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ARTICLES

FUZZY-BASED DECISION SUPPORT SYSTEM FOR PRIVATE BANKING

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ABSTRACT. Private banking is one of the most profitable services offered by financial institutions in Brazil. In this service, a client should consider the suitability of a range of banking product, during his or her life based on dynamic objectives. It is a very complex service that involves regulation from governmental agencies, financial institutions' interests, and clients' objectives. It is not uncommon for these aspects to be conflicting, hindering the decision-making process and leading to unwanted decisions. This paper describes the development of a decision support system (DSS) for private banking, called OptPrivate, that integrates the suitability assessment with the portfolio selection, enabling the investor to choose a portfolio based on several and conflicting objectives. The DSS considers the investment selection problem in two interconnected subproblems: suitability and capital allocation. In the former, the DSS is based on an integrated modeling approach that considers legal aspects and investors' preferences throughout a fuzzy multiple-attribute decision-making approach (MADM), called FTOPSIS-Class. In the latter, a fuzzy multi-objective optimization model uses the sorting results from the previous step to define a portfolio, simultaneously considering risk, return, and the investor's profile. To facilitate the DSS application, linguistic variables are used in several aspects of the decision-making process, including risk exposure. The DSS was validated using field tests at a well-respected private bank in Brazil. The DSS recommends more suitable portfolios, in line with the investor's profile, with greater profitability and less volatility than those recommended by the financial analysts for all test cases.

Keywords: decision support systems, portfolio selection, fuzzy logic.

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1 INTRODUCTION

Investment decision-making is becoming more complex since additional criteria exist beyond mean-variance (Aouni et al., 2018; de Almeida-Filho et al., 2020). Thus, several models have been developed to enhance the portfolio selection problem considering fuzzy approaches (Rahiminezhad Galankashi et al., 2020; Ferreira et al., 2018), risk measures (Kaucic et al., 2019; Righi & Ceretta, 2016; Righi, 2019; Righi & Borenstein, 2018). The effects of the financial crisis of 2008 on the individual investor are discussed in the literature (Ferreira et al., 2018; Subhash & Enke, 2019; de Almeida-Filho et al., 2020), requiring financial services to be innovative for dealing with investor's needs for a risk-informed decision process.

The primary targets of the private banking sector are high-income clients whose personalized service is essential for banks to obtain competitive advantage. Due to the amounts commonly involved in these transactions, as well as the complexities and uncertainties of financial markets, many investors demand rapid responses and security from their account managers. At the same time, these professionals need to be well versed in numerous products and their peculiarities, in addition to having the necessary skills to match each product to their clients' profile, since the sector is highly regulated. The Securities and Exchange Commission (SEC) of the United States and the Brazilian Securities Commission (CVM) are examples of regulatory agencies. These institutions usually establish guidelines that regulate private banking investments to ensure that (i) the products, operations or services offered meet the investor's goals; (ii) the financial situation of the investor is compatible with the products, operations or services acquired; and (iii) the client understands the risks of each transaction performed.

In this context, banks are responsible for following the guidelines imposed by the regulatory agencies to avoid being penalized, including establishing and formalizing the risk for each client. However, the processes and instruments used by banks are not standardized, nor is there a consensus on the risk levels inherent to each procedure. In Brazil, for example, financial institutions are normally responsible for categorizing products based on market and/or credit risk levels (when applicable), in line with Central Bank guidelines. Every institution must associate investors' risk profile with the possible investments. For example, CVM recently determined that an investor must be reimbursed around USD 31,000 for losses resulting from an investment that was not compatible with the individual's profile. In this case, the financial institution had not verified the client's investment portfolio risk profile (suitability) before making the transaction (Schincariol, 2018).

Another important aspect related to private banking is the issue of conflict of interest (Krausz & Paroush, 2002). Since the investor generally depends on financial advice to make an allocation decision, distorted instructions often occur as a result of conflicts of interest and asymmetric information between financial advisors and investors. Financial analysts may suggest investment funds that charge higher administration or performance fees, or that are more in line with the bank's commercial strategies, rather than what is best for the customer, even when complying with regulatory guidelines. This aspect has increasingly attracted the attention of the media (Siedle, 2010) and financial regulatory bodies (Crockett et al., 2003).

Given all the peculiarities presented, there is a consensus in the literature that financial decisions need flexibility, customizable platforms and the ability to consider several factors, variables, requirements, and restrictions, in order to provide clients with the best advice (Zopounidis & Doumpos, 2013; Gonzalez-Carrasco et al., 2012), making it an opportune environment to develop and use decision support systems (DSSs). Indeed, since the modern portfolio theory proposed by Markowitz (1952), there has been a growing interest in optimizing investment portfolios, extensively covered in the literature (Fabozzi et al., 2007; Metaxiotis & Liagkouras, 2012; Mansini et al., 2014).

Since then, several DSSs have been developed for financial investments (Weber, 2008), based mainly on portfolio optimization models. The models were initially developed considering the classic risk-return dichotomy, restricting the analysis to two criteria (Prigent, 2007). More recently, some studies extended the Markowitz model to consider cardinality restrictions (Chang et al., 2000), using different risk measures (Chang et al., 2009). Currently, multi-objective optimization (MOO) is a very important alternative in formulating and solving the problem (Steuer, 2013). Zopounidis & Doumpos (2013) present an interesting review on applying MOO approaches to the portfolio selection problem. Recognizing that several items of the portfolio selection process are vague prompted the use of fuzzy sets in some models, Bermúdez et al. (2012) combined MOO and fuzzy sets to model portfolio selection with cardinality restrictions. Perez & Gomez (2016) proposed a binary nonlinear model that uses MOO with fuzzy parameters, while Calvo et al. (2016) used fuzzy sets to represent a non-financial criterion in a bi-objective risk-return problem.

However, analysis of the literature conducted shows that applying optimization concepts, MOO, and DSS in the private banking sector remains scarce (Ferreira et al., 2018). The literature focuses on studying the need of financial advisors for wealth management (Cao et al., 2017) or clients' satisfaction with the service provided by financial institutions (Mihelis et al., 2001). One of the first studies dealing with the application of computational systems in private banking was conducted by Gonzalez-Carrasco et al. (2012) to determine the investor's risk profile and recommend the most suitable investment portfolio, given the investor's characteristics (sex, marital status and income, among others) and socio psychological profile. Although this paper uses semantic technologies and fuzzy logic, the system does not consider regulatory aspects or real features generally associated with this problem, such as the investor's budget, incompatibility between assets and the possible compromise between the client's conflicting goals and the attributes of the investments suggested by private banking. Another exception is the integrated decision analysis framework to support portfolio selection in private banking developed by Ferreira et al. (2018). The framework integrates fuzzy multi-attribute decision making (MADM) and fuzzy multi-objective optimization (FMOLP). The fuzzy MADM component makes it possible to measure the suitability for each type of asset class available considering a number of criteria (return, risk, liquidity, investment objectives, etc.) that are in keeping with investor's profiles (conservative, moderate, bold, or aggressive). The FMOLP model obtains an optimal proportion for each asset in the investors' portfolio, considering several objectives and real features as constraints. However, the framework focuses on investors' risk profiles, rather on specific clients, which restricts its use in the real world. Moreover, the framework is quite complex to be used without a proper computational environment.

The primary aim of this article is to present a fuzzy-based DSS developed to support investors and analysts in selecting the best private banking investment portfolio. The DSS focuses entirely on clients, defining customized portfolios, rather in investor profiles as in Ferreira et al. (2018). One of the objectives of the DSS proposed here is to offer an environment that increases bank/client interaction and improves their relationship. Thus, banks can encourage the participation of investors in the decision-making process, creating a collaborative environment in which they are better able to understand the trade-offs between the different parameters used in investment portfolios, thereby minimizing conflict of interest effects. The DSS also makes it possible to select the best portfolio in terms of profitability, risks inherent to the financial product and client profile in an explicit, dynamic and flexible manner, addressing the following two key points: (i) determining client preference in relation to the products, services and operations offered by the bank, considering a series of relevant attributes; (ii) support for determining investor objectives, restrictions and preferences in line with the regulatory rules imposed, allowing their risk profile to adhere to the investments offered by financial institutions. The DSS proposed was assessed using a database of investors from an investment bank in Brazil, with different investment objectives and risk profiles. The results were positive for the DSS when compared to the portfolios selected only by financial analysts.

This paper makes the following contributions: (i) to reformulate the framework proposed by Ferreira et al. (2018) to focus on the client rather on the client's risk profile, and on the involved regulatory aspects; (ii) to develop a DSS that automates and customizes the new framework, offering a friendly reliable environment in order to better identify the best investment portfolio for each client according to their goals and aspirations and based on the current regulation; and (iii) to validate the developed computational systems, evaluating its performance and acceptance in a real setting.

The outline of the paper is as follows. Section 2 describes the key elements to be considered in the decision-making process in private banking, focusing in the Brazilian context. Section 4 discusses the DSS developed, describing the main components of the computational system. The field tests carried out in a well-known Brazilian bank with a respected wealth management division are described in Section 5, which also analyzes results from the test fields. Finally, concluding remarks and future research are presented in Section 6.

