







# Isochrone fitting of Galactic globular clusters – V. NGC 6397 and NGC 6809 (M55)

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## ABSTRACT

We fit various colour–magnitude diagrams (CMDs) of the Galactic globular clusters NGC 6397 and NGC 6809 (M55) by isochrones from the Dartmouth Stellar Evolution Database (DSED) and Bag of Stellar Tracks and Isochrones (BaSTI) for  $\alpha$ -enhanced  $[\alpha/\text{Fe}] = +0.4$ . For the CMDs, we use data sets from *Hubble Space Telescope*, *Gaia*, *Visible and Infrared Survey Telescope for Astronomy*, and other sources utilizing 32 and 23 photometric filters for NGC 6397 and NGC 6809, respectively, from the ultraviolet to mid-infrared. We obtain the following characteristics for NGC 6397 and NGC 6809, respectively: metallicities  $[\text{Fe}/\text{H}] = -1.84 \pm 0.02 \pm 0.1$  and  $-1.78 \pm 0.02 \pm 0.1$  (statistic and systematic uncertainties); distances  $2.45 \pm 0.02 \pm 0.06$  and  $5.24 \pm 0.02 \pm 0.18$  kpc; ages  $12.9 \pm 0.1 \pm 0.8$  and  $13.0 \pm 0.1 \pm 0.8$  Gyr; reddenings  $E(B - V) = 0.178 \pm 0.006 \pm 0.01$  and  $0.118 \pm 0.004 \pm 0.01$  mag; extinctions  $A_V = 0.59 \pm 0.01 \pm 0.02$  and  $0.37 \pm 0.01 \pm 0.04$  mag; and extinction-to-reddening ratio  $R_V = 3.32^{+0.32}_{-0.28}$  and  $3.16^{+0.66}_{-0.56}$ . Our estimates agree with most estimates from the literature. BaSTI gives systematically higher  $[\text{Fe}/\text{H}]$  and lower reddenings than DSED. Despite nearly the same metallicity, age, and helium enrichment, these clusters show a considerable horizontal branch (HB) morphology difference, which must therefore be described by another parameter. This parameter must predominantly explain why the least massive HB stars (0.58–0.63 solar masses) are only found within NGC 6809. Probably they have been lost by the core-collapse cluster NGC 6397 during its dynamical evolution and mass segregation. In contrast, NGC 6809 has a very low central concentration and, hence, did not undergo this process.

**Key words:** Hertzsprung–Russell and colour–magnitude diagrams – globular clusters: general – globular clusters: individual: NGC 6397, NGC 6809 – dust, extinction – proper motions – stars: horizontal branch – stars: evolution.

## 1. INTRODUCTION

In Gontcharov, Mosenkov & Khovritchev (2019, hereafter [Paper I](#)), Gontcharov, Khovritchev & Mosenkov (2020, hereafter [Paper II](#)), Gontcharov et al. (2021, hereafter [Paper III](#)), and Gontcharov et al. (2023, hereafter [Paper IV](#)) we used theoretical stellar evolution models and their corresponding isochrones to fit colour–magnitude diagrams (CMDs) for the Galactic globular clusters (GCs) NGC 288, NGC 362, NGC 5904 (M5), NGC 6205 (M13), NGC 6218 (M12), NGC 6362, and NGC 6723.

This series of papers is inspired by the recent appearances and improvements for models/isochrones and photometric data sets of individual cluster members in ultraviolet (UV), optical, and infrared

(IR) bands. In particular, we use photometric data from the *Hubble Space Telescope* [*HST*, Piotto et al. (2015); Nardiello et al. (2018, hereafter [NLP18](#)); Simioni et al. (2018, hereafter [SBA18](#))]; *Gaia* Data Release 2 (DR2, Evans et al. 2018), Early Data Release 3 (EDR3, Riello et al. 2021), and Data Release 3 (DR3, Vallenari et al. 2023); *Wide-field Infrared Survey Explorer* (*WISE*, Wright et al. 2010) as the unWISE catalogue (Schlafly, Meisner & Green 2019); various ground-based telescopes by Stetson et al. (2019, hereafter [SPZ19](#)); and other sources. Moreover, the precise parallaxes and proper motions (PMs) from *HST* and *Gaia* EDR3<sup>1</sup> allow us an accurate selection of GC members. To fit CMDs we use theoretical models of stellar evolution, namely the Dartmouth Stellar Evolution

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<sup>1</sup>The photometry and astrometry of GCs are exactly the same in *Gaia* EDR3 and DR3.

**Table 1.** Reddening converted into  $E(B - V)$  (mag) and age (Gyr) estimates for NGC 6397 or NGC 6809 from recent isochrone fittings of CMDs.

Study	Isochrones	Data set and colour	NGC 6397		NGC 6809	
			$E(B - V)$	Age	$E(B - V)$	Age
Dotter et al. (2010)	DSED	<i>HST</i> /ACS, F606W – F814W	0.18	13.5 ± 0.5	0.11	13.5 ± 1.0
Di Criscienzo, D’Antona & Ventura (2010)	Own	<i>HST</i> /ACS, F606W – F814W		13.0 ± 1.0		
Siegel et al. (2011)	DSED	<i>HST</i> /ACS, F606W – F814W			0.12 ± 0.01	
VandenBerg et al. (2013)	VR	<i>HST</i> /ACS, F606W – F814W		13.0 ± 0.3		13.0 ± 0.3
Martinazzi et al. (2014)	DSED	Dedicated ground-based, $B - V$	0.12 ± 0.01	12.0 ± 0.5		
Chen et al. (2014)	PARSEC	<i>HST</i> /ACS, F606W-F814W	0.20	12.0		
Campos et al. (2016)	Own	<i>HST</i> /ACS, F606W – F814W	0.18 ± 0.01	12.9 ± 0.4		
Correnti et al. (2018)	VR	<i>HST</i> /WFC3, IR F110W – F160W	0.22 ± 0.02	12.6 ± 0.7		
Tailo et al. (2020)	Own	<i>HST</i> /WFC3/ACS, F438W – F814W		13.0 ± 0.5		13.3 ± 0.5
Valcin et al. (2020)	DSED	<i>HST</i> /ACS, F606W – F814W	0.15 ± 0.01	14.2 ± 0.7	0.08 ± 0.01	13.9 ± 0.6
Ahumada et al. (2021)	VR	Dedicated ground-based, $V - I$	0.19	13.3 ± 0.3		

Database (DSED, Dotter et al. 2007)<sup>2</sup> and a Bag of Stellar Tracks and Isochrones (BaSTI, Pietrinferni et al. 2021).<sup>3</sup> These models are presented by user-friendly online tools in order to calculate isochrones for low metallicity, various levels of helium abundance, and  $\alpha$ -enhancement, which are typical in GCs (Monelli et al. 2013; Milone et al. 2017). These isochrones reproduce different stages of stellar evolution, namely the main sequence (MS), turn-off (TO), subgiant branch (SGB), red giant branch (RGB), horizontal branch (HB), and asymptotic giant branch (AGB). Best-fitting isochrones provide us with age, distance, reddening, and metallicity [Fe/H] for a cluster dominant population or a mix of populations.

We cross-identify data sets to estimate systematic differences between them, convert the derived reddenings into extinction for each filter we consider, and draw an empirical extinction law (i.e. a dependence of extinction on wavelength) for each combination of cluster, data set, and model.

In this paper, we fit the pair of GCs NGC 6397 and NGC 6809 (also known as Messier 55 or M55). These clusters are considerably contaminated by foreground and background stars (in particular, those of Sagittarius dwarf galaxy, see Siegel et al. 2011). Hence, their data sets should be cleaned with PMs and parallaxes. Accordingly, we can fit isochrones directly to a bulk of certain cluster members in very clean CMDs, without calculation of any fiducial sequence. Since these clusters are similar in metallicity, age, helium enrichment, and reddening, it is fruitful to consider their relative estimates. Moreover, this similarity makes NGC 6397 and NGC 6809 interesting to find an explanation for their significant HB morphology difference besides metallicity, age, and helium enrichment.

Previous isochrone fittings of the clusters since Dotter et al. (2010) and corresponding reddening and age values are presented in Table 1. Their results can be compared with ours (see Section 5). Unfortunately, earlier fittings by Alcaino et al. (1997), Anthony-Twarog & Twarog (2000, hereafter AT2000), Gratton et al. (2003), and Richer et al. (2008) for NGC 6397 and by Piotto & Zoccali (1999) for NGC 6809 seem to be obsolete due to incomplete or too simple models.

NGC 6397 and NGC 6809 are among metal-poor GCs, with [Fe/H] < -1.7. They are valuable candidates to verify modern models/isochrones in such a low-metallicity regime, especially in application to the data sets never fitted before.

Most of the studies in Table 1 fit Victoria–Regina (VR, VandenBerg & Denissenkov 2018) or DSED isochrones to the

*HST*/Advanced Camera for Surveys (ACS) data sets.<sup>4</sup> Our study stands out, since it engages the BaSTI isochrones (together with the DSED ones) and many more data sets, which have appeared or improved recently. The number of data sets and photometric measurements, fitted in our study, is an order of magnitude higher than in any study before.

The outline of this paper is as follows. We present some properties of NGC 6397 and NGC 6809 in Section 2, theoretical models and isochrones used – in Section 3, and data sets used – in Section 4. The results of our isochrone fitting are introduced and discussed in Section 5. In Section 6, we summarize our main findings and conclusions.

