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MULTIVARIABLE LOW ORDER STRUCTURED-CONTROLLER DESIGN BY FREQUENCY RESPONSE APPROXIMATION

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Abstract - The method presented here offers an effective and time saving tool for multivariable controller design. The relation between controller complexity and closed loop performance can easily be evaluated. The method consists of five steps: 1. A desired behavior of the closed loop system is specified. Considering the nonminimum phase part of the process model the closed loop attainable performance is determined. 2. The process model and the attainable performance are scaled by the RPN-scaling procedure. 3. This defines an "ideal" scaled controller, which is usually too complex to be realized. 4. The frequency response of the ideal scaled compensator is approximated by a simpler one with structure and order chosen by the user. 5. Since the approximation in frequency response is performed with the scaled system, it is necessary to return to the original system's units.
Keywords: low order controllers, multivariable PID-controllers, decentralized controllers, model reduction, RPN

INTRODUCTION

Restrictions on the feedback compensator structure are often encountered in chemical plants, when several control stations are provided only with local measurements. Such decentralized information structures result in block-diagonal compensator matrices. Decentralized controllers are also attractive because the information about the feedback is concentrated in the diagonal blocks. This means they are easier to understand and to put into operation and more easily made failure tolerant than general multivariable control systems. The design of robust decentralized controllers remains a demanding problem; since standard methods for robust design can not be used for structured compensators. Here, we propose a procedure to design a low order (like PID) structured controller which can easily be implemented in any industrial control system.

In this paper it is presented a method for designing a fixed-order structured feedback controller for linear time-invariant (LTI) systems. The basic difficulty with the fixed-order structured controller design problem is that it is, in general, impossible to ensure that a desired or target closed loop transfer function is attainable by a controller of a given fixed order and structure. The basic idea of this approach is to approximate the high order full controller that achieves the desired attainable closed loop response by a low order structured controller. The next section introduces some of the ideas of the proposed method.

WHAT IS BEHIND A FEEDBACK CONTROLLER? (DIRECT SYNTHESIS METHOD APPLIED TO SISO CONTROLLER DESIGN)

Consider the block diagram of Figure 1, where $G(s)$ represents the consolidated transfer function for the process + measurement device + final control element. In this case the closed-loop transfer function is given by:

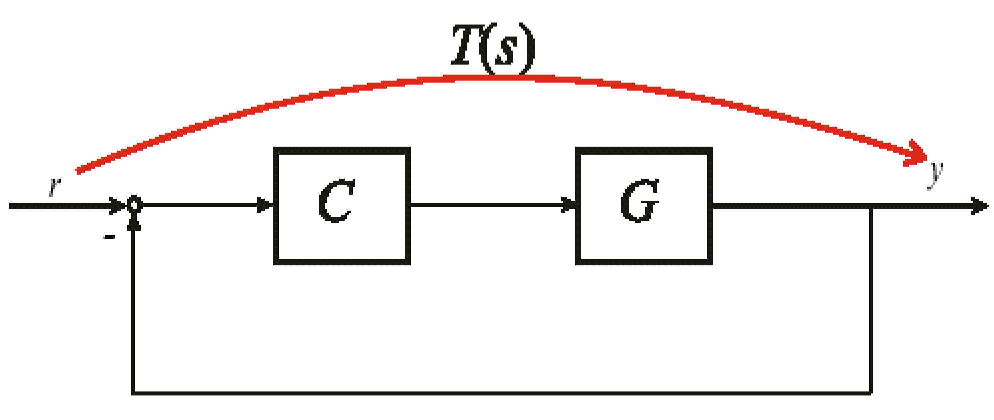

Figure 1: Standard feedback configuration

$$T(s) = \frac{y(s)}{r(s)} = \frac{GC(s)}{1 + GC(s)} \qquad (1)$$

Suppose now that we require the closed-loop behavior to be a predetermined transfer function of our choice. The choice of the reference trajectory *T(s)* depends on the type of closed-loop response desired and on the possible responses from the process. Some particular choices for the reference trajectory are shown in Figure 2. Note that for systems with neither time delays nor inverse response, reference trajectories of type (a) (cf. Figure 2) would be appropriate. However, if the process has a significant time delay, then a reference trajectory of type (c) is required because no controller can overcome the initial delay in the closed-loop response. Similarly, if the process has an inverse response, then the reference trajectory must allow inverse response too.

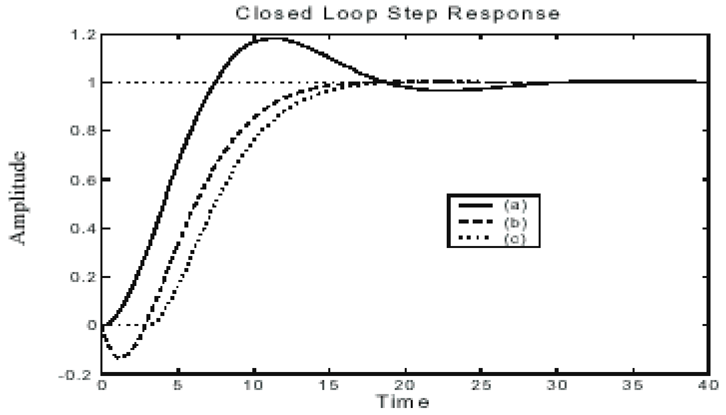

Figure 2: Reference response for $T(s)$

Having defined an appropriate form for the reference trajectory, the controller synthesis problem is now posed as follows: Given the process model *G*(*s*), what is the form of the controller *C*(s) required to produce the closed-loop behavior represented by the reference trajectory *T*(s)?

