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## Introduction

1 Urban built form has been traditionally seen as a result of a social, productive and reproductive process that generates demands for adapted space more than as an autonomous process. A remarkable exception is the work of Wheaton (1982), who suggested that changes in urban built form could emerge as a result of rent maximization instead of utility-maximization. Following this, Krafta (1994) proposed a model to explain those changes as profit seeking activity on space that optimizes relative location. However, those two propositions, although breaking new ground, do not succeed to fully explain how inner urban built form changes occur. The first one does not take space as a real variable – a typical shortcoming of urban economic approaches – while the second one aims rather at urban morphology (more specifically, only applicable to parts of the city where morphological uniformity is verified). A new explanation is conceived here relating variables of both an economic and spatial nature: one in which built form change does occur as a result of a profit seeking activity, expressed by differential values of land, built form and location.

2 The output is a process where built forms reach recursive points of criticality, go through phase transitions, and may progressively shift into entirely new patterns of form, activity, centrality, and value. Such possible paths of emergence are triggered by a simple mechanism: the tension between what we shall define as *expected* value and the *actual* value of an ageing built form associated with a location whose value increases in time, similar to Smith's (1978) rent gap. The paper explores the consequences of these ever growing tensions. It describes the dynamics of urban growth as a truly complex phenomenon, as change irrupts into the initial homogeneity in built forms, lead them into increasing heterogeneity, [and might lead them back into progressive alignment in a new phase of relative spatial homogeneity – only to start all over again]. That is a description of the diffusion of change throughout an urban system in most certainly *recursive* yet *unpredictable* sequences of urban self-destruction and reconstruction, in a syncopated rhythm relatable to catastrophic change in processes defined by self-organizing criticality.

3 There, triggering self-organizing criticality lay the tensions of actuality and expectation, form and value. The principle of rent maximization of a built form structure, fixed in space that belongs to a growing spatial system, suggests that the rent it generates could be different at each moment in time considered. It is certainly maximum at time *t0*, when the structure is actually built up and used for the first time. As time passes by, the city extends its boundaries making all pre-existing locations a little more central, the built forms age and lose functionality, all conspiring to make the expected rent to grow faster than the actual one. The tension between a location value increasing with urban growth and a built form value decreasing with time seems to be central at the process of inner city built form change.

## Our theoretical proposition

4 We assume that every intra-urban location has a composite market value in which two components are essential: the land and the built form structure. Land value is a function of its relative position within the urban spatial system and the web of activities going on around it. To this extent, location depends first of all directly from the configuration as well as the size of the urban system at each moment in time. Every new location added to the system modifies the location value of all pre-existing ones.

5 Secondly, location value depends upon the distribution of land uses and activities over the city. Similarly, every new activity added to the system is likely to modify the location values of all urban sites, changing the urban polarization and flow distribution. Built form structure values are also a function of factors such as age and activity purpose. Buildings are expected to lose value as their ages increase, although several circumstantial factors could retard or speed up

devaluating process. Losses in functionality also contribute to devaluation, due to changing requirements of adequacy to new social activities, technologies and equipment.

6 It would also be interesting to think of an *expected value*, that is, that financial revenue that each landowner would get when a new built form is produced, and the potentials of its location are fully explored: both location and built form values are at their peak. In a scenario of continuous urban growth, location values should increase, built form values should decrease and expected value should increase even more than location values, creating and nurturing a gap between real and expected revenues ([Smith, 1987).

7 Let us examine now how the tension between real and expected values affects an urban system. Urban growth does occur initially in the boundaries, where land value is lowest, forming rings of development. However, each new ring adds centrality to all preexisting rings, particularly the most central ones, where the gap between real and expected rents grows up to a point that the built form becomes unstable. Eventually the decision to replace the old structure with one compatible with the expected value is taken. In fact, the expected value may be seen as a virtual built form always already latent in the same location of the actual built form, gradually exceeding it and pressing it into its own breaking point. As the expected value grows, the pressure set by this virtual built form becomes a means through which the urban system *copes with* the stability enforced by current built forms, and *breaks through* the constraints to change they bring about. The system does so rearranging itself through sheer substitution: previously stabilized built forms are (economically) required to give place to others in order to make room for the very continuity of the urban system as a whole. Every built form in the city is prone to meet this *phase transition point* in time – and perhaps, over and over again in a same location.

8 Although each urban location, individually, undergoes a *cyclic process* (as a built structure offers maximum rent at the moment of its inception, and loses such capacity in time, as the city grows, and it does so up to the point when is substituted, restarting the whole cycle), the areas in the city where they are located experiment quite different dynamics. Indeed, the replacement of one first building suggests that the area may have reached a critical situation (Bak,1988), in which the phase transition or change affects the stability of other buildings, even though it momentarily stabilizes its own location, and may generate extensive and profound changes in urban morphology. Substitution does not lead into a previous state of stability, but rather pushes the system closer to new irruptions – which, once following each other, bring to system permanently to instability (Batty, 1998). In other words, replacement does not solve the original tension, as many other buildings are likely to be in a similar situation and could be replaced even faster, stimulated by that first replacement. It is not possible to predict exactly *when*, *where* and *how* many buildings will be replaced at each “avalanche”. Sometimes could be just one building, or sets of isolated ones, our small groups, up to entire zones.