## 2. PROPERTIES OF THE CLUSTERS

Table 2 presents some properties of NGC 6397 and NGC 6809.

Two or even three populations are known in the clusters (Di Criscienzo, D’Antona & Ventura 2010; VandenBerg & Denissenkov 2018). All the populations of both the clusters are  $\alpha$ -enriched with  $[\alpha/\text{Fe}] \approx 0.4$  (Carretta et al. 2010; Rain et al. 2019; Mészáros et al. 2020).

The populations of each cluster differ in helium abundance  $Y$ . Milone et al. (2017) estimated the fraction of the first (primordial) population of stars as  $0.345 \pm 0.036$  in NGC 6397 and  $0.311 \pm 0.029$  in NGC 6809. VandenBerg & Denissenkov (2018) found the fractions of three populations in NGC 6809 as 51 per cent, 41 per cent, and 8 per cent with  $Y = 0.25, 0.265, \text{ and } 0.28$ , respectively. Mucciarelli et al. (2014) derived a nearly primordial average  $Y = 0.24 \pm 0.02$  for NGC 6397 using a large data set of helium abundances obtained with a high-resolution spectrograph. Milone et al. (2018) found a small average helium difference between the populations, as well as a small maximum internal helium variation:  $\Delta Y = 0.006 \pm 0.009$  and  $\Delta Y_{\text{max}} = 0.008 \pm 0.011$  for NGC 6397 and  $\Delta Y = 0.014 \pm 0.008$  and  $\Delta Y_{\text{max}} = 0.026 \pm 0.015$  for NGC 6809. Lagioia et al. (2021) found no evidence of intrinsic broadening of the AGB due to helium abundance variation for both NGC 6397 and NGC 6809. However, such broadening of the RGB is seen in some of our CMDs and may suggest that NGC 6809 has some stars with rather high  $Y$ , as discussed in Section 4.4. Kaluzny et al. (2014) derived  $Y \approx 0.25$  for NGC 6809 from their consistent mass–radius, mass–luminosity, and CMD fitting of the cluster’s eclipsing binary V54. We verify in Section 5 that  $Y \approx 0.26$  also does not contradict to the properties of

<sup>2</sup><http://stellar.dartmouth.edu/models/>

<sup>3</sup><http://basti-iac.oa-abruzzo.inaf.it/index.html>

<sup>4</sup>The solar-scaled Padova–TRieste Stellar Evolution Code (PARSEC, Bressan et al. 2012) isochrones, used by Chen et al. (2014), seem to be inappropriate for GCs.

**Table 2.** Some properties of the clusters under consideration.

Property	NGC 6397	NGC 6809 (M55)
RA J2000 (h m s) from Goldsbury et al. (2010)	17 40 42	19 40 00
Dec J2000 (° arcmin arcsec) from Goldsbury et al. (2010)	−53 40 28	−30 57 53
Galactic longitude (°) from Goldsbury et al. (2010)	338.1650	8.7926
Galactic latitude (°) from Goldsbury et al. (2010)	−11.9595	−23.2716
Tidal radius (arcmin) from Moreno, Pichardo & Velázquez (2014)	44.5	15.3
Angular radius (arcmin) from Bica et al. (2019)	11.5	19.0
Truncation radius (arcmin) from this study	41.0	18.0
Distance from the Sun (kpc) from Harris (1996), 2010 revision <sup>a</sup>	2.3	5.4
Distance from the Sun (kpc) from Baumgardt & Vasiliev (2021)	2.482 ± 0.019	5.348 ± 0.052
[Fe/H] from Carretta et al. (2009)	−1.99 ± 0.02	−1.93 ± 0.02
[Fe/H] from Mészáros et al. (2020)	−1.89 ± 0.09	−1.76 ± 0.07
[α/Fe] from Carretta et al. (2010)	+0.36	+0.42
Mean differential reddening $\Delta E(B - V)$ (mag) from BCK13	0.019 ± 0.009	0.027 ± 0.010
Maximum differential reddening $\Delta E(B - V)_{\max}$ (mag) from BCK13	0.051	0.050
$E(B - V)$ (mag) from Harris (1996), 2010 revision	0.18	0.08
$E(B - V)$ (mag) from Schlegel, Finkbeiner & Davis (1998)	0.19	0.14
$E(B - V)$ (mag) from Schlafly & Finkbeiner (2011)	0.16	0.12
$E(B - V)$ (mag) from Meisner & Finkbeiner (2015)	0.31	0.15
$E(B - V)$ (mag) from Gontcharov et al. (2022)	0.17	0.12

Note.

<sup>a</sup>The commonly used data base of GCs by Harris (1996) (<https://www.physics.mcmaster.ca/~harris/mwgc.dat>), 2010 revision.

V54. In Section 3, we take this information into account to select appropriate  $Y$  for our isochrone-to-data fitting.

Table 2 demonstrates that both NGC 6397 and NGC 6809 have rather accurate metallicity estimates from spectroscopy by Carretta et al. (2009). However, later [Fe/H] estimates demonstrate some issues. For example, for NGC 6809, Rain et al. (2019) found very low [Fe/H] =  $-2.01 \pm 0.02$  from their analysis of Ultraviolet and Visual Echelle Spectrograph (UVES) spectra of 11 stars, while Wang et al. (2017) found [Fe/H] =  $-1.86 \pm 0.06$  using the same technique applied to UVES and GIRAFFE spectra. Moreover, Mészáros et al. (2020) derived average [Fe/H] =  $-1.89 \pm 0.09$  and  $-1.76 \pm 0.07$  for NGC 6397 and NGC 6809, respectively, from high-resolution spectra of a hundred RGB stars of each cluster. Mészáros et al. (2020) found that their [Fe/H] are about 0.15 dex systematically higher than those from Carretta et al. (2009) and from the compilation of Harris (1996), not only for NGC 6397 and NGC 6809, but for the bulk of GCs in their sample. They noted that most of the [Fe/H] difference can be explained through the choice of the reference solar abundance mixture,<sup>5</sup> while the remaining difference may be due to wrong calibrations because of wrong reddening estimates or due to a systematic difference in the temperature scales used, or due to some effects which are not modelled yet. Besides, [Fe/H] estimates from photometry are not always consistent with each other and do not always agree with the estimates of Carretta et al. (2009): For example, Correnti et al. (2018) found [Fe/H] =  $-1.88 \pm 0.04$  for NGC 6397 from isochrone fitting of IR photometry of the faint MS. Moreover, Lovisi et al. (2012) used high-resolution spectra of NGC 6397 stars at various stages and found [Fe/H] =  $-2.12 \pm 0.01$  for the TO, while

<sup>5</sup>Mészáros et al. (2020) and Carretta et al. (2009) are based on the solar metallicity estimates from Grevesse, Asplund & Sauval (2007) and Gratton et al. (2003), respectively. Gontcharov & Mosenkov (2018) show that models of the Galaxy combined with stellar evolution models make solar metallicity tightly related to estimates of reddening/extinction across the whole Galactic dust layer above or below the Sun. Namely, the higher the reddening (more dusty Galaxy) the lower the solar metallicity. In combination with the discussion of Mészáros et al. (2020), this shows a relation between [Fe/H] of GCs and the amount of dust in our Galaxy.

−1.20 ± 0.22 for blue stragglers and −1.94 ± 0.14 for the HB. To explain such discrepancies, Jain et al. (2020) analyse a variation of metallicity between the TO and RGB through the use of hundreds stellar spectra and conclude that at the low metallicity regime of [Fe/H] ≈ −2 both synthetic and empirical stellar spectra need to be improved against a considerable systematics. Also these discrepancies may be due to diffusive processes combined with convection affecting in different ways the stars in distinct evolutionary stages (Cassisi & Salaris 2020). Thus, we see that the current absolute accuracy of the iron scale seems to be no higher than ±0.1 dex and, hence, needs improvement and verification by different methods.

Photometry can provide [Fe/H] estimates for such a verification. The slopes of the RGB and faint MS (>3 mag fainter than TO), are sensitive to [Fe/H].<sup>6</sup> We get [Fe/H] as an isochrone fitting parameter (along with reddening, age, and distance) in CMDs with well-populated bright RGB or faint MS. The average values of the derived [Fe/H] estimates are used for fitting the remaining CMDs. We consider it separately for each model.

However, both the bright RGB and faint MS are affected by helium enrichment (Savino et al. 2018), crowding or poor astrometry at the cluster field centres, saturation and completeness effects, and systematic errors of photometry. These effects may result in a systematic uncertainty of about 0.2 dex in our [Fe/H] estimate obtained from the pair of a CMD and a model.

Table 2 indicates a relatively small foreground and differential reddening (DR) for NGC 6397 and NGC 6809. However, the reddening estimates in Table 2 are not fully consistent when taking into account their stated precision as a few hundredths of a magnitude, which is confirmed by Gontcharov & Mosenkov (2018). Namely, the outliers are a very high reddening estimate by Meisner & Finkbeiner (2015) for NGC 6397 and a very low estimate by Harris (1996) for NGC 6809. Interestingly, in Paper IV, the reddening estimate by Meisner & Finkbeiner (2015) is the higher outlier for NGC 6362, which is located in the same fourth Galactic quadrant as NGC 6397,

<sup>6</sup>Note that isochrones show large systematic errors in the faint MS domain. Hence, we consider this domain with caution (see Section 5.2).

while the reddening estimate by Harris (1996) is the lower outlier for NGC 6723, which is located in the same first Galactic quadrant as NGC 6809. This may be a result of spatial variations of dust medium properties.

