*ISF*).

The **complement** (**NEGATION**, **NOT**) of a CSF, f is a CSF, $h = f^c$, (h = !f), such that $h^{ON} = f^{OFF}$, and the $h^{OFF} = f^{ON}$. A *truth table* representation of the ***NOT*** operator is presented in Table 2.3.

Table 2.3: *Truth table* representation of ***NOT*** operator.

f	*!f*
0	1
1	0

The **union** (**SUM**, **OR**) of two CSF, f and g is a CSF, $h = f \cup g$, ($h = f + g$), such that h^{ON} is the union f^{ON} and g^{ON}. A *truth table* representation of the ***OR*** operator is presented in Table 2.4.

Table 2.4 *Truth table* representation of ***OR*** operator.

f	*g*	*h = f + g*
0	0	0
0	1	1
1	0	1
1	1	1

The **intersection** (**PRODUCT**, **AND**) of two CSF, f and g is a CSF, $h = f \cap g$, ($h = f * g$), such that h^{ON} is composed by the terms that are in both f^{ON} and g^{ON}. A *truth table* representation of the **AND** operator is presented in Table 2.5.

Table 2.5 *Truth table* representation of ***AND*** operator.

f	*g*	*h = f * g*
0	0	0
0	1	0
1	0	0
1	1	1

The symmetric **difference** (**exclusive-or**, **XOR**) between two CSF, f and g is a CSF, $h = f \Delta g$, ($h = f \oplus g$), such h^{ON} contains all elements which are in either f^{ON} or g^{ON} but not in their intersection. A *truth table* representation of the ***XOR*** operator is presented in Table 2.6.

Table 2.6 *Truth table* representation of ***XOR*** operator.

f	g	$h = f \oplus g$
0	0	0
0	1	1
1	0	1
1	1	0

We say that two CSF f and g are equal when $f \equiv g$. In this case, f^{ON} and g^{ON} must be the same. We define **equality** between two CSF f and g as follows:

$$f \equiv g \leftrightarrow f \equiv f \cup g \equiv g.$$

2.2.3 Operations on incompletely specified Boolean functions

In the following the operations for incompletely specified Boolean functions (ISF) are defined. In an analog way, operations over ISF domain are quite related to CSF, regardless when a *don't-care* term appears. A *don't-care* term is a term that we indeed do not care about its value. In this sense, the following operations on ISF are defined: complement, union, intersection and symmetric difference.

The **complement** (**NEGATION**, N**OT**) of an ISF, f is a ISF, $h = f^c$, (h = !f) defined as the following:

$$h^{ON} = f^{OFF} \tag{1.1}$$

$$h^{OFF} = f^{ON} \tag{1.2}$$

$$h^{\mathrm{DC}} = f^{\mathrm{DC}} \tag{1.3}$$

The **union** (**SUM**, **OR**) of two ISF, f and g is an ISF, $h = f \cup g$, ($h = f + g$) defined as the following:

$$h^{ON} = f^{ON} \cup g^{ON} \tag{1.4}$$

$$h^{OFF} = f^{OFF} \cap g^{OFF} \tag{1.5}$$

$$h^{\mathrm{DC}} = \mathbf{U} \setminus h^{ON} \setminus h^{OFF} \tag{1.6}$$

The **intersection** (**PRODUCT**, **AND**) of two ISF, f and g is an ISF, $h = f \cap g$, ($h = f$ * g) defined as the following:

$$h^{ON} = f^{ON} \cap g^{ON} \tag{1.7}$$

$$h^{OFF} = f^{OFF} \cup g^{OFF} \tag{1.8}$$

$$h^{\mathrm{DC}} = \mathbf{U} \setminus h^{ON} \setminus h^{OFF} \tag{1.9}$$

The symmetric **difference** (**exclusive-or**, **XOR**) between two ISF, f and g is an ISF, $h = f \Delta g$, ($h = f \oplus g$) defined as the following:

$$h^{ON} = (f^{ON} \cup g^{ON}) \setminus (f^{ON} \cap g^{ON}) \tag{1.10}$$

$$h^{OFF} = (f^{OFF} \cap g^{OFF}) \cup (f^{ON} \cap g^{ON}) \tag{1.11}$$

$$h^{DC} = f^{DC} \cup g^{DC} \tag{1.12}$$

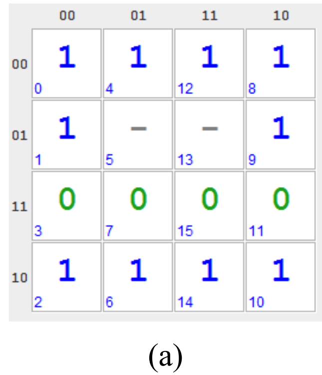
Table 2.7: Summary of all binary operations over ISF in *truth table* data structure.

f	***g***	$h = f + g$	$h = f * g$	$h = f \oplus g$
0	0	0	0	0
0	1	1	0	1
0	***X***	***X***	0	***X***
1	0	1	0	1
1	1	1	1	0
1	***X***	1	***X***	***X***
X	0	***X***	0	***X***
X	1	1	***X***	***X***
X	***X***	***X***	***X***	***X***

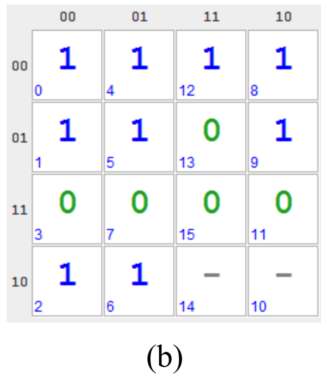
Table 2.7 summarizes all binary operations over ISF in a *truth table* data structure. Two ISF *f* and *g* are said equal when $f \equiv g$. More precisely, $(f^{ON} = g^{ON})$, $(f^{OFF} = g^{OFF})$ and $(f^{DC} = g^{DC})$. However, when in an ISF domain, it is needed often to verify if an ISF *f* is equivalent to another ISF *g*. **Equivalence** between two ISF is defined as follows:

$$f \approx g \leftrightarrow f^{ON} \cap g^{OFF} = \varnothing \text{ and } f^{OFF} \cap g^{ON} = \varnothing$$

In order to demonstrate the equivalence operator, let f=*11101X1011101X10*$_2$ and g=*1110111011X010X0*$_2$ be two ISF. As it is possible to see in Figure 2.4, there is no element in f^{ON} that is also in g^{OFF}, as well as no element in f^{OFF} that also belongs to g^{ON}. In this case we say that *f* is equivalent to *g* $(f \approx g)$.



(a)



(b)

Figure 2.4: Karnaugh map representation of f=*11101X1011101X10*$_2$ (a) and g=*1110111011X010X0*$_2$ (b). Dashes mean *don't-care* (***X***) terms.

2.2.4 Cofactor over Boolean functions

The cofactor operation is a very basic and significant operation over Boolean functions. Let us define cofactor as following.

Let $f : B^n \rightarrow B$ be a Boolean function and $x = [x_1,\ldots,x_n]$ the variables in support of *f*. The **cofactor** of *f* with respect to x_i is denoted as $f(x_1,\ldots,x_i{=}c,\ldots,x_n)$ where $c \in \{0, 1\}$ (BOOLE, 1854). It is also possible to define as **positive cofactor** the operation where a variable receive the Boolean constant 1. The opposite is defined as **negative cofactor**,

and is when a variable receives the Boolean constant 0. For presentation sake, let $f(x_1,\dots,x_i=c,\dots,x_n) \equiv f(x_i=c)$.

It is not a simple task to enumerate all methods that take advantage of the cofactor operation. One of the most important examples is the Shannon expansion, where a function can be represented as a sum of two sub-functions of the original (SHANNON, 1948):

$$f(x_1,\dots,x_n) = !x_i * f(x_i=0) + x_i * f(x_i=1) \quad (1.13)$$

Let us take as example a Boolean function f represented by the logic equation (1.14). Let the equation (1.15) and (1.16) represent, respectively, the negative and positive cofactor of variable x_2 over f. The expansion of f about x_2 is defined by equation (1.17). Notice that both equation (1.14) and (1.17) represent the same Boolean function.

$$f = !x_1\ !x_2\ !x_3 + x_1\, x_2 + x_2\, x_3 \quad (1.14)$$

$$f(x_2=0) = !x_1\ !x_3 \quad (1.15)$$

$$f(x_2=1) = x_1 + x_3 \quad (1.16)$$

$$f = !x_2(!x_1\ !x_3) + x_2(x_1+x_3) \quad (1.17)$$

2.2.5 Unateness

The unateness behavior is an intrinsic characteristic of Boolean functions. It provides the behavior of each variable, as well as the behavior of the entire function. Let f be a Boolean function. The unateness behavior of a variable x_i in f can be obtained according to the following relations:

$$\alpha = f(x_i=1) \quad (1.18)$$

$$\beta = f(x_i=0) \quad (1.19)$$

$$\gamma = \alpha + \beta \quad (1.20)$$

don't-care: $\alpha \equiv \beta$ (1.21)

positive unate: $(\alpha \equiv \gamma) \wedge (\alpha \neq \beta)$ (1.22)

negative unate: $(\beta \equiv \gamma) \wedge (\alpha \neq \beta)$ (1.23)

binate: $(\alpha \neq \beta) \wedge (\alpha \neq \gamma) \wedge (\beta \neq \gamma)$ (1.24)

Definition: We say that a Boolean function is unate if all its variables are either positive or negative unate. When all variables are positive (negative) unate, we say that the entire function is positive (negative) unate. In the case when at least one variable is binate, the Boolean function is considered binate.

Example: Let f be a Boolean function defined by the following equation:

$$f = !ab!cd+!abc!d+!abcd+a!bc!d+a!bcd+abc!d+abcd \quad (1.25)$$

which can be also represented by the bit string $f=1100110011100000_2$. In order to discover the unateness behavior of the variable "*a*", we need to perform the positive and negative cofactors with regard to the variable "*a*", as presented in Eq. 1.26 and Eq. 1.27, respectively. By applying a bitwise OR between the cofactors, the function γ_a is obtained, as shown in Eq. 1.28. As it is possible to see, the positive and negative

cofactors are not equals, as well as the function γ_a differs from both cofactors. This way, we determine that the variable "*a"* has a binate behavior in *f*.

$$\alpha_a = f(a=1) \equiv 1100110011001100_2 \quad (1.26)$$

$$\beta_a = f(a=0) \equiv 1110000011100000_2 \quad (1.27)$$

$$\gamma_a = \alpha + \beta \equiv 1110110011101100_2 \quad (1.28)$$

In a straightforward way, it is possible to discover the unateness behavior of the variable "*b*" in *f*. The equations Eq. 1.29 and Eq.1.30 represent the positive and negative cofactors of "*b*" in *f*, respectively. Through the bitwise OR between the cofactors, the function γ_b is obtained, as presented in Eq. 1.31. As it is possible to see, the positive and negative cofactors are not equals. However, the function γ_b (bitwise OR between cofactors of "*b*") is equivalent to the positive cofactor of "*b*". This means that the variable "*b*" has a positive unate behavior in *f*. The same process is applied to the variables "*c*" and "*d*", which have a positive unate behavior in *f*. For simplicity sake, this process was omitted, since it is straightforward from equations Eq. 1.26, 1.27 and 1.28.

$$\alpha_b = f(b=1) \equiv 1100110011101110_2 \quad (1.29)$$

$$\beta_b = f(b=0) \equiv 1100110000000000_2 \quad (1.30)$$

$$\gamma_b = \alpha + \beta \equiv 1100110011101110_2 \quad (1.31)$$

In mathematics, a monotonic function (or monotone function) is a function between ordered sets that preserves some given order (GRÄTZER, 1971). Monotonic functions are divided between monotonically increasing and monotonically decreasing functions. Unate Boolean functions are monotonic functions with Boolean domain and image, where positive (negative) unate Boolean functions are monotonically increasing (decreasing) functions. For more information, (GRÄTZER, 1971) is suggested.

Unate Boolean functions are of special interest in this work. Some characteristics and applications of the unate Boolean functions are presented in Chapter 3. An algorithm that split binate variables into two independent unate variables, in order to transform a binate function into an unate one, is presented in Chapter 5. This algorithm and the application of this task are also discussed.

2.3 Logic equations

It is possible to use the set of Boolean operators in order to generate more complex structures. One example of complex structure is the algebraic representation of a Boolean function, which is simply named **equation** herein.

The simplest equation represents a Boolean variable, e.g. $f = a$. This equation has only one literal. A **literal** is a variable or its complement (e.g. *a* or *!a*). In general, a Boolean function can be represented by several different equations. Let us take as example the CSF denoted by the bit string: $f(x1, x2, x3)=11001001_2$. It is possible to represent this function in **sum-of-products** (*SOP*) or in **product-of-sums** (*POS*) forms, as it is possible to see in Figure 2.5a and Figure 2.5b, respectively. We say that a SOP is an irredundant sum-of-products (*ISOP*) when no literal can be deleted from the SOP without changing the Boolean function. For instance, the SOP $f=!x_1\,!x_2\,!x_3+!x_1\,x_2\,x_3+x_1\,x_2\,!x_3+x_1\,x_2\,x_3$ represents the same Boolean function as the ISOP $f = !x_1\,!x_2\,!x_3 + x_1\,x_2 + x_2\,x_3$. Moreover, we can delete literals from the SOP until it reaches an ISOP, without

changing function. In an analog way, the same property occours for POS, and is called irredundant product-of- sums (*IPOS*).

There are some methods that produce more complex equations, known as factoring algorithms. **Factoring** is the process of deriving a parenthesized algebraic equation, or factored form, representing a given logic function, usually provided initially in SOP or in POS. The main target of factoring algorithms is to find an equation with the minimal number of literals as possible. For instance, let the input of a factoring algorithm be the SOP equation given by $f = !x_1\ !x_2\ !x_3 + x_1\ x_2 + x_2x_3$. This equation can be factored into the logically equivalent one with only 6 literals: $f = !x_2(!x_1\ !x_3) + x_2(x_1+x_3)$, while the original has 7 literals. Figure 2.6 shows a logic tree of the factored equation.