2 PROBLEM STATEMENT

Suitability is an issue in world-wide banking, and it is well discussed in CFA Standards of Practice Handbook. Each country has its specific bodies to control and regulate banking activities.

In Brazil this is verified at Chapter XII of the ANBIMA (Brazilian Association of Financial Market and Capital Entities) Code for Investment Fund Regulation and Better Practices entitled "Duty to Verify the Suitability of Recommended Investments" (see ANBIMA, 2016) addresses the non-transferable responsibility of financial institutions to conduct the suitability process of their clients. The legislation underscores the obligation of verifying the suitability of products, services and operations for the client's risk profile, prohibiting the recommendation of products without first determining the client's risk profile. Also according to current regulation, the following conditions should be verified:

1. Whether the product, service or operation is suitable for the client's investment goals, by determining: (i) The period in which the client wants to maintain the investment; (ii) The client's preferences in terms of risk; and (iii) The purposes of the investment.

2. Whether the client's financial situation is compatible with the product, service or operation, by assessing: (i) The value of the client's average declared income; (ii) The value and assets that make up the client's total worth; and (iii) The future resource needs declared by the client.

3. Whether the client has sufficient knowledge to understand the risks related to the product, service or operation, by analyzing: (i) The types of products, services and operations the client is familiar with; (ii) The nature, volume and frequency of the client's previous operations in the stock market, as well as the period in which these operations occurred; and (iii) The client's academic qualifications and professional experience.

Thus, before suggesting products, services, and operations, the bank is obliged to apply a questionnaire to each client to allocate them to predefined category depending on matters such as risk, the objective of the investment, grace period, and the client's knowledge of the market. Based on the answers given, the bank classifies the client into one of the categories, defined as investor profiles. Four profiles are defined by financial institutions, as follows:

Conservative Prioritizes security as the key aspect in investments. In this profile, it is advisable to keep a higher percentage of investments in low-risk products.

Moderate Emphasizes investment security, but also opts for products that can deliver greater long-term gains.

Bold Investors in this class look for possibilities of higher gains and therefore take greater risks. However, even for bolder strategies, it is advisable to keep part of the resources in lower risk products.

Aggressive The investor seeks a long-term return on investment and, thus, adapts his/her portfolio to short-term oscillations in the market. An appreciable portion of their investments are allocated to new sectors.

In addition to establishing the need to determine a client risk profile, the current legislation specifies the need to analyze and classify the categories of products the banking institutions offer, identifying the characteristics that may affect their client profile suitability, considering the following items: (i) the risks associated with the product and its underlying assets; (ii) the profile of service providers associated with the product; (iii) the existence of guarantees; and (iv) the grace periods.

The risks associated with the products are related to credit, market, and liquidity risks. Credit risk is the possibility of loss due to a borrower's defaulting on a loan or not meeting contractual obligations. Market risk is the potential loss of value in assets and liabilities due to changes in market variables, and liquidity risk is the risk that a business will have insufficient funds to meet its financial commitments in a timely manner. Investment funds in Brazil are generally classified into five categories in increasing order of risk, as follows:

Very Low-risk Funds Funds that have very low market risk, measured by the price variation of post-fixed bonds, that is, have very low credit risk.

Low-risk Funds Funds that have low market risk, measured by the price variation of post-fixed bonds, that is, have low credit risk.

Moderate-risk Funds Funds that have medium market risk, measured by the price variation of fixed-rate securities, pre-fixed and associated with inflation indices. They are also attributed this classification because they may contain medium credit risk

High-risk Funds Funds that may pose high market risk, measured by the price variation of fixed-rate, pre-fixed bonds, linked to inflation indices, foreign currencies, stock prices and derivative prices. They are also attributed this classification because they may contain high credit risk.

Very High-risk Funds Funds that may pose a very high market risk, measured by the price variation of fixed-rate, pre-fixed bonds, linked to inflation indices, foreign currencies, stock prices and derivative prices. This category includes external mirror funds and may contain high credit risk.

Thus, it is essential to consider adherence between the investor's risk profiles and the portfolio an important requirement to be represented here, given the regulatory aspects that are part of the private banking in any country following international standards.

The investment portfolio selection problem in the private banking sector can be divided into two interconnected subproblems: (i) measuring the suitability of each investment alternative for all the available risk profiles; and (ii) allocating resources in shares while taking a series of objectives into account, including the suitability of the shares to the investor's profile, investment return and risk, as well as the real restrictions applicable to each case. The DSS developed here presents a computer system that makes it possible to integrate these two subproblems, focused on the effective use of Operations Research and Artificial Intelligence based models to support the decision process and the interaction between investors and financial analysts. The regulatory aspects and the consideration of each individual client's needs and aspirations differentiate this DSS from other computational systems available in the area of finance (Weber, 2008; Alic et al., 2012), and in the specific private banking sector (Gonzalez-Carrasco et al., 2012; Ferreira et al., 2018).

3 METHOD

Given the lack of described methodologies for optimization portfolio for the private banking context, we based the development of our solving method on a related problem, the investment projects selection, a typical MADM problem (Fiala, 2018). The two problems present several analogous characteristics in the decision making process, such as dynamic multiple and conflicting objectives, normally of difficult measurements, uncertainties arising from incomplete and imprecise information, budget limitations, and regulation from governmental agencies. In situations where decision-makers (DMs) face many problems with incomplete, unqualifiable, vague, and unquantifiable information, fuzzy set theory (FST) was introduced on MADM (Zyoud et al., 2016). Several MADM methods and techniques, including hybrid approaches, were employed to help DMs to design project portfolios investment projects (Mohammed, 2021).

However, MADM techniques and methods have some shortcomings. Greiner et al. (2003) have identified that MADM techniques are not adequate to solve problems that involve the optimization of resources and interdependency between alternatives, requiring the application of complementary analysis techniques. MADM techniques are capable of determining priority measures for each of the alternatives under consideration. They are not, however, capable of determining the optimal mix of those development projects in light of a set of resource constraints or other constraints. These constraints usually impose a combinatorial nature to the problem. One of the most frequently used approaches found in the related literature to address these situations are characterized by a two-stage process (Kearns, 2004; Mavrotas et al., 2008). In the first stage, the relative benefits of each alternative are calculated by determining an individual score, which enables the ordering of alternatives through the application of methods of multiple-criteria analysis, such as AHP and TOPSIS. In the second stage, a mathematical model is built to optimize the overall value of the portfolio using individual scores calculated in the previous stage, including restrictions such as the factors related to interdependencies between alternatives.

The process was divided into three interconnected phases (see Fig. 1, reducing the complexity of the entire problem by solving smaller subproblems one at a time. The first phase comprises the definition of the objectives of the client, his/her risk profile, using the questionnaire of suitability in Appendix A, and possible investment alternatives. The portfolio selection problem is then broken down into two connected subproblems, as proposed by Ferreira et al. (2018): (i) to measure the suitability of each investment alternative for each investor profile; (ii) to allocate the resources available to banking products or services, taking into account several objectives, including the suitability of the asset for the investor's profile, and restrictions. The former was devised as a multi-attribute decision-making (MADM) problem, in which the main objective is to define the estimated suitability of an investment alternative $i \in I$ for each client $c \in C$, considering several different attributes, such as the investor's objectives, investment risk, and their knowledge of the product or service. The latter was modeled as a fuzzy multi-objective portfolio selection that consists of finding the proportions of various assets to be held in a portfolio that achieve a good compromise solution for the established objectives, subject to real life features, which are represented as constraints.

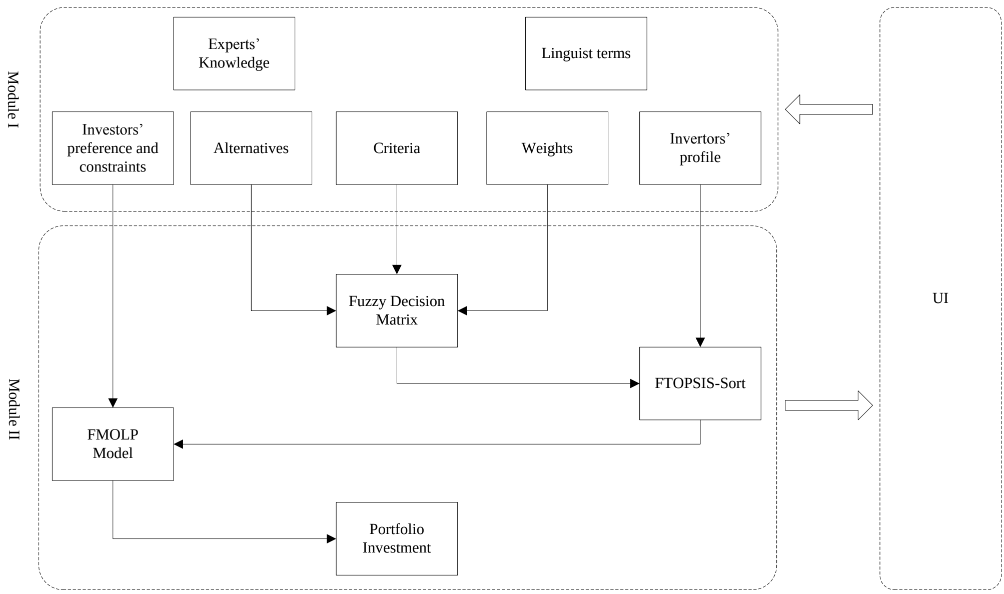

Figure 1 – Modeling approach.