Among the reddening estimates in Table 2, a pair is taken from the new version of our three-dimensional analytical model of dust spatial distribution (Gontcharov et al. 2022), whose predictions agree with isochrone-to-CMD reddening estimates for most middle- and high-latitude GCs.

A mild DR across the field of these clusters is shown by Alonso-García et al. (2012), Milone et al. (2012), and Bonatto, Campos & Kepler (2013, hereafter BCK13). We correct data sets with a large number of cluster members for DR in Section 4.5.

### 3. THEORETICAL MODELS AND ISOCHRONES

We use the following theoretical stellar evolution models and corresponding  $\alpha$ -enhanced isochrones to fit the CMDs of NGC 6397 and NGC 6809:

(i) BaSTI (Hidalgo et al. 2018; Pietrinferni et al. 2021) with various [Fe/H] and helium abundance,  $[\alpha/\text{Fe}] = +0.4$ , initial solar  $Z_{\odot} = 0.0172$  and  $Y_{\odot} = 0.2695$ , diffusion, overshooting, and mass-loss efficiency  $\eta = 0.3$ , where  $\eta$  is the free parameter in Reimers law (Reimers 1975). As in Paper III and in Paper IV, we also apply the BaSTI extended set of zero-age horizontal branch (ZAHB) models with the same mass for the helium core and the same envelope chemical stratification but different values for the total mass. This set seems to be a realistic description of stochastic mass-loss between the MS and HB.

(ii) DSED (Dotter et al. 2008) with various [Fe/H] and helium abundance,  $[\alpha/\text{Fe}] = +0.4$ , solar  $Z_{\odot} = 0.0189$ , and no mass-loss. DSED gives no realistic ZAHB with a stochastic mass-loss taken into account. However, DSED provides the HB and AGB isochrones for some filters. We do not use these isochrones for our fitting, following a recommendation by the DSED team (Dotter, private communication). Yet, we present these isochrones in some our CMD figures in order to show that an acceptable description of the HB and AGB is possible with DSED too.

In order to fix an appropriate  $Y$  for isochrone fitting of each cluster, we fit isochrones with different  $Y$  to all CMDs under consideration and conclude that BaSTI isochrones with  $Y > 0.25$  better fit the blue AGB domain, but only for NGC 6809 and only for data sets covering the whole cluster field, while both BaSTI and DSED isochrones with  $Y > 0.25$  better fit the faint RGB domain in all CMDs. As noted in Paper IV, this widening of the faint RGB may be due to a segregation into two populations, with higher and lower helium abundance. The remaining CMD domains are fitted by the  $Y = 0.267$  and  $0.25$  isochrones equally well, since these isochrones almost coincide. Thus, for our isochrone-to-CMD fitting, we adopt  $Y = 0.25$  for an unresolved mix of the populations in both the clusters, except the blue AGB and faint RGB, which are fitted with  $Y = 0.267$  (see Section 4.4). Only the isochrones with  $Y = 0.25$  are shown in our CMD figures for clarity.

We fit the isochrones for a grid within  $-2.4 < [\text{Fe}/\text{H}] < -1.5$  with a step of 0.1 dex, distances within  $\pm 0.8$  kpc from the Baumgardt & Vasiliev (2021) estimates with a step of 0.01 kpc, reddenings between zero and twice the highest reddening estimate from Table 2 with a step of 0.001 mag, and ages within 8–18 and 8–15 Gyr for BaSTI and DSED, respectively, with a step of 0.5 Gyr.

## 4. DATA SETS

### 4.1 Initial data sets

The following data sets (hereafter *twin* data sets, see Table 3) are used for both the clusters:

(i) The *HST* Wide Field Camera 3 (WFC3) UV Legacy Survey of Galactic Globular Clusters (the *F275W*, *F336W*, and *F438W* filters) and the Wide Field Channel of the ACS (the *F606W* and *F814W* filters) Survey of Galactic Globular Clusters (Piotto et al. 2015; NLP18),<sup>7</sup> with additional photometry of the same RGB, SGB, and MS stars of NGC 6397 in the WFC3 *F467M*, ACS *F435W*, and ACS *F625W* filters (Libralato et al. 2022),<sup>8</sup>

(ii) Parallel-Field Catalogues (the ACS *F475W* and *F814W* filters) of the *HST* UV Legacy Survey of Galactic Globular Clusters (SBA18),<sup>9</sup>

(iii) *UBVRI* photometry from various ground-based telescopes processed by SPZ19,<sup>10</sup> with the NGC 6397 and NGC 6809 data sets processed within the same pipeline and presented recently,<sup>11</sup>

(iv) *Gaia* DR3 photometry in the *G*, *G<sub>BP</sub>*, and *G<sub>RP</sub>* filters (Riello et al. 2021),<sup>12</sup>

(v) SkyMapper Southern Sky Survey (SMSS) DR3, i.e. SMSS DR3 photometry in the *v<sub>SMSS</sub>*, *g<sub>SMSS</sub>*, *r<sub>SMSS</sub>*, *i<sub>SMSS</sub>*, and *z<sub>SMSS</sub>* filters (Onken et al. 2019),<sup>13</sup>

(vi) *J<sub>VISTA</sub>* and *K<sub>S<sub>VISTA</sub></sub>* photometry of the *VISTA* Hemisphere Survey (VHS) with the VIRCAM instrument on the *Visible and Infrared Survey Telescope for Astronomy (VISTA)*, i.e. VHS DR5 (McMahon et al. 2013),<sup>14</sup> and

(vii) *WISE* photometry in the *W1* filter from the unWISE catalogue (Schlafly, Meisner & Green 2019).<sup>15</sup>

The following data sets are used for one of the clusters (see Table 3):

(i) *HST/ACS* photometry of NGC 6397 in the *F606W* and *F814W* filters (Richer et al. 2008),<sup>16</sup>

(ii) Photometry of NGC 6397 in the *F439W* and *F555W* filters from the *HST* Wide Field and Planetary Camera 2 (WFPC2, Piotto et al. 2002),<sup>17</sup>

(iii) Strömgren *uvby* photometry of NGC 6397 with the 1.54-m *Danish* telescope, European Southern Observatory (ESO), La Silla (Grundahl et al. 1999, hereafter GCL99),

(iv) CCD Strömgren *uvby* and *V* photometry of NGC 6397 with the 0.9-m telescope at Cerro-Tololo Inter-American Observatory (CTIO, AT2000),<sup>18</sup>

<sup>7</sup><http://groups.dfa.unipd.it/ESPG/treasury.php>

<sup>8</sup><https://archive.stsci.edu/hlsp/hacks>

<sup>9</sup><http://groups.dfa.unipd.it/ESPG/treasury.php>

<sup>10</sup><http://cdsarc.u-strasbg.fr/viz-bin/cat/J/MNRAS/485/3042>

<sup>11</sup>[https://www.canfar.net/storage/vault/list/STETSON/homogeneous/Latest\\_photometry\\_for\\_targets\\_with\\_at\\_least\\_BVI](https://www.canfar.net/storage/vault/list/STETSON/homogeneous/Latest_photometry_for_targets_with_at_least_BVI)

<sup>12</sup>In Paper IV, we checked that the DSED isochrones for DR2 are equally suitable for DR3 and, hence, they are shown in our CMDs with the DR3 data.

<sup>13</sup><https://skymapper.anu.edu.au>

<sup>14</sup><https://cdsarc.cds.unistra.fr/viz-bin/ReadMe/II/367?format=html&tex=true>; DSED does not provide isochrones for *VISTA* filters. However, as in Paper IV, *J<sub>VISTA</sub>* is substituted by *J<sub>UKIDSS</sub>*, a filter from the United Kingdom Infrared Telescope Infrared Deep Sky Survey (UKIDSS; Hewett et al. 2006), with a precision better than 0.01 mag.

<sup>15</sup><https://cdsarc.cds.unistra.fr/viz-bin/cat/II/363>

<sup>16</sup><https://cdsarc.cds.unistra.fr/viz-bin/cat/J/AJ/135/2141>

<sup>17</sup><http://groups.dfa.unipd.it/ESPG/hstphot.html>

<sup>18</sup><https://cdsarc.cds.unistra.fr/viz-bin/cat/J/AJ/120/3111>

**Table 3.** The effective wavelength  $\lambda_{\text{eff}}$  (nm), number of stars, and the median precision of the photometry (mag) for the data sets and filters under consideration. For data sets cross-identified with *Gaia* DR3, only *Gaia* cluster members are counted.