In other words, given *T*(*s*) and *G*(*s*), find the *C*(*s*) required to make the process closed-loop transfer function exactly equal to *T*(*s*). Note that the closed-loop behavior will be as we desire it to be if, and only if:

$$C(s) = \frac{1}{G(s)} \times \frac{T(s)}{1 - T(s)} \qquad (2)$$

This is the *controller synthesis formula* from which, given the process model *G*(*s*), we may derive the "ideal" controller *C*(*s*) required for obtaining the *attainable* closed-loop behavior represented by *T*(*s*).

Observe that there are no restrictions on the form this controller can take; for this reason, we may shall expect that there will be no guarantees that the controller will be implementable in all cases. However, we shall see that if $T(s)$ is chosen appropriately, the synthesized controller $C(s)$ will take on practical forms. In particular, we will show that in some important cases, the synthesized controller actually takes the familiar form of conventional PID controllers.

To illustrate this method, let us consider the first-order closed loop response $T(s) = \dfrac{1}{\tau_{cl}s + 1}$ for that the controller synthesis equation (2) simplifies to:

$$C(s) = \frac{1}{G} \times \frac{1}{\tau_{cl}s} \qquad (3)$$

Now, applying (3) to the second order process $G(s) = \dfrac{K}{\tau^2 s^2 + 2\zeta\tau s + 1}$ the final synthesized controller is given by:

$$C(s) = \frac{\tau^2 s^2 + 2\zeta\tau s + 1}{K\tau_{cl}s} = \underbrace{\frac{2\zeta\tau}{K\tau_{cl}}}_{K_P}\left(1 + \frac{1}{\underbrace{2\zeta\tau}_{\tau_I} s} + \underbrace{\frac{\tau}{2\zeta}}_{\tau_D} s\right) \qquad (4)$$

which is immediately recognizable as a PID controller with the indicated parameters.

Synthesis for Processes with Complex Dynamics

The fact that the general controller synthesis formula (2) involves the inverse of the process model should make us cautions in applying it for time delay and inverse response systems, since, in this case, the $exp(-\theta s)$ term in the time-delay system s transfer function will become $exp(+\theta s)$ in the controller, thereby requiring prediction; and for the inverse-response system, the relatively harmless right-half plane zero translates to a right-half plane pole for the controller, making it unstable. The primary reason for caution in applying (2) for open-loop unstable systems is not as obvious, but it is equally critical, as we will discuss later. In order to focus attention on essentials, consider the simple transfer function: $G(s) = K \exp(-\theta s)/(\tau s + 1)$.

Now, the best closed loop transfer function must include the pure time delay. In this case, closed loop transfer function must be changed as $T(s) = \exp(-\theta s)/(\tau_{cl}s + 1)$. It means that any control action applied to the plant will only produce any change in the measured process output after elapsed the time at least equal to the plant time delay, otherwise the controller must be able to "see the future". With this attainable closed loop transfer function, the controller is given by

$$C(s) = \frac{(\tau s + 1)}{K}\left(\frac{1}{\tau_{cl}s + 1 - \exp(-\theta s)}\right). \qquad (5)$$

Using a simple first-order Taylor series approximation: $\exp(-\theta s) \approx 1 - \theta s$ in (5), the direct synthesis controller equation simplifies to

$$C(s) = \underbrace{\frac{\tau}{K(\tau_{cl} + \theta)}}_{K_p}\left(1 + \frac{1}{\tau s}\right) \quad (6)$$

Of course, much better approximation can be obtained with the Padé approximation. Observe that the controller order will increase with the order of the Padé approximation. Section 4 shows how the "ideal" controller response given by (5) can easily be approximated with the frequency response approximation method by low order controller like a PID-controller. When the attainable performance $T(s)$ and the process model $G(s)$ are available, the "ideal" controller can easily be calculated by (2). In the next section, it is shown how the attainable performance $T(s)$ can be determined based on $G(s)$ and on the desired performance.

ATTAINABLE PERFORMANCE

Performance limitations due to sensor noise, input constraints, and model uncertainty are always present. In addition, the nonminimum phase elements of the system also limit the attainable performance. In this section, we discuss how these different limiting factors can be taken into account.

Specification of the Desired Performance

The desired performance can be specified in many different ways (see, e.g., Levine, 1996). Here we consider the specification of the *desired* (output) complementary sensitivity function T_d which relates the reference signal r and the output signal y in the 1 degree of freedom (DOF) control configuration (see Figure 1). For the SISO case, the time domain specifications (e.g., settling time, rise time, maximal overshoot, and steady-state error) can be mapped into the following second order transfer function

$$T_d \stackrel{\Delta}{=} \frac{1 - \varepsilon_\infty}{\left(\frac{s}{\omega_n}\right)^2 + 2\zeta\frac{s}{\omega_n} + 1} \quad (7)$$

where ε_∞ is the permitted offset (steady-state error). The parameters ω_n (undamped natural frequency) and ξ(damping ratio) of (7) can be easily calculated from the time domain specifications.

For the MIMO case, a straightforward extension of such a specification is to prescribe a decoupled or almost decoupled response, with possibly different parameters for each output, i.e., $T_d = diag(T_{d,1},...,T_{d,no})$, where each $T_{d,i}$ corresponds to a SISO time domain specification for each of the *no* output variables.

RHP-Zero and RHP-Pole Constraints

It is well known that there exists a symmetry between the RHP-zero and RHP-pole constraints; i.e., the role that the complementary sensitivity function T, $T(s) = GC\,(I+GC)^{-1}$, plays in RHP-zero constraints is played by sensitivity function S, $S = (I+GC)^{-1}$, in RHP-pole constraints cases, and vice-versa.

