

\textit{f_0(1370) Decay in the Fock-Tani Formalism}

Mario L. L. da Silva, Daniel T. da Silva, Cesar A. Z. Vasconcellos, 
Departamento de Física, Instituto de Física e Matemática, Universidade Federal de Pelotas, 
Campus Universitário, CEP 96010–900, Pelotas, RS, Brazil.

and Dimitar Hadjimichef 
Departamento de Física, Instituto de Física e Matemática, Universidade Federal de Pelotas, 
Campus Universitário, CEP 96010–900, Pelotas, RS, Brazil.

Received on 23 September, 2006

We investigate the two-meson decay modes for \( f_0(1370) \). In this calculation we consider this resonance as a glueball. The Fock-Tani formalism is introduced to calculate the decay width.

Keywords: Glueballs; Fock-Tani formalism; Meson decay

I. INTRODUCTION

The gluon self-coupling in QCD opens the possibility of existing bound states of pure gauge fields known as glueballs. Even though theoretically acceptable, the question still remains unanswered: do bound states of gluons actually exist? Glueballs are predicted by many models and by lattice calculations. In experiments glueballs are supposed to be produced in gluon-rich environments. The most important reactions to study gluonic degrees of freedom are radiative \( J/\psi \) decays, central productions processes and antiproton-proton annihilations.

Numerous technical difficulties have so far been present in our understanding of their properties in experiments, largely because glueball states can mix strongly with nearby \( g\bar{g} \) resonances [1], [2].

The best estimate for the masses of glueballs comes from lattice gauge calculations, which in the quenched approximation show [3] that the lightest glueball has \( J^{PC} = 0^{++} \) and that its mass should be in the range 1.45 – 1.75 GeV.

Constituent gluon models have received attention recently, for spectroscopic calculations. For example, a simple potential model, namely the model of Cornwall and Soni [4], [5] has been compared consistently to lattice and experiment [6], [7].

In the present we shall apply the Fock-Tani formalism [8] to glueball decay by defining an effective constituent quark-gluon Hamiltonian. In particular the resonance \( f_0(1370) \) shall be considered.

II. THE FOCK-TANI FORMALISM

Now let us to apply the Fock-Tani formalism in the microscopic Hamiltonian to obtain an effective Hamiltonian. In the Fock-Tani formalism we can write the glueball and the meson creation operators in the following form

\[
G_{\alpha} = \frac{1}{\sqrt{2}} \Phi_{\alpha}^{\mu} a_{\mu}^{\dagger}, \quad M_{\beta} = \Psi_{\beta}^{\nu} q_{\nu}^{\dagger}. \tag{1}
\]

The indexes \( \alpha \) and \( \beta \) are the glueball and meson quantum numbers: \( \alpha = \{ \text{space, spin} \} \) and \( \beta = \{ \text{space, spin, isospin} \} \).

The gluon creation \( a_{\mu}^{\dagger} \) and annihilation \( a_{\mu} \) operators obey the following commutation relations

\[
[a_{\mu}, a_{\nu}^{\dagger}] = \delta_{\mu\nu}, \quad [a_{\mu}, a_{\nu}] = \delta_{\mu\nu}.
\]

While the quark creation \( \bar{q}_{\nu}^{\dagger} \) and annihilation \( \bar{q}_{\nu} \) operators obey the following anticommutation relations

\[
[\bar{q}_{\mu}, \bar{q}_{\nu}^{\dagger}] = \{ \bar{q}_{\mu}, \bar{q}_{\nu}^{\dagger} \} = \delta_{\mu\nu}, \quad [\bar{q}_{\mu}, \bar{q}_{\nu}] = \{ \bar{q}_{\mu}, \bar{q}_{\nu} \} = \delta_{\mu\nu}.
\]

In (1) \( \Phi_{\alpha}^{\mu} \) and \( \Psi_{\beta}^{\nu} \) are the bound-state wave-functions for two-gluons and two-quarks respectively. The composite glueball and meson operators satisfy non-canonical commutation relations

\[
[g_{\alpha}, g_{\beta}^{\dagger}] = 0 \quad [g_{\alpha}, g_{\beta}] = \delta_{\alpha\beta} + \Delta_{\alpha\beta}, \quad [m_{\alpha}, m_{\beta}^{\dagger}] = \delta_{\alpha\beta} - \Delta_{\alpha\beta}, \quad [m_{\alpha}, m_{\beta}] = \delta_{\alpha\beta}, \quad [G_{\alpha}, G_{\beta}] = \delta_{\alpha\beta}, \quad [M_{\alpha}, M_{\beta}] = \delta_{\alpha\beta}.
\]

The “ideal particles” which obey canonical relations

\[
[g_{\alpha}, g_{\beta}^{\dagger}] = 0 \quad [g_{\alpha}, g_{\beta}] = \delta_{\alpha\beta}, \quad [m_{\alpha}, m_{\beta}^{\dagger}] = \delta_{\alpha\beta}, \quad [m_{\alpha}, m_{\beta}] = \delta_{\alpha\beta}.
\]