4 THE DECISION SUPPORT SYSTEM

OptPrivate is a prototype DSS for portfolio optimization in private banking. The DSS combines several methods from operations research/management science and artificial intelligence into one integrated software system that provides a friendly environment for the highly regulated investment process in private banking. The main objective of the computer system is to select an optimal portfolio for a particular investor, considering his/her objectives, risk profiles, budget and preferential constraints. It is important to note that the system does not provide a unique optimal portfolio, independently of the investor. The DSS was developed on the assumption that the best solution not only complies with all legislation and constraints, but it is also preferred, understood, accepted, supported and implemented with confidence by the investor/bank manager.

In addition to this main purpose, the system has the following generic objectives: (i) to help investors without an in-depth knowledge of finance to understand the main concepts related to private banking, such as regulation, risks, and risk profiles; (ii) to consider and evaluate the impact of several criteria on the investor's profiles; (iii) to provide a common environment for bank managers and investors to use in the investment process; and (iv) to allow clients to examine the sensitivity of the portfolios to changes in the investor's objectives. The DSS proposed in this paper follows the organization of a model-driven DSS, which uses data and parameters provided by users to assist decision makers in analyzing a specific decision-making process, and a friendly interface to promote and facilitate interaction between users and the system.

The DSS proposed was implemented in Java© language, given the numerous facilities and resources that it offers. The user interface allows the use of different decision models that are part of the new framework for uninitiated users, in addition to performing all communication needed with specialized mathematical software to solve the optimization models and databases to store parameters and personal information. Fig. 2 presents the DSS architecture and its main components.

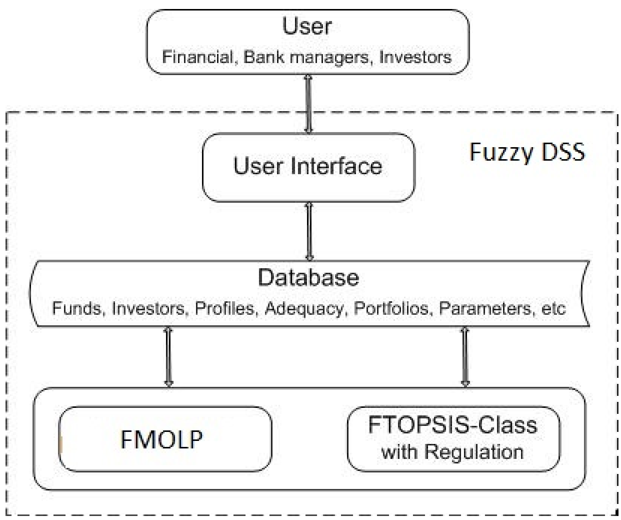

Figure 2 – Architecture of the proposed DSS.

4.1 Model Subsystem

4.1.1 FTOPSIS-Class with Regulation

The main objective of this method in the DSS is to measure the adequacy of an alternative of investment to an specific investor, considering both his/her risk profile obtained by the application of the Questionnaire of Suitability (see Appendix A), and the investment categorization defined by the bank. We modeled this problem as a Multiple Criteria Nominal Classification (MCNC) problem (Chen, Ye, 2006).

The main scope of this model within the DSS framework is to act in the following two activities:: (i) to assign the investment options into pre-defined homogeneous groups, specified by multiple characteristics (clients' profiles), performed by the financial analyst; and (ii) the categorization of the same set of investments directly by the client. The former is used for the regulation process. The coefficient defined by the client cannot be very different from that defined by the investor to avoid the investment's inadequacy to the client's profile.

Although several MADM methods have been developed that can be applied to an MCNC (Zopounidis & Doumpos, 2002), we modified the Fuzzy-TOPSIS as introduced by Chen (2000) to perform classification, following an intuitive idea. Let m be the cardinality of investment set I, and n the number of evaluation criteria. The score of an alternative $i \in I$ to profile $p \in P$ is the closeness coefficient, CC_i^p which is computed based on the distances of the alternative i to the positive ideal solution of profile $p\,(A_p^* = [\tilde{v}_{pi}^*]_m)$, and to the negative ideal solution of profile $p\,(A_p^- = [\tilde{v}_{pi}^-]_m)$. The positive and negative ideal solutions of profile p are computed using matrix $\tilde{R} = [\tilde{r}_{pj}]_{|P|\times n}$, where $\tilde{r}_{pj}$ is the linguistic term associated with the main reference to classify the evaluation criterion j by profile p. Fuzzy numbers are used to quantify these linguistic terms as presented in Table 1. The values of matrix $\tilde{R}$ should be defined by a finance expert. The Algorithm 1 describes in details FTOPSIS-Class to define CC_i^p. Note that Step 6 deals with the regulation issues involved in private banking.

Table 1 – Linguistic variables.

Ratings	Fuzzy numbers	Weights	Fuzzy numbers
Very Low (VL)	(0.0,0.0,0.1,0.2)	Unimportant (U)	(0.0,0.0,0.1,0.2)
Low (L)	(0.1.0.2,0.3,0.4)	Moderately Important (MI)	(0.1.0.2,0.3,0.4)
Medium (M)	(0.3,0.4,0.5,0.6)	Important (I)	(0.3,0.4,0.5,0.6)
High (H)	(0.5,0.6,0.7,0.8)	Very Important (VI)	(0.5,0.6,0.7,0.8)
Very High (VH)	(0.7,0.8,0.9,1.0)	Extremely Important (EI)	(0.7,0.8,0.9,1.0)

4.1.2 Fuzzy Multi-Objective Linear Programming (FMOLP)

Portfolio optimization has been traditionally modeled based on the standard Markowitz mean-variance approach (Prigent, 2007). However, in the last decades, multi-objective optimization (MOO) has become the predominant way of formulating and solving the problem (Steuer et al., 2007; Zopounidis et al., 2015). The main objective in MOO is to choose non-dominant solutions, based on different levels of trade-off among the different objectives, from the Pareto front. These methods aim to single out a specific solution, which is regarded as an "optimal" compromise solution, from the set of all non-dominated solutions of the problem. Particularly, the main objective of this model in the DSS is to find the optimal portfolio that optimizes simultaneous objectives of a client to a set of requirement constraints. The model uses the client's wishes and preferences, partially defined in the MCNC module.

Algorithm 1 FTOPSIS-Class with Private Banking Regulation.

Step 1: Structure the decision problem, by identifying DMs, the set of criteria and alternatives;

Step 2: Choose trapezoidal fuzzy linguistic terms to assess the relative importance of the criteria and to evaluate the rating of the alternatives. A trapezoidal fuzzy number $\tilde{a}$ can be defined as $\tilde{a} = (a_1, a_2, a_3, a_4)$ according to the membership function $\mu_{\tilde{a}}(x)$ defined as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} f^L_{\tilde{a}}(x), & a_1 \le x \le a_2 \\ 1, & a_2 \le x \le a_3 \\ f^R_{\tilde{a}}(x), & a_3 \le x \le a_4 \\ 0, & \text{otherwise} \end{cases}$$

where $f^L_{\tilde{a}}(x) : [a_1, a_2] \to [0,1]$ is a strictly increasing function and $f^R_{\tilde{a}}(x) : [a_3, a_4] \to [0,1]$ is a strictly decreasing function.

Step 3: Construct the normalized decision matrix $\tilde{R} = [\tilde{r}_{ij}]_{m\times n}$ as follows:

$$\tilde{r_{ij}} = \begin{cases} \left(\frac{a_{ij}}{d^*_j}, \frac{b_{ij}}{d^*_j}, \frac{c_{ij}}{d^*_j}, \frac{d_{ij}}{d^*_j}\right) & \text{if } j \in B, \text{ where set } B \text{ is associated with benefit criteria, and } d^*_j = \max_i d_{ij} \\ \left(\frac{a^-_j}{a_{ij}}, \frac{a^-_j}{b_{ij}}, \frac{a^-_j}{c_{ij}}, \frac{a^-_j}{d_{ij}}\right) & \text{if } j \in C, \text{ where set } C \text{ is associated with cost criteria, and } a^-_j = \min_i a_{ij} \end{cases}$$

Step 4: Construct the weighted normalized fuzzy decision matrix $\tilde{V} = [\tilde{v}_{ij}]_{m\times n}$ from $\tilde{R} = [\tilde{r}_{ij}]$ and $\tilde{W} = [\tilde{w}_j]$ as $\tilde{v}_{ij} = \tilde{r}_{ij} \otimes \tilde{w}_j$, where $\tilde{a} \otimes \tilde{b} \equiv (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3, a_4 \times b_4)$ (Chen et al., 2006)

Step 5: For each profile $p = 1, 2, \ldots, |P|$, do:

Step 5.1: Set the positive ideal solution regarding the profile p as $\tilde{A}^*_p = \{\tilde{v}^*_{p1}, \tilde{v}^*_{p2}, ..., \tilde{v}^*_{pn}\}$, where $\tilde{v}^*_{pj} = \tilde{q}_{pj}$, since the goal of the model is to maximize the adequacy of the alternative i in relation to category p, thereby minimizing the distance between $\tilde{A}^*_q$ and the reference values of each category;

Step 5.2: Set the negative ideal solution regarding the category p as $\tilde{A}^-_p = \{\tilde{v}^-_{p1}, \tilde{v}^-_{q2}, ..., \tilde{v}^-_{pn}\}$, where $\tilde{v}^-_{p'j}$ are the values of the farthest profile p' from p, and the distance to be maximized.

