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COMPARISON OF DIFFERENT IMPLEMENTATION STRUCTURES FOR EXTENDED KALMAN FILTER

N. P. G. Salau[1]; A. R. Secchi[2]; J. O. Trierweiler[3]

Departamento de Engenharia Química – Universidade Federal do Rio Grande do Sul
Campus Universitário: Centro – CEP: 90040-040– Porto Alegre – RS – Brasil
Telefone: (xx-51) 3316-3528 – Fax: (xx-51) 3316-3277 – Email: [1]ninas@enq.ufrgs.br, [2]arge@enq.ufrgs.br, [3]jorge@enq.ufrgs.br.

ABSTRACT – The Extended Kalman Filter application encloses four important areas, related directly with the advanced control strategies implantation to an industrial process: state estimation, unknown process parameters estimation, dynamic data reconciliation, and data filtering. The main goal of this paper is to evaluate the quality of four different Extended Kalman Filter formulations for these four areas. The Filters are applied in a studied case: the Sextuple Tank-Process. This process presents a high non-linearity degree and a RHP transmission zero, with multivariable gain inversion.

KEYWORDS: Extended Kalman Filters, State and Parameter Estimation, Dynamic Data Reconciliation, Nonlinear Models.

1. INTRODUCTION

It is well established that the Kalman filter is an optimal state estimator for unconstrained linear systems subject to normally distributed state and measurement noise. Many physical systems, however, exhibit nonlinear dynamics and have states subject to hard constraints, such as nonnegative concentrations or pressures. In these cases, Kalman filtering is no longer directly applicable. As a result, many different types of nonlinear state estimators have been proposed; Soroush (1998) provides a review of many of these methods. One reason for the popularity of the EKF is that its application encloses four important areas, related directly with the advanced control strategies implantation to an industrial process: state estimation, unknown process parameters estimation, dynamic data reconciliation, and data filtering. The use of dynamic data reconciliation techniques can considerably reduce the inaccuracy of process data due to measurement errors. In their paper, Abu-el-zeet et al. (2002) have shown that the overall performance of the model-based predictive controller improves considerably when the data is first reconciled prior to being fed to the controller. In their paper, Rao and Rawlings (2002) have considered the formulations of Kalman Filter and MHE to the problem of detecting the location and magnitude of a leak in a wastewater treatment process. While the constrained estimators provide a good estimate of the total losses when there is a leak, MHE and Kalman filter provide poor estimates when there are no leaks. The problem stems from an incorrect model of the process (the true model process has no leaks while the model assumes leaks) and, for solving this problem; they have just suggested a proper strategy where this problem is formulated as a constrained signal-detection problem. However, they had not implemented this proposal strategy.

In their work, Marcon et al. (2002) compared the quality in state estimation using two Kalman Filter formulations: Extended Kalman-Bucy Filter (EKF) and Constrained Extended Kalman Filter (CEKF). The Filters were applied in a quadruple cylindrical tank and compared for state estimation application. In this work, besides the Kalman filters formulations that had been used in Marcon et al. (2002), two others formulations for EKF are used. All the filters performances are evaluated not only for the state estimation, but also for the unknown process parameters estimation and to the dynamic data reconciliation. Furthermore, the comparison is carried out using a sextuple spherical tank process that exhibit higher dynamic nonlinearities, and also has a RHP transmission zero, with multivariable gain inversion.

2. EXTENDED KALMAN FILTER

The Kalman Filter is a set of mathematical equations that provides an efficient computational (recursive) solution of the least-square methods. The filter is very powerful in several aspects: it supports estimations of past, present, and even future states, and it can do so even when the precise nature of the modeled system is unknown (Welch and Bishop, 2000). In this work, four distinct formulations for the Extended Kalman Filter had been evaluated.

The process model is described by the equations below.

$$\frac{dx}{dt} = f(x,u,t) \qquad y = g(x,u,t) \qquad x(0) = x_0 \tag{1}$$

where x, u, y and t are, respectively, vectors of state variables, manipulated variables and measured variables, and time.