This way one can transform the composite state \( |\alpha\rangle \) into an ideal state \( |\alpha\rangle \), in the glueball case for example we have

\[
|\alpha\rangle = \frac{1}{\sqrt{2}} \left( -\frac{\pi}{2} G_{\alpha}^{\dagger} |0\rangle \right) = g_{\alpha} |0\rangle
\]

where \( U = \exp(tF) \) and \( F \) is the generator of the glueball transformation given by

\[
F = \sum_{\alpha} g_{\alpha}^{\dagger} \tilde{G}_{\alpha} - \tilde{G}_{\alpha} g_{\alpha} \tag{4}
\]

with

\[
\tilde{G}_{\alpha} = G_{\alpha} - \frac{1}{2} \Delta_{\alpha\beta} G_{\beta} - \frac{1}{2} G_{\beta}^{\dagger} \Delta_{\beta\gamma} G_{\gamma}.
\]

In order to obtain the effective potential one has to use (4) in a set of Heisenberg-like equations for the basic operators \( g, \tilde{G}, a \)

\[
\frac{dg_{\alpha}(t)}{dt} = [g_{\alpha}, F] = \tilde{G}_{\alpha}; \quad \frac{d\tilde{G}_{\alpha}(t)}{dt} = [\tilde{G}_{\alpha}(t), F] = -g_{\alpha}.
\]
The simplicity of these equations is not present in the equations for \( a \)
\[
\frac{da_{\mu}(t)}{dt} = -\sqrt{3} \Phi^\nu_{\mu \bar{\nu}} a_{\nu} g_{\bar{\nu}} + \frac{\sqrt{3}}{2} \Phi^\nu_{\mu \bar{\nu}} a_{\nu} \Delta q_{\nu} g_{\bar{\nu}}
+ \Phi^\nu_{\mu \bar{\nu}}'(G^a_{\mu} a_{\bar{\nu}} - g_{\nu} a_{\mu} G^a_{\bar{\nu}})
- \sqrt{3} \Phi^\nu_{\mu \bar{\nu}} \Phi^\mu_{\nu} a_{\bar{\nu}} G^a_{\nu}
\times G^a_{\mu} a_{\mu} G_{\bar{nu}}.
\]
The solution for these equation can be found order by order in the wave functions. For zero order one has \( a_{\mu}^{(0)} = a_{\mu} \), \( g_{\alpha}^{(0)}(t) = G_{\alpha} \sin t + g_{\alpha} \cos t \) and \( G_{\beta}^{(0)}(t) = G_{\beta} \cos t - g_{\beta} \sin t \). In the first order \( g_{\alpha}^{(1)} = 0 \), \( G_{\beta}^{(1)} = 0 \) and \( a_{\mu}^{(1)}(t) = \sqrt{2} \Phi a_{\mu} g_{\bar{\nu}} \). If we repeat a similar calculation for mesons let us to obtain the following equations solution: \( q_{\nu}^{(0)} = q_{\nu} \), \( \dot{q}_{\nu}^{(0)} = \dot{q}_{\nu} \), \( q_{\nu}^{(1)}(t) = \Phi^{\mu \nu} a_{\nu} m_{\bar{\nu}} \) and \( \dot{q}_{\nu}^{(1)}(t) = -\Phi^{\mu \nu} \dot{q}_{\nu} m_{\bar{\nu}} \).

### III. THE MICROSCOPIC MODEL

The microscopic model adopted here must contain explicit quark and gluon degrees of freedom, so we obtain a microscopic Hamiltonian of the following form
\[
H = g^2 \int d^3 x d^3 y \Psi^i(\bar{x}) \gamma^\mu A_i^\mu(\bar{x}) \frac{\lambda^a}{2} \Psi(\bar{y})
\times \Psi^j(\bar{y}) \gamma^\mu A_i^\mu(\bar{y}) \frac{\lambda^a}{2} \Psi(\bar{y})
\]
(5)
Where the quark and the gluon fields are respectively [9]
\[
\Psi(\bar{x}) = \sum \int \frac{d^3 k}{(2\pi)^3} [u(\bar{k}, s) q(\bar{k}, s) + v(-\bar{k}, s) \bar{q}(-\bar{k}, s)] e^{i \bar{k} \bar{x}}
\]
(6)
and
\[
A_i^\mu(\bar{x}) = \int \frac{d^3 k}{(2\pi)^3} \frac{1}{\sqrt{20 g}} [a_i^\mu(\bar{k}) + a_i^\mu(-\bar{k})] e^{i \bar{k} \bar{x}}
\]
(7)
We choose this Hamiltonian due to its form that allow to obtain a operators structure of this type \( q^i \bar{q}^j q^l \bar{q}^l aa \).