Step 5.3: Calculate the distances of each alternative i in relation to category p as follows:

$$\tilde{d}^{p^*}_i = \sum_{j=1}^{n} \delta(\tilde{v}_{ij}, \tilde{v}^*_{pj}),\ i = 1, 2, ..., m$$

$$\tilde{d}^{p^-}_i = \sum_{j=1}^{n} \delta(\tilde{v}_{ij}, \tilde{v}^-_{pj}),\ i = 1, 2, ..., m$$

where the vertex distance $\delta(\tilde{a}, \tilde{b})$ between two trapezoidal fuzzy numbers $\tilde{a} = (a_1, a_2, a_3, a_4)$ and $\tilde{b} = (b_1, b_2, b_3, b_4)$ is defined as follows (Chen, 2000):

$$\delta(\tilde{a}, \tilde{b}) = \sqrt{\frac{1}{4}[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2 + (a_4 - b_4)^2]}$$

Step 5.4: Calculate the closeness coefficient of each alternative i regarding profile p as $CC^p_i = \frac{\tilde{d}^{p^-}_i}{\tilde{d}^{p^*}_i + \tilde{d}^{p^-}_i}, i = 1, 2, ..., m$.

Step 6 (Regulation): If client c of profile p is satisfied with CC^p_i computed by financial analysts, set $CC^c_i = CC^p_i$. Otherwise, client c can define his/her value of CC^c_i, as follows:

Step 6.1: Apply Fuzzy-TOPSIS as presented by Chen (2000) to define the value of CC^c_i, using the same criteria structure and linguistic terms of Step 1.

Step 6.2: The value defined interactively by the client is only accepted if $\arg\min_{p\in P} \delta_p = p'$, where $\delta_p = |CC^p_i - CC^c_i|, \forall p$. Otherwise go to Step 6.1 to recompute CC^c_i.

The model presents three objective functions, namely, the suitability of the investment for the client's profile (1), the mean expected return (2), and the mean absolute deviation (MAD) (3). The MAD was selected to measure portfolio risk. This method was introduced by Konno & Yamazaki (1991) in portfolio optimization and has been extensively tested on various stock markets (Mansini et al., 2014). The FMOLP was formulated based on Ferreira et al. (2018), as follows:

$$\max Z_1 = \sum_{i \in I} CC_i^c x_i \tag{1}$$

$$\max Z_2 = \sum_{i \in I} R_i x_i \tag{2}$$

$$\min Z_3 = \frac{1}{2m} \sum_{i=1}^{m} \left| R_i - \left(\sum_{i=1}^{m} x_i R_i \right) \right| \tag{3}$$

st

$$x_i \geq \frac{I_i}{C} z_i \qquad \forall i \in I \tag{4}$$

$$x_i \leq z_i \qquad \forall i \in I \tag{5}$$

$$\sum_{i \in I} x_i \leq 1 \tag{6}$$

$$x_i \geq 0, z_i \in \{0,1\} \qquad \forall i \in I \tag{7}$$

where I is the set of assets, CC_i^c the suitability of asset i for client c, R_i the return of asset i, I_i the required initial investment of asset i, and C the available capital for investment. There are two decision variables, as follows: x_i the proportion of capital C invested in asset i, and z_i a binary decision variable, with $z_i = 1$ if asset i is the portfolio, and $z_i = 0$ otherwise. Constraints (4) ensure that the minimum amount demanded by each fund is respected. Constraint 6 avoids that the financial resources allocated by the client be disrespected. Constraints (5) ensure that resources are only allocated to assets selected by the model. The domains for the decision variables x and z are defined in constraints (7).

Scalar techniques and Pareto methods are very popular in solving MO portfolio optimization problems (Xidonas et al., 2010). The former is a set of *a priori* method, in which decisions are made before searching a solution, while the latter is *a posteriori* method, in which a search is performed before making decisions by the DM. Both methods have well known strengths and weaknesses (Ehrgott, 2008). Although Pareto methods, specially evolutionary algorithms are becoming very popular to solve MOO (Zitzler et al., 2000), they are well not suited to our specific problem, since they generate excessive non-dominate solutions, making the decision process a little bit confuse for average clients of a private banking. Scalar methods have the advantages finding solutions that are of interest to DMs. There are several ways of converting the MOO to a single-objective program described in the literature (Ehrgott, 2008), being the weighted sum the simplest and most used. However, as this technique have difficulties in finding Pareto optimal solutions, we choose the fuzzy weighted min-max method as introduced by Lin (2004). The key advantage of the weighted min-max method is that it is able to provide almost all the Pareto optimal points, even for a nonconvex Pareto front. Although this method requires the minimization of individual single-objective optimization problems to determine the utopia point, which can be computationally expensive (Chang, 2014), this is not a concern in our optimization by clients, since the involved mixed integer problems are quite simple to solve by contemporary commercial mathematical programming solvers.

In the developed DSS, the DM assumes a fuzzy perspective, in which membership functions $\mu_{Z_j}(x)$ are defined for each objective, and the compromise solution is the one that achieves all objectives given a certain tolerance limit under the system constraints. In this case, the choice of a portfolio for client c consists of finding a vector $\mathbf{x}^T$ to satisfy the following formulation (Amid et al., 2011):

$$\tilde{Z}_i \geq\sim Z_i^o \qquad i = 1, \ldots, k \tag{8}$$

$$\tilde{Z}_j \leq\sim Z_j^o \qquad j = k+1, \ldots, l \tag{9}$$

st

$$g_s(x) = \sum_{i=1}^{n} a_{si} x_i \leq b_s \qquad \forall s \tag{10}$$

$$x_i \geq 0 \qquad \forall i \tag{11}$$

where Z_k^o and Z_l^o are the aspiration levels that the DM wants to reach, a_{si} and b_s are crisp values, and symbol $\sim$ indicates a fuzzy environment.

To solve this problem, Lin (2004) expanded the max-min operator approach (Zimmermann, 1978) by proposing a weighted max-min model, in which the DM provides relative weights (θ_k) for the fuzzy goals with corresponding membership functions, as follows:

$$\max \lambda \tag{12}$$

st

$$\theta_k \lambda \leq f_{\mu_{Z_k}(x)} \qquad k = 1, 2, \ldots, l \tag{13}$$

$$g_s(x) \leq b_s \qquad \forall s \tag{14}$$

$$\lambda \in [0, 1] \tag{15}$$

$$\sum_{k=1}^{l} \theta_k = 1 \tag{16}$$

$$\theta_k \geq 0 \qquad k = 1, 2, \ldots, l \tag{17}$$

$$x_i \geq 0 \qquad i = 1, 2, \ldots, m \tag{18}$$

where the membership function for maximization objectives (Z_k), and for minimization one (Z_l) are as follows, respectively:

$$\mu_{Z_k}(x) = \begin{cases} 1, & Z_k \geq Z_k^- \\ 0, & Z_k \geq Z_k^+ \\ f_{\mu_{Z_k}(x)} = \frac{Z_k^+ - Z_k(x)}{Z_k^+ - Z_k^-}, & Z_k^- \leq Z_k(x) \leq Z_k^+ \end{cases} \tag{19}$$

$$\mu_{Z_l}(x) = \begin{cases} 1, & Z_l \geq Z_l^+ \\ 0, & Z_l \geq Z_l^- \\ f_{\mu_{Z_l}(x)} = \frac{Z_l(x)-Z_l^-}{Z_l^+-Z_l^-}, & Z_l^- \leq Z_l(x) \leq Z_l^+ \end{cases} \tag{20}$$

where Z_k^+ and Z_l^- are the optimal single objective functions of positive objective Z_k and negative objective Z_l, respectively.

Our specific multi-objective model for portfolio optimization can be stated as follows:

$$\max \lambda \tag{21}$$

st

$$\theta_1 \lambda \leq \frac{\mu_{Z_1}(\sum_{i \in I} CC_i^c x_i) - \mu_{Z_1}(Z_1^-)}{\mu_{Z_1}(Z_1^+) - \mu_{Z_1}(Z_1^-)} \tag{22}$$

$$\theta_2 \lambda \leq \frac{\mu_{Z_2}(Z_2^-) - \mu_{Z_2}(\sum_{i \in I} R_i x_i)}{\mu_{Z_2}(Z_2^-) - \mu_{Z_2}(Z_2^+)} \tag{23}$$

$$\theta_3 \lambda \leq \frac{\mu_{Z_3}(Z_3^-) - \mu_{Z_3}(\frac{1}{2m}\sum_{i=1}^m |R_i - (\sum_{i=1}^m x_i R_i)|)}{\mu_{Z_3}(Z_3^-) - \mu_{Z_3}(Z_3^+)} \tag{24}$$

$$\theta_k \geq 0 \qquad k = 1,2,3 \tag{25}$$

$$(4)-(7),(15)-(16) \tag{26}$$

Model (21)–(26) finds an optimal feasible solution, such that the ratio of the levels achieved is as close as possible to the ratio of the weights. We refer to Lin (2004) and Amid et al. (2006) for a detailed description of the solution method developed to cope with this problem. It should be noticed that this model can be easily expanded by the introduction of objectives and constraints, representing real features. The DSS has capabilities to facilitate this customization process.