Telescope, data set, and reference	Filter	$\lambda_{\text{eff}}$	Number of stars/median precision NGC 6397	NGC 6809
<i>HST</i> /WFC3 (NLP18)	<i>F275W</i>	285	5093/0.01	7007/0.01
<i>HST</i> /WFC3 (NLP18)	<i>F336W</i>	340	6514/0.01	9212/0.01
1.54-m <i>Danish</i> telescope, ESO, La Silla (GCL99)	Strömgren <i>u</i>	349	5207/0.02	–
0.9-m <i>CTIO</i> telescope (AT2000)	Strömgren <i>u</i>	349	2466/0.02	–
Various (SPZ19)	<i>U</i>	366	11 403/0.02	7529/0.02
SkyMapper Sky Survey DR3 (Onken et al. 2019)	<i>v</i> SMSS	385	3494/0.02	–
1.54-m <i>Danish</i> telescope, ESO, La Silla (GCL99)	Strömgren <i>v</i>	414	5619/0.01	–
0.9-m <i>CTIO</i> telescope (AT2000)	Strömgren <i>v</i>	414	2470/0.01	–
<i>HST</i> /ACS (Libralato et al. 2022)	<i>F435W</i>	434	11 678/0.01	–
<i>HST</i> /WFC3 (NLP18)	<i>F438W</i>	438	7339/0.01	11 207/0.01
<i>HST</i> /WFPC2 (Piotto et al. 2002)	<i>F439W</i>	452	5407/0.01	–
Various (SPZ19)	<i>B</i>	452	14 414/0.01	7813/0.01
<i>VLT</i> , ESO (Nardiello et al. 2015)	<i>B</i>	452	1006/0.01	–
1-m <i>Swope</i> telescope, Las Campanas (Narloch et al. 2017)	<i>B</i>	452	–	9892/0.01
0.9-m <i>CTIO</i> telescope (Kaluzny 1997)	<i>B</i>	452	4496/0.03	–
2.5-m <i>Du Pont</i> telescope, Las Campanas (Kaluzny et al. 2010)	<i>B</i>	452	–	7271/0.01
2.5-m <i>Du Pont</i> telescope, Las Campanas (MFR)	<i>B</i>	452	–	Fiducial
<i>HST</i> /WFC3 (Libralato et al. 2022)	<i>F467M</i>	467	7805/0.01	–
1.54-m <i>Danish</i> telescope, ESO, La Silla (GCL99)	Strömgren <i>b</i>	467	5714/0.01	–
0.9-m <i>CTIO</i> telescope (AT2000)	Strömgren <i>b</i>	467	2476/0.01	–
<i>HST</i> /ACS (SBA18)	<i>F475W</i>	475	4890/0.06	5627/0.07
Pan-STARRS (Chambers et al. 2016)	<i>g</i> PS1	496	–	4326/0.01
<i>Gaia</i> DR3 (Riello et al. 2021)	<i>G</i> BP	505	17 312/0.02	8828/0.03
SkyMapper Sky Survey DR3 (Onken et al. 2019)	<i>g</i> SMSS	514	7337/0.02	3332/0.02
1.54-m <i>Danish</i> telescope, ESO, La Silla (GCL99)	Strömgren <i>y</i>	548	5713/0.01	–
0.9-m <i>CTIO</i> telescope (AT2000)	Strömgren <i>y</i>	548	2476/0.01	–
<i>HST</i> /WFPC2 (Piotto et al. 2002)	<i>F555W</i>	551	5407/0.02	–
Various (SPZ19)	<i>V</i>	552	14 432/0.01	7816/0.01
<i>VLT</i> , ESO (Nardiello et al. 2015)	<i>V</i>	552	1020/0.01	–
1-m <i>Swope</i> telescope, Las Campanas (Narloch et al. 2017)	<i>V</i>	552	–	9892/0.01
0.9-m <i>CTIO</i> telescope (Kaluzny 1997)	<i>V</i>	552	4496/0.02	–
2.5-m <i>Du Pont</i> telescope, Las Campanas (Kaluzny et al. 2010)	<i>V</i>	552	–	7271/0.01
0.9-m <i>CTIO</i> telescope (AT2000)	<i>V</i>	552	2476/0.01	–
2.5-m <i>Du Pont</i> telescope, Las Campanas (MFR)	<i>V</i>	552	–	Fiducial
Two telescopes (Ahumada et al. 2021)	<i>V</i>	552	12 878/0.02	–
<i>HST</i> /ACS (NLP18)	<i>F606W</i>	599	12 386/0.01	21 417/0.01
<i>HST</i> /ACS (Richer et al. 2008)	<i>F606W</i>	599	2324/0.01	–
<i>Gaia</i> DR3 (Riello et al. 2021)	<i>G</i>	604	17 312/0.01	8828/0.01
SkyMapper Sky Survey DR3 (Onken et al. 2019)	<i>r</i> SMSS	615	8581/0.02	3805/0.02
<i>HST</i> /ACS (Libralato et al. 2022)	<i>F625W</i>	633	12 531/0.01	–
Various (SPZ19)	<i>R</i>	659	7730/0.01	7642/0.01
Pan-STARRS (Chambers et al. 2016)	<i>i</i> PS1	752	–	4326/0.01
<i>Gaia</i> DR3 (Riello et al. 2021)	<i>G</i> RP	770	17 312/0.01	8828/0.02
SkyMapper Sky Survey DR3 (Onken et al. 2019)	<i>i</i> SMSS	776	8387/0.02	4069/0.02
<i>HST</i> /ACS (NLP18)	<i>F814W</i>	807	12 346/0.01	21 417/0.01
<i>HST</i> /ACS (SBA18)	<i>F814W</i>	807	4890/0.03	5627/0.03
<i>HST</i> /ACS (Richer et al. 2008)	<i>F814W</i>	807	2324/0.01	–
<i>HST</i> /ACS (Libralato et al. 2022)	<i>F814W</i>	807	12 024/0.01	–
Various (SPZ19)	<i>I</i>	807	14 423/0.01	7816/0.01
<i>VLT</i> , ESO (Nardiello et al. 2015)	<i>I</i>	807	960/0.01	–
2.5-m <i>Du Pont</i> telescope, Las Campanas (MFR)	<i>I</i>	807	–	Fiducial
Two telescopes (Ahumada et al. 2021)	<i>I</i>	807	12 878/0.02	–
SkyMapper Sky Survey DR3 (Onken et al. 2019)	<i>z</i> SMSS	913	7502/0.02	3324/0.02
2MASS (Skrutskie et al. 2006)	<i>J</i> <sub>2MASS</sub>	1234	4652/0.08	–
<i>VISTA</i> VHS DR5 (McMahon et al. 2013)	<i>J</i> <sub>VISTA</sub>	1277	17 984/0.02	9292/0.02
<i>VISTA</i> VHS DR5 (McMahon et al. 2013)	<i>K</i> <sub>S</sub> VISTA	2148	16 182/0.05	6753/0.07
<i>WISE</i> , unWISE (Schlafly, Meisner & Green 2019)	<i>W1</i>	3317	1052/0.01	1116/0.01

**Table 4.** The results of our isochrone fitting for two models and some key CMDs for both the clusters. The colour is the abscissa and the magnitude in the redder filter is the ordinate in all the CMDs, except the *AT2000* CMDs where  $u - v$ ,  $v - b$ , or  $b - y$  are the abscissas and  $V$  is the ordinate. Each derived reddening is followed by its empirical systematic uncertainty and corresponding  $E(B - V)$ , given in parentheses and calculated using extinction coefficients from Casagrande & VandenBerg (2014), Casagrande & VandenBerg (2018a), Casagrande & VandenBerg (2018b), or Cardelli, Clayton & Mathis (1989, hereafter CCM89) with  $R_V = 3.1$ .  $[\text{Fe}/\text{H}]$  is given only for CMDs, which allow its calculation as a fitting parameter. The complete table is available online.

Quantity	NGC 6397		NGC 6809	
	DSED	BaSTI	DSED	BaSTI
		<i>NLP18</i>		
$E(F606W - F814W)$ (mag)	$0.199 \pm 0.04$ (0.19)	$0.184 \pm 0.04$ (0.17)	$0.128 \pm 0.03$ (0.12)	$0.113 \pm 0.03$ (0.11)
age (Gyr)	13.5	13.5	13.0	13.0
distance (kpc)	2.49	2.46	5.22	5.18
$[\text{Fe}/\text{H}]$	-2.0	-1.9	-1.8	-1.7
		<i>Gaia DR3</i>		
$E(G_{\text{BP}} - G_{\text{RP}})$ (mag)	$0.320 \pm 0.03$ (0.21)	$0.288 \pm 0.03$ (0.19)	$0.232 \pm 0.03$ (0.15)	$0.198 \pm 0.03$ (0.13)
age (Gyr)	12.5	13.0	12.5	13.0
distance (kpc)	2.39	2.34	5.04	5.00
$[\text{Fe}/\text{H}]$	-1.8	-1.8	-1.7	-1.7
		<i>SPZ19</i>		
$E(B - V)$ (mag)	$0.162 \pm 0.03$ (0.18)	$0.157 \pm 0.03$ (0.18)	$0.107 \pm 0.03$ (0.12)	$0.098 \pm 0.03$ (0.11)
age (Gyr)	13.0	13.0	13.5	13.5
distance (kpc)	2.43	2.44	5.12	5.18
$[\text{Fe}/\text{H}]$	-1.8	-1.8	-1.8	-1.8
...	...	...	...	...