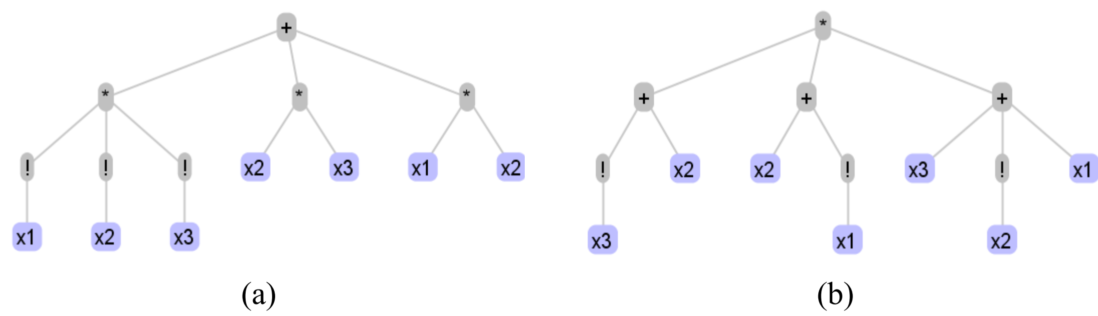


Figure 2.5: The same Boolean function represented by an equation in sum-of-products (a) and in product-of-sums form (b).

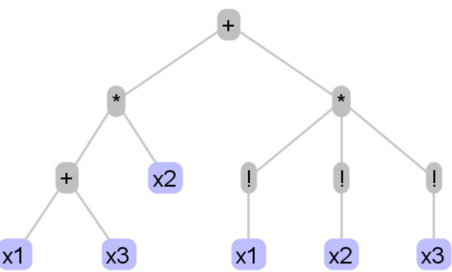


Figure 2.6: Arbitrary Boolean function represented by a factored form.

3 READ-ONCE BOOLEAN FUNCTIONS

The classes of read-once functions have interesting special properties (KARCHMER ET AL., 1993; BOROS; GURVICH; HAMMER, 1994; BOROS; IBARAKI; MAKINO, 1998; GOLUMBIC, 2004). The class of *read-once* functions is of special interest in several areas, including learning theory (ANGLUIN; HELLERSTEIN; KARPINSKI, 1993; BSHOUTY; HANCOCK; HELLERSTEIN, 1995), data bases (KANAGAL; LI; DESHPANDE, 2011) and digital circuit design (PEER; PINTER, 1995). In this chapter, the concept and applications of *read-once* Boolean functions will be presented.

3.1 Definition

A Boolean function is considered ***read-once*** (*RO*) if it can be represented in a factored form where each variable appears only once (GOLUMBIC; MINTZ; ROTICS, 2001).

For example, the Boolean function represented by

$$f = x_1 x_2 + x_1 x_3 x_4 + x_1 x_3 x_5$$

is a *read-once* function since it can be factored into

$$f = x_1 (x_2 + x_3(x_4 + x_5))$$

where each variable appears only once (GOLUMBIC; CRAMA; HAMMER, 2009).

If a given function f can be factored in a RO equation, then all variables of f are either positive or negative unate. If a function has some binate variable, this variable will appear in two phases, contradicting the *read-once* definition, where each variable appears at most once (LEE; WANG, 2007). This is a necessary but not sufficient condition, since there are functions composed by only unate variables that cannot be factored in a RO equation. For example, the unate function $f = x_1 x_2 + x_1 x_3 + x_2 x_3$ has $f = x_1(x_2 + x_3) + x_2 x_3$ as the minimal solution, in which variables x_2 and x_3 appear more than once.

3.2 Previous work

The class of *read-once* Boolean functions is known for a long time, and was first introduced by Hayes (HAYES, 1975) and was called *fanout-free* functions. The method proposed by Hayes suffers of high complexity, since the algorithm makes intensive calls to a procedure to perform equivalence checking of cofactors, which was originally an expensive computation.

Peer and Pinter have also proposed in (PEER; PINTER, 1995) an algorithm to synthesize *non-repeating literal trees*, another name to *read-once* functions. The main drawback of their method is that it runs in non-polynomial time. The main reason is that their method requires calling several times a procedure that converts a SOP to POS (and POS to SOP) form. This original routine requires a non-polynomial time to run, making the method very costly in runtime.

More recent work was proposed in order to overcome the limitations of the Hayes and Peer and Pinter's methods. Golumbic (GOLUMBIC; MINTZ; ROTICS, 2001) was the first to propose a polynomial time factoring algorithm for RO functions, called *IROF*. His method is based on Gurvich work (GURVICH, 1991). Another recent work was proposed by Lee (LEE; WANG, 2007) and is based on the Hayes work. His method, called herein as *JPHI*, replaced the equivalence checking of cofactors by a property called disappearance, turning the algorithm feasible in polynomial time. Both IROF and JPHI methods factorize RO functions in polynomial time. From this point on, the focus will be only on these two more recent approaches.

3.3 IROF method

In this section the basic idea of Golumbic's approach to factorize *read-once* functions will be presented. More specific details can be found in the paper (GOLUMBIC; MINTZ; ROTICS, 2001).

The IROF method receives a Boolean function f in an irredundant sum-of-products (ISOP) form. The first step is to build an undirected graph $G = (V, E)$, where $V = \{v_0, \dots, v_n\}$ represents the set of literals of the ISOP. An edge $(vi, vj) \in E$ exists if the literals vi and vj appears in the same product in the ISOP.

Let us take an ISOP $f = x_1 x_2 + x_1 x_3 x_4 + x_1 x_3 x_5$ as example. The first step of IROF algorithm is to transform the ISOP into a graph. The resulting graph is depicted in Figure 3.1.

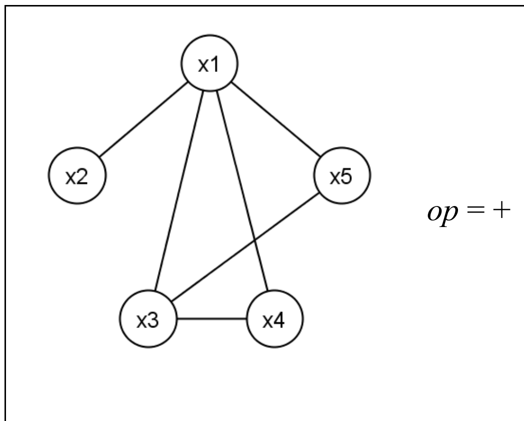


Figure 3.1: The resulting graph for the ISOP $f = x_1 x_2 + x_1 x_3 x_4 + x_1 x_3 x_5$.

After the graph generation task, the algorithm proceeds to the partition step. Let $op \in \{+, *\}$ be a Boolean operator. Given a graph G and an operator op starting with '+', the ***coGraph_Rec***(G, op) step first verifies if G contains only one node. In this trivial situation, the method stores the operator op on the node and returns. If G contains more than one node, the algorithm proceeds checking if G is connected or not. If G is connected, the algorithm complement the graph G and swap the operator op (e.g. if op is '+', op' will be '*'), making a recursive call ***coGraph_Rec***(G', op'), where G' is the

complement of graph G. However, if G is disconnected, the algorithm selects connected components from G, and for each connected component H, make a recursive call ***coGraph_Rec***(H', op'). The pseudo algorithm is presented in Figure 3.2

```
coGraph_Rec(G, op) {
        if (G contains only one node v_i) {
                return op, v_i;
        if (G is connected)
                return partition(G', op');
        H = connected components of G;
        foreach (H_i ∈ H)
                partition(H_i', op');
}
```

Figure 3.2: Pseudo algorithm for the ***coGraph_Rec*** step.

Let us apply the ***coGraph_Rec*** step in the graph G represented by Figure 3.1. Since G contains more than one node and is a connected graph, a complemented graph G' is created (Figure 3.3), and a recursive call ***coGraph_Rec**(G', op')* is called**.** The method is called recursively until there are no elements to be processed. The figures Figure 3.1, Figure 3.3, Figure 3.4, Figure 3.5 and Figure 3.6 show a complete example of the IROF algorithm. The algorithm successfully recognized the *read-once* equation $f = x_1 (x_2+x_3(x_4+x_5))$ after the entire flow.

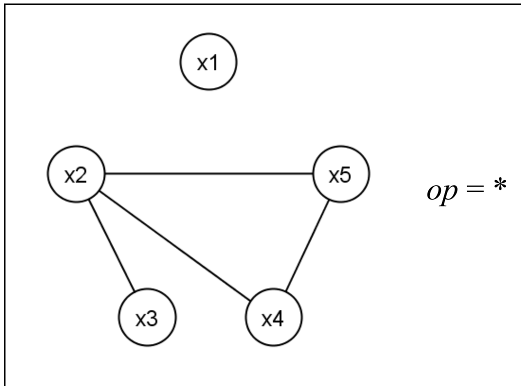


Figure 3.3: Complement graph of Figure 3.1. In this step, the ***coGraph_Rec*** build the partial equation: $f=x_1*f_1(x_2, x_3, x_4, x_5)$.

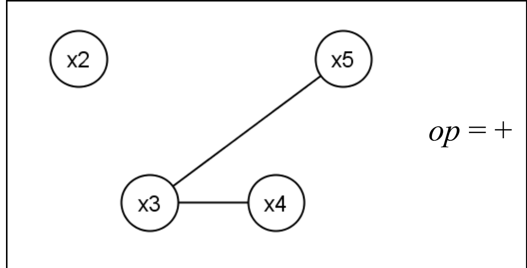


Figure 3.4: Complement graph of Figure 3.3. The node x_1 was removed. In this step, the ***coGraph_Rec*** build the partial equation: $f_1=x2+f_2(x_3, x_4, x_5)$.

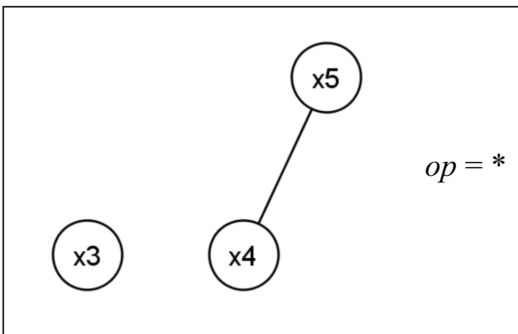


Figure 3.5: Complement graph of Figure 3.4. The node x_2 was removed. In this step, the ***coGraph_Rec*** build the partial equation: f_2=$x3$*$f_3(x_4, x_5)$.

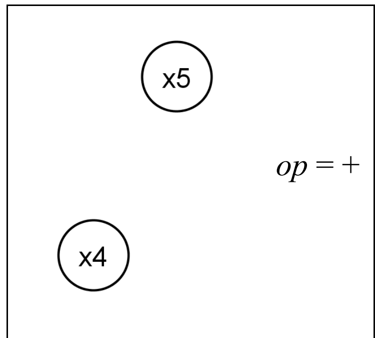


Figure 3.6: Complement graph of Figure 3.5. The node x_3 was removed. In this step, the ***coGraph_Rec*** build the equation: $f_3 = x4 + x5$.

The IROF algorithm was the first method to factorize RO functions in a polynomial time. The IROF algorithm can be implemented in a time complexity $O(n|f|)$, where $|f|$ denotes the length of the ISOP equation of a function f, and n is the number of variables in f (GOLUMBIC; MINTZ; ROTICS, 2001). Despite the efficiency of the IROF algorithm, it cannot be modified to deal with incompletely specified Boolean functions, since it depends on an ISOP as input. Moreover, if the entire function is not *read-once*, the IROF method is not able to recognize subfunctions that are *read-once*. The JPHI method is able to find *read-once* subfunctions, even if the input function were not *read-once*. Furthermore, it is possible to modify the JPHI method to accept incompletely specified Boolean functions as input. JPHI method will be described in the next section.

3.4 JPHI method

This section will introduce the factoring algorithm for RO functions proposed by Lee (LEE; WANG, 2007), called herein as John P. Hayes Improved (*JPHI*). The goal of this section is to give a brief introduction about the method, and more details can be found in the paper.

The JPHI method receives a Boolean function f in an irredundant sum-of-products (ISOP) form. We will consider ISOPs where all variables are positive unate, without losing generality. If a function has a negative unate variable, it is possible to substitute a positive unate variable for this negative unate variable. For example, the function $g(x_1, x_2) = x_1 + !x_2$ can be viewed as $f(x_1, !x_2) = x_1 + x_2$.

The JPHI method is highly based on cofactors. The basic idea is to tie variables when their cofactors are equal. Let us take as example the function $f = x_1 x_2 x_3 x_4 + x_1 x_2 x_3 x_5 + x_4 x_6 + x_5 x_6$. The negative cofactor list is presented in Figure 3.7.

$$f(x_1 = 0) = x_4 x_6 + x_5 x_6$$

$$f(x_2 = 0) = x_4 x_6 + x_5 x_6$$

$$f(x_3 = 0) = x_4 x_6 + x_5 x_6$$

$$f(x_4 = 0) = x_1 x_2 x_3 x_5 + x_5 x_6$$

$$f(x_5 = 0) = x_1 x_2 x_3 x_4 + x_4 x_6$$

$$f(x_6 = 0) = x_1 x_2 x_3 x_4 + x_1 x_2 x_3 x_5$$

Figure 3.7: Equations of negative cofactors of $f = x_1 x_2 x_3 x_4 + x_1 x_2 x_3 x_5 + x_4 x_6 + x_5 x_6$.

As it is possible to see, the negative cofactors of variables x_1, x_2 and x_3 have the same equation. In this way, we can tie these variables through an **AND** operator, e.g. $\varphi = x_1x_2x_3$, since the equations are from negative cofactors (Figure 3.8). When positive cofactors are equal, then the variables are tied through **OR** operators. After replacing f by φ, the algorithm continues dealing with φ until reaching a RO form.

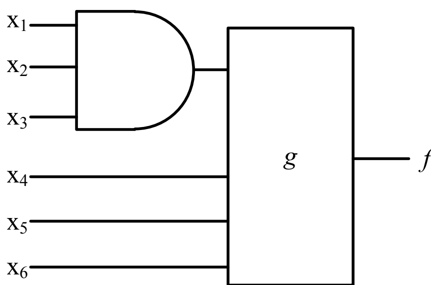


Figure 3.8: Partial decomposition after tying variables into $\varphi = x_1 x_2 x_3$.