4.2 Database

The "Database" module stores all the information needed to select a private banking portfolio, including client data such as objectives, criteria evaluation for each investment, and investment constraints, as well as information about possible assets, such as return, liquidity, and minimum required investment. The database subsystem also allows querying, recovering, and controlling data. This module allows bank managers and investors to create, maintain and update all the information available through the user interface subsystem. Moreover, this subsystem enables investors and bank managers to obtain/deliver information or data that guarantees the effectiveness of the decision making process, automatically updating all changes in attributes during the operation of the system.

A relational database was designed to store all the information needed to process and store the data required for portfolio selection in private banking. The database was modeled in MySQL. Communication in Java© was performed using the Java Database Connectivity (JDBC) API.

4.3 User Interface

This subsystem is a visual interactive tool for portfolio optimization in private banking, and includes the following functions: (i) controlling the flow of information between the modules within the software system, (ii) running the different models within the DSS, (iii) to interactively build the databases, and (iv) presenting the results. The module uses extensive graphical facilities and menu-driven interfaces to achieve a satisfactory level of interaction, present the output in a meaningful way and provide smooth and reliable communication with the user. One of the most important interfaces is the suitability questionnaire presented in Appendix A, which establishes a standard client profile definition, closely following CVM regulations.The user interface provides a meaningful framework within which information can flow in both directions between the user and the DSS, so that the user can take full responsibility for the decision. Fig. 3 presents a UML diagram that illustrates how the DSS is executed in a complete session.

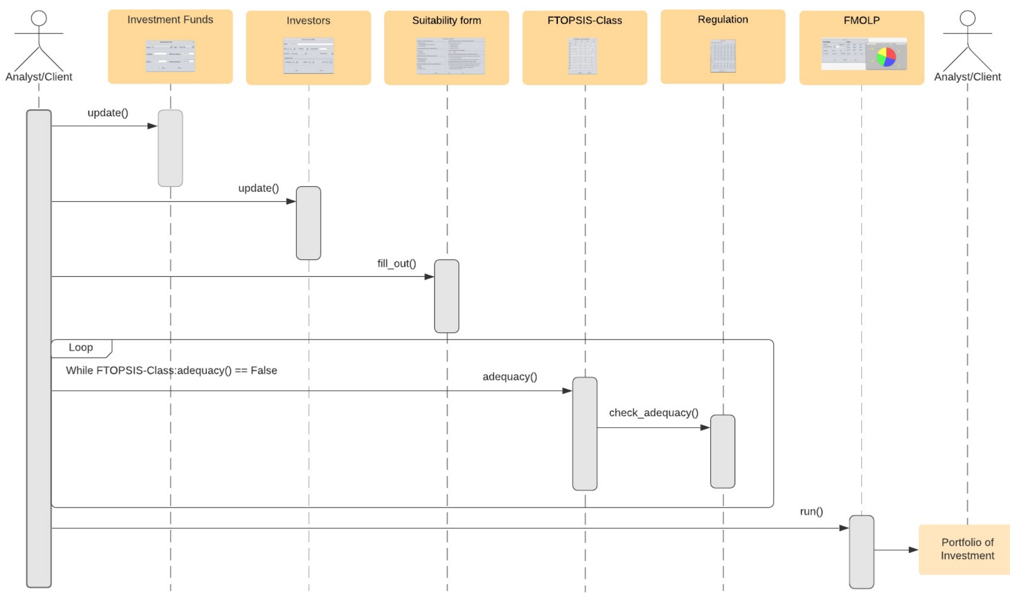

Figure 3 – Schematic use of the DSS in a complete session.

5 DSS VALIDATION

This section presents the results obtained after validating the proposed DSS. A field test, consisting of simulating operational use of the system *in situ* (Borenstein, 1998), was carried out to validate OptPrivate. Experiments were conducted at BTG Pactual, a Brazilian bank with substantial involvement in investment banking, wealth management and global asset. The validation process for this DSS was also presented for the CFA Society Brazil Innovation Award, which was granted to the presented DSS (https://cfasociety.org.br/advocacy/premio-cfa-society-brazil-de-inovacao-financeira/). Thus, a sample of thirty private investors, clients from a BTG Pactual branch effectively took part in the validation process. The "ideal characteristics" for participants were: being a private investor who seeks to invest its personal funds. Participation in the experiments was by spontaneous adhesion and by commitment to appear on the dates stipulated for the experiments. Further, we did not impose any conditions on clients to participate, other than interest and willingness to collaborate in the research. We compared the portfolios selected with and without applying the DSS, using several performance measures. Also, we observed how the investors and bank analysts reacted to the DSS, in terms of acceptance of their results and facility of use.

As an initial step to use the DSS proposed, the investor's basic information contained on the investor information form, should be filled out. This initial information enables access to the suitability questionnaire and can be used to help define the investor's risk profile and cross-check the socioeconomic data with the DSS results obtained, as well as the statistical analyses presented to demonstrate the robustness of the results.

Table 2 presents the most relevant data of investors involved in the experiments, including their preferences for the FMOLP module, represented by variables θ_1, θ_2 and θ_3. The risk profile (RP) of each investor is represented by categories C (conservative), M (moderate), B (bold), or A (aggressive). Based on these data, it was estimated that (i) over 90% of the investors have a university degree; (ii) 86.7% own their own home; (iii) 40.0% exhibited a moderate risk profile (RP = M); (iv) 46.7% monthly income of less than US\$25,000.00; (v) 23.3% between US\$25,000.00 and US\$50,000.00; (vi) 20% between US\$50,000.00 and US\$87,500.00; and (vii) 10% more than US\$87,500.00.

In the experimental period, the investors were generally offered ten investment funds by the bank analysts of the BTG-Pactual. The monthly profits of each fund for year 2017 are described in Table 3, while Table 4 contains complementary data, such as accumulated returns, administration fees, and risk classification, among others. Table 1 presents the linguistic terms used by FTOPSIS-Class to rate the alternatives and weights of the criteria. Following Ferreira et al. (2018), trapezoidal fuzzy numbers were used.

Once all information and parameters needed to apply the models are included, FTOPSIS-Class can be fired to calculate the adherence of each investment fund for each pre-established investor's RP (Conservative, Moderate, Bold and Aggressive). Table 5 shows the assessment criteria of each fund to define the suitability of each fund to each RP. The criteria used are divided into two groups. The first encompasses those related to fund performance (C_1, C_2, and C_3), while the second includes those associated with the investor's goals (C_4), and their knowledge of financial instruments (C_{51}, C_{52}, and C_{53}).

Initial assessment of investment funds in terms of the criteria for each RP was conducted by professionals certified to operate in the financial market, in line with the DSS interface. Fig. 4 shows the results obtained to classify each fund in relation to the RPs considered. The greater the adherence, the higher the CC_i^p. For example, fund F_1 is be better classified as conservative, while F_8 falls under bold. Complementary, the DSS can be used by the financial analysts to fit the initial

Table 2 – Investors information.

#	Age	θ_1	θ_2	θ_3	Higher education	Homeowner	Income (month/$)	Profile	Children	Properties
1	29	0.00	0.90	0.10	Yes	Yes	7,500.00	A	1	2
2	33	0.10	0.70	0.20	Yes	Yes	37,500.00	A	2	3
3	33	0.11	0.61	0.28	Yes	No	12,500.00	A	0	0
4	28	0.28	0.61	0.11	Yes	No	5,000.00	B	0	0
5	67	0.28	0.11	0.61	Yes	Yes	50,000.00	M	3	6
6	57	0.00	1.00	0.00	Yes	Yes	75,000.00	M	3	8
7	58	0.61	0.11	0.28	Yes	Yes	50,000.00	B	2	4
8	29	0.00	0.80	0.20	Yes	Yes	20,000.00	A	0	2
9	63	0.20	0.00	0.80	Yes	Yes	17,500.00	C	3	1
10	56	0.50	0.20	0.30	Yes	Yes	12,500.00	M	1	1
11	30	0.80	0.10	0.10	Yes	Yes	37,500.00	M	1	2
12	33	0.80	0.10	0.10	Yes	Yes	35,000.00	M	1	1
13	52	0.00	0.00	1.00	No	Yes	75,000.00	M	2	5
14	50	0.40	0.00	0.60	Yes	Yes	17,500.00	M	2	2
15	57	0.61	0.11	0.28	Yes	Yes	25,000.00	B	1	3
16	51	0.00	0.00	1.00	Yes	Yes	25,000.00	M	2	2
17	37	0.61	0.11	0.28	Yes	Yes	20,000.00	M	1	4
18	53	0.40	0.30	0.30	Yes	Yes	50,000.00	B	2	4
19	48	0.50	0.00	0.50	Yes	Yes	7,500.00	B	2	1
20	61	0.28	0.61	0.11	Yes	No	125,000.00	B	3	0
21	60	0.20	0.50	0.30	Yes	Yes	100,000.00	B	3	6
22	45	0.00	0.00	1.00	Yes	Yes	150,000.00	B	2	3
23	64	0.80	0.10	0.10	Yes	Yes	12,500.00	C	2	2
24	40	0.70	0.20	0.10	Yes	Yes	62,500.00	C	0	3
25	66	0.50	0.20	0.30	Yes	Yes	25,000.00	M	2	3
26	63	0.00	0.90	0.10	Yes	Yes	37,500.00	A	1	4
27	64	0.70	0.00	0.30	No	Yes	17,500.00	C	1	2
28	46	0.00	0.00	1.00	Yes	No	10,000.00	C	1	0
29	69	0.30	0.40	0.30	Yes	Yes	15,000.00	M	0	2
30	48	0.20	0.70	0.10	Yes	Yes	12,500.00	M	1	1