9 The theory outlined above is very similar to those developed by Bak et al (1988) and generally addressed under the label “self-organizing criticality”. The term refers to dynamic systems whose evolution is prone to reach critical states, when phase transitions come into being, and keep them around criticality – a form of dynamic instability. Bak et al describe such properties evoking the telling imagery of the sand pile, as sand grains are added over a same point in a perfectly flat surface. As grains are added, the pile takes form. Inclination of its own surface tends to increase up to the point that grains may not find a place in the top and rolls down to a level below, ant then to the next, all the way to the bottom, in a sort of micro-avalanche, which in turn reduce the inclination angle of the pile’s surface a little below the previous critical point, in a way that grains added bring the pile once again to the critical inclination and new avalanche, *ad infinitum*. According to Bak, the precise point in time and magnitude of any avalanche cannot be foreseen. Remarkably, its frequency may be described through a power law.

10 Other dimensions and dynamics of urban systems may be described in similar terms. The traffic system comes easily to mind. As most daily travels and commuting within a city tend to concentrate on a rather small set of streets, there is a natural tendency to traffic jams and gridlocks – and on a regular basis. Jams may lead car drivers to search alternative routes, which reduces jams – a situation once again attractive to other drivers, leading the traffic system to a

new gridlock. Massive changes in road width have the effect of initially reducing traffic levels, only to bring the system yet again to the previous critical point.

11 However, the criticality of urban growth is not only dependent upon building replacement, but also on the internal transformations that follow. As the notion of real-expected rent gap suggests, building replacement involves always changes in density and activity patterns. Wheaton (1982) shows that changes in built forms in a set of American cities are likely to involve change in built form densities (increasing about seven times) or unitary value (about three times). Changes in density are of course followed by changes in occupation and actual use, which take the form of either more dwellers or more users. Changes in value mean changes in socioeconomic standards for new dwellers and users.

12 Therefore, densification, gentrification or substitution of residential for retail activities are at the bottom of urban structural transformation. In this sense, if Bak's proposition is to attain any sense in terms of a description of urban processes, the city may be well seen as a system geared to instability as long as its growth rate is kept.

13 That includes changes in patterns of built forms through densification. New buildings are economically geared to be larger than their predecessors, and house different activities, which cause qualitative changes in the city, i.e. actually affect land value, spatial interaction and centrality patterns. Different from boundary growth, which only increases existing centralities and reinforces land use patterns, inner growth is qualitative, and transforms the very structure of the city. Considering the large amount of built forms that shapes the city, once the first one reaches instability, the cycles of criticality of the city may never cease.

14 In a fluid world, urban built form would change continuously and would be the mirror of economic expectancies; nevertheless, as it is durable and involves destruction in order to be adapted to economic requirement, it creates the conditions for criticality. To this extent, urban built form works out as a resistance to be surpassed if the social nature of the urban space production is to continue. It also introduces a *tempo*, that is, a complex yet specific rhythm to urban development; depending on capital devaluation, inner urban changes are firstly retarded and then, when the tension between real and expected rent becomes unbearable, accelerated and overwhelming.

## The diffusion of change: a Simple Simulation Model of Built Form Growth

15 Two simple relationships are proposed: the first one evolves from the fact that expenses in land, as one important component of built space production, should not exceed a certain proportion of the whole investment. For simplicity, we assume it to be 20%, a fairly realistic proportion. This implies that, keeping the 1:4 relationship between land and built structure, the amount of built area per land area unit increases with land value. In turn, places where land value is higher will be more densely occupied; alternatively, the higher land valued zones would take built form units with higher unit values, reflecting higher standards, or find some mixed area in between these possibilities.