(v) *BVI* photometry of NGC 6397 with the FOcal Reducer and low-dispersion Spectrograph 2 mounted at the *Very Large Telescope* (*VLT*) UT1 of the ESO (Nardiello et al. 2015),<sup>19</sup>

(vi) *BV* photometry of NGC 6397 with the 0.9-m *CTIO* telescope (Kaluzny 1997),<sup>20</sup>

(vii) *VI* photometry of NGC 6397 with the 1.54-m telescope of the Bosque Alegre Astrophysical Station of the Córdoba Observatory, National University of Córdoba, Argentina and 1-m *Swope* telescope of Las Campanas Observatory, Chile (Ahumada et al. 2021),

(viii) The Two Micron All Sky Survey (2MASS) photometry of NGC 6397 in the  $J_{2\text{MASS}}$  filter (Skrutskie et al. 2006),

(ix) *BV* photometry of NGC 6809 with the 2.5-m *Du Pont* telescope at Las Campanas Observatory (Kaluzny et al. 2010),<sup>21</sup>

(x) *BV* photometry of NGC 6809 with the 1-m *Swope* telescope of Las Campanas Observatory (Narloch et al. 2017),<sup>22</sup>

(xi) The fiducial sequences for NGC 6809 in the *BVI* filters derived by Mandushev et al. (1996, hereafter *MFR*) and Mandushev (1998) from the photometry with the 2.5-m *Du Pont* telescope of the Las Campanas Observatory, and

(xii) The Panoramic Survey Telescope and Rapid Response System (Pan-STARRS) Data Release I, i.e. PS1 (Chambers et al. 2016) photometry of NGC 6809 in the  $g_{\text{PS1}}$  and  $i_{\text{PS1}}$  filters.<sup>23</sup>

Most of these data sets have never been isochrone-fitted before, since they appeared recently.

Three data sets with the *HST/ACS* photometry, i.e. presented by (i) Richer et al. (2008), (ii) *SBA18*, and (iii) *NLP18* together with the photometry from Libralato et al. (2022) for some of the same

stars, have little, if any, common stars. The Strömgren photometry data sets of *GCL99* and *AT2000* are independent. All of the *BVI* data sets from *SPZ19*, Nardiello et al. (2015), Kaluzny (1997), Kaluzny et al. (2010), Narloch et al. (2017), and *MFR* are independent. The *SPZ19* data sets contain photometry from various initial data sets, but not from the others under consideration.

Each star has photometry in some but not all filters. In total, 32 and 23 filters are used for NGC 6397 and NGC 6809, respectively, spanning a wavelength range between the UV and middle IR. Table 3 presents the effective wavelength  $\lambda_{\text{eff}}$  in nm, number of stars, and the median photometric precision (after the cleaning of the data sets, which is described below) for each filter. We calculate the median precision from the precision statements by the authors of the data sets. Then we apply it to evaluate the uncertainties of our results (see appendix A of Paper II and Section 5).

To clean the data sets, we generally follow the recommendations of their authors to select single star-like objects with reliable photometry. Typically, stars with a photometric uncertainty  $< 0.12$  mag are selected, while for some data sets we apply a higher or lower cut level between 0.08 and 0.2 mag. For the *HST* WFC3 and ACS photometry, we use stars with  $|sharp| < 0.15$ , membership probability  $> 0.9$  or  $-1$ , and quality fit  $> 0.9$ . For the *SPZ19* data sets, we use stars with DAOPHOT parameters  $\chi < 3$  and  $|sharp| < 0.3$ . For the SMSS DR3 data sets, we select star-like objects (i.e. with ClassStar  $> 0.5$ ) and with flags  $< 8$ . For the data set of *GCL99*, we select stars with  $\chi < 3$  and  $|sharp| < 0.3$ . For the data set of Nardiello et al. (2015), we select stars with the quality of point spread function fit parameter  $< 0.5$ . For the data set of Kaluzny (1997), we select stars with all quality flags 0. For the Narloch et al. (2017) data set, we use stars with a cluster member probability higher than 0.5.

#### 4.2 *Gaia* DR3 cluster members

Similar to Paper IV, accurate *Gaia* DR3 parallaxes and PMs are used to select cluster members and derive systemic parallaxes and PMs. The distribution of the *Gaia* DR3 data sets, selected within the truncation radii of the clusters, over the PM components is presented

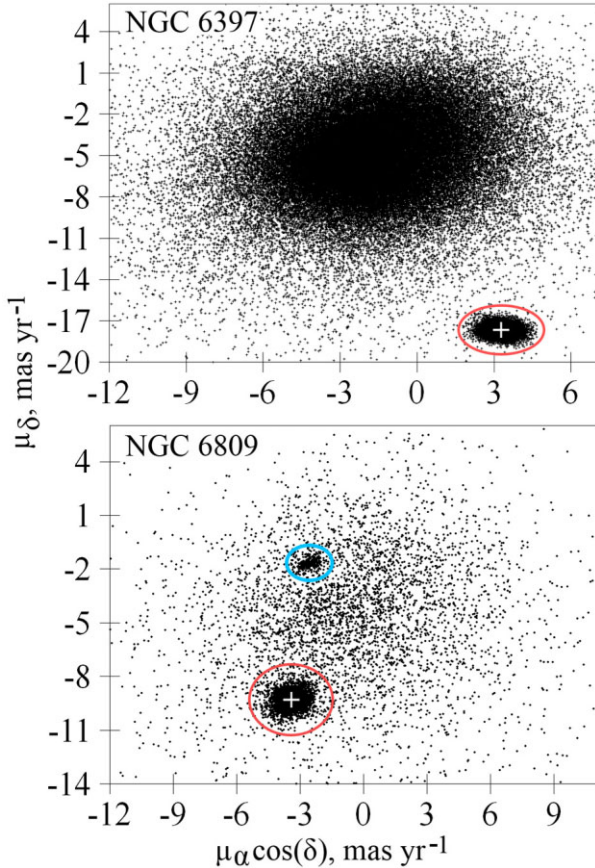
<sup>19</sup><http://groups.dfa.unipd.it/ESPG/followup.html>. We do not use the photometry in the  $U$  filter, since the BaSTI and DSED isochrones cannot fit its faint MS with reliable parameters, while Nardiello et al. (2015) provide few stars with the  $U$  photometry in the remaining CMD domains.

<sup>20</sup><https://cdsarc.cds.unistra.fr/viz-bin/cat/J/A+AS/122/1>

<sup>21</sup><https://case.camk.edu.pl/results/Photometry/M55/index.html>

<sup>22</sup><https://cdsarc.cds.unistra.fr/viz-bin/cat/J/MNRAS/471/1446>

<sup>23</sup>NGC 6809 is near the declination limit of PS1 at about  $-30^\circ$  and, hence, we use only photometry in  $g_{\text{PS1}}$  and  $i_{\text{PS1}}$ , which has the best quality.



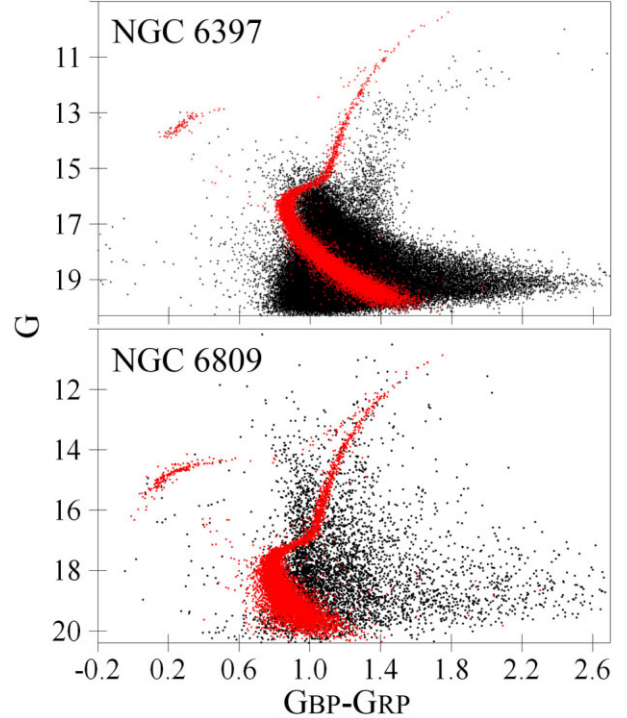
**Figure 1.** The distribution of the *Gaia* DR3 data sets, selected within the truncation radii of the clusters, over the PM components ( $\text{mas yr}^{-1}$ ), after the remaining cleaning of the sample. To make the figure clearer, we only show stars with precise PM components ( $<1.5 \text{ mas yr}^{-1}$ ) and photometry in all filters ( $<0.07$  and  $<0.1$  mag for NGC 6397 and NGC 6809, respectively). The weighted mean PM and the selection area for the clusters are shown by the crosses and circles around them, respectively. The smaller circle shows a concentration of the Sagittarius dwarf galaxy members behind NGC 6809.

in Fig. 1. The cluster members are those inside the red circles. It is seen that members can be separated from the foreground and background stars.

We now briefly describe the selection of the members. As seen in Table 2, Moreno, Pichardo & Velázquez (2014) and Bica et al. (2019) provide different estimates for the tidal radii of these GCs. Therefore, first, we consider initial *Gaia* DR3 samples within initial radii which exceed any previous estimate. The cluster centre coordinates are taken from Goldsbury et al. (2010).

Second, we find empirical truncation radii of 41 and 18 arcmin for NGC 6397 and NGC 6809, respectively, as the radii where the star count surface density drops to the Galactic background. All the data sets, except for the MFR data set with fiducial sequences only, are truncated at these radii to reduce contamination from non-members.