The JPHI method uses the *disappearance* property to accelerate the JPH's method (HAYES, 1975). The *disappearance* property can be defined as follows: Let x_i and x_j be two distinct variables of f and c be a Boolean constant. If xj disappears in the function $f(xi = c)$ and x_i disappears in the function $f(x_j = c)$, then $f(xi = c) \equiv f(x_j = c)$. A formal proof is given in (LEE; WANG, 2007).

In order to explore the disappearance property, a matrix can be build. This matrix has rows and columns labeled by $f(x_i = c)$ and x_j. The first step to build the matrix is to list the cofactors. The matrix is then filled with element 1 or 0 in position ($f(x_i = c)$, x_j) according to whether x_j appears in $f(x_i = c)$. If x_j appears $f(x_i = c)$, 1 is filled into ($f(x_i = c)$, x_j); otherwise, 0 is filled into ($f(x_i = c)$, x_j). This matrix has 0s on the diagonal because x_i always disappears in $f(x_i = c)$. Through the disappearance property it is not more necessary to perform equivalence checking over cofactors, which was the main bottle neck of the JPH approach. The matrix based on the Figure 3.7 is depicted in Figure 3.9.

$$\begin{array}{c} \\ f(x_1=0) \\ f(x_2=0) \\ f(x_3=0) \\ f(x_4=0) \\ f(x_5=0) \\ f(x_6=0) \end{array} \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{matrix} \\ \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

Figure 3.9: Disappearance matrix of $f = x_1 x_2 x_3 x_4 + x_1 x_2 x_3 x_5 + x_4 x_6 + x_5 x_6$. Source: LEE; WANG, 2007.

In a straightforward way, the positive cofactor list is evaluated and a disappearance matrix for the positive cofactors is built. After examining the adjacency relations among variables, it is possible to recognize the function $f = (x_1x_2x_3 + x_6)(x_4 + x_5)$. Figure 3.10 shows the iterations when recognizing the *read-once* function $f = (x_1x_2x_3 + x_6)(x_4 + x_5)$.

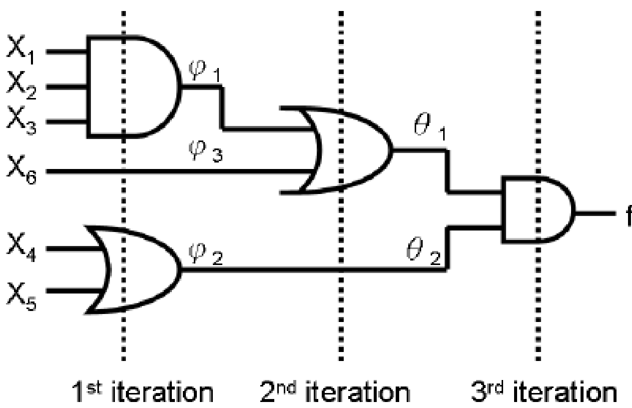


Figure 3.10: Iterations of recognizing $f = (x_1x_2x_3 + x_6)(x_4 + x_5)$. Source: LEE; WANG, 2007.

Besides the efficiently approach for recognizing *read-once* functions, the JPHI algorithm can also find *read-once* subfunctions even if the input function was not a *read-once* one. This is possible since the disappearance property tie variables in subfunction regardless tying the entire set of variables. Consider the function $k=(x_1x_2x_3+x_6)(x_4+x_5)(ab+bc+ac)$ which is not *read-once* due to the term ($ab+bc+ac$). JPHI method will produce a partially *read-once* equation, as it is shown in a circuit description in Figure 3.11. Despite the efficiency, the IROF algorithm is not able to recognize subfunctions that are *read-once*. If the same function k were used as input of IROF algorithm, it will just return that k is not a *read-once* function.

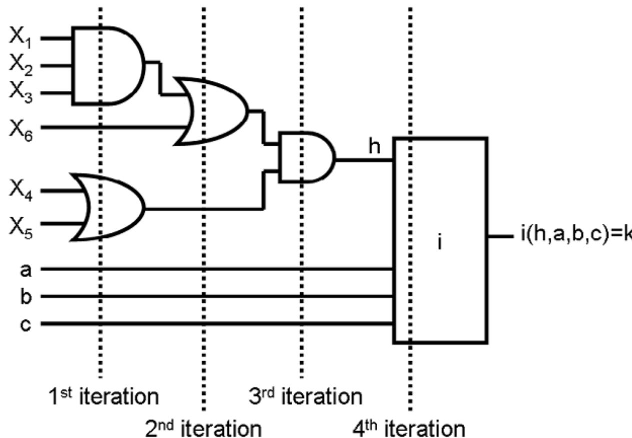


Figure 3.11: Iterations of recognizing $f = (x_1x_2x_3 + x_6)(x_4 + x_5)$. Source: LEE; WANG, 2007.

The time complexity of the JPHI algorithm is $O(n^2K)$, where K denotes the number of products in the ISOP equation of a Boolean function f, and n is the number of variables in f. In spite of the fact that the JPHI algorithm has higher complexity than the IROF algorithm, it is possible to modify the JPHI method in order to factorize incompletely specified Boolean functions as *read-once* solutions.

3.5 IROF and JPHI comparison

We will now briefly compare the IROF (GOLUMBIC; MINTZ; ROTICS, 2001) and JPHI (LEE; WANG, 2007) methods. Table 3.1 compare the runtime (in seconds) required to synthesize an arbitrary benchmark of 9 *read-once* functions. The table was compiled with results regarding papers by Golumbic (IROF) and Lee (JPHI).

Table 3.1: Comparison regarding runtime (in seconds) required to synthesize an arbitrary benchmark of 9 *read-once* functions.

	SOP*	**N****	**JPH**	**IROF**	**JPHI**
l2_b10	10240	20	4	0.3	0.11
l4_b3	3072	24	6	0.1	0.08
l4_b6	7290	24	277	0.21	0.18
l6_b4	672	20	3	0.02	0.02
l6_b8a	132	52	>1hr	0.02	0.29
l6_b8b	24192	52	>1hr	0.74	2.08
l8_b5	3380	29	>1hr	0.09	0.11
l10_b3	2160	30	>1hr	0.07	0.16
l14_b3	6720	42	>1hr	0.2	0.72

Source: LEE; WANG, 2007.

*SOP: Number of literals in the SOP equation.
**N: Number of variables in the SOP equation.

In order to create a fairest comparison, both methods were implemented and tested over 3503 *read-once* functions from GenLib44-6 benchmark (SENTOVICH et al., 1992). This benchmark has functions from 1 to 16 inputs. The functions were grouped regarding input count, and then factorized by IROF and JPHI algorithms. The total runtime (in *ms*) to factorize each group of functions is presented in Table 3.2. It is possible to see in the table that both algorithms are equivalent in runtime from functions up to 8 inputs. From 9 inputs onwards, the runtime of the JPHI algorithm starts to grow quickly than the IROF. This behavior was expected, since the time complexity of JPHI is greater than IROF algorithm. It is possible to see in Figure 3.12 the average (in *ms*) runtime to synthesize the benchmark of 3503 read-once functions, grouped by number of inputs. The platform was a Linux system on Intel Core i5 2400 processor with 2GB main memory.

Table 3.2: Comparison of runtime between IROF and JPHI to synthesize the benchmark of 3503 read-once functions, grouped by number of inputs.

Inputs	Functions	IROF* (ms)	JPHI** (ms)
1	1	0	4
2	2	1	1
3	4	0	1
4	10	2	5
5	22	3	4
6	54	20	16
7	116	31	41
8	228	83	81
9	374	115	144
10	530	162	188
11	612	168	247
12	604	171	305
13	468	145	310
14	300	107	297
15	130	41	184
16	48	19	100
$\sum$	**3,503**	**1,068**	**1,928**

*In-house implementation of the IROF algorithm presented in (GOLUMBIC; MINTZ; ROTICS, 2001).

**In-house implementation of the JPHI algorithm presented in (LEE; WANG, 2007).

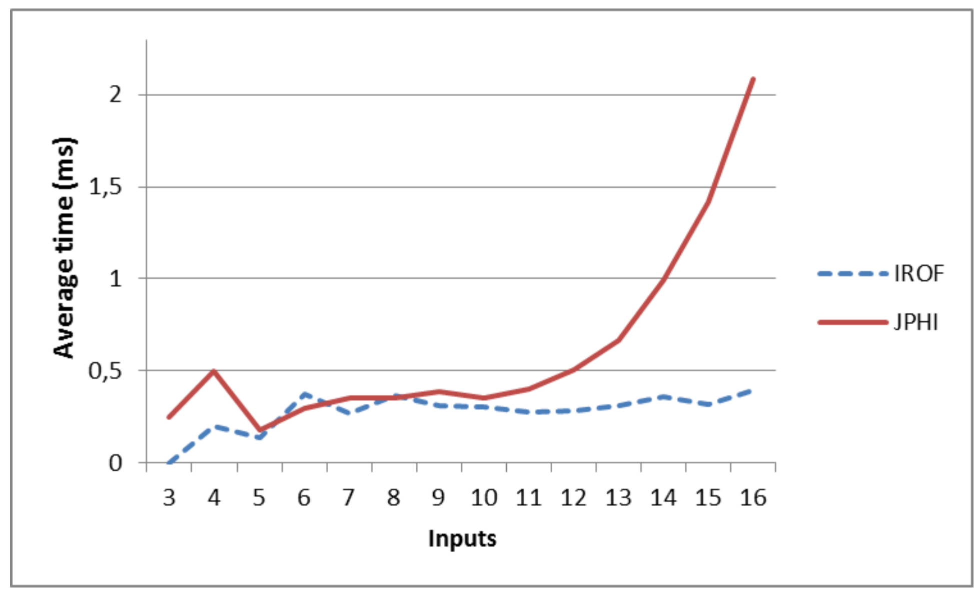


Figure 3.12: The average (in *ms*) runtime to synthesize the benchmark of 3503 read-once functions, grouped by number of inputs.

The IROF algorithm was the first method to factorize RO functions in a polynomial time. The IROF algorithm can be implemented in a time complexity $O(n|f|)$, where $|f|$ denotes the length of the ISOP equation of a function f, and n is the number of variables in f (GOLUMBIC; MINTZ; ROTICS, 2001). As mentioned before, the IROF algorithm cannot deal with incompletely specified Boolean functions. Moreover, if the entire function is not *read-once*, the IROF method is not able to recognize subfunctions that are *read-once*. The JPHI method is able to find *read-once* subfunctions, even if the input function were not *read-once*. Furthermore, it is possible to modify the JPHI method to accept incompletely specified Boolean functions as input. JPHI method will be described in the chapter 4. Table 3.3 summarizes behavior comparison between IROF and JPHI algorithm.

Table 3.3: behavior comparison between IROF and JPHI algorithm.

	Time complexity	**Fail fast**	**Partial RO**	**Work with ISF***
IROF	$O(n\|f\|)$	Yes	No	No
JPHI	$O(n^2K)$	No	Yes	Yes

n: number of variables in the SOP equation.
$|f|$: number of literals in the SOP equation.
K: number of products in the SOP equation.
*ISF: Incompletely Specified Boolean Functions

4 FACTORING INCOMPLETELY SPECIFIED BOOLEAN FUNCTIONS INTO READ-ONCE EQUATIONS

Efficient algorithms exist to perform factoring of *read-once* formulas. Most of them readily discard functions containing binate variables, as a *read-once* function is always unate. This work will extend the approach proposed by (LEE; WANG, 2007). The original Lee's approach is described in the section 3.4. The modification for treating incompletely specified functions is described in this chapter. Several approaches (HAYES, 1975; GURVICH, 1991; PEER; PINTER, 1995; LEE; WANG, 2007; GOLUMBIC; MINTZ; ROTICS, 2001) have been proposed to identifying *read-once* functions (RO). Golumbic described the current state-of-the-art algorithm. The IROF algorithm needs an irredundant-sum-of-product (ISOP) as input and produces *read-once* formulae when this is possible or reports a failure otherwise. Despite the efficiency of the IROF algorithm, it cannot be modified to deal with incomplete-specified functions (ISF), since it depends on an ISOP as input. This is the reason why the Lee's approach (LEE; WANG, 2007) was chosen to be extended. The original approach proposed by Lee was described in the section 3.4. The modification for treating incompletely specified functions is described as follows.

Let $B=\{0,1\}$ and $Y=\{0,1,X\}$. Let f be an incompletely specified Boolean function (ISF) of n-input variables $x_1,x_2,\ldots,x_n$:

$$f : B^n \rightarrow Y$$

where $x = [x_1,x_2,\ldots,x_n] \in B^n$. Notice that ISF differs from completely specified functions (CSF) in the fact that the former may also assume *don't-care* (***X***) values, besides the binary values *0* and *1*.

Methods for factoring ISF have been proposed in the literature (YOSHIDA; FUJITA, 2011; MARTINS ET AL., 2010), but only the *Exact Factor* approach guarantees exactness in the result (YOSHIDA; FUJITA, 2011). However, the method can take more than 10 minutes to synthesize any equation with 12 literals, even for RO functions (considering the computing platform Linux system on AMD Athlon 64 X2 4400 processor with 2 GB main memory (YOSHIDA; FUJITA, 2011).

Definition 4.1: If an ISF has a RO representation, there is at least one proper assignment of the *don't-care* values that transforms this ISF into a CSF which is trivially synthesized through RO algorithms.

Unfortunately, to identify such CSF is not a straightforward task. An ISF can present a huge number of CSF forms, and only the CSFs that lead to unate functions are of interest. Hence, it is necessary that this CSF represents an unate function. However, such condition is not sufficient because the CSF must be successfully synthesized by RO algorithms.