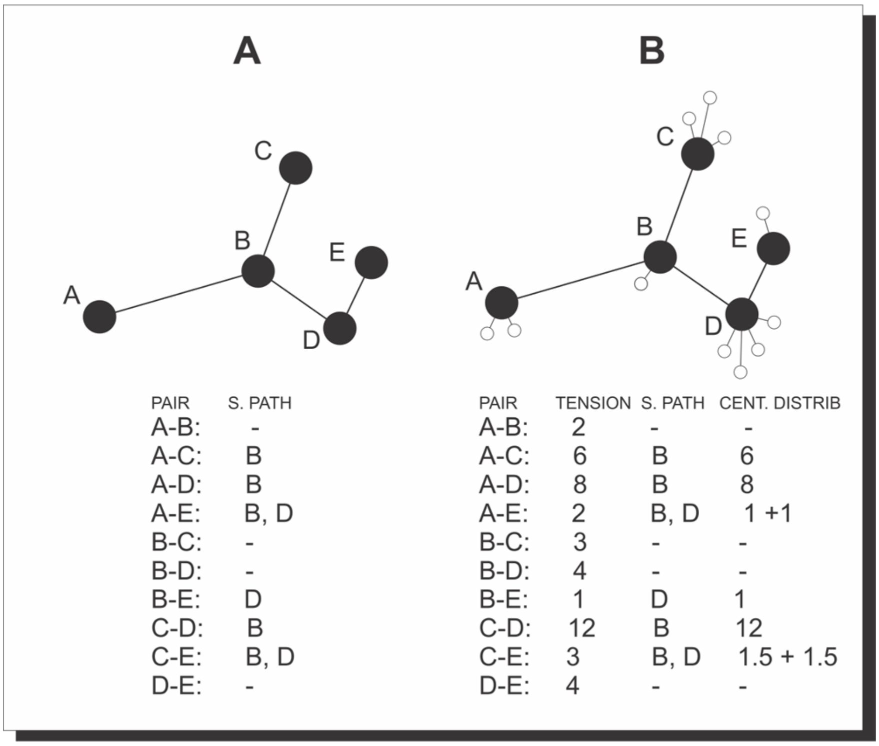
16 The second relationship refers to the addition of new built form units to the city, and states that they will replace another pre-existing one only if the new built structure price is less than the difference between real and expected rent observed in that site; otherwise they will be placed in the boundary of the city. In fact, the relationship of inner growth to peripheral growth as two different yet mutually dependent processes is a largely unresolved theoretical problem. We suggest that this relationship leads every decision to place a new built form in the city into a *bifurcation* where one of the possibilities is to renew the built form stock in the central areas and therefore, for sufficiently central and old areas, instability, built form replacement and structural changes.

17 Simulation has been kept in the simplest possible terms: an initial linear urban "seed" of ten cells, each cell bearing a single built form unit, all units occupied by residential and service activities, on the 9:1 proportion. The city grows both externally, adding cells to the right and left of the seed, and internally, replacing existing built form units for new, denser ones, at a constant rate. Spatial growth occurs as a response to populational growth. Denser built form

units are in fact superimposed units (say, density level 3 represents a structure containing three built form units).

18 In order to keep track of the inner dynamics, four indicators have been established, all of them derived from a basic *centrality* measure. Centrality is a usual way of depicting spatial differentiation. Several centrality measures are available and described in the literature (see Crucitti et al, 2006). Our centrality measure is derived from Freeman (1977) *betweenness centrality*. Freeman's original measure defines centrality when an entity (an individual in his model, a spatial unit here in the present model) falls into the shortest path between other two entities, being central to their interaction. For a system of many individuals, the most central individual is the one more often found in the shortest route between all possible pairs. In the adaption of this model to the representation of urban structure, two elements were added. First, there is a hypothetical tension between every pair of nodes generated by differences in their socioeconomic contents. Tension is the configurational interpretation of the attraction between locations, due to the quantity, intensity and nature of activities taking place in them. Second, a distance grade is added – one able to weight the tension between pairs as a function of the amount of nodes falling in the shortest path between pairs (Krafta, 1994). The model resembles classic gravitational approaches, and allows to break through certain limitations of usual configurational approaches to the urban, as it includes variables such as distribution of built forms and activities in a spatial system. This way, the measure of centrality is responsive to spatial configuration, as well as distribution of stocks and activities throughout the system.

**Fig 1: Betweenness Centrality and its adaptation to measure spatial differentiation**




The diagram A is Freeman's betweenness centrality; for all pairs, nodes B and D are central, B is the most central, as falls in the shortest path of five pairs. Diagram B is our proposed spatial differentiation measure based on Freeman's, plus gravity. Nodes are discrete public spaces, each pair of nodes develops a tension, computed as the product of their weights (this represents built forms and activities located in each node), every tension is distributed among nodes placed along the shortest path. For all pairs, as in Freeman's, nodes B and D are central. Node B score is 28.5, whereas D is 3.5. Centrality values evolve with change in a single built form unit (small hollow circles), as well as in the public space grid (large black circles).

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central. Node B score is 28.5, whereas D is 3.5. Centrality values evolve with change in a single built form unit (small hollow circles), as well as in the public space grid (large black circles).

20 The *centrality measure* is the *first indicator* of built form dynamics, and it is used as a proxy for land value, as suggested previously elsewhere (see Spinelli and Krafta, 1999). Centrality values are updated after every iteration and represent the structure and evolution in the system of land values.

21 The *second indicator* is *built structure value*, fixed at four times the respective original land value (first indicator), according to our primary system relationship. Thus, the initial parameter for built form value in the original cells are their centrality values, times four. The built structure value is devaluated at a specific rate after every iteration, in order to capture the progressive loss of market value.

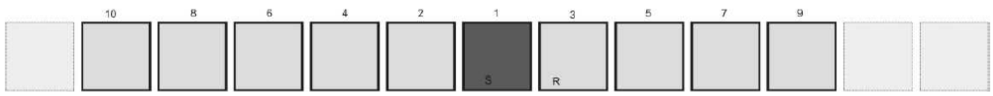
22 *The third indicator* is the *real value*, computed as the sum of first and second ones, land plus built form structure.