(a) RHP-Zero Constraints

If $G(s)$ has a RHP-zero at z with output direction y_z, then for internal stability of the feedback system the controller must not cancel the RHP-zero. Thus $L=GC$ must have also a RHP-zero in the same direction as G, i.e., $y_z^H G(z) = 0 \Rightarrow y_z^H G(z)C(z) = 0$. It follows from $T=LS$ that the interpolation constraints

$$y_z^H T(z) = 0\,; y_z^H S(z) = y_z^H \qquad (8)$$

must be satisfied. So T must have a RHP-zero z with the same direction as $G(s)$ and $S(z)$ has an eigenvalue of 1 with corresponding left eigenvector y_z.

(b) RHP-pole constraints

If $G(s)$ has a RHP-pole at p with output direction y_p, then for internal stability of the feedback system the controller must not cancel the RHP-pole. Thus $L^{-1}=C^{-1}G^{-1}$ must have a RHP-zero in the same input direction as the output direction of the pole p, i.e., $\Rightarrow$ $C^{-1}(p)G^{-1}(p)\, y_p = 0$. It follows from $S=TL^{-1}$ that the interpolation constraints

$$S(p)y_p = 0; T(p)y_p = y_p \qquad (9)$$

must be satisfied. S must have a RHP-zero p with the same input direction as the output pole direction y_p and that $T(p)$ has an eigenvalue of 1 with corresponding right eigenvector y_p.

In the frequency domain, the RHP-zeros constraint the bandwidth up to which effective disturbance attenuation is possible, i.e., where S can be made small. The largest achievable bandwidth in the SISO case is determined by the RHP-zero closest to the origin. RHP-poles limit the minimal bandwidth of the complementary sensitivity function T. In the SISO case the smallest possible bandwidth is determined by the RHP-pole furthest from the origin. If the plant has both RHP poles and zeros the difficulties are aggravated. Specially, if the RHP poles and zeros are close in magnitude and direction to each other, considerable peaking of the sensitivity and complementary sensitivity functions occurs (see, e.g., Engell, 1988, Skogestad and Postlethwaite, 1996, Levine, 1996).

Attainable Performance Considering the Internal Stability Constraints

The output Blaschke factorization for the zeros $B_{O,z}(s)$ and the input Blaschke factorization for the poles $B_{I,p}(s)$ can be used to satisfy the internal stability constraints (8) and (9). For the definition of the Blaschke factorization and an algorithm to calculate it, see, e.g., Zhou et al. (1996) or Trierweiler (1997).

(a) RHP-Zeros

When the plant $G(s)$ is asymptotically stable and has at least as many inputs as outputs, $G(s)$ can be factored as $G(s) = B_{O,z}(s)G_m(s)$. The possible complementary sensitivity functions T can be then factored to satisfy the interpolation constraint (8) as

$$T(s) = B_{O,z}(s)\, B_{O,z}^{\dagger}(0)\, T_d(s) \tag{10}$$

where $T_d(s)$ is the ideal desired closed-loop transfer function, $B_{O,z}^{\dagger}$ denotes the pseudo-inverse of $B_{O,z}$, and $B_{O,z}(0)\ B_{O,z}^{\dagger}(0) = I$. It is easy to verify that (10) implies (8), since $y_{z,i}^H B_{O,z}(z_i) = 0 \Rightarrow y_{z,i}^H T(z_i) = 0$.

$T(s)$ is different from the original desired transfer function $T_d(s)$, but has exactly the same singular values, since $B_{O,z}(s)$ is allpass so that specified robustness properties are preserved at the plant output. The factor $B_{O,z}$ (0) ensures that $T(0) = T_d(0)$ so that steady-state characteristics (most commonly, $T_d(0) = I$) are preserved.

(b) RHP-Poles

When the plant $G(s)$ has no RHP-zero and at least as many outputs as inputs, $G(s)$ can be partitioned as $G(s) = G_m(s)\ B_{I,p}(s)$, and the possible complementary sensitivity functions T can be factored to satisfy the interpolation constraint (9) as

$$T(s) = I - \left(I - T_d(s)\right) B_{I,p}^{\dagger}(0)\, B_{I,p}(s) \tag{11}$$

where $T_d(s)$ is an ideal desired closed-loop transfer function, $B_{I,p}^{\dagger}$ denotes the pseudo-inverse of $B_{I,p}$, and $B_{I,p}^{\dagger}(0)\ B_{I,p}(0)= I$. It is easy to verify that (11) implies (9), since

$$\begin{aligned} &B_{I,p}(p_i)\, y_{p,i} = 0 \Rightarrow \\ &T(p_i) y_{p,i} = \left[I - \left(I - T_d(p_i)\right) B_{I,p}^{\dagger}(0)\, B_{I,p}(p_i)\right] y_{p,i} \Leftrightarrow \\ &T(p_i) y_{p,i} = y_{p,i}. \end{aligned} \tag{12}$$

Again, although $T(s)$ is different from the original desired transfer function $T_d(s)$, it has exactly the same singular values, since $B_{I,p}(s)$ is all-pass. It means that $\sigma_i[T(j\omega)] = \sigma_i[T_d(j\omega)]$ so that specified robustness properties are preserved at the plant output. The presence of the factor $B_{I,p}$ (0) ensures that $T(0) = T_d(0)$ so that steady-state characteristics are preserved.

Pure Time Delays

Pure time delays (deadtimes) are an inherent characteristic of many chemical processes. Sources of time delays are transportation lags due to the flow of material (e.g., in the piping of a heat exchanger network distributed in the plant) and measurement lags (e.g., composition analysis of the product of a distillation column from a chromatograph). In addition, it is common practice to approximate higher-order dynamics by a low-order model plus a time delay. System models in terms of partial differential equations are another source of time delays.