parameters to the profile of each client (CC_i^c) seeking personalized service, as Fig. 3. In this case, the DSS allows a certain freedom to change the parameters according to the investor's interests. However, if the changes are significant enough to compromise the investor's profile, assessment must be corrected or the client must sign an assumption of risk form. The DSS monitors this regulation issue by comparing CC_i^p (defined by the investor analyst) and CC_i^c (defined by the client), as described in Section 4.1.

Table 3 – Profitability of the funds.

Fund/Month	Dec/16	Jan/17	Feb/17	Mar/17	Apr/17	Mai/17	Jun/17	Jul/17	Aug/17	Sep/17	Oct/17	nov/17
F_1	1.09%	1.07%	0.86%	1.04%	0.78%	0.93%	0.90%	0.81%	0.78%	0.63%	0.64%	0.55%
Accumulated	1.09%	2.17%	3.05%	4.12%	4.93%	5.91%	6.86%	7.73%	8.57%	9.25%	9.95%	10.56%
F_2	1.14%	1.10%	0.90%	1.19%	0.81%	0.94%	0.83%	0.81%	0.81%	0.65%	0.65%	0.58%
Accumulated	1.14%	2.25%	3.17%	4.40%	5.25%	6.24%	7.12%	7.98%	8.86%	9.57%	10.28%	10.92%
F_3	1.14%	1.31%	1.19%	1.11%	0.77%	0.83%	0.88%	1.11%	0.80%	0.71%	0.58%	0.57%
Accumulated	1.14%	2.46%	3.68%	4.84%	5.64%	6.52%	7.46%	8.65%	9.52%	10.30%	10.94%	11.57%
F_4 I	1.20%	1.21%	1.13%	1.25%	0.90%	1.04%	1.04%	0.92%	0.89%	0.72%	0.72%	0.65%
Accumulated	1.20%	2.42%	3.58%	4.88%	5.82%	6.92%	8.03%	9.03%	10.00%	10.79%	11.59%	12.31%
F_5	1.29%	1.60%	1.05%	1.10%	0.88%	0.48%	1.10%	1.35%	0.87%	0.87%	0.60%	0.68%
Accumulated	1.29%	2.91%	3.99%	5.14%	6.06%	6.57%	7.74%	9.20%	10.15%	11.10%	11.77%	12.53%
F_6	1.15%	2.27%	1.47%	1.57%	1.44%	-0.09%	1.65%	3.14%	1.97%	2.37%	0.40%	0.91%
Accumulated	1.15%	3.45%	4.97%	6.61%	8.15%	8.05%	9.84%	13.28%	15.52%	18.25%	18.73%	19.81%
F_7	0.00%	0.00%	-0.20%	2.08%	1.08%	0.49%	-0.58%	0.88%	2.44%	1.12%	1.91%	-0.09%
Accumulated	0.00%	0.00%	-0.20%	1.88%	2.98%	3.48%	2.88%	3.79%	6.32%	7.51%	9.56%	9.46%
F_8	-0.25%	3.70%	3.49%	0.71%	2.38%	-1.80%	1.61%	4.21%	4.44%	3.90%	-1.42%	-1.82%
Accumulated	-0.25%	3.44%	7.05%	7.81%	10.38%	8.39%	10.14%	14.77%	19.87%	24.54%	22.77%	20.54%
F_9	2.31%	1.76%	1.24%	1.85%	1.71%	0.81%	1.76%	2.15%	0.73%	0.49%	0.09%	-0.79%
Accumulated	2.31%	4.11%	5.40%	7.35%	9.19%	10.07%	12.01%	14.42%	15.25%	15.82%	15.92%	15.01%
F_{10}	-4.10%	-2.85%	-1.33%	0.77%	1.57%	1.92%	2.23%	-5.62%	1.02%	0.68%	3.80%	0.00%
Accumulated	-4.10%	-6.83%	-8.07%	-7.36%	-5.91%	-4.10%	-1.97%	-7.47%	-6.53%	-5.90%	-2.32%	-2.32

Table 4 – Alternatives of investment.

Fund	Administration Fee	Risk	Liquidity	Minimum balance ($)	Initial investment($)	Return
F_1	0.20%	VL	D+0	250.00	750.00	10.56%
F_2	0.30%	VL	D+0	250.00	750.00	10.91%
F_3	0.80%	VL	D+1	1,250.00	6250.00	11.57%
F_4	0.50%	L	D+31	250.00	1,250.00	12.31%
F_5	1.75%	L	D+5	250.00	1,250.00	12.53%
F_6	2.00%	M	D+31	250.00	1,250.00	19.80%
F_7	0.50%	H	D+61	1,250.00	1,250.00	12.21%
F_8	2.00%	H	D+33	-	1,250.00	20.54%
F_9	0.75%	H	D+60	6,250.00	6,250.00	15.01%
F_{10}	1.00%	H	D+1	250.00	1,250.00	-2.32%

Table 5 – Criteria and sub-criteria definition.

Criteria	Sub-criteria
Risk (C_1)	
Number of days to withdraw the money (C_2)	
Net Return (C_3)	
Compatibility of investor's level of knowledge on financial markets (C_4)	
Investment objectives	Wealth preservation (C_{51}) Wealth generation in the short term (C_{52}) Wealth accumulation in the long term (C_{53})

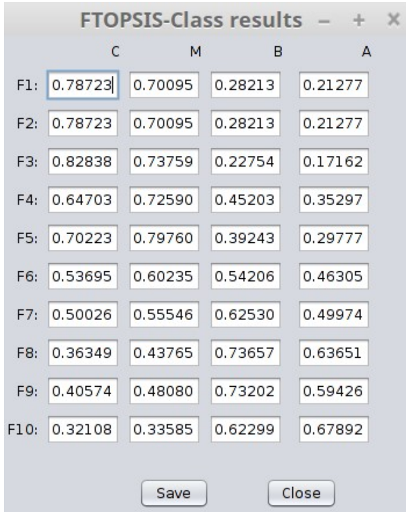

Figure 4 – Results from FTOPSIS-Class.

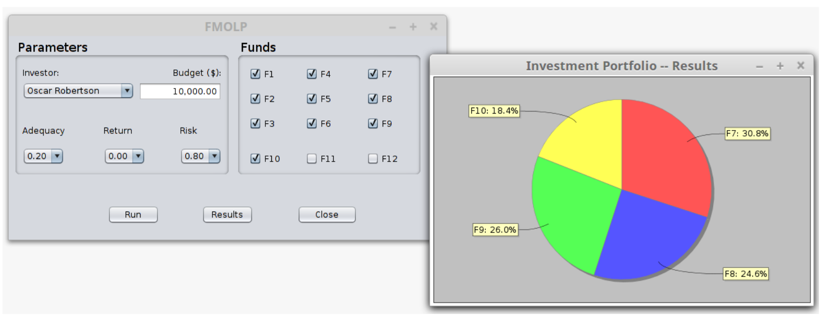

Figure 5 – Output illustration.

Next, the FMOLP module is executed towards defining the investment portfolio according to the goals and weights interactively established by client/financial analyst. The mixed integer linear programming (MILP) methods generated during the process of solving multi-objective model (21)–(26) were solved using the interface offered by the AMPL Java API, and Gurobi as the MILP solver.

Fig. 5 shows an example of how the selected portfolio is presented to the users. All clients were satisfied in using the values of CC_i^p as their own CC_i^c. Table 6 presents the portfolio obtained by the DSS for each investor, while Table 7 describes the portfolios defined without the DSS, combining the legal aspects, general private banking guidelines and the relationship manager's knowledge about the funds and the macroeconomic situation at the time of allocation. Tables 6 and 7 show the returns computed for each of the portfolios obtained with and without the use of the DSS, respectively, over the next 12 months (Dec/17 to Nov/18).