23 Finally the *fourth indicator* is the *expected value*, taken as five times the updated centrality value.

24 Within a scenario of continuous growth, the first indicator is expected to increase continuously along the iterations, whereas the second does the opposite, loosing small parts of a fixed value. The third indicator is also upwards, although its performance depends upon the rate of growth. The fourth indicator is proportional to the first one. We suggest that the logics of an economic tension latent in urban form (or conversely, an urban form cut across by economic tension) correspond to a *local rule* in complex systems dynamics, i.e. a rule active in localized phenomena or processes; one through which larger scale, spatially and temporally unpredictable change may come into being, and wholly new patterns emerge.

25 Simulation is performed with an *exogenous rate of growth*, applied over the urban seed and subsequent configurations, according to *iterations* representing one year each. The state of the system at time [t0] is ten cells in a row, each one bearing a corresponding built form unit. The central cell contains service activities; the nine others are entirely residential, as depicted in figure 2.

**Fig 2: The system's description at the initial state**




Thick lines are cell boundaries, dark grey is service (s), light grey is residential (r) and dotted lines are expansion cells' boundaries.

26 In the system's description depicted in figure 2 thick lines are cell boundaries, dark grey is service (s), light grey is residential (r) and dotted lines are expansion cells' boundaries. Starting from [t0] – population of 9 plus 1 service, all placed in 10 cells, and considering a population and associated service structure growth rate of 5%, no new cell will be added to the system in the first iteration; the population increase of 0.45 and service of 0.05 will wait in a queue until whole units are formed, enough to make the spatial system do to grow one new cell. A single new cell with a built form unit and residential activity will be added in the boundary of the system, in the third iteration, making the cell line grow into one direction (see table 2). This happens as a result of accumulated population growth of two accumulated iterations. The new cell will be placed in the boundary, since other built forms are still generating income equal or very similar to that expected, so preventing existing built forms to be replaced.

**Table 1: Progress on population and spatial growth of 5%**

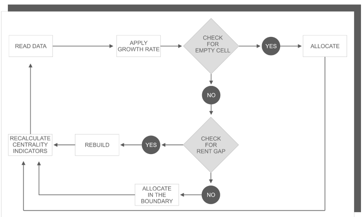
| | T0 | Cells | T1 | Cells | queue | T2 | Cells | queue | T3 | cells | queue |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Population | 9 | 9 | 9.45 | 9 | 0.45 | 9.92 | 9 | 0.92 | 10.4 | 10 | 0.4 |
| Service | 1 | 1 | 1.05 | 1 | 0.05 | 1.1 | 1 | 0.1 | 1.15 | 1 | 0.15 |

27 After the first allocation, as after all others, indicators are updated and the second fundamental relationship of the system checked up to determine whether the next allocation should occur in boundary once more or in the centre, as illustrated in the model flow chart. After the inclusion of the new cell, all indicators are updated. The following changes occur: (i) the

indicator of location values has changed for every cell, and their parameters are increased due to urban growth; (ii) the indicator of built form value has decreased at the rate of depreciation calculated every iteration; (iii) the third indicator has increased, in accordance with previous transformations, along with the fourth indicator. The second iteration takes place calculating increases in indicators, verifying activity in the new built form or forms (fractions of growth less than one are accumulated and allocated only when they amount to one unit or more), and checking the position or positions where growth shall occur (i.e. either at the boundary if the second conditions is not reached, the cell in which the imposition of the two state condition).

28 Retail and residential growth are kept at the same rate, service growth occurs always at the most central cells, expelling previous residential use to the boundary. Retail growth at the centre increases centrality geometrically, due to its polarization power. The algorithmic procedure for simulating such process is shown in figure 3, in which *Data* is population, service and cell quantities existing in the system prior to the iteration. It also includes the queue, that is, fractions of population and service units left from previous iterations and still not allocated in the spatial system.

**Fig 3: Model's schematic flow chart**




## Experimentation

29 The model has been applied to the system of figure 2 thirty iterations over, and the indicators results registered for cells 1, 2 and 3. Table 2 shows what happens to cell 3, the first one to get instable, as the expected rent value grows bigger than the built form value. That happens in the eighth iteration, although the replacement of cell's existing built form only takes place two iterations later, as the ninth one is zero allocation. The relationship between original and actual land values by then is around two, suggesting that cell occupation should double, leading to a substantial change in built form structure. Once the replacement takes place, the rent gap returns to zero, as the new cell occupation is giving full rent. This relationship can find negative values at the time of replacement (see figure 4), since investment can be made in a way to surmount differences in rent values, anticipating a more intense concentration. Table 2 shows partial results for simulation over ten iterations, for cell 3, describing: a) steady growth of land value caused by continuous increase of cell centrality (column A), b) steady decrease of built form values, up to the moment it is replaced and its value updated (column B), c) steady growth of the gap between real and expected rent values, leading to a situation of instability and eventually built form replacement (column E).