	0000	0001	0011	0010	0110	0111	0101	0100	1000	1001	1011	1010	1110	1111	1101	1100
000	0 (0)	0 (8)	0 (24)	0 (16)	0 (48)	0 (56)	0 (40)	0 (32)	0 (64)	0 (72)	0 (88)	0 (80)	1 (112)	1 (120)	0 (104)	– (96)
001	0 (1)	0 (9)	– (25)	– (17)	– (49)	– (57)	0 (41)	0 (33)	0 (65)	0 (73)	– (89)	– (81)	1 (113)	1 (121)	0 (105)	1 (97)
011	0 (3)	0 (11)	– (27)	– (19)	– (51)	– (59)	– (43)	– (35)	0 (67)	0 (75)	– (91)	– (83)	1 (115)	1 (123)	– (107)	1 (99)
010	0 (2)	0 (10)	0 (26)	0 (18)	– (50)	– (58)	– (42)	– (34)	0 (66)	0 (74)	0 (90)	0 (82)	1 (114)	1 (122)	– (106)	– (98)
100	0 (4)	0 (12)	0 (28)	0 (20)	0 (52)	0 (60)	0 (44)	0 (36)	– (68)	– (76)	– (92)	– (84)	1 (116)	1 (124)	– (108)	– (100)
101	0 (5)	0 (13)	– (29)	– (21)	– (53)	– (61)	0 (45)	0 (37)	– (69)	– (77)	– (93)	– (85)	1 (117)	1 (125)	– (109)	1 (101)
111	1 (7)	0 (15)	– (31)	1 (23)	1 (55)	– (63)	– (47)	1 (39)	1 (71)	– (79)	– (95)	1 (87)	1 (119)	1 (127)	– (111)	1 (103)
110	0 (6)	0 (14)	0 (30)	0 (22)	– (54)	– (62)	– (46)	– (38)	– (70)	– (78)	– (94)	– (86)	1 (118)	1 (126)	– (110)	– (102)

Figure 4.1: Example of an ISF that can be factorized into a *read-once* form.

Let us take as example the ISF f represented by the Karnaugh map depicted in Figure 4.1 (the *don't-care* terms appear as dashes). As an initial strategy, it is possible to apply a two-level minimizer method, like the Quine-McCluskey (MCCLUSKEY, 1956), in order to transform this ISF into a CSF that can be factorized into a *read-once* equation. However, this is a wrong strategy. Applying a two-level minimizer in the ISF f leads us to the SOP:

$$f = x3\ x4\ x5\ x6 + x0\ x1\ x3 + x0\ x1\ x2 \tag{4.1}$$

which is functional represented by the Karnaugh map presented in Figure 4.2. Unfortunately, this SOP equation cannot be represented by a *read-once* formula. A naïve approach can assign all *don't-care* terms to constant zero or even to constant one. Unfortunately, these two resulting functions cannot be represented by *read-once* equations. However, with a proper assignment of the *don't-care* terms, we can obtain another SOP equation:

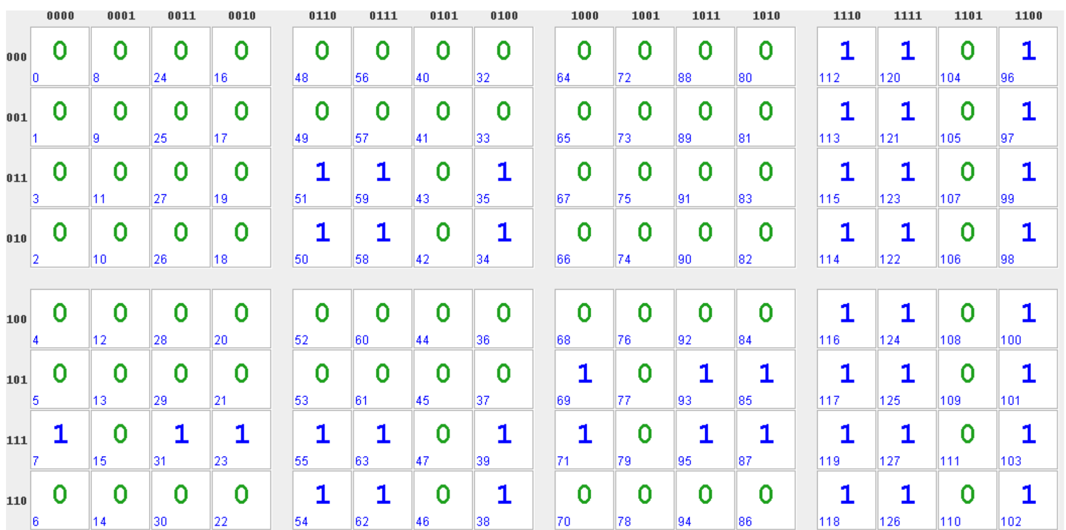
$$f=x0x1x2+x0x1x3+x0x2x4x6+x0x3x4x6+x1x2x5+x1x3x5+x2x4x5x6+x3x4x5x6 \tag{4.2}$$

which can be successfully factorized into a *read-once* equation presented in Eq. 4.3. The correct assignment of the *don't-care* terms is depicted in the Figure 4.3.

$$f= (x0 + x5) * (x3 + x2) * (x1 + x6\ x4) \tag{4.3}$$

	0000	0001	0011	0010	0110	0111	0101	0100	1000	1001	1011	1010	1110	1111	1101	1100
000	0 0	0 8	0 24	0 16	0 48	0 56	0 40	0 32	0 64	0 72	0 88	0 80	1 112	1 120	0 104	1 96
001	0 1	0 9	0 25	0 17	0 49	0 57	0 41	0 33	0 65	0 73	0 89	0 81	1 113	1 121	0 105	1 97
011	0 3	0 11	0 27	0 19	0 51	0 59	0 43	0 35	0 67	0 75	0 91	0 83	1 115	1 123	0 107	1 99
010	0 2	0 10	0 26	0 18	0 50	0 58	0 42	0 34	0 66	0 74	0 90	0 82	1 114	1 122	0 106	1 98
100	0 4	0 12	0 28	0 20	0 52	0 60	0 44	0 36	0 68	0 76	0 92	0 84	1 116	1 124	0 108	1 100
101	0 5	0 13	0 29	0 21	0 53	0 61	0 45	0 37	0 69	0 77	0 93	0 85	1 117	1 125	0 109	1 101
111	1 7	0 15	0 31	1 23	1 55	0 63	0 47	1 39	1 71	0 79	0 95	1 87	1 119	1 127	0 111	1 103
110	0 6	0 14	0 30	0 22	0 54	0 62	0 46	0 38	0 70	0 78	0 94	0 86	1 118	1 126	0 110	1 102

Figure 4.2: Karnaugh map for the SOP *f = x3 x4 x5 x6 + x0 x1 x3 + x0 x1 x2*. This function was synthesized by the Quine-McCluskey's method and cannot be represented by a *read-once* formula.



	0000	0001	0011	0010	0110	0111	0101	0100	1000	1001	1011	1010	1110	1111	1101	1100
000	0 0	0 8	0 24	0 16	0 48	0 56	0 40	0 32	0 64	0 72	0 88	0 80	1 112	1 120	0 104	1 96
001	0 1	0 9	0 25	0 17	0 49	0 57	0 41	0 33	0 65	0 73	0 89	0 81	1 113	1 121	0 105	1 97
011	0 3	0 11	0 27	0 19	1 51	1 59	0 43	1 35	0 67	0 75	0 91	0 83	1 115	1 123	0 107	1 99
010	0 2	0 10	0 26	0 18	1 50	1 58	0 42	1 34	0 66	0 74	0 90	0 82	1 114	1 122	0 106	1 98
100	0 4	0 12	0 28	0 20	0 52	0 60	0 44	0 36	0 68	0 76	0 92	0 84	1 116	1 124	0 108	1 100
101	0 5	0 13	0 29	0 21	0 53	0 61	0 45	0 37	1 69	0 77	1 93	1 85	1 117	1 125	0 109	1 101
111	1 7	0 15	1 31	1 23	1 55	1 63	0 47	1 39	1 71	0 79	1 95	1 87	1 119	1 127	0 111	1 103
110	0 6	0 14	0 30	0 22	1 54	1 62	0 46	1 38	0 70	0 78	0 94	0 86	1 118	1 126	0 110	1 102

Figure 4.3 Karnaugh map for the SOP *f = x0 x1 x2 + x0 x1 x3 + x0 x2 x4 x6 + x0 x3 x4 x6 + x1 x2 x5 + x1 x3 x5 + x2 x4 x5 x6 + x3 x4 x5 x6*. This function was synthesized by the ISF2RO method (section 4.1) and can be represented by a read-once formula *f= (x0 + x5) * (x3 + x2) * (x1 + x6 x4)*.

As it is possible to see, it is not an easy task assigning the *don't-care* terms in order to obtain a *read-once* equation. For example, consider the following ISF:

$$f = \texttt{11101X1011X00000} \qquad (4.4)$$

We can assign the two *don't-care* (***X***) in four different ways, as seen in Table 4.1.

Table 4.1: *Don't-care* assignments and results.

Assignment	ISOP	RO
00	x1*x4+x2*x3	x1*x4+x2*x3
01	x1*x4+x2*x3+x2*x4	**Error**
10	x1*x4+x1*x3+x2*x3	**Error**
11	x1*x4+x1*x3+x2*x3+x2*x4	(x1 + x2) * (x3 + x4)

All *don't-care* assignments lead to unate functions, but only two of them result in RO equations. In this sense, a method to assign the *don't-care* terms in a RO driven way, called *ISF2RO,* is proposed. The algorithm is based on the principle discussed in (LEE; WANG, 2007), that compares cofactors in order to group variables in a RO fashion way. The *ISF2RO* algorithm is presented in the follow section.

4.1 ISF2RO method

In this section some basic data structures used in *ISF2RO* method will be presented.

Definition 4.2: An *assignment* is a data structure used to represent the state when a variable x_i was assigned to a Boolean constant c. An assignment is represented by a tuple $<x_i,c>$. This structure will be used for cofactoring functions.

Definition 4.3: A *logic arrangement* is a data structure used to store the information necessary to run the algorithm, i.e., the grouping state, as illustrated in Figure 4.4

LogicArrangement
String expression Set<Assignment> on_spec Set<Assignment> off_spec ISF pos_cube_cof ISF neg_cube_cof

Figure 4.4: *LogicArrangement* class diagram.

The algorithm, called *ISF2RO*, receives an ISF f as input. Initially, consider that all variables in f are positive unate, without losing generality. The method starts by inserting the input variables into the main list of logic arrangements. These logic arrangements are cofactored and compared to each other. Let x_i be a variable in f. The logic arrangement la_i of x_i is defined as follows:

$$la_i.expression = \text{“}x_i\text{”} \quad (4.5)$$

$$la_i.on_spec = \{ (x_i = 1) \} \quad (4.6)$$

$$la_i.off_spec = \{ (x_i = 0) \} \quad (4.7)$$

$$la_i.pos_cube_cof = f(la_i.on_spec) \quad (4.8)$$

$$la_i.neg_cube_cof = f(la_i.off_spec) \quad (4.9)$$

The proposed pseudo-algorithm is shown in Figure 4.5. The method starts by filling the main list with logic arrangements representing the input variables of the function *f*, as observed in the line 1 in Figure 4.5. The next step consists in finding two logic arrangements that have the same cube cofactor function.

In order to illustrate the idea behind the method, let *f*=`11101X1011X00000`. After filling the main list with the input variables, the expected list is shown in Table 4.2.

Table 4.2: Expected initial main list.

LA	Exp	On	Off	Positive cube cofactor	Negative cube cofactor
la_1	x_1	$x_1=1$	$x_1=0$	11101X1011101X10	11X0000011X00000
la_2	x_2	$x_2=1$	$x_2=0$	1110111011X011X0	1X101X1000000000
la_3	x_3	$x_3=1$	$x_3=0$	11111X1X11110000	10101010X0X00000
la_4	x_4	$x_4=1$	$x_4=0$	1111111111XX0000	1100XX0011000000

In Table 4.2, it is possible to see that la_1 has the same positive cofactor as la_2. Similar situation occurs between la_3 and la_4. Then, according to the lines (8-10, 17-19) in Figure 4.5, these logic arrangements could be grouped through an **OR** operator. There are also logic arrangements with equivalents negative cofactors: $la_1 \approx la_4$ and $la_2 \approx la_3$. Such logic arrangements are grouped by an **AND** operator in accordance to the lines (12-14, 17-19) in Figure 4.5. In Table 4.3, it is possible to see the result of first iteration of the proposed algorithm.

Table 4.3: Grouped logic arrangements after first iteration.

LA	Exp.	On	Off	Positive cube cofactor	Negative cube cofactor
la_5	x_1+x_2	$x_1=1$	$\{x_1=0; x_2=0\}$	11101X1011101X10	0000000000000000
la_6	x_3+x_4	$X_3=1$	$\{x_3=0; x_4=0\}$	1111111111XX0000	0000000000000000
la_7	x_1*x_4	$\{x_1=1; x_4=1\}$	$x_1=0$	1111111111111111	1100XX0011000000
la_8	x_2*x_3	$\{x_2=1; x_3=1\}$	$x_2=0$	1111111111111111	10101010X0X00000

The algorithm continues grouping logic arrangements whenever it is possible. The stopping criteria of the algorithm is defined when a RO equation is recognized, as it is possible to see in the line (26) in Figure 4.5.

Table 4.4: Grouped logic arrangements after the second iteration.

Exp.	On	Off	Pos. cube cofactor	Neg. cube cofactor
(x1 * x4) + (x2 * x3)	{x1=1; x4=1}	{x1=0; x2=0}	1111111111111111	0000000000000000
(x1 + x2) * (x3 + x4)	{x1=1; x3=1}	{x1=0; x2=0}	1111111111111111	0000000000000000

```
1  list = create_logic_arrangements_from_input_variables(f);
2  for (;;) {
3          for (i=0; i < |list|-1; i++) {
4                  f1 = list[i];
5                  for (j=i+1; j < |list|; j++) {
6                          f2 = list[j];
7                          if (f1.pos_cube_cof ≈ f2.pos_cube_cof) {
8                                  f3.exp = f1.exp + f2.exp;
9                                  f3.on_spec=min(f1.on_spec,f2.on_spec);
10                                 f3.off_spec = f1.off_spec ∪ f2.off_spec;
11                         } elsif (f1.neg_cube_cof ≈ f2.neg_cube_cof) {
12                                 f3.exp = f1.exp * f2.exp;
13                                 f3.on_spec = f1.on_spec ∪ f2.on_spec;
14                                 f3.off_spec = min(f1.off_spec, f2.off_spec);
15                         }
16                         if (f3 != null) {
17                                 f3.pos_cube_cof = cubeCof (f, f3.on_spec);
18                                 f3.neg_cube_cof = cubeCof (f, f3.off_spec);
19                                 temp_list.add(f3);
20                         }
21                 }
22                 if (|temp_list| == 0)
23                         return FAILURE;
24                 ro_instances = find_read_once_expressions(temp_list);
25                 if (|ro_instances| != 0)
26                         return ro_instances;
27                 list = list ∪ temp_list;
28                 clear(temp_list);
29         }
30 }
```

Figure 4.5: ISF2RO pseudo-algorithm.