2.1 Extended Kalman Filter (EKF)

In this work, a filter known as Continuous-Discrete Extended Kalman-Bucy Filter (Krebs, 1980) is used. The basic equations of this filter can be divided in two stages: I) Prediction and II) Correction.

Prediction:

$$\frac{dx_e}{dt} = f(x_e,u,t), \qquad t_{k-1} \le t \le t_k \tag{2}$$

$$\frac{dP_e}{dt} = F(t)P_e(t) + P_e(t)F(t)^T + Q(t) \tag{3}$$

where $F(t) := \left.\frac{\partial f(x(t),u,t)}{\partial x}\right|_{x = x_e}$

x_e: estimated state variables vector
P_e: estimated error covariance matrix

Correction:

$$K(t_k) = P_e(t_k)H^T(t_k)\left[H(t_k)P_e(t_k)H^T(t_k) + R(t_k)\right]^{-1} \tag{4}$$

$$x_c(t_k) = x_e(t_k) + K(t_k)\{y(t_k) - g(x_e,u,t)\} \tag{5}$$

$$P_c(t_k) = \left[I - K(t_k)H(t_k)\right]P_e(t_k) \times \left[I - K(t_k)H(t_k)\right]^T + K(t_k)RK(t_k)^T \tag{6}$$

where $H(t) := \left.\frac{\partial g(x,u,t)}{\partial x}\right|_{x = x_e}$

K: Kalman Filter gain matrix for $t = t_k$
H: matrix that connects process outputs with state variables
x_c: corrected state variables vector

P_c: corrected error covariance matrix

The implementation of the prediction stage of the EKF consists basically of the integration of differential equations, from the dynamic model, and the differential equations related to the covariance matrix P, which are made between the sampling times. With the values predicted in the sampling time t_k and with the measured values $y(t_k)$, the values estimated for the states, x_e, and for covariance matrix, P_e, can be corrected using the equations listed in the Correction stage. The corrected value of states, x_c, and the corrected value of covariance matrix, P_c, are used as the new initial conditions of differential equations system for the next time interval. The matrices Q and R are, respectively, the covariances of the process and the measurements noises (random errors). They consist, in this way, in the filter adjustment basic parameters and reflect the reliable degree in the modeling and the measurements, respectively.

2.2 Constrained Extended Kalman Filter (CEKF)

CEKF is an alternative state estimator based on optimization, originated from Moving Horizon Estimation (MHE), introduced by (Gesthuisen et al., 2001; Muske and Rawlings, 1994; Muske et al., 1993). MHE advantages (Robertson et al., 1996) over classic estimators, as the Extended Kalman Filter (EKF), are the possibility to consider physical constraints of the states (e.g., concentrations are always greater or equal to zero) and the fact that over the considered horizon no information about the non-linear system is lost. The disadvantage is the necessity of solving a non-linear dynamic program. The CEKF formulation follows from MHE for a horizon length equals to zero. In this way, the CEKF formulation is very similar to the conventional EKF, therefore the horizon of measures considered in the correction stage of both cases are identical. The difference is the fact that the system constrains directly appears in the optimization – filter correction stage.

The process model is described by the equations below:

$$\begin{aligned} \dot{x} &= f(x,u) + \xi(t) \\ x(0) &= x_0 + \xi_0 \\ y(t) &= h(x) + \omega(t) \end{aligned} \qquad (7)$$

where x, u, y, ξ and ω are, respectively, vectors of state variables, manipulated variables, measured variables, modeling and measurement errors.

The basic equations of CEKF can be divided, like in the EKF, in two stages: I) Prediction and II) Correction. In prediction stage it is performed only the integration of differential equations from the dynamic model. As results, it is obtained the estimated states and/or parameters (x_e) of the system. In contrast to EKF, in this stage the integration of covariance matrix, P, is not carried through with the differential equations of the system, although this could also be made in another EKF formulation (not presented here due to lack of space).