**Table 2. Partial results for simulation over ten iterations, for cell 3**

| | A | B | C | D | E |
|---|---|---|---|---|---|
| cell 3 | Land Value | Built Form Value | Actual Total Value | Expected Value | GAP For Cell 3 |
| t0 | 46,04 | 184,16 | 230,20 | 230,20 | 0,00 |
| t1 | 46,04 | 174,95 | 220,99 | 230,20 | 9,21 |
| t2 | 46,04 | 166,20 | 212,24 | 230,20 | 17,96 |
| t3 | 52,79 | 157,89 | 210,68 | 263,95 | 53,27 |
| t4 | 52,79 | 150,00 | 202,79 | 263,95 | 61,16 |
| t5 | 54,83 | 142,50 | 197,33 | 274,15 | 76,82 |
| t6 | 60,99 | 135,37 | 196,36 | 304,95 | 108,59 |
| t7 | 60,99 | 128,61 | 189,60 | 304,95 | 115,35 |
| t8 | 63,12 | 122,18 | 185,30 | 315,60 | 130,30 |

| | | | | | |
|---|---|---|---|---|---|
| t9 | 63,12 | 116,07 | 179,19 | 315,60 | 136,41 |
| t10 | 69,10 | 276,40 | 345,50 | 345,50 | 0,00 |

Table shows: a) steady growth of land value caused by continuous increase of cell centrality, b) steady decrease of built form values, up to the moment it is replaced and its value updated, c) steady growth of the gap between real and expected rent values, leading to a situation of instability and eventually built form replacement.

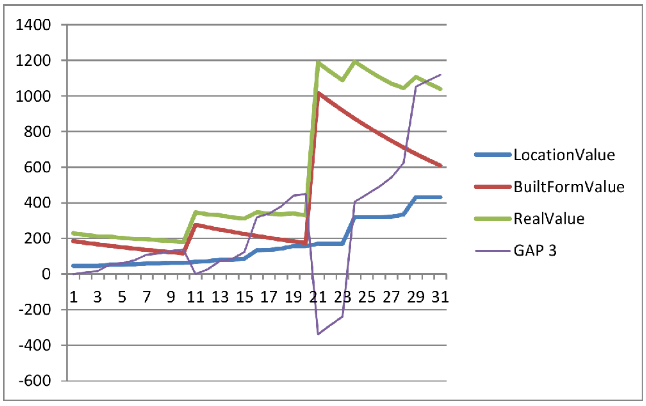
30 Location value increases every time a new cell is added to the system, leading to changes in the centrality levels of pre-existing cells – in particular the most central, which become even more central. The initial value of 46.04 changes after the third iteration, as a result of the addition of a first new cell, and in the next iterations as the system grows. On the other hand, the built form value loses value continuously, following to the stipulated devaluation rate. Real rent value changes in accordance with the sum of these two previous variables. In turn, the expected value grows proportionally to the land value, thereby creating the rent gap (Gap for Cell 3 in Table 2). At the eighth iteration, the difference is already larger than the built form value, triggering its replacement.

31 Figure 4 shows the four indicators' performances along the thirty iterations. The thin (purple) line representing the gap between real and expected rents, showing a general tendency to grow up and to lead to rupture, suggests that the urban built form has a cyclic behaviour. It returns to the original situation (zero or a negative value around zero), only to grow again in the direction of a future disruption. Every sudden break in this line means that there is a new built form in the cell.

32 Figure 5 shows the performance of cells 1 and 2, and they are, as expected, very similar to the one already observed in cell 3; however with an important difference, that is the specific times the replacements take place, different for each cell. In cell one three replacements, at iteration 11, 18 and 28, do occur, in cell 2 they happen at iteration 12 and 23; in cell 3 the events go on at 10 and 20, conforming a first group of 3 changes in straight sequence, a second group of another three changes in an interrupted, irregular sequence, and finally a large, isolated single change (that should repeat itself if the simulation goes on).

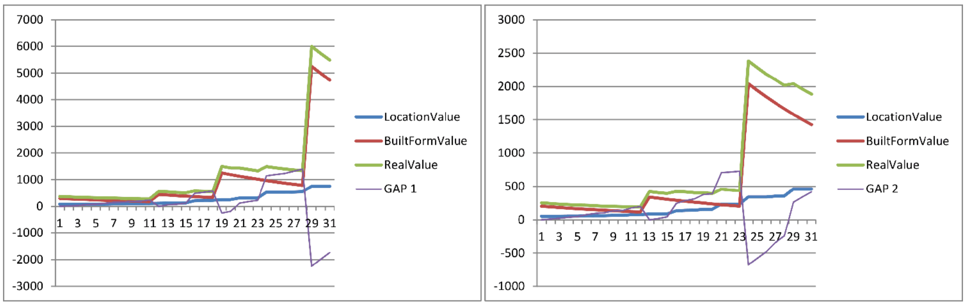
33 It is also clear that, after an initial period of peripheral growth, the system gets involved in an almost continuous process of internal changes.

**Fig 4: Performance of cell 3 indicators over 30 iterations**




Showing a) steady growth of land value due to increase of centrality, b) general tendency of built form value decrease, eventually corrected by replacement with denser units, c) general tendency of increasing gap between real and expected rents, eventually rebalanced by built form replacement. Events of built form replacement occur on iterations 10 and 20.