After the second iteration (Table 4.4), two new logic arrangements can be found with the following equations:

$$f = (x1 * x4) + (x2 * x3) \tag{4.10}$$

$$f = (x1 + x2) * (x3 + x4) \tag{4.11}$$

Since both (Eq. 4.10) and (Eq. 4.11) are RO equations, the test in the line (26) in Figure 4.5 will return both solutions.

The worst case time complexity of *ISF2RO* algorithm is $O(2^{2^n})$, where n is the number of variables in f. However, our empirical results are encouraging. The worst runtime observed in our results for functions up to 16 inputs was 500 seconds. The runtime is still lower than *Exact Factor* (YOSHIDA; FUJITA, 2011), even being able of dealing with more input variables. The *Exact Factor* algorithm takes 600 seconds to factorize an equation with 12 literals, while ours *ISF2RO* algorithm finds minimal equations for functions of 16 literals in less than 500 seconds.

In order to show the time complexity of the algorithm, an ISCAS85 benchmark of functions was synthesized. The functions obtained from the benchmark (72423 in total) were grouped regarding input count, from 3 up to 16 input variables. The worst case runtime for each group is depicted in Figure 4.6. The vertical axis is presented in logarithm scale. This way, it is easy to observe that the algorithm has an exponential time complexity. However, for this benchmark of functions, the worst case runtime was 20 seconds to synthesize a function with 16 inputs. The runtime which *ISF2RO* algorithm takes to factorize a function in 16 literals is still lower than the runtime of the *Exact Factor* algorithm (YOSHIDA; FUJITA, 2011) while factoring functions with only 12 literals.

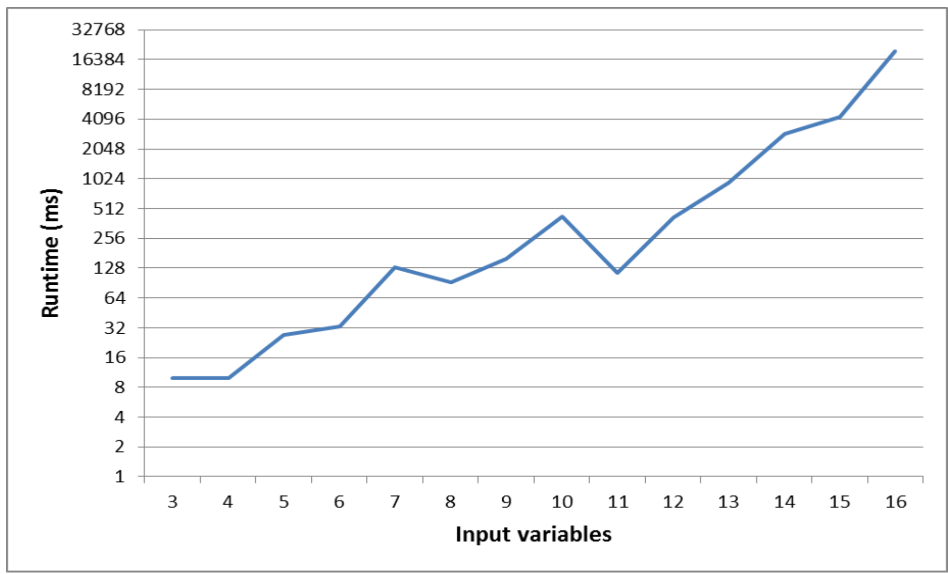


Figure 4.6: Worst case runtime (in *ms*) to synthesize functions from *ISCAS'85* benchmark grouped by input count.

5 READ-POLARITY-ONCE FUNCTIONS

Efficient exact algorithms exist for a sub-class of functions known as *read-once* functions (GOLUMBIC; MINTZ; ROTICS, 2001; LEE; WANG, 2007). A Boolean function is considered read-once (RO) if it can be represented in a factored form where each variable appears only once, e.g. *f=a*(b+c*(d+e))*. The class of RO functions is of special interest in logic synthesis, since they are quite frequent in circuit applications (PEER; PINTER, 1995).

However, exact algorithms for RO functions present two important limitations: (1) they do not consider incompletely specified Boolean functions (ISF); and (2) they are not suitable for functions with binate variables. In order to overcome the first constraint, I proposed an algorithm (ISF2RO) to find RO formulas for ISF, whenever possible (section 4.1). With respect to the second limitation, I propose a domain transformation, called here as a *unatization* process, that splits existing binate variables into two independent unate variables. Such domain transformation leads to ISF, which can be factored efficiently by the ISF2RO algorithm (section 4.1).

The combination of both contributions gives exact factoring results for a novel broader class of functions called *read-polarity-once* (RPO) functions, where each polarity (positive or negative) of a variable appears at maximum once in the factored expression. For instance, RO algorithms fail when factoring *f=!a*b*d+b*c+a*c*, since the variable *a* is binate. The proposed RPO algorithm can factorize such function into an exact expression *f=(!a*d+c)*(a+b)*, which presents only 5 literals.

Moreover, a study was made about the occurrence of RPO functions in circuits considering some ISCAS'85 benchmarks (IWLS, 2005). Experimental results have shown that RPO functions are significantly more frequent than RO functions. The entire flow comprising the unatization of RPO functions and the factoring of ISF in RO expressions has been validated. The implementation was able to efficiently find optimal solutions of functions up to 8 binate variables (i.e., up to 16 literals).

The remainder of this chapter is organized as follows. Section 5.1 presents definitions for the RPO class. Section 5.2 presents the proposed domain transformation, i.e., the unitization process of RPO functions. In section 5.3 is presented the complete algorithm to perform the factoring of ISF in RPO equations. Experimental results are shown in section 5.4.

5.1 Definition

Definition 5.1: A Boolean function is called *read-polarity-once* (RPO) if each polarity (positive or negative) of a variable appears at maximum once in the minimum factored equation (CALLEGARO et al., 2012).

Lemma 5.1: a positive (negative) unate variable contributes with at least one positive (negative) literal in a factored form.

Lemma 5.2: a binate variable contributes with at least two literals (one positive and one negative) in a factored form.

Theorem 5.1: a function represented by an RPO equation is in minimum form, if each unate variable contributes with exactly one literal and each binate variable contributes with exactly two literals (one positive and one negative).

Proof: straightforward by lemmas 5.1 and 5.2.

Corollary: The proposed algorithm gives optimal results in literal count for RPO functions as it uses at most one literal per unate variable and at most two literals (one positive and one negative) per binate variables, while generating an RPO expression.

Definition 5.2: The RPO class is a superset of the RO class.

Corollary: Every RO function is also a RPO function, while a RPO function is not necessarily a RO function. For instance, $f = a*(b+c*(d+e))$ is both RO and RPO function, while $f = (a+b)*(!a+!b)$ is only a RPO function.

The class of RPO functions is slightly different from the class of RO functions. The RPO class contains binate functions as elements (of the class), while the RO class contains only unate functions. Figure 5.1 shows a comparison between the universe of RO and RPO functions.

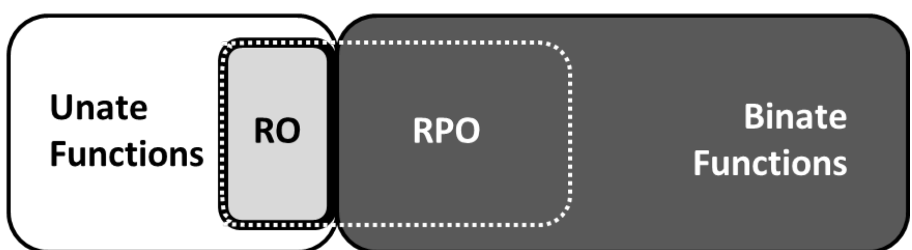


Figure 5.1: Comparison between the universe of *read-once* and *read-polarity-once* functions.

According to definition 5.1, if a function can be factored in a RPO expression, each polarity (positive or negative) of a variable appears at maximum once in the factored equation. Thus, an interesting point of investigation is if it is possible to separate the positive and negative literals, and transform the function into an unate function. Another point, equally interesting, is if that resulting transformation could be treated successfully by RO factoring algorithms.

In this sense, I propose a domain transformation (***unatization***) that splits existing binate variables into two independent unate variables. This domain transformation leads to ISF, which can be factored efficiently by the algorithm proposed in chapter 4. The combination of both contributions provides exact results for the class of RPO functions.

5.2 Unatization process

The *unatization process* is a fundamental step when recognizing *read-polarity-once* functions. This step could be seen as the major contribution of this work. Without the unatization process, the factoring flow herein presented would not exist. The starting point to obtain optimal equations for *read-polarity-once* functions was the very basic

idea of the domain transformation, where binate variables were split into two independent unate variables. The idea behind this process is that, if an unate function could be represented in a *read-once* form, this form will result in minimal literal count. By returning to an original domain of variables name, the resulting form will also still in optimal form regarding literal count. This is correct since each unate variable will appears once in the equation, while binate variables will appear twice (one literal to the positive and another one to the negative polarity).

Although been a critical step when recognizing *read-polarity-once* functions, it is important to notice that the unatization process can also be applied in other logic synthesis methods. It is possible to apply the unatization process on arbitrary Boolean functions in order to take advantage of methods that are specific designed for unate functions, e.g. *unate recursive paradigm* on ESPRESSO (BRAYTON, 1987). Another application example is the functional decomposition methods (BERTACCO; DAMIANI, 1997; MISHCHENKO; STEINBACH; PERKOWSKI, 2001), where the idea of splitting binate variables could result in better decomposed circuits.

Definition 5.3: The **unatization** method receives as input an ISF and split all binate variables into two independent unate variables.

Akers (AKERS, 1961) proposed a similar domain transformation that modifies a Boolean function into a *logically passive function*. However, the transformation proposed by Akers is devoted to find a single SOP representation (out of the many possible SOP representations). According definition 4.1, the specific SOP representation has to be chosen wisely to derive a RO expression. Our transformation into ISF exploits a broader space of solutions, where a given completely specified function satisfying the ISF can lead to a RO function (while others cannot).

Definition 5.4: Let f be an ISF, where f^{ON} represents the on-set of f and f^{DC} represents the *don't-care* set of f.

The pseudo-algorithm for the unatization process is shown in Figure 5.2. The basic idea behind the process is to split the binate variables into independent unate variables. Let x_i be a binate variable of f. In order to unatize x_i, a variable not_x_i is inserted into f, as shown in the line 4 in Figure 5.2. It is important to notice that both variables x_i and not_x_i cannot have the same value at the same time. When a Boolean constant c is assigned to input x_i, the complemented value has to be assigned to input not_x_i. In this sense, the lines in the *truth table* where both variables are assigned to the same constant are set to *don't-care* (see lines 5-6 in Figure 5.2), as these lines represent impossible input conditions. The next step is to guarantee that both variables (x_i and not_x_i) became positive unate after the domain transformation.

Table 5.1: Two states that must be fixed in the unatization process.

$f(x_i=1)$	$f(x_i=0)$
0	X
X	1

The ***fix_positive_unate*** method is based on the definition of unateness shown in subsection 2.2.5. Let x*i* be the variable to be fixed, $Y=\{0,1,X\}$ and $\{tp_i,tn_i\} \in Y$ be a term

in the line *i* of the *truth table* when $f(x_i=1)$ and $f(x_i=0)$, respectively (Table 5.1). The following two states must be fixed:

$$tp_i = 0 \text{ and } tn_i = X \quad (5.1)$$

$$tp_i = X \text{ and } tn_i = 1 \quad (5.2)$$

In Equation (5.1), if tn_i receives the value *1* the function becomes binate. The same happens in Equation (5.2) if tp_i receives the value *0*. In order to avoid both situations, the method ***fix_positive_unate*** (see lines 14 to 25 in Figure 5.2) properly assign values to the *don't-care* terms that are responsible for these cases.

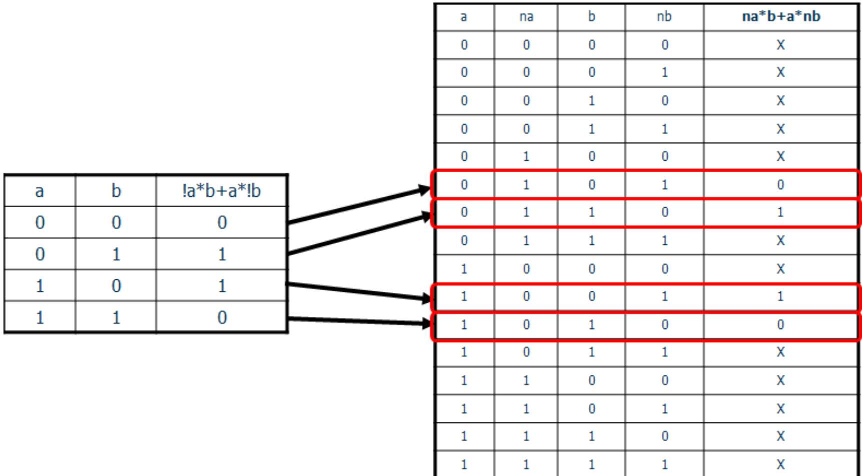
```
1   unatization(f^ON, f^DC) {
2       for (int i = 0; i < n; i++)
3           if (is_binate(x_i)){
4               createVariable(not_x_i);
5               XNOR = !( x_i ^ not_x_i);
6               f^DC = f^DC + XNOR;
7               fix_positive_unate(f^ON, f^DC, x_i);
8               fix_positive_unate(f^ON, f^DC, not_x_i);
9           }
10      }
11  }
12
13  fix_positive_unate(f^ON, f^DC, x_i) {
14      PD = positive_cofactor(f^DC, x_i);
15      ND = negative_cofactor(f^DC, x_i);
16      PC = positive_cofactor(f^ON, x_i);
17      NC = negative_cofactor(f^ON, x_i);
18      state_0x = !PD * !PC * ND;
19      f^DC = f^DC * !state_0x;
20      f^ON = f^ON * !state_0x;
21      state_x1 = PD * !ND * NC;
22      f^DC = f^DC * !state_x1;
23      f^ON = f^ON + state_x1;
24  }
```

Figure 5.2: Pseudo-algorithm for the unatization process.