Third, we leave only stars with PMs; duplicated\_source = 0 (Dup = 0), i.e. sources without multiple source identifiers; astrometric\_excess\_noise < 1 ( $\epsilon_i < 1$ ); a renormalized unit weight error not exceeding 1.4 (RUWE < 1.4); available data in all three *Gaia* filters with a precision  $<0.12$  mag; and a corrected excess factor phot\_bp\_rp\_excess\_factor (i.e. E(BP/RP) Corr) between  $-0.14$  and  $0.14$  (Riello et al. 2021). Note that this cleaning



**Figure 2.**  $G_{BP} - G_{RP}$  versus  $G$  CMDs for the stars included (red symbols) and excluded (black symbols) by their parallaxes and PMs after the remaining cleaning of the *Gaia* DR3 samples. For clearer figure, only stars with precise PM components ( $<1.5 \text{ mas yr}^{-1}$ ) and photometry in all filters ( $<0.07$  and  $<0.1$  mag for NGC 6397 and NGC 6809, respectively) are shown. A significant background of the clusters is the MS stars (at  $G_{BP} - G_{RP} \approx 1$  and  $G < 16$  mag) and giants (at  $G_{BP} - G_{RP} \approx 1.4$  and  $G < 16$  mag) of Sagittarius dwarf galaxy.

removes almost all stars of the *Gaia* DR3 data sets within a central arcminute of both the cluster fields.

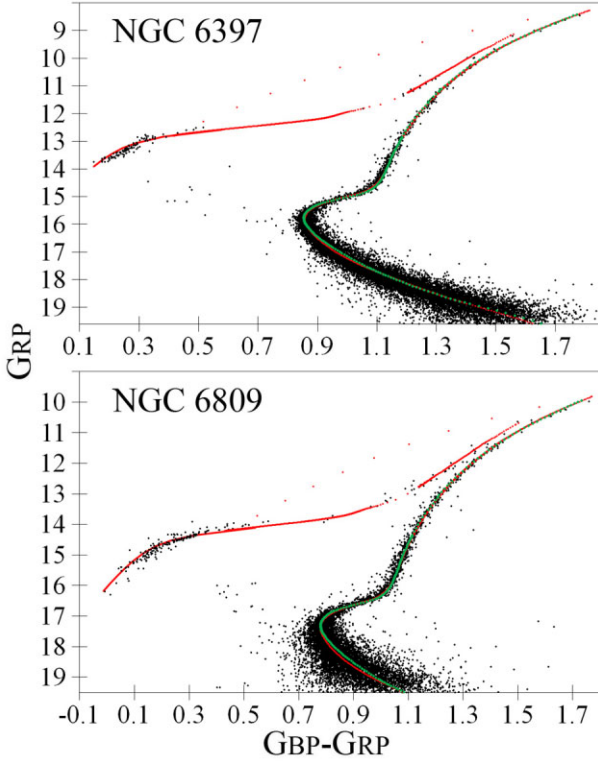
Fourth, foreground and background stars are rejected as those with an inappropriate parallax (see Paper IV).

Fifth, we begin with the initial systemic PM components  $\overline{\mu_\alpha \cos(\delta)}$  and  $\overline{\mu_\delta}$  from Vasiliev & Baumgardt (2021, hereafter VB21), calculate the standard deviations  $\sigma_{\mu_\alpha \cos(\delta)}$  and  $\sigma_{\mu_\delta}$  of the PM components for the cluster members, cut off the sample at  $3\sigma$ , and recalculate the weighted mean systemic PM components. We repeat this procedure iteratively until it stops losing stars in the  $3\sigma$  cut. Faint cluster members with less certain PMs make a negligible contribution to the weighted mean systemic PMs.

The final empirical standard deviations  $\sigma_{\mu_\alpha \cos(\delta)} = 0.38$  and  $\sigma_{\mu_\delta} = 0.33 \text{ mas yr}^{-1}$  for NGC 6397 and NGC 6809, respectively (averaged for the PM components), are reasonable, but significantly higher than the mean stated PM uncertainties (0.15 and 0.21  $\text{mas yr}^{-1}$ , respectively), which may mean an underestimation of the latter.

The CMDs of the *Gaia* DR3 stars from Fig. 1 are shown in Fig. 2. It is seen that Sagittarius dwarf galaxy is a major background contaminant for both the clusters. For NGC 6809 it is clearly seen in Fig. 1. Fig. 2 shows that the galaxy's members also dominate among bright non-members ( $G < 16$  mag) of NGC 6397: The RGB of the galaxy is seen as a bulk of stars several magnitudes fainter than the RGB of NGC 6397.

Fig. 3 presents the final  $G_{BP} - G_{RP}$  versus  $G_{RP}$  CMDs for the *Gaia* DR3 clusters members after correction for DR described in Section 4.5.



**Figure 3.**  $G_{BP} - G_{RP}$  versus  $G_{RP}$  CMDs for the *Gaia* DR3 clusters members after correction for DR. The isochrones from BaSTI (red) and DSED (green) for  $Y = 0.25$  are calculated with the best-fitting parameters from Table 4.

**Table 5.** The cluster systemic PMs ( $\text{mas yr}^{-1}$ ). The statistic uncertainties are given for the PMs from this study and from Vitral (2021), while the total (statistic plus systematic) uncertainty is given for the PMs from VB21. The latter is adopted by us as the most realistic estimate of the PM uncertainties.

Cluster	Source	$\mu_{\alpha} \cos(\delta)$	$\mu_{\delta}$
NGC 6397	This study	$3.260 \pm 0.010$	$-17.660 \pm 0.010$
	VB21	$3.260 \pm 0.023$	$-17.665 \pm 0.022$
	Vital (2021)	$3.256 \pm 0.003$	$-17.654 \pm 0.003$
NGC 6809	This study	$-3.430 \pm 0.010$	$-9.310 \pm 0.010$
	VB21	$-3.431 \pm 0.025$	$-9.311 \pm 0.024$
	Vital (2021)	$-3.431 \pm 0.003$	$-9.315 \pm 0.003$

Our final weighted mean systemic PMs are presented in Table 5 in comparison to those from VB21 and Vitral (2021). Being obtained from *Gaia* DR3 by different approaches, these estimates are, nevertheless, consistent within  $\pm 0.01 \text{ mas yr}^{-1}$ , i.e. well beneath the *Gaia* DR3 PM systematic errors (about  $0.02 \text{ mas yr}^{-1}$ ), which are estimated by VB21. Since only the statistic uncertainties are evaluated for ours and Vitral (2021)'s estimates, we adopt the dominating systematic uncertainties as the final ones of our PMs.

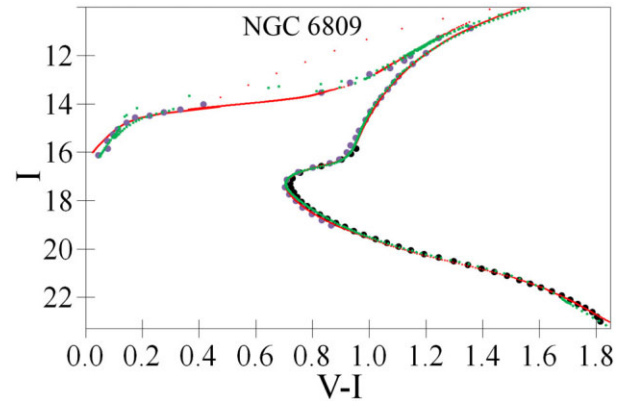
Similarly, we adopt the total uncertainty of *Gaia* DR3 parallaxes, found by VB21 as 0.01 mas, for our median parallaxes of cluster members. We correct them for the parallax zero-point following Lindegren et al. (2021) and present them in Table 6 for comparison with other estimates in Section 5.4. Table 7 contains the final lists of the *Gaia* DR3 cluster members.

**Table 6.** Various parallax estimates (mas) for NGC 6397 and NGC 6809 with their total (statistic and systematic) uncertainties.

Parallax	NGC 6397	NGC 6809
VB21, <i>Gaia</i> astrometry	$0.414 \pm 0.010$	$0.206 \pm 0.010$
This study, <i>Gaia</i> astrometry	$0.416 \pm 0.010$	$0.203 \pm 0.010$
This study, isochrone fitting	$0.408 \pm 0.014$	$0.191 \pm 0.007$

**Table 7.** The list of the *Gaia* DR3 members of NGC 6397 and NGC 6809. The complete table is available online.

NGC 6397	NGC 6809
5921306965291992704	6751230058605062400
5921306999650258688	6751240954939935872
5921307553705108864	6751241053721334144
5921307789925746304	6751241058019152384
5921308137820609792	6751241500397868928
...	...



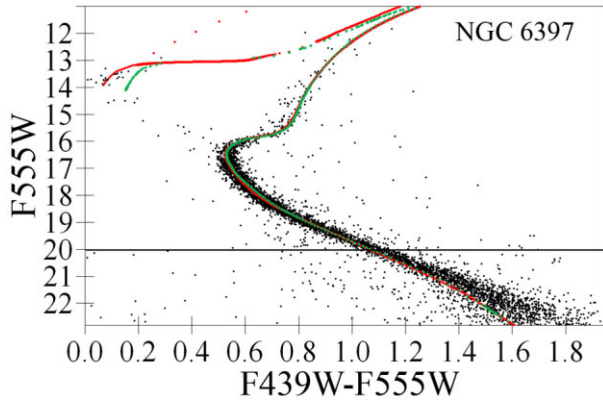
**Figure 4.**  $V - I$  versus  $I$  CMD with the fiducial sequences from the MFR (darker circles) and Mandushev (1998) (lighter circles) observational run for NGC 6809. The isochrones from BaSTI (red) and DSED (green diamonds) for  $Y = 0.25$  are calculated with the best-fitting parameters from Table 4. Appropriate DSED predictions for the HB and AGB are shown by the green squares for illustration purposes.