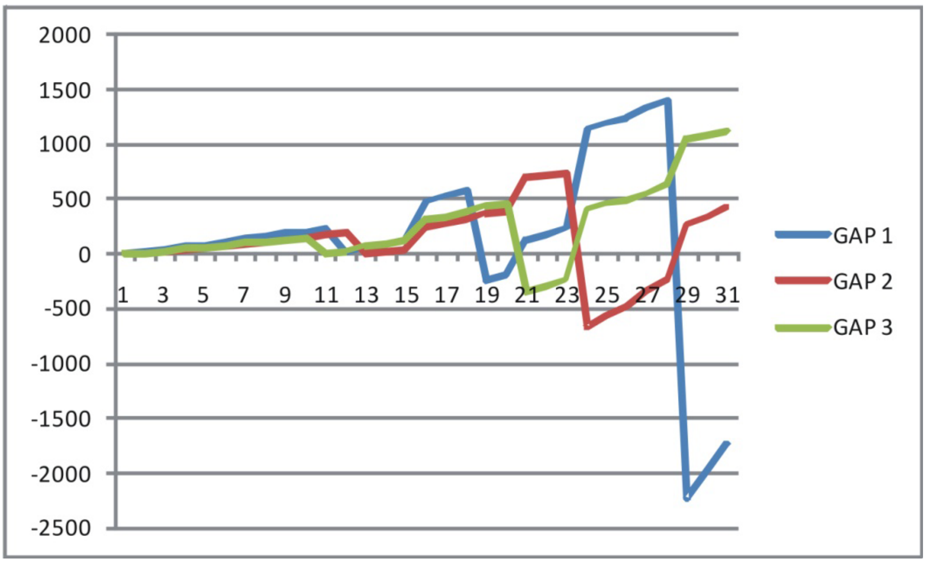
**Fig 5: Performance of cells 1 and 2 indicators**




Showing similar development as for cell 2, except that the events leading to rupture and transformation are differently timed; in cell 1 they occur at iterations 11, 18 and 28, whereas in cell 3 at 12 and 23.

34 Figure 6 depicts the three observed cells' respective rent gaps, where it can be seen that, while each cell goes through a cyclic process, the system is under constant change. Changes in individual built forms trigger further cycles of change, one leading to another, irrupting in a complex temporal sequence, albeit not perfectly overlapped or simultaneous. We shall see below that such sequences of both isolated and wave-like changes progressively break through the original homogeneity in form and lead the urban system into increasing heterogeneity – and a constant state of self-organizing criticality.

**Fig 6: Rent gaps of the three observed cells along thirty iterations**




Showing that, although each cell performs a cyclic-like process of development, the system as a whole is virtually always under criticality.

## Discussion on Criticality in Urban Systems

35 Despite somehow oversimplified constraints, the experiment has revealed a relevant relationship between outer and inner urban growth, in which built form changes are a key factor. Considering the effects that built form changes have on the urban systems, it can be devised that, while the outer growth is conservative, meaning that it does not modify the city's structure, the built form changes in the inner city are innovative, as they imply also changes in centrality, land uses and activities already established. Moreover, the experiment has suggested that the urban systems, starting from a seed that shows early peripheral growth, rapidly evolves up to a critical point, where a structural change is very likely. Once reached this point, the system develops a dynamics characterized by oscillations around the critical point, showing critical events of different scales distributed in time and in space. The experiment has also shown change taking place in waves, starting on in a cell and subsequently spreading to neighbouring ones, suggesting that those adjacent cells reach their threshold more rapidly, influenced by the change occurring next door.

36 Even through a simplified simulation, our findings point to a set of potentially interesting observations:

37 a) *The existence of an organic relationship between internal and peripheral urban growth:* traditionally, centrality models have been applied to the analysis of states of spatial systems, as tools to capture spatial differentiation in a given time. A variation of this classic application is the proposal by Krafta (1994), which captures spatial differentiation as a bias that leads to change. The higher centrality levels of the locations best positioned in the system become a centrifugal force, encouraging development in less privileged locations in that system. That would be the case due to the comparative advantages for land acquisition. That previous work suggests that the relationship between centre and periphery may be synthesized in the form of a vector into peripheral growth. Such approach to growth includes intra-urban transformations, and takes into account that urban built form is composed by different patterns, each area consisting of a pattern of built form with its own core and periphery. However, we take this relationship one step further and consider that peripheral development may have effects of actually inducing a kind of centripetal force. This vector can be identified in the evolution of the urban system, as peripheral growth increases the centrality of pre-existing locations, particularly more central ones.

38 We may only grasp the form of virtual centrality latent in built forms, here defined as “expected value” if we consider configurational change in time. Thus, if it is reasonable to suppose that the existence of a centre stimulates the growth of peripheries, now we may also infer that the development of a periphery stimulates changes in the centre. The dual system of centrifugal and centripetal forces considered here is similar to models of the New Economic Geography (Fujita & Thisse, 2009), particularly the center-periphery model of Krugman (1991), assuming opposing forces and spatial differentiation (in regional scale).