In order to compare the portfolios selected with and without using the DSS, the following classical performance measures were computed, as defined in Sharpe (1964), Markowitz (1952) and Fama & MacBeth (1973): (i) mean return (the higher the mean, the greater the gain during the period), (ii) standard deviation (the lower the standard deviation, the greater the loss or the worse the return for the period — called tail in the tables); (iii) and the Sharpe ratio, a well-known measure of risk-adjusted return, computed as $S = \frac{E[R_a - R_f]}{\sigma_a}$, where R_a is the portfolio return, R_f is the risk-free return, $E[R_a - R_f]$ is the expected value of the excess of the asset return over the benchmark return, and σ_a is the standard deviation of the asset excess return. The higher the ratio, the better the ratio between the average return and the risk-free rate, risk reward and standard deviation. The analysis considers both variability and loss (Righi & Ceretta, 2016; Righi, 2019).

Table 6 – DSS Portfolio results.

Portfolio	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	Mean	Dev.	Tail	Sharpe
1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	0.00%	0.00%	1.49%	4.74%	7.30%	0.12
2	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	0.00%	0.00%	1.49%	4.74%	7.30%	0.25
3	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	0.00%	0.00%	1.49%	4.74%	7.30%	0.13
4	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	0.00%	0.00%	1.49%	4.74%	7.30%	0.08
5	0.00%	0.00%	0.00%	22.00%	28.00%	6.00%	8.00%	0.00%	27.00%	9.00%	0.63%	0.60%	0.24%	0.10
6	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	0.00%	0.00%	1.49%	4.74%	7.30%	0.03
7	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	0.00%	0.00%	1.49%	4.74%	7.30%	0.12
8	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	0.00%	0.00%	1.49%	4.74%	7.30%	0.23
9	0.00%	43.00%	46.00%	0.00%	0.00%	5.00%	0.00%	6.00%	0.00%	0.00%	0.48%	0.45%	0.38%	0.17
10	0.00%	0.00%	0.00%	0.00%	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.33%	0.42%	0.78%	0.12
11	0.00%	0.00%	0.00%	0.00%	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.33%	0.42%	0.78%	0.05
12	0.00%	0.00%	0.00%	0.00%	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.33%	0.42%	0.78%	-0.15
13	10.00%	8.00%	0.00%	16.00%	17.00%	0.00%	13.00%	24.00%	0.00%	12.00%	0.83%	0.77%	0.99%	-0.29
14	0.00%	16.00%	0.00%	0.00%	62.00%	0.00%	7.00%	10.00%	0.00%	5.00%	0.54%	0.48%	0.47%	0.80
15	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	0.00%	0.00%	1.49%	4.74%	7.30%	0.16
16	10.00%	8.00%	0.00%	16.00%	17.00%	0.00%	13.00%	24.00%	0.00%	12.00%	0.83%	0.77%	0.99%	-0.14
17	0.00%	0.00%	0.00%	0.00%	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.33%	0.42%	0.78%	0.00
18	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	54.00%	46.00%	0.00%	1.18%	3.89%	5.10%	0.08
19	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	91.00%	9.00%	0.88%	2.62%	1.68%	0.22
20	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	0.00%	0.00%	1.49%	4.74%	7.30%	0.08
21	0.00%	0.00%	0.00%	0.00%	0.00%	12.00%	0.00%	88.00%	0.00%	0.00%	1.35%	4.27%	6.71%	-0.01
22	10.00%	8.00%	0.00%	15.00%	15.00%	0.00%	15.00%	25.00%	0.00%	12.00%	0.84%	0.83%	1.10%	0.80
23	0.00%	0.00%	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.33%	0.42%	0.78%	0.80
24	0.00%	0.00%	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.33%	0.42%	0.78%	0.00
25	0.00%	0.00%	0.00%	0.00%	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.33%	0.42%	0.78%	-0.04
26	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	0.00%	0.00%	1.49%	4.74%	7.30%	0.10
27	0.00%	0.00%	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.33%	0.42%	0.78%	0.00
28	10.00%	8.00%	0.00%	16.00%	16.00%	0.00%	13.00%	24.00%	0.00%	12.00%	0.83%	0.77%	0.98%	0.80
29	0.00%	0.00%	0.00%	0.00%	22.00%	78.00%	0.00%	0.00%	0.00%	0.00%	0.34%	1.27%	2.38%	-0.03
30	0.00%	0.00%	0.00%	0.00%	0.00%	18.00%	0.00%	82.00%	0.00%	0.00%	1.28%	4.03%	6.42%	-0.01

It is important to underscore that some funds were selected not following the risk profiles of some clients, when not using the DSS. For example, Investor 9 opted for multimarket funds (F_5 and F_6) and a variable income fund (F_8) that are outside his/her RP. This fact is in contradiction with his/her desire of minimizing risks by setting a weight of 80% for the minimization risk goal (θ_3), when using the DSS. As previously explained, this situation may result in sanctions being imposed by the regulatory agencies, except if the client signed an assumption of risk form for funds that are incompatible with his/her profile.

Table 8 shows a comparison between the two portfolios, DSS and real, for each client, as well as the difference in each performance metric. Moreover, it exhibits the proportion of portfolios

Table 7 – Real Portfolio result.

Portfolio	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	Ret.	Dev.	Tail	Sharpe
1	0.00%	30.80%	0.00%	0.00%	32.00%	14.00%	0.00%	23.20%	0.00%	0.00%	0.66%	1.29%	1.90%	0.12
2	0.00%	30.00%	0.00%	0.00%	7.00%	0.00%	0.00%	57.50%	0.00%	5.50%	1.12%	2.49%	3.68%	0.25
3	0.00%	13.00%	0.00%	0.00%	0.00%	38.00%	0.00%	29.00%	20.00%	0.00%	0.79%	2.25%	3.47%	0.13
4	0.00%	25.00%	0.00%	0.00%	25.00%	30.00%	0.00%	20.00%	0.00%	0.00%	0.61%	1.31%	2.08%	0.08
5	32.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	68.00%	0.00%	0.72%	2.22%	1.56%	0.10
6	0.00%	49.00%	0.00%	9.00%	20.00%	5.00%	12.00%	5.00%	0.00%	0.00%	0.52%	0.45%	0.40%	0.03
7	0.00%	43.20%	21.30%	11.00%	13.50%	0.00%	0.00%	11.00%	0.00%	0.00%	0.57%	0.58%	0.55%	0.12
8	10.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	70.00%	14.60%	5.40%	1.29%	3.47%	5.06%	0.23
9	0.00%	0.00%	0.00%	15.00%	17.00%	21.00%	0.00%	47.00%	0.00%	0.00%	0.92%	2.44%	3.87%	0.17
10	0.00%	0.00%	0.00%	39.10%	17.00%	26.70%	0.00%	17.20%	0.00%	0.00%	0.64%	1.13%	1.70%	0.12
11	0.00%	0.00%	0.00%	18.80%	11.40%	32.60%	24.00%	9.80%	0.00%	3.40%	0.55%	0.98%	1.57%	0.05
12	0.00%	0.00%	0.00%	0.00%	29.30%	49.70%	21.00%	0.00%	0.00%	0.00%	0.34%	1.06%	1.77%	-0.15
13	0.00%	64.90%	19.70%	0.00%	15.40%	0.00%	0.00%	0.00%	0.00%	0.00%	0.46%	0.16%	-0.05%	-0.29
14	0.00%	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.53%	0.03%	-0.48%	0.80
15	4.00%	0.00%	0.00%	0.00%	24.00%	13.00%	0.00%	47.00%	12.00%	0.00%	0.94%	2.69%	4.05%	0.16
16	0.00%	0.00%	0.00%	0.00%	30.80%	69.20%	0.00%	0.00%	0.00%	0.00%	0.33%	1.17%	2.20%	-0.14
17	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.50%	0.03%	-0.46%	0.00
18	0.00%	9.20%	7.40%	15.10%	23.30%	9.40%	19.90%	15.70%	0.00%	0.00%	0.58%	1.06%	1.60%	0.08
19	0.00%	0.00%	0.00%	32.35%	0.00%	0.00%	0.00%	67.65%	0.00%	0.00%	1.21%	3.21%	4.76%	0.22
20	19.10%	0.00%	0.00%	0.00%	17.70%	39.90%	0.30%	23.00%	0.00%	0.00%	0.63%	1.55%	2.57%	0.08
21	0.00%	13.00%	0.00%	0.00%	0.00%	48.60%	68.40%	0.00%	0.00%	0.00%	0.49%	1.39%	2.25%	-0.01
22	0.00%	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.53%	0.03%	-0.48%	0.80
23	0.00%	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.53%	0.03%	-0.48%	0.80
24	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.50%	0.03%	-0.46%	0.00
25	0.00%	0.00%	0.00%	0.00%	49.61%	39.83%	0.00%	10.56%	0.00%	0.00%	0.45%	1.13%	1.77%	-0.04
26	7.60%	0.00%	0.00%	0.00%	0.00%	30.00%	34.30%	28.10%	0.00%	0.00%	0.69%	1.92%	3.31%	0.10
27	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.50%	0.03%	-0.46%	0.00
28	0.00%	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.53%	0.03%	-0.48%	0.80
29	0.00%	22.50%	0.00%	0.00%	8.40%	33.00%	17.40%	0.00%	18.70%	0.00%	0.48%	1.01%	1.45%	-0.03
30	0.00%	0.00%	23.40%	0.00%	0.00%	43.10%	11.10%	6.30%	16.10%	0.00%	0.49%	1.31%	2.11%	-0.01

where the DSS approach performed better in each criterion, in addition to the Wilcoxon nonparametric test (due to the low number of observations) of the significance (difference of zero) of the means of these differences.