The last situation that must be observed is when both tp_i and tn_i carry the value *X*. By definition, this situation does not turn the variable x_i to a binate behavior. However, depending on the assignments of both *don't-care* terms, the behavior of the variable x_i can be transformed into a binate function. This assignment is quite uncommon in the *ISF2RO* algorithm. When such case occurs, the logic arrangement cannot be grouped anymore. In order to avoid these cases, sets of pairs of functions to be distinguished (SPFD) could be considered to speed-up the algorithm (YAMASHITA; SAWADA; NAGOYA, 2000).

Example: Let *a* and *b* be binate variables from $f=(a+b)*(!a+!b)$. In order to *unatize f*, we introduce independent variables to represent the negative unate literals. Hence, by introducing variables *na* and *nb*, a domain transformation is performed and the function

becomes a 4-input function, with most of the terms appearing as *don't-cares*, as seen in Figure 5.3. This is the first step of the unatization process.



a	b	!a*b+a*!b
0	0	0
0	1	1
1	0	1
1	1	0

a	na	b	nb	na*b+a*nb
0	0	0	0	X
0	0	0	1	X
0	0	1	0	X
0	0	1	1	X
0	1	0	0	X
0	1	0	1	0
0	1	1	0	1
0	1	1	1	X
1	0	0	0	X
1	0	0	1	1
1	0	1	0	0
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Figure 5.3: First step of unatization process: split binate variables.

The function represented by the *truth table* shown in Figure 5.3 is not positive unate. The second step of the unatization process is to turn all the variables to positive unate ones. Let us take as example the variable *a*. Figure 5.4 shows positive and negative cofactors of variable *a*. The lines in the *truth table* where the squares appear means that these lines must be fixed (Eq. 5.1 and Eq. 5.2) in order to turn the variable *a* in positive unate.

a	na	b	nb	**na*b+a*nb**	f(a=1)	f(a=0)
0	0	0	0	X	X	X
0	0	0	1	X	1	X
0	0	1	0	**X=0**	0	X
0	0	1	1	X	X	X
0	1	0	0	X	X	X
0	1	0	1	0	X	0
0	1	1	0	1	X	1
0	1	1	1	X	X	X
1	0	0	0	X	X	X
1	0	0	1	1	1	X
1	0	1	0	0	0	X
1	0	1	1	X	X	X
1	1	0	0	X	X	X
1	1	0	1	X	X	0
1	1	1	0	**X=1**	X	1
1	1	1	1	X	X	X

Figure 5.4: Second step of the unatization process: turn the variables into a positive unate behavior.

By computing the cofactors of the function and setting the *don't-care* values to force the function to become positive unate in all of its variables, a new function is obtained. The computation of the cofactors and the new function obtained is presented in Figure 5.5.

a	na	b	nb	**na*b+a*nb**	f(a=1)	f(a=0)	f(na=1)	f(na=0)	f(b=1)	f(b=0)	f(nb=1)	f(nb=0)
0	0	0	0	**X=0**	X	X	x	x	0	x	0	0
0	0	0	1	**X=0**	1	X	0	x	x	0	0	0
0	0	1	0	**X=0**	0	X	1	0	0	x	x	0
0	0	1	1	X	X	X	x	x	x	0	x	0
0	1	0	0	**X=0**	X	X	x	x	1	x	0	x
0	1	0	1	0	X	0	0	x	x	0	0	x
0	1	1	0	1	X	1	1	0	1	x	x	1
0	1	1	1	**X=1**	X	X	x	x	x	0	x	1
1	0	0	0	**X=0**	X	X	x	x	0	x	1	0
1	0	0	1	1	1	X	x	1	x	1	1	0
1	0	1	0	0	0	X	1	0	0	x	1	0
1	0	1	1	**X=1**	X	X	x	x	x	1	1	0
1	1	0	0	X	X	X	x	x	1	x	1	x
1	1	0	1	**X=1**	X	0	x	1	x	1	1	x
1	1	1	0	**X=1**	X	1	1	0	1	x	1	1
1	1	1	1	**X=1**	X	X	x	x	x	1	1	1

Figure 5.5: Cofactors that are set to force the function to become positive unate.

Notice that two unspecified lines remain unspecified after this process (dashed rectangles presented in Figure 5.5). This is related to the unate flexibility of the function. Any of the four possible ways to assign these two lines will remain unate functions.

It is also worth to mention that the algorithm does not depend on the *truth table* data structure. In fact, our algorithm is based on reduced and ordered binary decision diagram (ROBDD) data structure (BRYANT, 1986).

The unatization algorithm can be implemented in time complexity of $O(n \times |m|)$, where n is the number of variables in f and $|m|$ is the number of nodes in the final ROBDD. Empirical results have shown that the unatization runtime is irrelevant in the entire flow for synthesize RPO functions. The runtime of the *ISF2RO* algorithm is currently the main bottleneck.

5.3 Factoring incompletely specified boolean functions into read-polarity-once equations

Finally, it is possible to describe the entire flow of the RPO factoring algorithm, called *ISF2RPO*. The complete algorithm proceeds in two main steps. The first step reads an ISF f and computes the polarity of the variables. Every binate variable is split into two separate positive unate variables according to the unatization process (section 5.2).

The ISF function returned by the unatization process is then used in the second step, where the *ISF2RO* algorithm (section 4.1) performs a search for a RO equation. If *ISF2RO* returns a RO output, the equation is then rewritten considering the original variables as they were presented before the domain transformation. An entire flow for recognize read-polarity-once functions is described in Figure 5.6.

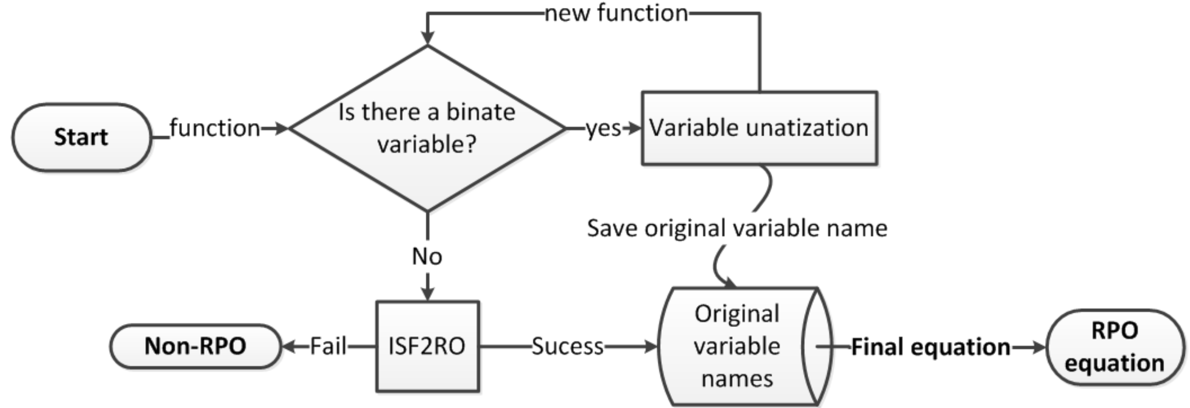


Figure 5.6: ISF2RPO flow chart.

Example: Let the input of the *ISF2RPO* algorithm be *f=a !b + !a b*. The first step of the *ISF2RPO* is to split binate variables into unate ones through the unatization process (section 5.2). The ISF *g* obtained after the unatization process is presented in Figure 5.7. Notice that the negative polarities *!a* and *!b* are renamed to *na* and *nb*, respectively.

a	na	b	nb	*g*
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	X
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	X
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Figure 5.7: The ISF *g* obtained after the unatization process of the function *f=a !b + !a b.*

After the unatization process, the function *g* is then used as input of the *ISF2RO* algorithm presented in section 4.1. Notice that two unspecified lines remain unspecified after the unatization process. This is directly related to the following two different equations obtained by the *ISF2RO* method: (Eq. 5.3) and (Eq. 5.4). Figure 5.8 shows how *ISF2RO* method set the remaining *don't-care* terms to produce different equations. Notice that both equations are in the minimal form, e.g. in a RO form.

$$g = na * b + a * nb \tag{5.3}$$

$$g = (na + nb) * (a + b) \tag{5.4}$$

a	na	b	nb	na*b+a*nb
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

a	na	b	nb	(na+nb)*(a+b)
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Figure 5.8: Transformation of the remaining *don't-care* terms and how it produces different read-once equations.

The last step of the *ISF2RPO* flow is to rename variables with their original names. The equations Eq. 5.3 and Eq. 5.4 are renamed and are presented in Eq. 5.5 and Eq. 5.6, respectively. Notice that both equations are in the minimal form. They are structurally distinct and represent the same Boolean function

$$f = (!a * b) + (a * !b) \quad (5.5)$$

$$f = (!a + !b) * (a + b) \quad (5.6)$$

The time complexity of the *ISF2RPO* algorithm is bounded by the complexity of the *ISF2RO* algorithm (see Figure 5.6). Experimental results demonstrate that the RPO algorithm can efficiently find optimal solutions in the number of literals for functions up to 8 binate variables (i.e., up to 16 unate literals).

5.4 Experimental Results

This section presents a study about the occurrence of RPO functions over the set of 5-input NPN-class, as well as the occurrence of RPO functions over the ISCAS'85 benchmark circuits (IWLS, 2005). The last experiment was carried out over a benchmark of important functions for logic brick design (MOTIANI; KHETERPAL; PILEGGI, 2010).

5.4.1 NPN equivalence class up to 5 inputs

The first experiment was carried out over the set of all 5-input NPN-class (negation-permutation-negation) functions, grouped in 616,125 classes by equivalence through input permutation, negation of its inputs and/or negation of the output (CORREIA; 2001). Instead of running the algorithm for all Boolean space of 5 variables (2^{32} functions), the 5-input NPN-class benchmark was chosen. The benchmark can represent the functionality of all Boolean space of 5 variables in a more compact set, without losing generality. To run the algorithm for all the 616,125 functions four minutes of

execution time were needed. The worst case optimization was 800 *ms* and the average case was less than 1ms. The platform was a Linux system on Intel Core i5 2400 processor with 2GB main memory.

For the universe of the 5-input NPN-class functions, there are 1,462 functions that are classified as RPO, while only 21 functions are classified as RO. This means that there are approximately 70x more RPO functions than RO functions, for the set of 5-input functions. Our results demonstrate that the universe of RPO is quite broader than the universe of RO functions, for which many works have been devoted (HAYES, 1975; GURVICH, 1991; PEER; PINTER, 1995; GOLUMBIC; MINTZ; ROTICS, 2001; LEE; WANG, 2007).

Comparative results evaluating the efficiency of the proposed algorithm are shown in Table 5.2, considering the set of 1,462 RPO functions. Our algorithm presented better results in terms of number of literals than *Quick Factor* (QF) (SENTOVICH et al., 1992), *Good Factor* (GF) (SENTOVICH et al., 1992), *ABC* (BERKELEY, 2012) and *X-Factor* (MINTZ; GOLUMBIC, 2005) tools. The proposed algorithm is still slow when compared to existing factoring methods (QF and GF methods). However, it guarantees minimal factored forms in literal count (theorem 5.1). It is important to notice that the *ISF2RPO* algorithm (section 5.3) factorize only RPO functions, while the other methods are of general factoring purposes.

Table 5.2: Total number of literals obtained and runtime when factoring process of 1,462 RPO functions using different approaches.

	QF	**GF**	**ABC**	**X-Factor***	**ISF2RPO (this work)**
Literals	16,086	15,671	15,981	13,253	13,064
Runtime	1.9s	2.3s	2.0s	7.1s	5.7s

* Results of an in-house implementation of the X-Factor algorithm.

5.4.2 ISCAS'85 benchmark suite

A study about the occurrence of RPO functions over some ISCAS'85 benchmarks (IWLS, 2005) was performed. Such analysis has been carried out in order to figure out the frequency of RPO functions in comparison to RO functions in mapped circuits. We have extracted functions up to 8 inputs from the benchmarks through K-Cuts method (MARTINELLO, 2010). These functions were grouped in P-classes, by equivalence through input permutations. The term ‘occurrences’ represents the number of functions counted before grouping them into P-classes of equivalence.

Table 5.3: Analysis of functions up to 8 inputs identified in ISCAS'85 benchmarks.

	RO		RPO	
Inputs	**P Classes**	**Occurrences**	**P Classes**	**Occurrences**
2	67%	84%	100%	100%
3	53%	66%	90%	88%
4	44%	54%	85%	71%
5	37%	42%	69%	53%
6	33%	36%	57%	46%
7	34%	36%	52%	47%
8	32%	34%	46%	44%

We have extracted the functions in two ways. Table 5.3 summarizes the results regarding all possible functions up to 8 inputs in the circuits. In Table 5.4 the functions were extracted from the circuits using an AIG greedy covering algorithm (MARTINELLO, 2010).

Similarly to RO functions, the number of RPO functions decreases as the number of variables increases. This is an expected result, since the more inputs the Boolean function have, the more complex the function become.

Table 5.4: Functions selected from the ISCAS'85 benchmarks using a greedy covering algorithm.

	RO		RPO	
Inputs	**P Classes**	**Occurrences**	**P Classes**	**Occurrences**
2	78%	93%	100%	100%
3	59%	63%	87%	94%
4	49%	53%	78%	81%
5	35%	36%	79%	81%
6	41%	39%	63%	60%
7	45%	43%	62%	57%
8	14%	15%	27%	27%

In Table 5.5, it is possible to see the runtime for synthesize all K-cuts functions up to K=8. In Table 5.6, it is shown the runtime of the algorithm for synthesizing the functions extracted from the covered circuit.

Table 5.5: Runtime for synthesizing all K-Cuts functions (K=8).

Circuit	P Classes	Time (s)	Avg. time (s)
C1355	680	123.455	0.18
C17	12	0.01	0.01
C1908	1224	133.162	0.11
C2670	6345	284.626	0.04
C3540	9275	123.945	0.01
C432	844	3.259	0.01
C499	432	98.187	0.23
C5315	11350	806.489	0.07
C6288	142	0.634	0.01
C7552	17888	8045.137	0.45
C880	1691	86.301	0.05

Experimental results over ISCAS'85 benchmarks have demonstrated that RPO functions are significantly more frequent in circuit application than RO functions. The entire flow to factorize RPO functions was validated and our implementation was able to find optimal solutions of functions up to 8 binate variables in a reasonable runtime.