### IV. THE FOCK-TANI FORMALISM APPLICATION

Now we are going to apply the Fock-Tani formalism to the microscopic Hamiltonian
\[
H_{FT} = U^{-1} H U
\]
(8)
which gives rise to an effective interaction \( H_{FT} \). To find this Hamiltonian we have to calculate the transformed operators for quarks and gluons by a technique known as the equation of motion technique. The resulting \( H_{FT} \) for the glueball decay \( G \rightarrow mm \) is represented by two diagrams which appear in Fig. (1).

Analyzing these diagrams, of Fig. (1), it is clear that in the first one there is no color conservation. The glueball’s wavefunction \( \Phi \) is written as a product
\[
\Phi^\nu_{\mu} = \chi_{\mu \nu} C^{\mu \nu} \Phi^\nu_{\mu},
\]
(9)
\( \chi_{\mu \nu} \) is the spin contribution, with \( A_{\nu} \equiv \{ S_\alpha, S_\alpha^3 \} \), where \( S_\alpha \) is the glueball’s total spin index and \( S_\alpha^3 \) the index of the spin’s third component; \( C^{\mu \nu} \) is the color component given by \( \frac{1}{\sqrt{3}} \delta^{\mu \nu} \) and the spatial wave-function is
\[
\Phi^\nu_{\mu} = \chi_{\mu \nu} C^{\mu \nu} \Phi^\nu_{\mu} \left( \frac{1}{\alpha^2} \right)^{\frac{1}{2}} e^{-\frac{1}{\alpha^2} (\rho_\mu - \rho_\nu)^2}.
\]
(10)
The expectation value of \( r^2 \) gives a relation between the rms radius \( r_0 \) and \( \beta \) of the form \( \beta = \sqrt{1.5/r_0} \). The meson wave
The function $\Psi$ is similar with parameter $b$ replacing $\beta$. To determine the decay rate, we evaluate the matrix element between the states $|i\rangle = g_{a}^{i}|0\rangle$ and $|f\rangle = m_{5}^{a}m_{4}^{a}|0\rangle$ which is of the form

$$
\langle f | H_{FT} | i \rangle = \delta(\vec{p}_{\alpha} - \vec{p}_{\beta} - \vec{q})h_{fi}.
$$
(11)

The $h_{fi}$ decay amplitude can be combined with a relativistic phase space to give the differential decay rate [10]

$$
\frac{d\Gamma_{\alpha-\beta|f}}{d\Omega} = 2\pi \frac{P_{E_{f}}}{} \left| h_{fi} \right|^{2}
$$
(12)

After several manipulations we obtain the following result

$$
h_{fi} = \frac{8 \alpha_{s}}{3\pi} \frac{1}{\bar{n}d^{2}} \left( \frac{a^{2}}{q^{2} + m_{g}^{2}} \right)^{3/4} \int dq \frac{q^{2}}{\sqrt{q^{2} + m_{g}^{2}}} \times \left( 1 - \frac{q^{2}}{4m_{q}^{2}} - \frac{q^{2}}{4m_{s}^{2}} \right) e^{-\left( \frac{1}{\bar{n}x} + \frac{1}{\bar{n}y} \right) q^{2}}
$$
(13)

Finally one can write the decay amplitude for the $f_{0}$ into two mesons

$$
\Gamma_{f_{0} \rightarrow M_{1}M_{2}} = \frac{512 \alpha_{s}^{2}}{9} \frac{P_{E_{M_{1}}E_{M_{2}}}}{M_{f_{0}}} \left( \frac{1}{\bar{n}d^{2}} \right)^{3/2} I^{2}
$$
(14)

where

$$
I = \int dq \frac{q^{2}}{\sqrt{q^{2} + m_{g}^{2}}} \left( 1 - \frac{q^{2}}{4m_{q}^{2}} - \frac{q^{2}}{4m_{s}^{2}} \right) e^{-\left( \frac{1}{\bar{n}x} + \frac{1}{\bar{n}y} \right) q^{2}}
$$
(15)

with $m_{q}$ the $u$ and $d$ quark mass and $m_{s}$ the mass of the $s$ quark. The decays that are studied are for the following processes $f \rightarrow \pi\pi$, $f \rightarrow KK$ and $f \rightarrow \eta\eta$. The parameters used are $b = 0.34$ GeV, $m_{q} = 0.33$, $m_{q}/m_{s} = 0.6$, $\alpha_{s} = 0.6$. Experimental data is still uncertain for this resonance. There is a large interval for the full width $\Gamma = 200$ to 500 MeV and the studied decay channels are seen, but still with no estimation.

V. CONCLUSIONS

The Fock-Tani formalism is proven appropriate not only for hadron scattering but for decay. The example decay process $f_{0}(1370) \rightarrow \pi\pi$, $KK$ and $\eta\eta$ in the Fock-Tani formalism is studied. The same procedure can be used for other $f_{0}(M)$ and for heavier scalar mesons and compared with similar calculations which include mixtures.

Acknowledgments

The author M.L.L.S. acknowledges support from the Conselho Nacional de Desenvolvimento Científico e Tecnológico - CNPq and D.T.S. acknowledges support from the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - CAPES.