<u>Prediction:</u>

$$\frac{dx_e}{dt} = f(x_e, u, t), \qquad t_{k-1} \le t \le t_k \qquad (8)$$

In the correction stage is solved the following optimization problem:

<u>Correction:</u>

$$\min_{\hat{\xi}_{k-1,k},\ldots,\hat{\xi}_{k-1,k}} \Psi_k = \hat{\xi}_{k-1,k}^T P_k^{-1} \hat{\xi}_{k-1,k} + \hat{\omega}_{k,k}^T R^{-1} \hat{\omega}_{k,k} \qquad (9)$$

$$\hat{x}_{k,k} = \hat{x}_{k,k-1} + \hat{\xi}_{k-1,k} \qquad \hat{x}_{min} \le \hat{x}_{k,k} \le \hat{x}_{máx}$$
$$y_k = h(\hat{x}_{k,k}) + \hat{\omega}_{k,k} \qquad \hat{\xi}_{min} \le \hat{\xi}_{k-1,k} \le \hat{\xi}_{máx} \qquad \hat{\omega}_{min} \le \hat{\omega}_{k,k} \le \hat{\omega}_{máx} \quad (10)$$

The aim of the optimization procedure is to achieve the correction of the states (x_c) calculated during the Prediction stage, so that the dynamic model trajectory is as closest as possible to the real process behavior. This procedure is done for each time interval, from t_{k-1} to t_k. The corrected states (x_c) are calculated from x_e and the optimization results. Those values are used as the new initial states in prediction stage when $t = t_k$.

$$x_c(t_k) = x_e(t_k) + \xi(t_k) \quad (11)$$

The discrete covariance error matrix is updated using the equation below:

$$P_e(t_k) = Q_d + F_d P(t_{k-1}) F_d^T - F_d P(t_{k-1}) H(t_k)^T \begin{bmatrix} H(t_k) P(t_{k-1}) H(t_k)^T \\ + R_d \end{bmatrix}^{-1} H(t_k) P(t_{k-1}) F_d^T \quad (12)$$

where, F_d is the Jacobian matrix for the discrete system. According to Brown and Hwang (1996), to make the transition from the discrete to continuous case, the relations below between Q_d and R_d and the corresponding Q and R for a small step size Δt were used.

$$Q_d = Q\Delta t \quad (13)$$

$$R_d = \frac{R}{\Delta t} \quad (14)$$

2.4 Discrete Extended Kalman Filter (DEKF)

The basic difference between the DEKF and the EKF is that this formulation uses only the discrete form. In this way, the equations were modified.

Prediction:

$$\frac{dx_e}{dt} = f(x_e, u, t), \qquad t_{k-1} \le t \le t_k \quad (15)$$

$$P_e(t_k) = F_d P_c(t_{k-1}) F_d^T - Q_d \quad (16)$$

Correction:

$$K(t_k) = P_e(t_k) H^T(t_k) \begin{bmatrix} H(t_k) P_e(t_k) H^T(t_k) \\ + R_d(t_k) \end{bmatrix}^{-1} \quad (17)$$

$$x_c(t_k) = x_e(t_k) + K(t_k)\{y(t_k) - g(x_e, u, t)\} \quad (18)$$

$$P_c(t_k) = [I - K(t_k) H(t_k)] P_e(t_k) \quad (19)$$

2.3 MODIFIED DISCRETE EXTENDED KALMAN FILTER (MDEKF)

In the MDEKF, the covariance error matrix, P, is not corrected. During the prediction stage, like in CEKF, the integration of matrix P is not carried through with the differential equations of the system. This matrix is estimated and updated in discrete form, using the Equation 12 of CEKF.