### 4.3 Cluster members in other data sets

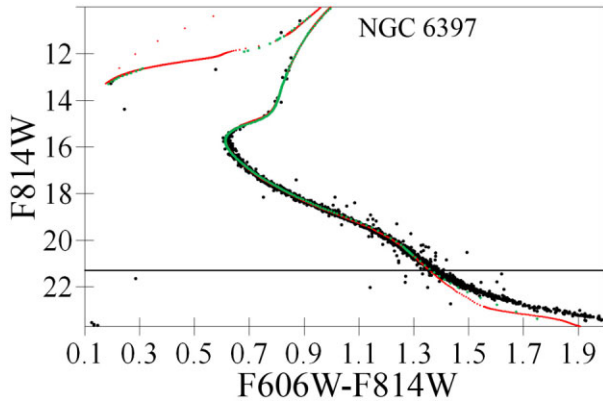
Almost all authors of the data sets under consideration made an effort to select cluster members. This cleaning is acceptable for the MFR and Piotto et al. (2002) data sets. The original star-by-star data for the MFR data set are not available (we use its fiducial sequences). Anyway, we cannot cross-identify both the data sets with *Gaia* to improve cluster member selection. This may introduce some additional systematic errors into corresponding results. The level of these errors is estimated from the comparison of our results for various data sets in Section 5. The CMDs for these data sets are presented in Figs 4 and 5, respectively. Two observational runs of MFR and Mandushev (1998), shown by different colours, indicate an insignificant systematic difference of 0.019 mag between their TO colours. We cannot fit the Piotto et al. (2002) faintest MS stars by any reliable BaSTI or DSED isochrone and, hence, ignore these stars.

Richer et al. (2008), NLP18, and Libralato et al. (2022) have cleaned their *HST* data sets from non-members by use of dedicated *HST* PMs: Their CMDs are presented in Figs 6, 7, and 8, respectively. Although imperfect, their membership selection cannot be





**Figure 5.** *HST*/WFPC2  $F439W - F555W$  versus  $F555W$  CMD from the Piotto et al. (2002) data set for NGC 6397. The isochrones from BaSTI (red) and DSED (green diamonds) for  $Y = 0.25$  are calculated with the best-fitting parameters from Table 4. Appropriate DSED predictions for the HB and AGB are shown by the green squares for illustration purposes. The horizontal line shows the cut of the faintest MS stars without a reliable isochrone fitting.

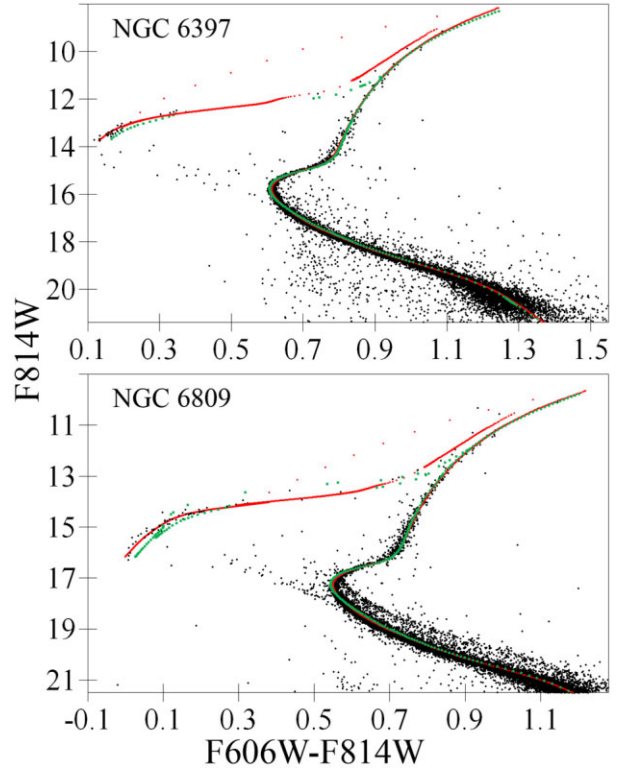


**Figure 6.** The same as Fig. 5 but for the *HST*/ACS  $F606W - F814W$  colour from the Richer et al. (2008) data set for NGC 6397. The black horizontal line shows the cut of the faintest MS stars without a reliable isochrone fitting.

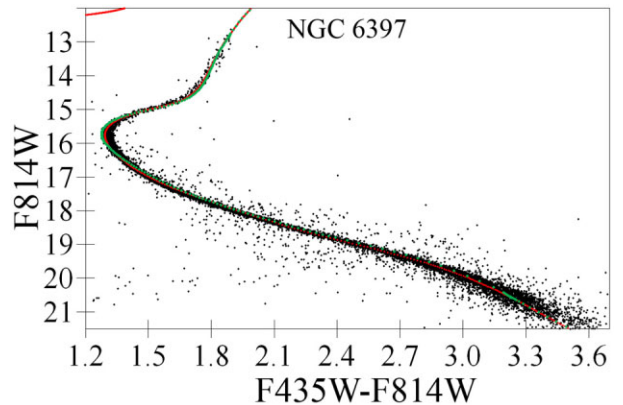
significantly improved through the use of the *Gaia* data, since these data sets cover only small fields with few *Gaia* stars.

Similar to the Piotto et al. (2002) faintest MS stars, we cannot fit those of the Richer et al. (2008) by any reliable BaSTI or DSED isochrone and, hence, ignore the faintest MS stars. For the Richer et al. (2008) data set, this magnitude limit is about  $F814W < 21.3$ . The other *HST*/ACS data sets of NLP18 and Libralato et al. (2022) are cut at the same magnitude due to observational limit. However, the *HST*/ACS data sets of SBA18, whose CMDs are presented in Fig. 9, allows a precise isochrone-to-data fitting by BaSTI and DSED down to a fainter  $F814W \approx 23$  mag. This may mean a systematic difference between the Richer et al. (2008) and SBA18 faintest MS stars probably due to systematic errors in the former (however, see discussion in Di Criscienzo, D’Antona & Ventura 2010). Anyway, the faint MS slope within about  $19 < F814W < 21.3$  allows us to derive  $[Fe/H]$  estimates for the Richer et al. (2008), NLP18, Libralato et al. (2022), and SBA18 data sets (see Table 4).

The *Gaia* DR3 cluster members are among only bright stars of the data sets of SBA18, while faint stars of these data sets draw rather clear CMDs. Therefore, we decide to derive cluster parameters from combined CMDs: We use only *Gaia* DR3 cluster members among



**Figure 7.** *HST*/ACS  $F606W - F814W$  versus  $F814W$  CMD from the (NLP18) data set. The isochrones from BaSTI (red) and DSED (green diamonds) for  $Y = 0.25$  are calculated with the best-fitting parameters from Table 4. Appropriate DSED predictions for the HB and AGB are shown by the green squares for illustration purposes.

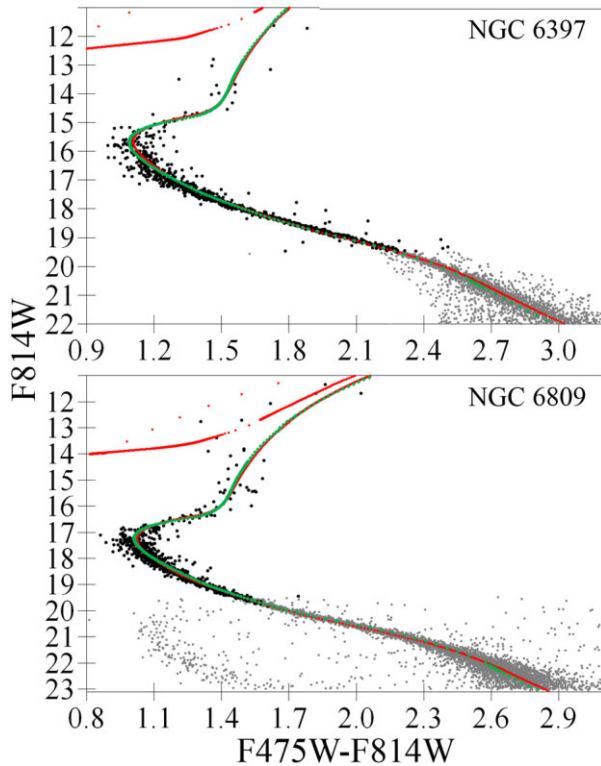


**Figure 8.** The same as Fig. 3 but for the *HST*/ACS  $F435W - F814W$  colour for NGC 6397 from Libralato et al. (2022).

bright SBA18 stars (about  $F814W < 19.5$ ) together with all faint SBA18 stars, as shown in Fig. 9.

Cluster members in the Ahumada et al. (2021) data set are reliably found by its authors using the method of Bustos Fierro & Calderón (2019) after cross-identification with *Gaia* DR2.