39 b) *The power of urban growth to change the internal urban spatial structure:* the structural modification suggested here attends two conditions: the first refers to changes in patterns of built form substitution. Wheaton observed that a building would never be replaced by another of similar characteristics. In fact, substitution necessarily brings significant densification and increases in value, or both combined. Also, densification and gentrification involve re-ordering of relations between origins and destinations of trips, locations of services, and patterns of use of public spaces, among others. The second condition, not fully proved by the simulation, is the emergence of new centres spatially separated from the original one. Simulation showed that substitutions occur initially near or in the original centre, on a more or less regular basis. Nevertheless it is possible to discern important variations in this pattern, once we consider variables not included in this model, such as limitations imposed by urban regulations, costs of relocation of retail activities, technical limitations, etc., which help to define the urban physical structure and transfer the effect of centripetal forces to locations away from the original centre.

40 Indeed, the regularity of the rent differential growth process and the recursive replacement of old buildings with new ones in our simulation only occurs due to the simplicity of equations, geared to grasp the main aspects of the process, although not ignoring or denying other variables, such as those suggested above. Assuming that central locations reach a threshold that would slow or even prevent the renewal of built stocks, substitution and pattern change tends to occur inevitably in other locations, namely those where this threshold has not yet been reached.

41 c) *Intra-urban growth would have characteristics of self-organized criticality.* Our theory suggests that the dynamics of urban growth brings more spatial and temporal tension to its most central locations, from the micro scale of its spatial unit (the plot itself) to the urban fabric as a whole. Such tension is the output of the opposition between two forces: one describes the potentials of a location to generate rent value; the other describes their actual performance in generating such value. The opposition is expressed in the growing difference between the two, as the former grows continuously, while the latter has limits imposed by the built form itself. This tension has a space-time dimension as it depends on the position of every plot and the size of the spatial system as much as it depends on the evolution of that system. Evolution brings about changes in parameters of growth vectors. From the point of view of each location,

evolution leads into a critical situation defined by a growing rent deficit. However, even if guided by a critical state, such dynamics does not qualify yet as self-organizing criticality, since the process is essentially cyclical. From the viewpoint of the spatial system as a whole, on the other hand, the replacement of a single building in a single plot adds complexity to the process, as it generates increases in centrality (in and around itself) as a relational property.

42 Thus, a new built form speeds up the differential rent tension between neighbours, increasing probabilities of substitution in the short term. Thus, an individual critical situation has properties of diffusion, and brings a localized transformation to a systemic level. As we have seen, once the urban system reaches such critical state, it stays there as older built forms give place to newer ones. Such fluctuations may be microscopic (e.g. in areas already renewed or already dense) or be felt as waves in larger scales (e.g. in obsolete urban areas). In between, there is a range of transformation effects yet to be described, which may well follow a power law.

43 The critical state to which every built form converges to may be compared to Bak's "minimally stable state," the one-dimensional counterpart of self-organizing criticality. Bak's simulation of a one-dimensional sand pile uses cellular automata, where cells pile up until a specified limit; then they roll down to a certain level of stability (the bottom is the first level or plateau). Sequences of micro-avalanches create a number of plateaus equally subject to a limit. Thus, the system propagates avalanches which range from a single cell to larger scales, according to the sequence of plateaus created during its evolution. Systems would have spatio-temporal "signatures:" intermittence in time, and self-similarity in space (a fractal property).

44 In turn, cities consist of a reversed formation: "piles" tend to grow in central positions. Nevertheless, they are likely to have the same spatio-temporal signature and may well respond to the same dynamics of criticality found in Baks simulation. Our approach also suggests some particularities and allows some critical self-observations.

45 i) *The cyclic process of built form substitution would also have a physical threshold, which in turn leads to a theoretical limit*. Cycles of individual built form substitution finds limits in urban legislation, technology, and cultural tradition; which amount to temporal and spatial limits to change. Such limits may be broken through technological and cultural evolution or the passage of time itself; this possibility is nevertheless inessential to the present discussion. What relates urban spatial dynamics to self-organizing criticality is not exactly what happens within each spatial unit in the system, or how many changes its units undergo. It is the fact that the whole system reaches a critical state in which change becomes a possibility, and once it reaches such state, as described above, the urban system remains there, in constant transformation – from isolated to chained streams of change.

46 ii) *A one-dimensional system is less then realistic representation*. The simulation performed above is certainly a simplified one (as much as Bak's one-dimensional sand pile!), and as such cannot embrace the whole set of possibilities engendered by the phenomenon of built form growth and, more broadly, criticality in complex urban systems. In other words, a linear urban model has little power to represent more complex patterns of change, as its growth is limited to one cell at each side at each time. Bi-dimensional systems, having geometric rates of spatial growth, would probably provide a more realistic pattern of structural change, as several cells would reach their limits at the same time, prompting the system to perform choice between equally possible transformations.

47 Although changes cannot be easily predictable in such bi-dimensional system, two lines of growth could be possible: a first, longitudinal movement stemming from the centre to the boundaries of the system, like the one simulated above; and a second, transversal movement brought about by the multiplication of centre-boundary lines of tension engendered by the bi-dimensional matrix. That would also allow discontinuities found in the one-dimensional system to emerge in the centre-boundary lines of tension. It has the advantage of allowing chains of change in more directions with potential overlapping (like sound waves generated in the air by different sources), portraying more tellingly both the non-deterministic nature of the process and the limits to systemic effects of every event or single change in the system.