Prediction:

$$\frac{dx_e}{dt} = f(x_e, u, t), \qquad t_{k-1} \le t \le t_k \quad (20)$$

$$P_e(t_k) = Q_d + F_d P(t_{k-1}) F_d^T - F_d P(t_{k-1}) H(t_k)^T \begin{bmatrix} H(t_k) P(t_{k-1}) H(t_k)^T \\ + R_d \end{bmatrix}^{-1} H(t_k) P(t_{k-1}) F_d^T \quad (21)$$

Correction:

$$K(t_k) = P_e(t_k) H^T(t_k) \begin{bmatrix} H(t_k) P_e(t_k) H^T(t_k) \\ + R_d(t_k) \end{bmatrix}^{-1} \quad (22)$$

$$x_c(t_k) = x_e(t_k) + K(t_k)\{y(t_k) - g(x_e, u, t)\} \quad (23)$$

3. PROCESS MODEL

The different filter formulations presented in the previous section have been implemented in the Sextuple-Tank Process Model (Escobar, 2006) depicted in Figure 1.

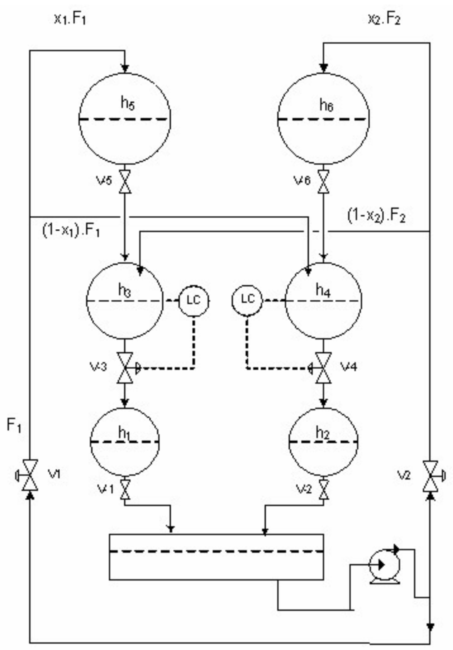

Figure 1 – The Sextuple-Tank Process

The proposed unit consists of six interacting spherical tanks with different diameters D_i. The objective consists in controlling the levels of the lower tanks (h_1 and h_2), using as manipulated variables the flow rates (F_1 and F_2) and the valve distribution flow factors of these flow rates ($0 \le x_1 \le 1$, $0 \le x_2 \le 1$) that distribute the total feed among the tanks 3, 4, 5 and 6. The complemental flow rates feed the intermediary tank on the respective opposite side. The levels of the tanks 3 and 4 are controlled by means of SISO PI controllers around the set-points given by h_{3s} and h_{4s}. The manipulated variable in each loop is the discharge coefficients R_i of the respective valve. Under these assumptions, the system can be described by the following equations:

$$\begin{aligned}
&A_5(h_5)\frac{dh_5}{dt} = x_1 \cdot F_1 - R_5\sqrt{h_5} \\
&A_3(h_3)\frac{dh_3}{dt} = R_5\sqrt{h_5} + (1-x_2)\cdot F_2 - R_3\sqrt{h_3} \\
&A_1(h_1)\frac{dh_1}{dt} = R_3\sqrt{h_3} - R_1\sqrt{h_1} \\
&A_6(h_6)\frac{dh_6}{dt} = x_2 \cdot F_2 - R_6\sqrt{h_6} \\
&A_4(h_4)\frac{dh_4}{dt} = (1-x_1)\cdot F_1 + R_6\sqrt{h_6} - R_4\sqrt{h_4} \\
&A_2(h_2)\frac{dh_2}{dt} = R_4\sqrt{h_4} - R_2\sqrt{h_2} \\
&\frac{dI_3}{dt} = \frac{1}{T_{I3}}(h_{3s} - h_3) \\
&\frac{dI_4}{dt} = \frac{1}{T_{I4}}(h_{4s} - h_4) \\
&R_3 = R_{3s} + K_{P3}(h_{3s} - h_3) + K_{P3}I_3 \\
&R_4 = R_{4s} + K_{P4}(h_{4s} - h_4) + K_{P4}I_4 \\
&R_{3s} = \frac{x_{1s}\cdot F_{1s} + (1-x_{2s})\cdot F_{2s}}{\sqrt{h_{3s}}} \\
&R_{4s} = \frac{x_{2s}\cdot F_{2s} + (1-x_{1s})\cdot F_{1s}}{\sqrt{h_{4s}}} \\
&A_i(h_i) = \pi(D_i h_i - h_i^2) \quad i = 1,2,3,4,5,6
\end{aligned} \quad (24)$$