Table 5.6: Total runtime for synthesizing functions selected by a greedy covering algorithm.

Circuit	P Classes	Time (s)	Avg. time (s)
C1355	16	3.70	0.23
C17	3	0.01	0.01
C1908	44	2.37	0.05
C2670	68	39.53	0.58
C3540	129	1.29	0.01
C432	25	0.15	0.01
C499	10	0.89	0.09
C5315	108	58.07	0.54
C6288	38	0.20	0.01
C7552	130	8.42	0.06
C880	36	18.08	0.50

5.4.3 Patent US7784013

According to (MOTIANI; KHETERPAL; PILEGGI, 2010), a set of logic functions can be added to a cell library to significantly improve specific designs. In one of the examples given by Motiani, a set of 12 distinct functions were added to the library. Out of the twelve functions added, 6 were RO, 10 were RPO (including the 6 RO that are also RPO) and only 2 are not RPO functions, as it is possible to see in Table 5.7. This observation highlights the importance of the RPO class for different technologies, including the logic brick methodology proposed by (MOTIANI; KHETERPAL; PILEGGI, 2010).

Table 5.7: Set of 12 distinct functions given by Motiani where 10 were RPO functions.

Original	Read-polarity-once equations
f01 = p0p1!p3+p2!p3+p4p5	*f01 = ((((p1 * p0) + p2) * !p3) + (p5 * p4))*
f02 = p0p1!p3+!p3!p4+p1p2!p3	*f02 = ((((p2 + p0) * p1) + !p4) * !p3)*
f03 = p1p3p6+p0!p2p5+p3p4p6+!p1!p2!p4	*f03 = (!p4 * !p1 + p0 * p5) * !p2 + p6 * p3 * (p4 + p1)*
f04 = !p1!p2p3+!p0!p1p3+p1!p3+p0p2!p3	*f04 = ((((p2 * p0) + p1) * !p3) + ((!p1 * p3) * (!p0 + !p2)))*
f05 = p1!p4+!p0!p3+p1!p3+!p2!p4+!p0!p4+!p2!p3	*f05 = (((!p0 + p1) + !p2) * (!p3 + !p4))*
f06 = p0p2+p1p2+p3p4	*f06 = (((p1 + p0) * p2) + (p4 * p3))*
f07 = !p1!p2+p4p5+!p6!p7+p0p3	*f07 = (((!p1 * !p2) + (p3 * p0)) + ((!p6 * !p7) + (p4 * p5)))*
f08 = p1p3p4+p0p2p3p4+!p1!p2!p3p4+!p0!p1!p3p4+ !p1!p2p3!p4+p1!p3!p4+p0!p1p2!p4	*f08 =* ***Non RPO***
f09 = p0!p3!p5+!p0!p1p2+p2p3p4+p1!p4!p5+ p1!p3!p5+p0!p4!p5	*f09 = (p4 * p3 + !p1 * !p0) * p2 + (!p3 + !p4) * !p5 * (p0 + p1)*
f10 = !p0!p1p2+!p0p1!p3+p0p1p2+p0!p1!p3	*f10 =* ***Non RPO***
f11 = !p1!p2!p3+p0!p3+!p1!p2!p4+p0!p4	*f11 = (((!p1 * !p2) + p0) * (!p4 + !p3))*
f12 = !p0!p1p4+!p2p4+p0p2!p3+p1p2!p3	*f12 = ((((p1 + p0) * p2) + p4) * ((!p1 * !p0) + (!p3 + !p2)))*

6 CONCLUSIONS AND FUTURE WORK

The main contribution of this work was the introduction of the concept of *read-polarity-once* functions. Besides introducing the class of RPO functions, several related contributions were also introduced: (1) an algorithm for factoring incompletely specified functions into read-once equations; (2) a domain transformation that splits existing binate variables into two independent unate variables; and (3) a complete algorithm for exact factoring the class of read-polarity-once functions.

The first contribution was the proposal of an algorithm (*ISF2RO*) that is able to factorize incompletely specified Boolean functions into *read-once* forms, when this is possible. To the best of the author's knowledge, the unique method that guarantees exactness in the result is the *Exact Factor* approach, proposed by (YOSHIDA; FUJITA, 2011). However, the method is feasible with functions with up to 10 variables. The *ISF2RO* proposed in this work was tested and the results show that it is able to factorize functions with up to 16 input variables.

The second contribution was the proposal of a method for domain transformation that splits existing binate variables into two independent unate variables, called *unatization process*. The *unatization process* is a fundamental step when recognizing *read-polarity-once* functions. However, it can be applied in other logic synthesis methods, like functional decomposition methods.

The third contribution was the development of a complete algorithm for exact optimal factoring the class of *read-polarity-once* functions called *ISF2RPO*. Such algorithm was implemented and compared to existing factoring algorithms. The proposed algorithm guarantees minimal factored forms for the class of *read-polarity-once* functions. Results show that, out of the set of 612,125 NPN-class functions with up to 5-inputs, 1,462 functions were identified as *read-polarity-once*, while only 21 functions can be considered *read-once*. Moreover, results taking into account ISCAS'85 benchmarks have shown that *read-polarity-once* functions are quite more frequent in circuit application than *read-once* functions. The *read-polarity-once* class of functions is also important for different technologies, including the logic brick methodology proposed (MOTIANI; KHETERPAL; PILEGGI, 2010), as demonstrated by the large number of *read-polarity-once* functions (10 out of 12) given as example in the (United States Patent number 7784013).

It is worthy to highlight that both algorithms were implemented in a tool, called SwitchCraft (CALLEGARO et al., 2010). SwitchCraft framework provides a set of tools for switch network and logic gate generation. The tool has been applied at Federal University of 'Rio Grande do Sul' (UFRGS), in Brazil, in undergraduate and graduate courses about fundamentals of IC digital design.

There is still much work that needs to be carried on. First of all, the runtime of the *ISF2RO* must be improved. A new version of the algorithm based on branch-and-bound is already in development. Another point to be carried is with regard to the scalability of the *ISF2RO* method. The current version of the algorithm scales up to 16 variables. I believe that will be possible to scale to 32 variables with the new version of the algorithm.

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ANEXO A <FUNÇÕES *READ-POLARITY-ONCE*>

A.1 Introdução

O fluxo de projeto de síntese de circuitos é normalmente dividido em três etapas principais: síntese arquitetural, síntese lógica e síntese física (MICHELI, 1994). A síntese de arquitetural, muitas vezes chamada de síntese de alto nível, consiste em transformar uma descrição algorítmica de um comportamento desejado em um formato de hardware que implementa esse comportamento, como no formato RTL (*Register Transfer Level*). Normalmente, essas descrições algorítmicas são representadas em um formato semelhante à linguagem C, como (por exemplo, *System C*) ou formato HDL (*Hardware Description Language*) comportamental (por exemplo, VHDL e Verilog).

O processo de síntese lógica tem sido uma das áreas de maior sucesso comercial na automação de projetos eletrônicos (EDA). Este sucesso comercial indica que todos os dispositivos digitais que usamos no nosso dia-a-dia foram projetados com um conjunto de ferramentas de síntese lógica. A tarefa de síntese lógica é composta por várias etapas. Estas etapas podem ser diferentes de acordo com a natureza do circuito, por exemplo, como circuito sequencial ou de combinacional. O objetivo da síntese lógica é determinar a estrutura primitiva de um circuito, ou seja, a sua representação em nível de portas lógicas primitivas. A síntese lógica é normalmente dividida em três fases: otimizações independente de tecnologia, mapeamento tecnológico e otimizações dependentes de tecnologia. A primeira aplicam-se transformações que não dependem da tecnologia, mas sim do comportamento funcional de uma rede Booleana, por exemplo, algoritmos de fatoração. Em seguida, a fase de mapeamento tecnológico combina porções do circuito para uma célula informação de tecnologia. A fase dependente de tecnologia aplica otimizações no circuito mapeado, tais como redimensionamento de células e duplicação lógica.

A síntese física consiste principalmente em duas tarefas principais: o posicionamento de blocos e roteamento dos fios que os conectam. O primeiro distribui fisicamente as células enquanto o posterior executa as interligações de sinal.

Este trabalho aborda a síntese de funções Booleanas no âmbito de um fluxo de projeto de circuito digital, mais precisamente na fase de síntese lógica. No entanto, o foco pode ser considerado mais amplo, já que este trabalho propõe uma nova classe de funções Booleanas que podem ter aplicação em diferentes outras áreas além da síntese de circuitos.

A.1.1 Motivação

O processo de fatoração de funções booleanas é uma operação fundamental na síntese lógica algorítmica (BRAYTON, 1987; HACHTEL; SOMENZI, 2006). Fatoração é o processo de derivação de uma equação algébrica, ou forma fatorada, que

representa uma determinada função lógica, normalmente descrita inicialmente em forma de soma de produtos (SOP) ou produto-de-somas (POS). Por exemplo, *f=a*b+a*c*d+a*c*e* pode ser fatorado em uma equação logicamente equivalente *f=a*(b+c*(d+e))*.

Qualquer função lógica pode ser representada por diferentes equações fatoradas. A tarefa de fatorar funções Booleanas fórmulas menores, mais compactas e logicamente equivalentes é uma das operações básicas nos estágios iniciais da síntese lógica algorítmica (HACHTEL; SOMENZI, 2006). Na maioria dos estilos de projeto, como portas lógicas CMOS convencional, o custo de implementação elétrica de uma função Booleana corresponde diretamente à sua equação fatorada em relação a número de literais e contagem de dispositivos (transistores). A geração de uma forma fatorada exata, ou seja, a equação de menor comprimento é um problema NP-difícil (GOLUMBIC; MINTZ, 1999). Assim, algoritmos heurísticos foram desenvolvidos a fim de obter boas soluções fatoradas (BRAYTON, 1987; STANION; SECHEN, 1994; MINTZ; GOLUMBIC, 2005; HACHTEL; SOMENZI, 2006; YOSHIDA; FUJITA, 2011). Alguns algoritmos heurísticos conhecidos incluem XFactor (MINTZ; GOLUMBIC, 2005), que proporciona bons resultados, mas não garante as equações mínimas. Em (LAWLER, 1964), o autor pretende fornecer um algoritmo para fatoração exata. No entanto, o método de Lawler não é escalável e torna-se impraticável mesmo para funções com apenas quatro variáveis. Recentemente, novas abordagens têm melhorado o processo de fatoração de soluções exatas, mas a escalabilidade e tempo de execução continuam a serem os principais gargalos (YOSHIDA; IKEDA; ASADA, 2006; YOSHIDA; Fujita, 2011; MARTINS ET AL, 2012).

Algoritmos exatos e eficientes existem para uma subclasse de funções Booleanas conhecidas como *read-once* (RO) (LEE; WANG, 2007; GOLUMBIC; MINTZ; ROTICS, 2008). Uma função Booleana é considerada RO se ela pode ser representada por uma forma fatorada onde cada variável apareça apenas uma vez (GOLUMBIC; MINTZ; ROTICS, 2001). Como no exemplo acima mencionado, a função *f=a*(b+c*(d+e))* é RO.

No entanto, algoritmos exatos para funções RO apresentam duas limitações importantes: (1) eles não consideram funções booleanas incompletamente especificadas (ISBF), e (2) eles não são adequados para as funções com variáveis binate (ver Subseção 2.2.5). Neste contexto, uma questão pode ser levantada: "*Uma vez que as funções RO são importantes em aplicações reais de circuitos, suponhamos que estendem os limites desta classe, criando uma nova classe de funções, a fim de ultrapassar a limitação (1) e (2). O quanto mais abrangente seria esta nova classe no que diz respeito à classe RO? Quanto mais importante serão as funções dessa nova classe em aplicações reais de circuitos?*" Essas perguntas resumem a motivação desta dissertação.

A.1.2 Objetivos

O objetivo desta dissertação é a de transcender os limites da classe RO de funções. Uma maneira de fazer isso é criar uma nova e ampla classe de funções Booleanas, que pode lidar com funções binate e onde formas fatoradas são mínimas. Uma abordagem é a de dividir as fases positivas e negativas de variáveis binate em duas variáveis independentes. Desta forma, é possível representar a fase positiva em uma variável e a fase negativa em outra variável, sendo essas duas variáveis unate e independentes umas das outras. Esta transformação leva a um domínio de funções Booleanas

incompletamente especificadas (ISBF). Assim, é importante que se desenvolva um método que seja capaz de fatorar ISBF em formas RO, que são conhecidos por resultar em formas fatoradas mínimas. A combinação de ambas as estratégias podem levar a uma nova classe mais ampla de funções chamadas funções *read-polarity-once* (RPO), onde cada uma das polaridades (positiva ou negativa) de uma variável aparece no máximo uma vez na forma fatorada mínima, como por exemplo, *f=(!a*d+c)*(a+b)*. Neste sentido, este trabalho tem como objetivo fornecer um algoritmo eficiente que garanta formas fatoradas mínimas para a classe de funções Booleanas RPO.

A.2 Funções Read-Polarity-Once

Existem algoritmos exatos para uma subclasse de funções conhecidas como funções *read-once* (GOLUMBIC; MINTZ; ROTICS, 2001; LEE; WANG, 2007). Uma função Booleana é considerada *read-once* (RO), se ela pode ser representada numa forma onde cada variável apareça uma única vez, por exemplo, *f=a*(b+c*(d+e))*. A classe de funções RO é de especial interesse na síntese lógica, uma vez que eles são bastante frequentes em aplicações de circuitos (PEER; PINTER, 1995).

No entanto, algoritmos exatos para funções RO apresentam duas limitações importantes: (1) eles não consideram funções Booleanas incompletamente especificadas (ISF), e (2) não são adequados para as funções com variáveis *binate*. A fim de superar a primeira restrição, um algoritmo (ISF2RO) será aqui proposto para encontrar fórmulas RO para ISF, sempre que possível. No que diz respeito à segunda limitação, uma transformação de domínio será aqui proposta, que será chamada como um processo *unatization*, que divide variáveis *binate* em duas variáveis *unate* independentes. Esta transformação leva a domínio ISF, que pode ser fatorada de forma eficiente pelo algoritmo ISF2RO proposto.

