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Analytical Simulations for Spill Point Source Localization Using Inverse Problems in Closed Form

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ABSTRACT

This work delves into the realm of inverse problems in the context of aquatic pollution. Specifically, it presents a novel analytical approach for pinpointing the origins of leaks in underwater pipelines, circumventing the need for data regularization and auxiliary partial differential equation solving. This method streamlines the process, significantly reducing processing time compared to traditional numerical approaches. The approach involves identifying local concentration maxima in a known distribution, tracing streamlines connecting them, and utilizing parametric equations to describe these streamlines. Points of intersection between these parametric curves and the submerged pipe network define potential leak origins. Validation of these origins is achieved through simulations, using the same concentration distribution to compute the source term. This method not only minimizes computational time but also remains effective in scenarios with uncertain experimental data. The results demonstrate that this approach outperforms traditional techniques in terms of processing time, making it particularly advantageous in decision-making processes related to leak containment and environmental recovery in underwater environments. This analytical method offers a valuable tool for efficient and accurate leak source localization, enhancing environmental protection efforts.

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1. Introduction

There is growing concern about the environment in situations of oil spillage. There are several ways to identify the position of the source of a leak – in fact, techniques involving remote sensing (such as satellites [1] or buoy sensor systems) can be employed, as indicated by recent literature [2]. One way that can be employed to seek the solution to this problem is computational modelling, employing some form of computational simulation.

Among the manners to obtain very realistic simulations, an accurate one comes through differential equations modelling analytical models. Nevertheless, the computing procedure takes too long, expending resources and revealing itself to a very hard task, even when symbolic or algebraic optimized systems of computing are employed, such as Maple V or Octave.

Working inverse problems is a common issue for analytical simulations. For instance, it is a direct problem to simulate the oil pouring amplitude in subsea piping, given the rupture point and considering as canonical functions the direction and the force of winds and ocean currents. The inverse scenario constitutes an inverse problem: given the measured origin of the pouring and considering the velocity and the wind and ocean currents functions, one wishes to determine the pouring position. However, the numerical calculation demands a strong computational power, which can make unfeasible the decision process to contain the pouring and recovering the environment.

The treatment of inverse problems for aquatic pollution is commonly based on the analysis of previously regularized pollutant concentration distributions, aiming to localize the source of pouring coordinates. This approach often yields simulation systems for which the source code in the symbolic calculation turns out to be extensive and hard to improve, especially when employing regularization algorithms based on integral operators in treating experimental data for the concentration distribution [3].

These problems can be solved by direct simulation with iterative processes - that is trial and error processes in which are defined source coordinates and calculated concentration distributions generated by solving the Advection-diffusion equations. This procedure is repeatedly performed until the concentration distribution obtained matches the experimental data. It happens unfortunately the simulation systems based on numerical methods - such as finite differences [4,5] and finite elements - present the drawback of excessive time of computation. New analytical formulations have been developed with the shortage of time processing as a goal. These methods constructed upon Lie Symmetries considerations counted for advection and diffusion equations [6–9], despite the shortage of time processing, do not allow for solving inverse problems via direct simulation, due to the high number of simulated scenarios necessary to establish a reasonable agreement between experimental and calculated concentration distributions.

In the proposed work, a new analytical formulation is employed to locate the pouring origin coordinates in submerged pipelines, dismissing the regularization of experimental data as well as the resolution of the partial differential equations [10,11] - hence exhibiting a more efficient way to obtain results than the one through the use of Lie symmetries. To its development, it has been employed the Maple V.

The method consists of obtaining parameterized equations for the streamlines, then the point localization for which the derivative (taken in transversal directions to these streamlines) vanishes. Once the trajectories of the pollutant are parameterized, the trajectories will be traced back in the inverse direction until be found intersection points with the pipe net in the neighbourhood, so identifying the pour point.

2. Research significance

The problem of locating a spill point source – influencing water pollution – is not so simple. There are variables to consider. Furthermore, the slowness of mathematical models that demand time and/or computational capacity can be critical.

The model proposed here does not require high computational capacity, and presents the solution in a matter of minutes – which, in practice, can be considered in real time. In this way, it presents a scenario, which enables decision-making – which can minimize the effects of the leak.

3. Methods

We shall analyze the method in what follows. From a previously known concentration distribution, it is possible to identify the local maxima taken in the transversal direction to the flow line in the hydric body. Once identified the maximal points one selects the flow lines binding them together. Thereafter the parametrical equations describing these same flow lines are adjusted.