In general, the portfolios obtained by the DSS tend to exhibit greater average returns, with a higher risk level, measured by the standard deviation and worse return. Thus, the Sharpe ratio, based on the standardized risk return, is similar for the two approaches. The null hypothesis of no difference cannot be rejected, at the 5% significance level, in any of the cases. The variables that exhibit smaller p-values in the test, and then have been closest to null hypothesis rejection, are the return and risk, measured by the standard deviation and the worst case (called tail in the tables). The Sharpe ratio displayed the least difference, corroborating the results presented in Tables 6 and 7. A possible explanation for this behavior is that the portfolios obtained by the DSS have less funds, reducing diversification and raising portfolio risk, concomitantly with a higher return (see Righi & Borenstein (2018) for further details). Similar results were found in Ferreira et al. (2018) for similar numerical experiment. As such, the DSS proposed maintains the efficient frontier notion, located only on a more pronounced indifference curve in relation to the benchmarking used. This perspective is compatible with the financial theory.

Investors/financial analysts responded positively to OptPrivate. The main advantage of the DSS was to offer a friendly environment for a more objective and integrated analysis between these two important players in private banking. The use of the system avoids the evaluation of in-

Table 8 – Comparison between DSS and real portfolios performance.

Portfolio	Mean	Dev.	Tail	Sharpe
1	0.83%	3.45%	5.40%	0.09
2	0.37%	2.25%	3.63%	-0.04
3	0.70%	2.49%	3.83%	0.08
4	0.88%	3.42%	5.22%	0.12
5	-0.08%	-1.62%	-1.32%	0.12
6	0.97%	4.29%	6.90%	0.18
7	0.92%	4.15%	6.75%	0.09
8	0.20%	1.26%	2.24%	-0.02
9	-0.43%	-1.98%	-3.49%	-0.21
10	-0.31%	-0.71%	-0.92%	-0.54
11	-0.22%	-0.56%	-0.79%	-0.46
12	-0.01%	-0.64%	-0.99%	-0.26
13	0.38%	0.61%	1.04%	0.72
14	0.01%	0.45%	0.95%	-0.73
15	0.55%	2.05%	3.25%	0.05
16	0.50%	-0.40%	-1.21%	0.57
17	-0.17%	0.39%	1.24%	-0.41
18	0.60%	2.82%	3.50%	0.10
19	-0.33%	-0.59%	-3.08%	-0.08
20	0.86%	3.19%	4.73%	0.12
21	0.86%	2.88%	4.47%	0.21
22	0.32%	0.80%	1.58%	-0.39
23	-0.20%	0.39%	1.26%	-1.22
24	-0.17%	0.39%	1.24%	-0.41
25	-0.13%	-0.71%	-0.99%	-0.37
26	0.80%	2.82%	3.99%	0.11
27	-0.17%	0.39%	1.24%	-0.41
28	0.30%	0.74%	1.46%	-0.38
29	-0.14%	0.27%	0.93%	-0.10
30	0.79%	2.72%	4.31%	0.20
Proportion	60.00%	26.70%	26.70%	50.00%
Mean	0.28%	1.17%	1.88%	-0.11
P-value	0.13	0.09	0.07	0.81

vestment options being based only on either simple operational and financial measures or based on the bank staff's recommendations. Analysis and evaluation of possible portfolio alternatives, through our model-based DSS, provides a means to study each alternative with respect to several conflicting measures, simultaneously making more objective decisions and respecting the regulation issues involved in private banking.

Another aspect recognized by users, is the facility that the DSS allows investors to easily conduct a sensitivity analysis of several parameters of the problem, interactively assessing the consequences of each change. The possibility to alter input data interactively, to evaluate them in an efficient and effective manner, and to include real-world features, make the DSS an effective tool for private banking. In terms of recommended portfolios, the comparative analysis showed that the DSS achieved its main objective, since it was able to recommend the portfolios that best suited the investor profiles, with similar performance to real portfolios that did not focus on the peculiarities of each investor. However, there were some concerns about the interface of the system. Some of them complained about some bad design of the menus and the lack of some better graphical interfaces.

Overall, on one hand, the investors praised the DSS by the possibility of actively participating in the portfolio decision-making process, gaining insights of how the process happens, mainly in terms of using linguistic variables to express their evaluations. On the other hand, the bank analysts felt that the DSS can help them to better capture the investors needs and preferences, allowing a more interactive process. Further, both type users have recognized that the DSS can make the portfolio selection and updated process more objective, and less subject to biases and unwanted noises.

6 CONCLUSIONS

This paper describes OptPrivate, a DSS developed to optimize portfolios in private banking, a highly regulated environment. OptFinance contributes to the current literature by introducing an user oriented, systematic, and integrated approach to analyze and evaluate portfolios in private banking, which constitutes a prescriptive decision aid. The DSS facilitates the difficult and complicated problem of wealth management, offering a computer environment where clients and bank analysts can interact, considering regulatory, financial, and socioeconomic issues. In particular, we strongly believe that the DSS can improve the relationship between banks and investors, increasing their confidence in one another and representing an important mitigation tool for possible conflicts of interest. Although the DSS was developed based on Brazilian regulations, it can be easily customized for other countries, just altering some parameters and adding/eliminating some rules from the database.

The system was validated using field tests with clients from the BTG Pactual, one of the most competent asset management services in Latin America, with a multiple award-winning team. We compared portfolios recommended with and without the use of the DSS for thirty clients, using different performance measures, such as return, risk, and adherence to the client's risk profile. The computational system was able to offer portfolios with better adherence to the risk profile of the investor, without compromising portfolio performance in terms of risk and return. The results confirm that the DSS is a competitive environment for selecting investment portfolios in private banking.

Notwithstanding the short experimentation time, the tests carried out clearly demonstrate that OptPrivate has a good potential as an effective prescriptive tool in real-world private banking. Further, tests using quantitative techniques are forthcoming towards assessing the range of its capabilities. In addition, future research is directed towards: (i) the expansion of the modeling capabilities with the inclusion of evolutionary multi-objective methods (Coello et al., 2020), capable of automatically generating a representative subset of the Pareto optimal solutions; and (iii) the introduction of more accurate measures for risk. Further, the DSS is being reshaped to be used in the cloud, incorporating the state of the art in terms of user interface and internet resources usage. After this stage is complete, the DSS is ready to be used in day-to-day services of private banking in Brazil.

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APPENDIX A. QUESTIONNAIRE OF SUITABILITY

1. What is your main investment goal? (Weight 3)
   - (a) Asset preservation (1 point).
   - (b) Combination of preservation and appreciation (2 points).
   - (c) Gain maximization (3 points).
2. How long will the resources remain invested? (Weight 2)
   - (a) Up to 1 year (1 point).
   - (b) 1 to 5 years (2 points).
   - (c) More than 5 years (3 points).
3. What is the purpose of the invested resources? (Weight 2)
   - (a) Supplementary income (1 point).
   - (b) Possible future need (2 points).
   - (c) Not currently need (3 points).
4. What percentage of your income is regularly invested? (Weight 1)
   - (a) Up to 10% (1 point)
   - (b) 10 to 20% (2 points)
   - (c) More than 20
5. What would you do if you lost 10% of your total investment? (Weight 2)
   - (a) I do not know what I would do (1 point).
   - (b) I would sell all my shares (2 points).
   - (c) I would keep my shares (3 points).
   - (d) I would buy more shares (4 points).
6. How would you describe your expected income for the next 5 years? (Weight 2)
   - (a) My income should decrease because of retirement, change of job, or reduced revenue (1 point)
   - (b) My income should remain stable (2 points)
   - (c) My income should increase because of a promotion, new job or increased revenue (3 points)

7. Do you intend to invest in derivatives? (Weight 3)
   (a) No (1 point)
   (b) Yes (2 points)
8. Which investments were part of your portfolio in the last 5 years? (Weight 3)
   (a) Savings account, interbank deposit funds, bank deposit certificates, fixed income funds (1 point)
   (b) Multimarket funds, public securities, real estate credit bills, agribusiness credit bills (2 points)
   (c) Stock funds, shares, real estate funds, debentures, exchange funds (3 points)
   (d) Equity investment funds, derivatives (4 points)

The client's risk profile score is defined by the sum of the multiplication of each alternative by the weight of the respective question, when considering the $a-g$ questions. Since the last question is multiple-choice, the largest value of the user's answers was multiplied by the weight of the question. The RP of each client is classified as follows:

Conservative if the score is below or equal to 28 points.

Moderate if the score is between 28 and 37 points.

Bold if the score is between 37 and 47 points.

Aggressive if the score is above 47 points.