(a) Factorization

Some sources of deadtimes imply a natural factorization, e.g., measurement lags give rise to a diagonal matrix of pure delay terms at the output, whereas transportation lags of manipulated flows cause a diagonal matrix at the input. When the time delays result from the approximation of high-order dynamics, different delays may occur in each channel. The analysis of the effect of the delays on the attainable performance can be done by a factorization of the delay terms in an input diagonal deadtime matrix D_I, an output diagonal deadtime matrix D_O, and a transfer function matrix with minimal deadtimes in each channel G_{MD}, i.e., $G=D_O\,G_{MD}\,D_I$. As

$$\begin{aligned}\det(G(s)) &= \det(D_O(s)G_{MD}(s)D_I(s)) \\ &= \det(D_O(s))\det(G_{MD}(s))\det(D_I(s))\end{aligned} \qquad (13)$$

where $D_O = exp\{\, diag\,(-\alpha_1, \ldots, -\alpha_{no})\, s\,\}$ and $D_I = exp\{\, diag\,(-\beta_1, \ldots, -\beta_{ni})\, s\,\}$, the analysis can be split into two issues:

The Effect of the Diagonal Matrices D_I And D_O

Large deadtimes in these matrices are obviously detrimental to control performance. Roughly, the bandwidth limitations are the same as for a RHP zero at (2 /deadtime).

If the plant $G(s)$ can be partitioned as $G(s) = D_O(s)G_m(s)D_I(s)$, where $G_m(s)$ is a minimum phase transfer function, then the closed-loop transfer function T can be factored as

$$T(s) = D_O(s)T_d(s)D_I(s) \qquad (14)$$

where $T_d(s)$ is an ideal desired closed-loop transfer function. So if the system has input delays, the attainable performance cannot be faster as our capacity to manipulate the inputs. Equivalently, if the system has output delays, T cannot be faster as our capacity to measure the outputs.

The Delay Terms in G_{MD}

Here the analysis can be done by a computation of the zeros of G_{MD}, i.e., the values where $det(G_{MD}(z)) = 0$. The zeros of G_{MD} and G are the same, since $det(D_I(z)) \neq 0$ and $det(D_O(z)) \neq 0$. Therefore, the discussion in the last subsection about RHP-zeros carries over. A problem introduced by the irrational system is to determine the zeros which are infinite in number. Moreover, G can have a bounded or unbounded number of zeros which lie in the RHP (Jerome and Ray, 1991).

In the general case, $G(s)$ can first be factored as $G=D_O\,G_{MD}\,D_I$ and if G_{MD} is stable and can be approximated by a Padé approximation, $G_{MD}(s)$ can be partitioned as $G_{MD}(s) \approx B_{O,z}(s)G_m(s)$ and all closed-loop transfer functions T have the form:

$$T(s) = D_O(s)B_{O,z}(s)\,B_{O,z}^{\dagger}(0)\,T_d(s)\,D_I(s) \qquad (15)$$

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FREQUENCY RESPONSE APPROXIMATION

In this section, the approximation of a high-order controller which may result from any design method is considered. In the first step, each controller column is approximated independently using an iterative least squares technique. The most important aspect of this procedure is the use of a frequency-dependent weighting function which takes the sensitivity of the closed-loop system to changes of the compensator into account. The controller resulting from the column-by-column approximation is improved by minimizing a mixed L_2/L_∞ -criterion for the complete controller. This design method is in detail discussed by Engell and Müller (1993) and Müller (1996).

Column-By-Column Optimization (The Least Squares Problem)

For feedback control system shown in Figure 1 with the plant G and controller C, the reference-to-output transfer matrix is

$$T = (I + GC)^{-1} GC = GC\,(I + GC)^{-1} . \qquad (16)$$

The aim of the approximation is to minimize ΔT, the deviation of the closed-loop system frequency response from its desired value. ΔT can be brought to the simple form

$$\Delta T = SG\Delta C S_0 , \qquad (17)$$

where S_0 and S are the nominal and the actual sensitivity functions, respectively. Replacing S by S_0 yields a linear relation of ΔT and ΔC:

$$\Delta T \approx S_0 G \Delta C S_0 . \qquad (18)$$

As S_0 is approximately diagonal, each element of ΔC affects only the corresponding column of ΔT and the controller columns can be approximated independently. The nonlinear problem of optimizing the numerator and the denominator polynomials simultaneously is solved iteratively in a least squares sense. As the orders of the numerator polynomials can be different and each coefficient can be set to zero, arbitrary structures as e.g. blockdiagonal controllers can be optimized (for more details see Müller (1996)).

The Overall Optimization (The Non-Convex Problem)

If the high order nominal controller C_0 is approximated by a relatively simple transfer matrix, the resulting sensitivity function S differs significantly from S_0 and (18) does not hold. Hence, the solution of the column-by-column optimization can be improved by a nonlinear minimization of the overall quadratic error

$$J = \sum_{k=1}^{r} \sum_{i=1}^{r} \sum_{\lambda=1}^{N} \left| \frac{\Delta t_{ik}(j\omega_\lambda)}{\omega_\lambda} \right|^2 \quad (19)$$

where r and N are the numbers of reference inputs and frequency points, respectively. ΔT is divided by ω to minimize the error between the step responses rather than between the impulse responses.

In order to avoid unstable approximations of stable controllers, the denominators are parametrized by dampings and natural frequencies. Stability of the closed-loop system follows from the small gain theorem if

$$\|S_0 G \Delta C\|_\infty < 1 \quad (20)$$

is satisfied. To emphasize values greater than one, (20) is squared which leads to the criterion

$$J = \sum_{k=1}^{r} \sum_{i=1}^{r} \sum_{\lambda=1}^{N} \left| \frac{\Delta t_{ik}(j\omega_\lambda)}{\omega_\lambda} \right|^2 + \|S_0 G \Delta C\|_\infty^2 \quad (21)$$

The normalization with ω^-, the smallest frequency value used for the approximation, is necessary to make the first term, and thus the minimizing argument, independent of the considered frequency range.