Notwithstanding, the general lines of a complex process could be captured and reproduced in a one-dimensional simulation shown above.

48 iii) *Internal growth obtained through simulation operates as an alternative – a secondary option – to external growth*. Internal growth seems dependent on the expansion of the urban system, as the rent differentials in central locations only grow with peripheral growth. However, our simulation by no means implied that internal change is comparatively less important in urban dynamics. In fact, it showed that internal changes may lead to structural transformation, i.e. generate enough internal differentiation to rearrange polarities, spatial interdependencies, and land use patterns. Furthermore, simulation suggests a certain balance between the internal and external production of built forms, although such proportionality should be further pursued and corroborated by empirical work.

## Concluding Remarks

49 Let us address some final observations in a research agenda on criticality and urban form.

50 a) *The question of sustainability of urban systems*. Notions usually associated with sustainability, such as stability, durability and scale, seem seriously endangered by an approach based on criticality, as the one proposed here. The essence of the dynamics portrayed above is dissipative, i.e. a process that requires energy and resource consumption, including layers of its own materiality, in order to come into being. In this sense, the continuity of urban systems involves "creative destruction", i.e. the systematic devaluation of part of its own fixed capital and the continuous destruction of part of its own material form. Of course these observations are not new in spatial studies, but the present approach lends them renewed focus and role. Furthermore, it suggests that stability, durability and local scales are features rather related to stagnation. Now, urban stagnation is substantively replicated and entrenched in other dimensions of social and economic life.

51 b) *Sprawl*. Spatial dispersion is a hot subject in recent research and debate on cities, and is usually related to fragmentation of urban form and spread of urban functions across a territory. So how it relates to a process of growth intimately connect to interdependent processes of internal and peripheral change in the terms described above? Generally, dispersion has to do with one of vectors considered in this theory – peripheral growth – and the discontinuity it entails does not collide with that centrifugal force. The opposite force, centralization, overseen outside economic approaches, may be taken into account at least in two scales: first, encompassing the spatial system formed around historical cores and centres; second, encompassing fragments generated through sprawl. In the first case, some regeneration or renewal of historical cores may occur along with fragmentation at their boundaries; in the second case, progressive substitution of built forms within dispersed fragments may occur, just like in the cities they originated from. Either case is yet to be theoretically addressed.

52 c) *Urban Morphology*: the present approach allows inferences on emergent patterns of urban form as strongly immersed in heterogeneity. Spatial heterogeneity would be the output of processes of substitution in accumulated isolated or connected, avalanche-like changes brought about at different times. Substitution involves change of density and socioeconomic patterns. One may easily add changes in scale and technology. Scale means the size of every individual irruption, likely to increase with concentration of fixed capital. Technology brings along questions of form, materials, architectural lexicons and evolving patterns of human activity.

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## *Pour citer cet article*

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## *Droits d’auteur*



## *Résumés*

Our paper suggests that changes of the inner urban built form could be explained as a result of a process of self-organized criticality. Similar to the general behaviour of such kind of a system, urban built form grows discretely up to a point in which new additions could occur both in the boundaries and in inner areas. Once reached this critical point, the urban system stays around it, combining outer and inner growth. While outer growth is quantitative and does not modify the system’s structure, inner growth is qualitative as well as quantitative and can transform urban centrality. Replacement of old structures by new ones always occurs with extraordinary capital concentration, developing new polarization inside the urban fabric. The hypothesis is tested through the comparative evolution of different land, built form and location values within a period of continuous urban growth. Comparative indicator values are obtained by simplified simulation

## La forme urbaine, comme processus d'auto-organisation critique

Cet article explore la possibilité d'expliquer les changements de forme construite urbaine comme le résultat d'un processus d'auto-organisation critique (AOC). Pour le comportement observé dans ce type de systèmes, la croissance urbaine se fait par addition successive discrète de nouvelles formes construites sur les bords, à un point où de nouveaux bâtiments peuvent se produire aussi bien au bord comme à l'intérieur. En ayant atteint ce point, les nouvelles additions se produisent de façon combinée à la périphérie et à l'intérieur du tissu urbain. Alors que la croissance dans la périphérie est de nature quantitative et ne modifie pas la structure du système, la croissance interne est qualitative et peut modifier la centralité urbaine. Des remplacements de vieilles par de nouvelles structures se produisent toujours avec une concentration extraordinaire de capital, en développant une nouvelle polarisation dans la ville. Ces changements structurels seraient comparables à des changements catastrophiques caractéristiques des

Systèmes d'AOC. L'hypothèse est testée à travers l'évolution comparative des valeurs de la terre et des zones urbaines dans le cadre de la croissance urbaine. Le suivi du processus se fait par des indicateurs comparatifs extraits par simulation de la dynamique socio-spatiale.

***Entrées d'index***

***Mots-clés :*** développement urbain, auto-organisation, Modélisation
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