Next, the intersection points are identified between the curves described by the parametric equations and the lines related to the submerged pipe lattice. The intersection points draw a region containing the likely coordinates of the pour origin. Aiming to confirm these coordinates by making use of a pollutant propagation model, one proceeds as follows:

i) The very concentration distribution from which the maxima points have been calculated led to the advection-diffusion equation

$$\mu\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}-D\left(\frac{\partial^2 c}{\partial x^2}+\frac{\partial^2 c}{\partial y^2}\right)=Q \tag{1}$$

furnishing the source Q. At this equation, C stands for the pollutant concentration, μ and v are the x and y components of the velocity vector - previously known - and D is the diffusion coefficient.

ii) The source Q reconstructed from the equation above is traced inside the region previously searched to verify if the local maxima belong there. Otherwise, the maxima are searched for in the neighborhood up to be found.

An alternative way to probe the validity of results is furnished by obtaining a set of maxima points for the source Q. Since the problem variables are given by a discrete set of points, the Q may be interpreted as a linear combination of Dirac delta functions each one representing an individual point of pouring. Hence the equation may be treated as an inverse problem in terms of Green's functions. This treatment involves looking for the local minima for source Q. Initially, one assumes those points as a first approximation for the pour origins. Indeed, the operator

$$D(\frac{\partial^2 c}{\partial x^2}+\frac{\partial^2 c}{\partial y^2}) = A \quad (2)$$

converts local maxima for the concentration into minima for the source function Q. Thus, identifying the pour origins is prompted once the minima point for the submerged piping lattice stands for the places with maxima pollutant concentration.

Should the submerged piping be located in deep waters, then the origin points would not fit the source term inflection. The explanation for this arises from the previous pollution diffusion that may eventually make up the concentration distribution at the surface.

To confirm the pollution deposition coordinates, the parametric equations describing the isosurfaces for the current function. The intersections between these functions and the pipes define the starting points for the radioactive tracer emission that can be visually traced back. The very checking over the intersection between the radioactive tracers' trajectories and the null transversal derivative points overflow lines constitutes a qualitative confirmation of the polluting origin.

4. Results

As disposed in Fig.1, in an inverted mode, two pollution sources placed in the coordinates $P_0(x_0,y_0)$ as well as its respective stains arisen in a permanent regime, - i.e., after a long enough time interval such as it is enabled to spread all over the region within the boundaries given by the coordinates x_{min} e x_{max}, y_{min} e y_{max} (with t_{max} = 4h). The stain is described as follows**:**

$$f=\frac{1}{2}\frac{e^{\left(-\frac{\omega^2}{4\,Di\,\varphi}\right)}}{\sqrt{p\,Di\,\varphi}} \quad (3)$$

where Φ stands for the velocity potential and Ψ stands for current function for a non-viscous flow, using curved coordinates for the proposed problem. This approach does not means a corresponding non-viscous flow, rather the coordinates system suits the domain geometry - a characteristic that dismiss a discrete method.

A glance at Fig.1 reveals the existence of two pouring points whose stains bind together into a sole one, making up the origin. Since the surface exhibits only a central line that defines the points of maximum at about 80 percent of the domain extension, a preliminary analysis based only on the extrapolation of the line joining the points of maximum could possibly return a wrong result for the origin of the leaking, due to a sampling problem. Figure 2 shows the isolines presented in Figure 1 seen from above.

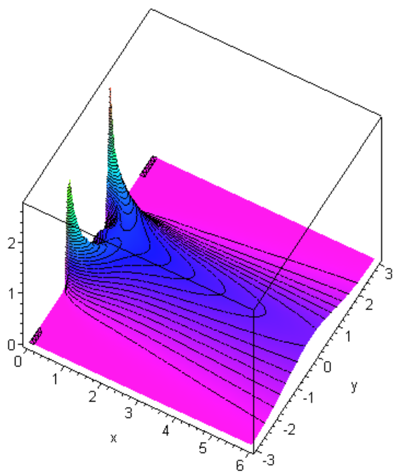

Fig. 1. Concentrations distributions.

However, once the region around the supposed dumping point is delimited, a direct simulation is carried out in order to verify the validity of the result. The scanning is carried out in the transverse direction to the current lines, along the dotted line indicated in Figure 3 within the delimited region – in blue and purple colors.

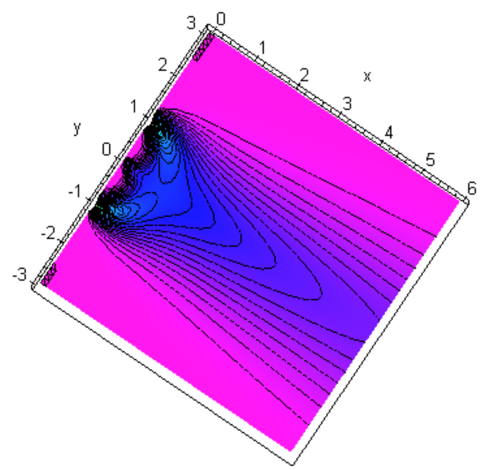

Fig. 2. Concentration isolines.