The trade-off between closed-loop stability and small approximation errors can be influenced by λ . A greater value of λ reduces the L_2-error but increases the risk of instability. If an approximation with the default value of 1000 is not satisfactory, λ can be decreased to achieve stability or increased to get a smaller L_2-error. Especially for the decentralized approximation of full-feedback controllers, λ is a valuable knob to maintain closed-loop stability.

To minimize the cost functional, a BFGS Quasi-Newton algorithm is used, starting from the solution of the independent optimization of the controller columns. Solving the overall optimization without this starting point is not feasible. Furthermore, the column-by-column approximation is an effective and computationally cheap indicator whether the solution is feasible with the chosen controller structure and the specified orders.

CONTROLLER DESIGN USING THE ATTAINABLE CLOSED-LOOP FUNCTION *T*

In the last section an efficient method for order reduction in the frequency domain was presented. Here, we show how the same procedure can be used to design a low order controller directly from the attainable closed loop performance determined in section 3. Since for multivariable systems the scaling inputs and outputs are crucial for a good controller design, it will firstly be presented a scaling procedure base on the Robust

Performance Number (RPN). The RPN-Scaling Procedure is the main responsible for the success of the controller design method presented in this paper.

Robust Performance Number

It is well known that if a plant has a large condition number of the frequency response matrix in the critical frequency range around the bandwidth of the closed loop system, i.e. strong directionality of its gain for these frequencies, the closed loop system is very sensitive to unstructured multiplicative input uncertainty of the plant model if a decoupling controller is used. The standard approach in the design via frequency response approximation is to choose a decoupled desired closed loop system and hence the resulting closed loop system may exhibit poor robustness against certain plant perturbations if the condition number is large around the gain crossover frequency. If the condition number of the frequency response matrix is small in this region, the design of a robust decoupling controller usually is simple. This consideration motivated the introduction of the Robust. Performance Number (RPN). The RPN is defined as (Trierweiler 1997, Trierweiler and Engell 1997)

$$\mathrm{RPN} \triangleq \Gamma_{\sup}\left(G,T,\omega\right) = \sup_{\omega \in \mathrm{R}}\left\{\Gamma\left(G,T,\omega\right)\right\} \qquad (22a)$$

$$\Gamma\left(G,T,\omega\right) \triangleq \sqrt{\bar{\sigma}\left(\left[I - T(j\omega)\right]T(j\omega)\right)\left(\gamma^{*}\left(G(j\omega)\right) + \frac{1}{\gamma^{*}\left(G(j\omega)\right)}\right)} \qquad (22b)$$

where $\gamma^{*}(G(j\omega)$ is the *minimized* condition number of $G(j\omega)$ and σ;$^{-}$([*I-T*] *T*) is the maximal singular value of the transfer function [*I-T*] *T*. *T* is the attainable desired output complementary sensitivity function which was determined for the nominal model *G* as discussed in section 3.

The term σ;$^{-}$([*I-T*] *T*) provides a frequency dependent weighting for g*(*G*)+1/g*(*G*), which has its peak value in the crossover frequency range. RPN can be understood as a measure of how difficult it is to compute a decoupling robust controller for a given system. A decoupling controller will achieve good performance robustness only when the RPN is small.

RPN-Scaling Procedure

Let L_S and R_S be diagonal, nonsingular, constant scaling matrices, then the condition numbers of the matrices $G(s)$ and $G_S(s) = L_S G(s) R_S$ may differ considerably. Although we can calculate the scaling matrices L_S and R_S that make $\gamma(L_S G(j\omega R_S)$ minimal for each frequency ω we cannot do it in general for the system *G*. Here, we have the possibility to chose only one set of matrices L_S and R_S to be used for all frequencies. The crossover frequency range (or equivalently the desired performance) plays a very important role for the determination of the optimal scaling. As proposed in (Trierweiler, 1997, Trierweiler and Engell, 1997), L_S and R_S are chosen to minimize the condition number at the frequency that define the RPN, i.e., the following RPN-scaling procedure must be applied:

1) Determine the frequency ω_{sup} where $\Gamma(G,T,\omega)$ achieves its maximal value.

2) Calculate the scaling matrices L_S and R_S , such that $\gamma(L_S\, G(j\omega_{sup})R_S)$ achieves its minimal value $\gamma^*(G(j\omega))$.

3) Scale the system with the matrices L_S and R_S , i.e., $G_s(s) = L_s G(s) R_s$.

The controller design should be performed with the scaled system $G_S(s)$. To implement the resulting controller $C_S(s)$ it is necessary to return to the original system units, what can be done by $C(s)=R_S C_S(s) L_S$.

Controller Design Method

The proposed controller design procedure consists of five main steps:

1) A desired behavior of the closed loop system $T_d(s)$ is specified. The closed loop attainable performance, $T(s)$, is determined as a function of nonminimum phase elements, sensor noise, input constraints, model uncertainty, and the desired performance.

2) The process model $G(s)$ and the attainable performance $T(s)$ are scaled by the RPN-scaling procedure, i.e., $G_s(s) = L_s G(s) R_s$ and $T_s(s) = L_s T(s) L_s^{-1}$.