A combinação de ambas as contribuições dá resultados exatos para a fatoração de uma nova classe de funções chamadas de funções *read-polarity-once* (RPO), onde cada uma das polaridades (positiva ou negativa) de uma variável aparece no máximo uma vez na expressão. Por exemplo, os algoritmos para RO falham ao fatorar *f=!a*b*d+b*c+a*c*, uma vez que a variável '*a*'é *binate*. O algoritmo RPO proposto pode fatorar essa função em uma expressão exata: *f=(!a*d+c)*(a+b)*, que apresenta apenas 5 literais.

Além disso, um estudo foi feito sobre a ocorrência de funções RPO em circuitos considerando *benchmarks* ISCAS'85 (IWLS, 2005). Os resultados experimentais mostraram que funções de RPO são significativamente mais frequentes do que as funções RO. Todo o fluxo compreendendo o processo de *unatization* de funções RPO e a fatoração de funções ISF em expressões RO foi validado. A implementação foi capaz de encontrar soluções exatas de forma eficiente para funções de até 8 variáveis *binate* (ou seja, até 16 literais).

A.2.1 Definição de funções Read-Polarity-Once

Definição 5.1: Uma função Booleana é chamada de *read-polarity-once* (RPO), se cada uma das polaridades (positiva ou negativa) de uma variável aparece, no máximo, uma vez na equação fatorada mínima (CALLEGARO et al, 2012).

Lema 5.1: uma variável *unate* positiva (negativa) contribui com, pelo menos, um literal positivo (negativo) em uma forma fatorada.

Lema 5.2: uma variável *binate* contribui com pelo menos dois literais (um positivo e um negativo) em uma forma fatorada.

Teorema 5.1: a função representada por uma expressão RPO está na forma mínima, se cada variável *unate* contribui com exatamente um literal e cada variável *binate* contribui com exatamente dois literais (um positivo e um negativo).

Prova: Direta dos lemas 5.1 e 5.2.

Corolário: O algoritmo proposto fornece resultados exatos na contagem de literais para funções RPO, pois ele usa no máximo um literal por variável *unate* e no máximo

dois literais (um positivo e um negativo) por variáveis *binate*, gerando uma expressão RPO.

Definição 5.2: A classe RO é um subconjunto da classe RPO.

Corolário: Cada função RO também é uma função RPO, enquanto que uma função RPO não é necessariamente uma função RO. Por exemplo, $f = a*(b+c*(d+e))$ é tanto uma função RO quanto RPO, enquanto que $f = (a+b)*(!a+!b)$ é apenas uma função RPO.

A definição da classe de funções de RPO é ligeiramente diferente da classe de funções de RO. A classe RPO contém funções *binate* como elementos (da classe), enquanto que a classe RO contém apenas funções *unate*. A Figura A.1 mostra uma comparação entre o universo de funções RO e funções RPO.

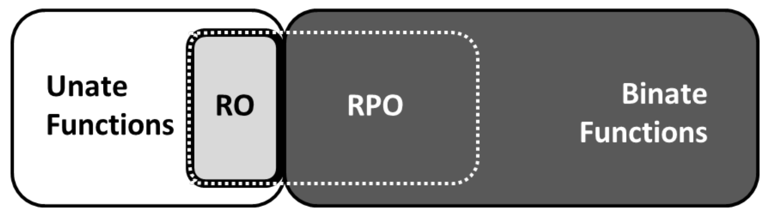

Figura A.1: Comparação entre o universo de funções *read-once* e *read-polarity-once*.

De acordo com a Definição 5.1, se a função pode ser fatorada numa expressão RPO, cada polaridade (positiva ou negativa) de uma variável aparece, no máximo, uma vez na forma fatorada. Assim, um aspecto interessante para investigação é se é possível separar os literais positivos e negativos, e transformar a função a uma função *unate*. Outro ponto, igualmente interessante, é se essa transformação resultante pode ser tratada com sucesso por algoritmos de fatoração RO.

É possível descrever todo o fluxo do algoritmo de fatoração RPO, chamado *ISF2RPO*. O algoritmo completo procede em duas etapas principais. A primeira etapa lê um ISF f e calcula a polaridade das variáveis. Cada variável *binate* é dividida em duas variáveis *unate* positivas de acordo com o processo de *unatization* (seção 5.2).

A função ISF devolvida pelo processo *unatization* é então utilizado na segunda etapa, em que o algoritmo *ISF2RO* (secção 4.1) realiza a fatoração para uma equação RO. Se o algoritmo *ISF2RO* retorna uma saída RO, a equação é então reescrita considerando as variáveis originais que foram apresentadas antes a transformação de domínio. Um fluxo inteiro para reconhecer as funções *read-polarity-once* é descrito na Figura A.2.

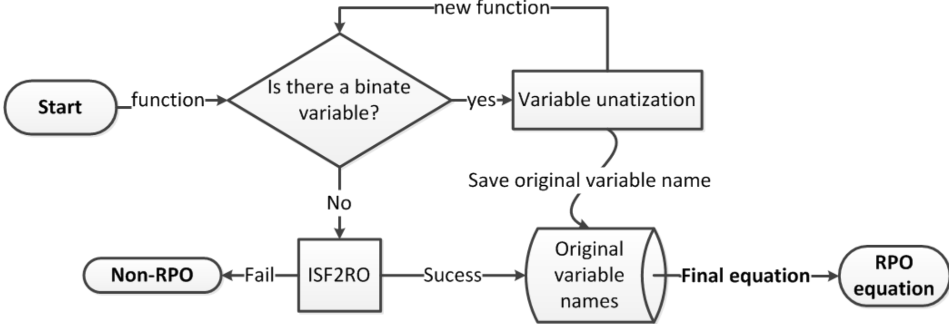

Figura A.2: Fluxograma do algoritmo *ISF2RPO*.

A.2.2 Resultados experimentais

Esta seção apresenta um estudo sobre a ocorrência de funções RPO sobre o conjunto de funções de 5 entradas da classe NPN, bem como a ocorrência de funções RPO em circuitos ISCAS'85 (IWLS, 2005).

A.2.2.1 Classe de equivalência NPN de 5 entradas

O primeiro experimento foi realizado sobre o conjunto de todas as classes de funções NPN (negação - permutação - negação) de 5 entradas. Estas funções são agrupadas em 616.125 classes de equivalência através permutação de entradas, negação de entradas e / ou negação de saída (CORREIA; 2001). O benchmark pode representar a funcionalidade de todo o espaço Booleano de cinco variáveis (2^{32} funções) em um conjunto mais compacto, sem perder generalidade. Para executar o algoritmo para todas as 616.125 funções foram necessários quatro minutos de tempo de execução. O pior caso foi a otimização de 800ms e o caso médio foi inferior a 1ms . A plataforma de execução é um sistema Linux em processador Intel Core i5 2400 com 2GB de memória principal.

Para o universo das funções NPN de 5 entradas , existem 1.462 funções que são classificadas como RPO , enquanto que apenas 21 funções são classificadas como RO . Isto significa que existem cerca de 70 vezes mais funções RPO do que funções RO, para o universo das funções NPN de 5 entradas . Nossos resultados demonstram que o universo da RPO é muito mais amplo do que o universo de funções RO, ao quais muitas obras foram dedicadas (HAYES, 1975; GURVICH, 1991; PEER; PINTER, 1995; GOLUMBIC; MINTZ; ROTICS, 2001; LEE; WANG, 2007).

Os resultados comparativos obtidos avaliando a eficiência do algoritmo são apresentados na Tabela A.2.1, considerando o conjunto as 1.462 funções RPO. Nosso algoritmo apresentou melhores resultados em termos de número de literais que as ferramentas *Quick Factor* (QF) (SENTOVICH et al., 1992), *Good Factor* (GF) (SENTOVICH et al., 1992), *ABC* (BERKELEY, 2012) and *X-Factor* (MINTZ; GOLUMBIC, 2005). O algoritmo proposto ainda é lento quando comparado aos métodos existentes de fatoração (QF e GF). No entanto, ele garante formas fatoradas mínimas em relação à contagem de literais (Teorema 5.1).

Tabela A.2.1: Número total de literais obtidos e tempo de execução, ao fatorar 1.462 funções RPO considerando diversas abordagens.

	QF	**GF**	**ABC**	**X-Factor***	**ISF2RPO (este trabalho)**
Literais	16,086	15,671	15,981	13,253	13,064
Tempo	1.9s	2.3s	2.0s	7.1s	5.7s

A.2.2.1 Benchmark de circuitos ISCAS'85

Um estudo sobre a ocorrência de funções RPO sobre circuitos do benchmark ISCAS'85 (IWLS, 2005) foi realizado. A análise foi realizada, a fim de descobrir a frequência de funções RPO, em comparação com funções RO em circuitos mapeados. Foram extraídas funções até 8 entradas dos circuitos através do método *K-Cuts* (MARTINELLO, 2010). Estas funções foram agrupadas em P-classes, por equivalência

através de permutações de entrada. O termo 'ocorrências' representa o número de funções contadas antes do agrupamento em P-classes.

Tabela A.2.2: Análise de funções até 8 entradas identificadas em circuitos ISCAS’85.

	RO		RPO	
Entradas	**P-Classes**	**Ocorrências**	**P-Classes**	**Ocorrências**
2	67%	84%	100%	100%
3	53%	66%	90%	88%
4	44%	54%	85%	71%
5	37%	42%	69%	53%
6	33%	36%	57%	46%
7	34%	36%	52%	47%
8	32%	34%	46%	44%

As funções foram extraídas de duas maneiras. A Tabela A.2.2 resume os resultados em relação a todas as funções possíveis de até 8 entradas nos circuitos. Na Tabela A.2.3 foram extraídas as funções dos circuitos que utilizam um algoritmo de cobertura guloso de AIGs (MARTINELLO, 2010).

Da mesma forma que as funções de RO, o número de funções RPO diminui à medida que o número de variáveis aumenta. Este é um resultado esperado, já que quanto maior o número de entradas, maior é a complexidade de implementação da função Booleana.

Tabela A.2.3: Funções selecionadas de circuitos ISCAS’85 utilizando um algoritmo de cobertura gulosa.

	RO		RPO	
Entradas	**P Classes**	**Ocorrências**	**P Classes**	**Ocorrências**
2	78%	93%	100%	100%
3	59%	63%	87%	94%
4	49%	53%	78%	81%
5	35%	36%	79%	81%
6	41%	39%	63%	60%
7	45%	43%	62%	57%
8	14%	15%	27%	27%

Na Tabela A.2.4, é possível ver o tempo de execução para sintetizar todos os K-cuts (para K = 8). Na Tabela A.2.5, mostra-se o tempo de execução do algoritmo para sintetizar as funções extraídas do circuito coberto.

Tabela A.2.4: Tempo de execução para a síntese de todos os K-Cuts (K=8).

Circuit	P Classes	Time (s)	Avg. time (s)
C1355	680	123.455	0.18
C17	12	0.01	0.01
C1908	1224	133.162	0.11
C2670	6345	284.626	0.04
C3540	9275	123.945	0.01
C432	844	3.259	0.01
C499	432	98.187	0.23
C5315	11350	806.489	0.07
C6288	142	0.634	0.01
C7552	17888	8045.137	0.45
C880	1691	86.301	0.05

Os resultados experimentais sobre circuitos ISCAS'85 demonstraram que as funções RPO são significativamente mais frequentes do que funções RO. Todo o fluxo para fatorar funções RPO foi validado, onde nossa implementação foi capaz de encontrar as soluções exatas para funções de até 8 variáveis *binate* em um razoável tempo de execução.

Tabela A.2.5: Tempo de execução para a síntese de funções selecionadas através da algoritmos de cobertura gulosa.

Circuit	P Classes	Time (s)	Avg. time (s)
C1355	16	3.70	0.23
C17	3	0.01	0.01
C1908	44	2.37	0.05
C2670	68	39.53	0.58
C3540	129	1.29	0.01
C432	25	0.15	0.01
C499	10	0.89	0.09
C5315	108	58.07	0.54
C6288	38	0.20	0.01
C7552	130	8.42	0.06
C880	36	18.08	0.50

A.3 Conclusões

A principal contribuição deste trabalho foi a definição do conceito de funções *read-polarity-once* (RPO). Além da definição da classe de funções RPO, várias contribuições relacionadas também foram apresentadas: (1) um algoritmo para fatoração de funções incompletamente especificadas em equações *read-once* (RO), (2) uma transformação de domínio, que divide variáveis *binate* em duas variáveis *unate* independentes e (3) um algoritmo completo para a fatoração exata de funções da classe RPO.

A primeira contribuição foi a proposta de um algoritmo (*ISF2RO*) que é capaz de fatorar funções Booleanas incompletamente especificadas em formas RO, sempre que possível. O único método conhecido que garante a exatidão do resultado é a abordagem *Exact Factor*, proposto por (YOSHIDA; Fujita, 2011) . No entanto, o método só é viável para funções com até 10 variáveis. O método *ISF2RO* aqui proposto é capaz de fatorar funções com até 16 variáveis de entrada.

A segunda contribuição foi a proposta de um método para a transformação de domínio que divide variáveis *binate* em duas variáveis *unate* independentes, chamado processo de *unatization*. O processo de *unatization* é um passo fundamental ao reconhecer funções RPO. No entanto, ele pode ser aplicado em outros métodos de síntese de lógica, como métodos de decomposição funcional.

A terceira contribuição foi o desenvolvimento de um algoritmo completo para a fatoração exata de funções da classe RPO, chamado *ISF2RPO*. Tal algoritmo foi implementado e comparado com algoritmos de fatoração existentes. O algoritmo proposto garante formas fatoradas mínimas para a classe de funções RPO. Os resultados mostram que, para o conjunto de 612.125 funções da classe NPN com até 5 entradas, 1.462 funções RPO foram identificadas, enquanto que para a classe RO apenas 21 funções foram identificadas. Além disso, os resultados levando em conta circuitos ISCAS'85 mostraram que as funções RPO são muito mais frequentes em relação às RO.

Vale a pena ressaltar que ambos os algoritmos foram implementados em uma ferramenta, chamada Switchcraft (CALLEGARO et al, 2010). O ambiente Switchcraft fornece um conjunto de ferramentas para a geração de redes de chaves e de portas lógicas. A ferramenta foi utilizada e aplicada em aula na Universidade Federal do Rio Grande do Sul (UFRGS) no Brasil, em cursos de graduação e pós-graduação.