where A_i, h_i, D_i, R_i are, respectively, cross section area, level, diameter, outlet flow coefficient of tank i; h_{3s} and h_{4s} are the intermediary tanks (3 and 4) level set points; R_{3s} and R_{4s} are the steady state outlet flow coefficient of intermediary tanks (3 and 4); F_1 and F_2 are the manipulated inlet flow rates; x_1 and x_2 are the valve distribution flow factors;

K_{P3} and K_{P4} are the proportional gain of the level PI Controllers for the intermediary tanks (3 and 4); T_{i3} and T_{i4} are the integral time constant of the level PI Controllers for the intermediary tanks (3 and 4).

4. KALMAN FILTERS ALGORITHMS

The Kalman filters formulations were implemented in MATLAB and applied in the process dynamic model presented in Section 3 using SIMULINK. The goal is the estimation of intermediary tanks (3 and 4) level, since these levels are the controlled variables of this process. The estimation of superior tanks (5 and 6) levels also is verified. In this case, the inferior tanks (1 and 2) levels are measured directly in the process, and the filtration of these variables is carried out by the Kalman filters. The system initial condition is an operating point that presents a minimum phase behavior ($1<x_1+x_2<2$). However, due to step changes in the valve distribution flow factors during the process simulation the system moves to an operating region presenting non-minimum phase behavior ($1<x_1+x_2<0$).

Three cases had been simulated to evaluate the quality of prediction and robustness for all the filters formulations, considering the afore-mentioned important areas related directly with the advanced control strategies implantation to an industrial process: state estimation, unknown process parameters estimation, dynamic data reconciliation, and data filtering. In all studied cases, the following conditions and settings were imposed:

- A PRBS (Pseudo-Random Binary Signal) is used in the manipulated inlet flow rates (F1 and F2).
- A step change is carried out in the valve distribution flow factors in t = 50 min. The values of these variable becomes: x_1=0.4 and x_2=0.5, corresponding to a minimal-phase operating region.
- The filter design parameter Q was considered as a diagonal matrix (the errors in the state variables are uncorrelated): $Q = I_{n\,x\,n}$, where n is the number of states.
- The filter design parameter R was considered as a diagonal matrix (the errors in the measured variables are uncorrelated) with an uncertainty in the measurements: $R = (10) I_{m\,x\,m}$, where m is the number of measured variables.
- In CEKF formulations, constrains were not imposed in the state variables.
- The measured variables are the inferior tanks (1 and 2) levels. These variables are generated from the model simulation with a band-limited white noise addition.
- All the evaluated simulations were made in 100 minutes.

4.1 Case 1: Parameters Estimation

In this case, errors of 10% in the outlet flow coefficients of tank 1 (R_1) and tank 2 (R_2) were considered. The filters performances are evaluated using an error criterion: Integral Time Absolute Error (ITAE). In Table 1 is shown the filters performance, obtained in the simulations.

Table 1 – Filters Performance to Case 1.