3) With T_S and G_S, the scaled ideal controller frequency response $C_S^{id}(j\omega)$ is calculated by $C_S^{id}(j\omega) = G_S^{-1}(j\omega)\left(T_S^{-1}(j\omega) - I\right)^{-1}$, when number of inputs (ni) = number of outputs (no). If $no>ni$, G_S^{-1} must be replaced by a right inverse. Note that C_S^{id} is usually a very high order controller. For example, in the case of systems with pure time delays, the resulting ideal controller is even not rational.

4) $C_S^{id}(j\omega)$ is approximated by a low order block-decentralized controller $C_S(s)$ using the frequency response approximation method presented in section 4. The most important aspect of this step is the use of a frequency dependent weighting function in an iterative approximation process, which takes the sensitivity of the closed loop system to changes of the controller into account. Because of the large number of parameters in a multivariable controller, the approximation is initially done sequentially using an iterative least squares technique, then the resulting controller is optimized using nonlinear minimization algorithms.

5) Conversion of the controller to the original system units, i.e., $C(s)=R_S\, C_S(s) L_S$ where $C_S(s)$ is the designed controller based on $G_S(s)$.

Figure 3 shows how this general procedure can be applied for controller design. If the resulting behavior is not satisfactory, either the controller order can be increased or the controller structure can be modified. When changing on the controller order and structure do not given good results, the desired performance can be modified. Of course, since the "ideal" controller used in our procedure is based on the process inverse, it will not ever produce good results. An inverse-based controller will have potentially good performance robustness only when the RPN is small. Based on our experience, our procedure based on the ideal controller will produce good practical results, if RPN < 5. This criterion should be understood more as a guideline not as absolute value. For example, RPN = 10 means that we need to start to pay attention on the input uncertainty, since an inverse-based controller becomes to be much sensible to input uncertainty. For this case, a more time consuming procedures (e.g., DK-iteration, H_∞ , etc.) is recommended. In those cases, the final controller can be reduced using the order reduction method presented in section 4.

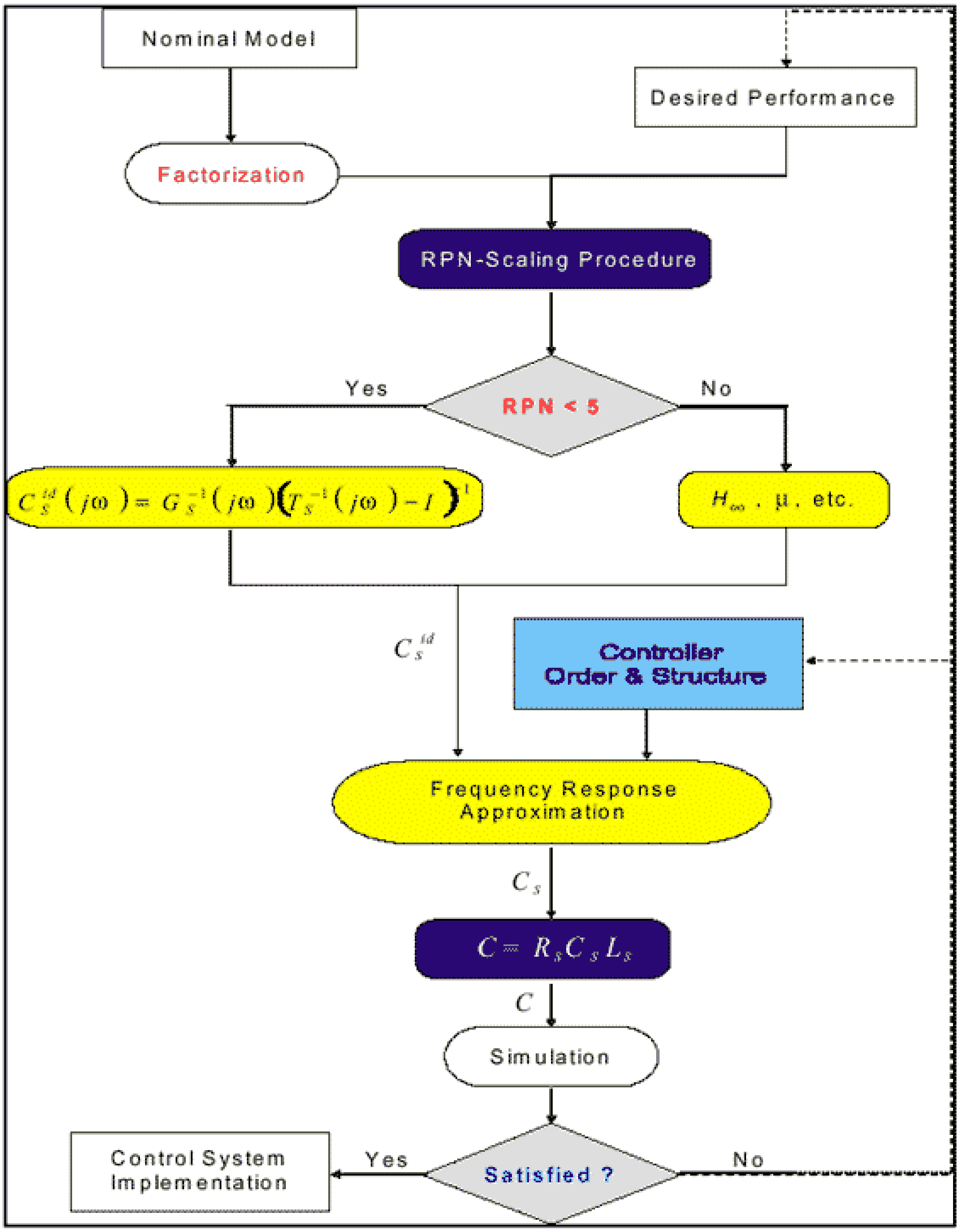

Figure 3 : Controller design methodology for low order controllers

CONTROLLER DESIGN OF A HEAT INTEGRATED DISTILLATION COLUMN

In this section we will design a controller for the low-purity[1] benzene/toluene/m-xylene (BTX) heat integrated distillation column system proposed by *Ding and Luyben* (1989). This system (Figure 4) consists of two columns, where the first acts as a prefractionator to provide an initial split of the feed. The prefractionator must produce an overhead product that contains little xylene and a bottoms product that contains little benzene. These two streams are fed into different feed trays in the second column, where the fine split occurs.