To each point on the yellow dotted line in Figure 3 the pollutant dump is simulated until the concentration distribution is compatible with the observed stain. In this way, the original points of dumping are easily traced, concluding the searching process.

If there are three or more points, the scanning process will automatically identify each dump point on the dotted line. The following question then arises: since all the found points lie on the dotted line, can some of these points eventually be misplaced on the phi coordinate such as each original dumping point may eventually be upstream or downstream of the estimated point? In spite this could happen often, it suffices to keep in mind the true dumping points at the intersection between the current-function isolines that pass through the estimated points and the pipe lattice that conducts the fluid – as shown in the green dashed line in Figure 3.

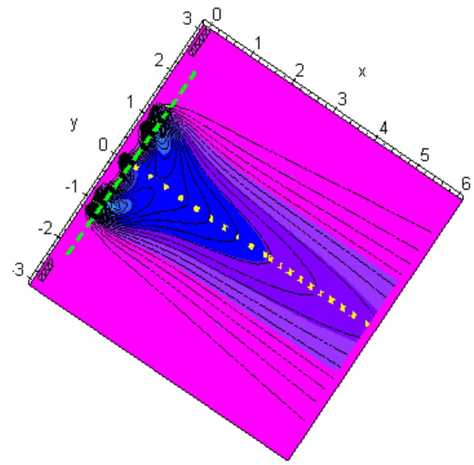

Fig. 3. Localization of the discharge points.

The searching process for these intersections prevents the two-dimensional region containing the first estimated point from be fully scanned in its whole extension – significantly reducing the processing time required to verify the results via direct simulation.

5. Discussion and conclusion

Oil spills are a major environmental problem, and the ability to quickly and accurately detect and locate them is critical to minimizing their impact. Various technologies have been employed to detect oil spills, including satellites, buoys, and sensors. However, even with these technologies, the precise location of the spill can be difficult to determine. This is because the spill can spread over a wide area, and the data collected by the sensors can be noisy or incomplete. The definition of the point of origin of the leak requires large computational resources, if interactive numerical methods are employed. The main contribution of the solution proposed here is to obtain the problem of the location of the dump in an extremely short time, which enables effective decision-making.

The analogous simulation performed through exact solutions based on Lie symmetries yields a concentration distribution identical to that shown in Figure 3. Although the exact solution yields a more accurate result - that is, the points $P_i(x_i, y_i)$ and $P_0(x_0, y_0)$ are closer when this solution is used - the processing time resulting from the use of the proposed solution is appreciably lower. While the solution obtained through Lie symmetries requires about 15 minutes of processing for direct simulation - employing AMD Sempron 3100++ 1.8 GHz processor, 1 GB of RAM - the solution obtained through the proposed method requires approximately 2 seconds of processing. Both results were obtained using the Maple V software.

It is a matter of importance that the reduced processing time for the inverse problem solving constitutes the main advantage from the computational point of view since it requires a high number of direct simulations to obtain the approximate coordinates of the dump origin. In the previous example, about 50 attempts have been performed to trace the point $P_0(x_0, y_0)$ for both solutions tested, so that the total processing time of the inverse problem therefore results in about 1 minute and 40 seconds for the proposed method and 1 hour and 15 minutes for the solution based on Lie symmetries. This difference can be critical in decision-making, minimizing expenses with subsea operations to locate the point in question.

In addition, taking into account the uncertainty of the experimental data with which the calculated concentration distributions must be compared - to identify the local maxima that delineate the current lines -, the accuracy of the obtained approximation becomes an irrelevant aspect for the search of the dump origin since both solutions belong to the same region delimited by the uncertainty.

It is possible to identify the source directly by visual inspection, since it consists in a compact support function, whose dispersion around the center is virtually negligible. In this sense, the proposed method is, in a sense, equivalent to a fast solver to an inverse problem in Greens function.

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Conflicts of interest

The authors declare no conflict of interest.

Authors contribution statement

VGR, JRZ, ES, JAM: Conceptualization; VGR, JRZ, ES, JAM: Formal analysis; VGR, JRZ, ES, JAM, SRS, AMS: Investigation; JRZ, ES, JAM: Methodology; JRZ: Project administration; VGR, ES: Resources; VGR, JRZ, ES: Software; JRZ: Supervision; JAM, VGR: Validation; VGR, SRS, AMS: Visualization; VGR: Roles/Writing – original draft; VGR, JRZ, ES, JAM: Writing – review & editing.

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