	ITAE			
	H_1	H_2	R_1	R_2
EKF	3050.2	3505.9	-	-
CEKF	6543.9	7553.6	-	-
DEKF	6543.7	7553.2	-	-
MDEKF	6544.3	7554.1	-	-
EKFest	77.5	88.1	3069.1	3639.8
CEKFest	36.2	41.0	4206.9	5097.9
DEKFest	36.3	41.1	4211.2	5104.8
MDEKFest	36.1	40.8	4106.6	4964.4

According to Table 1, the EKF presented the best performance when only state estimation was applied. Furthermore, when unknown process parameters were also estimated (subscript "est" in Table 1), the EKF also presented the best performance with a faster convergence to the parameters correct values and, due to a higher error in the simulation beginning, the EKF presented an ITAE higher than the other filters in state estimation (H_1 e H_2). The three other filters presented a similar performance in both situations.

4.2 Case 2: Dynamic Data Reconciliation

In this case, supposing a leak in the process, it was considered an error of 1000 $cm^3.min^{-1}$ in the manipulated inlet flow rate 1 (F_1). The filters performances are shown in Table 2.

Table 2 – Filters Performance to Case 2.

	ITAE			
	H_1	H_2	ΔF_1	ΔF_2
EKF	2158.5	2491.3	-	-
CEKF	4833.7	5586.1	-	-
DEKF	4833.2	5585.8	-	-
MDEKF	4833.7	5586.2	-	-
EKFrec	152.9	119.7	642366	498180
CEKFrec	362.7	342.6	1109591	823808
DEKFrec	362.5	342.6	1109411	823538
MDEKFrec	361.6	342.1	1107404	822985

According to Table 2, as in the case 1, the EKF presented the best performance when only state estimation was applied. Moreover, when applying dynamic data reconciliation, the EKF also presented the best performance, providing a good estimate of the total losses when there is a leak. Besides, like in case 1, the three other filters presented a similar performance in both situations.

4.3 Case 3: Parameters Estimation and Dynamic Data Reconciliation

In this case, supposing a leak in the process similar to case 2, errors of 10% in the outlet flow coefficients of tank 1 (R_1) and tank 2 (R_2) were also considered. The filters performances are shown in Table 3.

Table 3 – Filters Performance to Case 3.

	ITAE			
	H_1	H_2	ΔF_1	R_1
EKF	1023.6	1137.5	-	-
CEKF	2185.8	2489.0	-	-
DEKF	2185.8	2488.8	-	-
MDEKF	2185.9	2489.3	-	-
EKFrec	284.1	184.2	609617.0	2922953.7
CEKFrec	774.2	556.0	756488.6	3749536.5
DEKFrec	773.1	556.1	755276.4	3745489.4
MDEKFrec	773.0	556.8	754914.6	3744539.4

Again, according to Table 3, as in the previous cases, the EKF presented the best performance when only state estimation was applied. Furthermore, when unknown process parameters estimation and dynamic data reconciliation were applied, all the formulations presented a bad performance to estimate the correct values of the parameter (outlet flow coefficients of tank 1 and 2) and the leak (flows variations 1 and 2). Even so, the EKF has presented the best performance.

For all evaluated cases, the estimation of intermediary tanks (3 and 4) level has presented a good quality and, thereby, their simulations results were not shown. The situations where different initial conditions were used in the filters were also evaluated and all the filters formulations presented a good performance in state estimation, with a fast converge to the corrected values of the state variables. In order to simulate a dynamic uncertainty in the process modeling, it was used a cylindrical tanks model in the filters, instead of spherical tanks used in the process, and all the formulations estimated the states with very small errors and, therefore, their results also were not shown here.

5. CONCLUSIONS

In the studied cases, all filters formulations presented a good performance in data filtering. For the situations where only state estimations were applied, the EKF presented the best performance when only state estimation was applied. It was shown that when the unknown process parameters estimation and/or dynamic data reconciliation were implemented together with the state estimation, the EKF also presented the best performance. However, in these case studies no constrains were imposed to the state variables, which could improve the relative performance of CEKF against the others evaluated filters. Moreover, the effects of the filters design parameters (Q and R matrices) and the state observability analysis were not evaluated in this work. In future studies, these possible improvements in filters performances will be considered.

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