It was concluded in (Ding and Luyben, 1989) that one-point composition control on the prefractionator column is enough to reduce the adverse effects on sidestream purity XS2 produced by variations of the feed composition. Figure 4 shows the manipulated and controlled variables for this configuration. It is assumed in (Ding and Luyben, 1989) that the compositions can be measured with one minute time delay using a composition analyzer. Due to the low purities of these columns, the system dynamics can be captured well by the following linear model:

$$\begin{bmatrix} XB1 \\ XD2 \\ XS2 \\ XB2 \end{bmatrix} = \begin{bmatrix} \frac{-7.39e^{-s}}{(11s+1)(s+1)} & 0 & 0 & 0 \\ \frac{-0.11(200s+1)e^{-5s}}{(20s+1)^3} & \frac{10.1e^{-s}}{(28s+1)(4s+1)} & \frac{1.18e^{-11s}}{(31s+1)(6s+1)} & \frac{-18.3e^{-s}}{(28s+1)(5s+1)} \\ \frac{1.9e^{-2s}}{(4s+1)^2} & \frac{1.7(200s+1)e^{-1.4s}}{(108s+1)(s+1)^2} & \frac{-3.15e^{-s}}{(3s+1)(0.3s+1)} & \frac{-1.27(188s+1)e^{-s}}{(68s+1)(s+1)} \\ \frac{4.9e^{-16s}}{(40s+1)(3s+1)} & \frac{-8.21e^{-2.5s}}{(24s+1)(3s+1)} & \frac{12e^{-1.5s}}{(29s+1)(3s+1)} & \frac{19.4e^{-s}}{(26s+1)(3s+1)} \end{bmatrix} \begin{bmatrix} Q1 \\ R2 \\ S2 \\ Q2 \end{bmatrix} \quad (23a)$$

where the time constants are in minutes. Ding and Luyben (1989) considered different feed composition disturbances. Typical transfer functions from the disturbance to the outputs are

$$Z_1(s) = \begin{bmatrix} 0 & \frac{-2.42e^{-5s}}{(3s+1)(26s+1)^2} & \frac{-0.592e^{-5s}}{(7s+1)^2} & \frac{1.51e^{-19s}}{(45s+1)(5s+1)^2} \end{bmatrix}^T \quad (23b)$$

$$Z_2(s) = \begin{bmatrix} 0 & \frac{-2.47e^{-5s}}{(3s+1)(22s+1)^2} & \frac{1.83e^{-6s}}{(25s+1)(2s+1)} & \frac{-4.52e^{-8s}}{(50s+1)(7s+1)^2} \end{bmatrix}^T \quad (23c)$$

.

To determine the attainable performance T for this problem, we use the factorization (15), i.e., $T(s) = D_O(s)\, B_{o,z}(s)\, B^{\dagger}_{o,z}(0)\, T_d(s)$. Due to the composition analyzer deadtime of one minute, D_O is given by *diag* { *exp*(-1*s*), *exp*(-1*s*), *exp*(-1*s*), *exp*(-1*s*) } and $B_{O,z}$ depends on the order of the Padé approximation used to represent the time delays that cannot be factored out. For example, the transfer matrix (23a) without time delays has a RHP zero at 0.7814 with the output direction given by y_z = [0.015, -0.5418, 0.1638, 0.8243]^T. For a second order Padé approximation, the RHP zeros are (1.2 ± 1.5*i*, 9.9) with the output directions affecting mainly the last output (XB2) and to a lesser extent the second output (XD2). This indicates that the 'nonfactorable' time delays have a favorable, but not significant, influence on the system controllability. For the choice of the desired performance, the output zero direction gives valuable information about how fast each output can be made. This information is specially important, if we intend to use a low order controller. In this example, if we want to achieve small coupling with low order controller for setpoint changes, the output zero direction indicates that the outputs XD2 and XB2 should be regulated slower than the outputs XB1 and XS2. Therefore, we have chosen the desired performance as

$T_d = diag\{\ T_{d1}[\ 4,\ 10\%],$

$T_{d1}[\ 6,\ 10\%],\ T_{d1}[\ 4,\ 10\%],\ T_{d1}[\ 6,\ 10\%]\ \},$

where

$T_{d1}[\ 4,\ 10\%] = 1/(3.91s^2+2.34s+1)$

and

$T_{d1}[\ 6,\ 10\%] = 1/(8.8s^2 + 3.5s + 1).$

Figure 5 shows the RPN plot for T using the above desired performance and $B_{O,z}$ obtained by a second order Padé approximation of the nonfactored time delays. As the RPN is not large (RPN=3.17), an inverse-based controller can be used. The following PI controller was obtained by the application of the procedure presented in this paper.

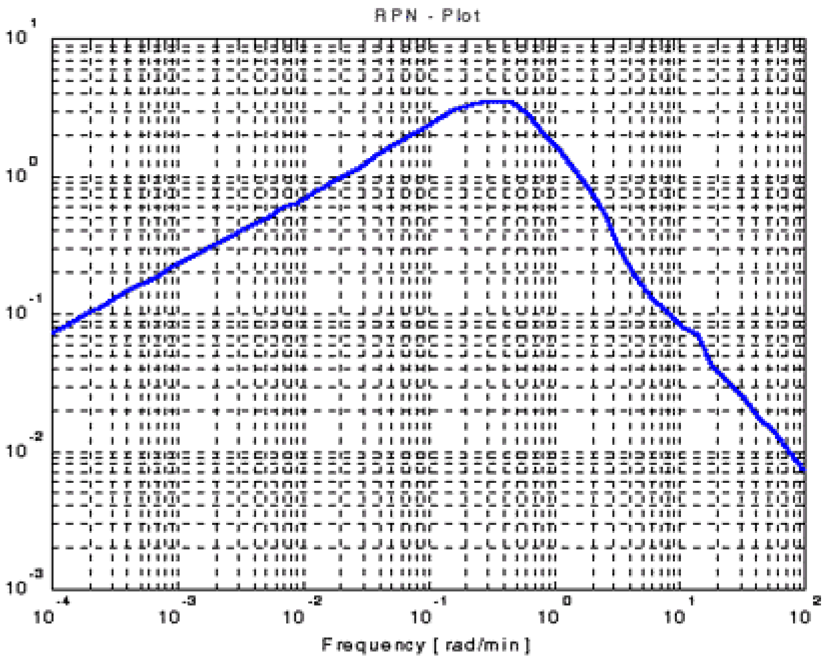

Figure 5 : RPN-plot

$$C_1 = \frac{1}{s}\begin{bmatrix} -0.4459s-0.0406 & 0 & 0 & 0 \\ 0 & 0.2482s+0.0537 & 0.08271s+0.134 & 0.3519s-0.0162 \\ 0 & 0.4654s+0.00468 & -0.2648s-0.0256 & 0.4245s+0.0160 \\ 0 & -0.1704s+0.0197 & 0.07207s+0.0731 & 0.1768s-0.00746 \end{bmatrix}.$$

Figures 6 and 7 show the regulatory and servo performance of the controller C_1, respectively. The controller C_1 presents already good servo and regulatory performances. Only for the step in XB2_ref a relatively strong coupling is observed. This problem could be solved by increasing the controller order or by making the desired performance for XB2_ref a little slower. Another possibility that preserves (or improves) the regulatory performance of C_1, could be a two-degree-of-freedom control configuration.

Incresing the controller order yields:

$$C_{3\times3} = \begin{bmatrix} \frac{0.424s^2+0.484s+0.0388}{s^2+1.121s} & \frac{0.147s^2+0.203s+0.0704}{s^2+0.4671s} & \frac{112.9s^2+267.5s+11.17}{s^2+452s} \\ \frac{1.273s^2+0.521s+0.0104}{s^2+1.121s} & \frac{-0.377s^2-0.161s-0.013}{s^2+0.4671s} & \frac{-265.1s^2+116.6s+3.79}{s^2+452s} \\ \frac{-0.570s^2-0.110s+0.010}{s^2+1.121s} & \frac{0.076s^2+0.142s+0.0380}{s^2+0.4671s} & \frac{15.43s^2+175.5s+6.49}{s^2+452s} \end{bmatrix}$$

$$C_2 = diag\left\{-0.4456 - \frac{0.04056}{s}, C_{3\times3}\right\}.$$

Figure 8 shows the simulation of the second order controller C_2. C_2 improves the servo performance, but the regulatory performance is almost the same as for the controller C_1.

Many different controller orders and structures can be synthesized. For example, the controller C_3

$$C_3 = \frac{1}{s}\begin{bmatrix} -0.4459s-0.0406 & 0 & 0 & 0 \\ 0 & 0.3393s+0.01565 & 0.1454s+0.1155 & 0 \\ 0 & 0.3069s+0.01181 & -0.1895s-0.02249 & 0 \\ 0 & 0 & 0.09521s+0.06392 & 0.1622s+0.007054 \end{bmatrix}.$$

is a good trade-off between *controller order* and *structure* versus *closed loop performance* (cf. Figure 9). Note that the controllers C_1 and C_3 have almost the same performance, but the controller C_3 is much more simple to implement and to make failure tolerant than controller C_1. Therefore, in a practical sense C_3 should be the selected controller for this process.

CONCLUSIONS

It was presented a fast and efficient method to design and to evaluate alternative multivariable control structures. The method consists of five steps and is based on the same ideas of the direct synthesis method. Since the synthesized "ideal" controller is usually too complex to be realized, it is approximated by a simpler one with structure and order chosen by the user. The most important aspect of this step is the use of a frequency dependent weighting function in an iterative approximation process, which takes the sensitivity of the closed loop system to changes of the compensator into account. Because of the large number of parameters in a multivariable controller, the approximation is initially done sequentially using an iterative least squares technique, then the resulting controller is optimized using nonlinear minimization algorithms. The potential of the method was demonstrated by the design of three controllers with different orders and structures for a heat integrated distillation column with 4 manipulated and 4 controlled variables.

With the proposed method, the trade-off *controller order* and *structure* versus *closed loop performance* can systematically be analyzed. If the resulting behavior is not satisfactory, either the controller order can be increased or the controller structure can be modified. Of course, our method will not always work. A criterion based on Robust Performance Number (RPN) is stated to identify when the procedure will produce good results. For the cases, where the procedure is not applicable, more time consuming procedures (e.g., DK-iteration, H_∞ , etc.) must be used. In those cases, the final controller can be reduced using the order reduction method presented in section 4. In general, any controller design procedure becomes much more easy to tune, if the RPN-scaling procedure is applied to scale the system. In the proposed controller design procedure, the RPN concept is used both for quantify the system's controllability as for scaling the system.

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Note

[1]95 mol% pure benzene produtct (XD2), 90 mol% pure toluene produtct (XS2), and 95 mol% pure xylene product (XB2),

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