The remaining data sets are cross-identified with those of *Gaia* DR3 to reveal cluster members. The improvement is seen from a typical Fig. 10, where contaminated CMDs of NGC 6397 and NGC 6809 for the SPZ19 data sets are compared with the same CMDs for only *Gaia* DR3 members of these data sets. However, this improvement comes at the expense of a few faint magnitudes lost.



**Figure 9.** The same as Fig. 3 but for the *HST*/ACS  $F475W - F814W$  versus  $F814W$  CMDs with the *Gaia* DR3 cluster members from the SBA18 data sets (black) and remaining faint stars from the SBA18 data sets (grey).

The *Gaia* membership identification is especially important for NGC 6397 and NGC 6809 in order to overcome a bias due to a non-uniform distribution of the Sagittarius dwarf galaxy stars over their CMDs, as seen in Fig. 10.

#### 4.4 Isochrone-to-data fitting

Owing to the accurate selection of the cluster members, our CMDs are better defined than typical CMDs in the pre-*HST* and pre-*Gaia* era. Therefore, we can fit isochrones directly to a bulk of cluster members, without needing to calculate a fiducial sequence. In this case, the best solution corresponds to a minimal sum of the residuals between isochrone’s and data set points. We select the best isochrone among those calculated for the parameter grid mentioned in Section 3.

We have to exclude three CMD domains from the direct fitting: the blue HB, RR Lyrae variables, and blue stragglers, marked I, III, and IV in Fig. 11, respectively. The blue HB, i.e. the area bluer than the turn of the observed HB downward, is excluded, since even its best prediction deviates from the observations in typical CMD when its other domains are fitted well, as seen in Fig. 11. We fit the HB stars between the areas I and II by the BaSTI HB models with  $Y = 0.25$ , while the stars in the area II (blue AGB) are better fitted by the AGB isochrones with higher  $Y = 0.267$ , as noted in Section 3. Another CMD domain fitted with higher  $Y = 0.267$  is the faint RGB, marked V in Fig. 11 and mentioned in Section 3. All the remaining stars are fitted by isochrones with  $Y = 0.25$ .

To balance the contributions of different CMD domains, we assign a weight to each data point. The weight is inversely proportional to the number of stars of a given magnitude for a given data set, i.e. it reflects the luminosity function of a given data set.

Since we fit a zigzag pattern of an isochrone to a zigzag pattern of the bulk of stars, different parts of them are more sensitive to different parameters. Namely, reddening and distance correlate with the overall shift of the pattern along the abscissa (i.e. colour) and ordinate (i.e. magnitude), respectively. Therefore, nearly vertical and nearly horizontal parts of the pattern are more sensitive to the determination of reddening and distance, respectively. Similarly,  $[\text{Fe}/\text{H}]$  is more sensitive to the slopes of the RGB and faint MS. Finally, age correlates with the length of the SGB, as well as with the HB–SGB and SGB–MS magnitude differences, although different definitions of each of these quantities are possible (e.g. the SGB–MS magnitude difference can be defined as the one between the middle of the SGB and the MS of the same colour).

We have checked that the results of the isochrone-to-data fitting obtained by two methods, with and without fiducial sequences, almost coincide. Namely, the derived  $[\text{Fe}/\text{H}]$ , ages, distances, and reddenings [converted into  $E(B - V)$ ] agree within 0.1 dex, 0.5 Gyr, 80 pc, and 0.01 mag, respectively. The results obtained without fiducial sequences are presented in Table 4.

#### 4.5 Differential reddening

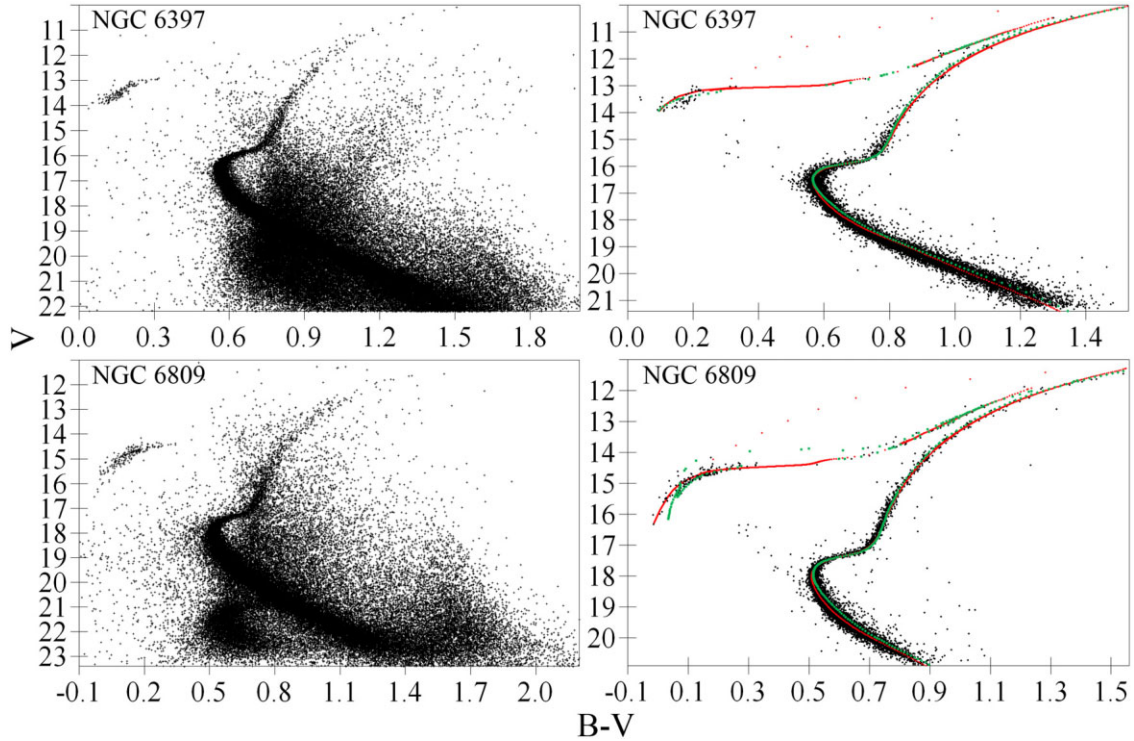
DR across the fields of both clusters is taken into account following the method of BCK13. Briefly, the cluster field is divided into a cell grid, with the angular resolution being higher in regions containing more stars. Then, the stellar-density Hess diagram (including photometric errors) of each cell is matched to the average (whole field) diagram by applying shifts along the reddening vector that are subsequently converted into DR in the cell. By design, the method assumes that differences in the CMDs can be accounted for entirely by DR. However, other variations of CMD over cluster field, such as photometry zero-point variations, point-spread function variations, telescope focus change, distortion, telescope breathing, stellar population variations, and other reasons, discussed by Anderson et al. (2008), are difficult to separate from DR.

It appears that only data sets with at least 3000 stars provide sufficient coverage of the cluster fields and, hence, draw rather precise DR maps, i.e. the data sets of NLP18, GCL99, *Gaia* DR3, SPZ19, SMSS, Libralato et al. (2022), Narloch et al. (2017), Kaluzny (1997), Kaluzny et al. (2010), PS1, 2MASS, VISTA, and all their cross-identifications, with the exception of the NGC 6397 data sets of Piotto et al. (2002) and Ahumada et al. (2021) with insufficient information about stellar coordinates.

Four examples of DR maps for NGC 6397 are shown in Fig. 12. All DR maps of both the clusters show that:

- (i) the DR maps have little to do with each other as for different data sets, as for different CMDs/colours of the same data set,
- (ii) DR variations are mostly small: within  $\Delta E(B - V) = \pm 0.04$  mag after conversion by use of any reliable extinction law, and
- (iii) some CMDs/colours show strong gradients over the fields (e.g. for GCL99  $b - y$  in Fig. 12) or sharp extremes of DR in a small area (e.g. red peaks for SPZ19  $B - V$  in Fig. 12).

The peaks in the SPZ19 DR maps can be explained as a manifestation of initial observational data sets (SPZ19 data set combines them) covering small parts of the field and having significant systematics over the field. Given that DR is not large across the fields of NGC 6397 and NGC 6809, these findings show that the effects mentioned above are more important than DR itself in the CMDs of both the clusters. Anyway, our correction of the data sets for DR reduces the scatter of their stars around their ridge lines or best-fitting isochrones in CMDs, e.g. in Fig. 13. Note that the mean DR



**Figure 10.**  $B - V$  versus  $V$  CMDs with the initial SPZ19 data sets (left panel) and the *Gaia* DR3 cluster members from the SPZ19 data sets (right panel). The isochrones from BaSTI (red) and DSED (green diamonds) for  $Y = 0.25$  are calculated with the best-fitting parameters from Table 4. Appropriate DSED predictions for the HB and AGB are shown by the green squares for illustration purposes.

correction for all the CMDs is exactly zero. This leads to a negligible shift of bulk of stars in the CMDs and does not change an average reddening over the field, which is presented in Table 4.

Since each data set and CMD/colour draw its own DR map, it is not surprising that our DR maps differ from those of Alonso-García et al. (2012).

## 5. RESULTS

As in our previous papers, we fit isochrones to a hundred CMDs with different colours. The results for adjacent CMDs appear consistent. We present some interesting CMDs with isochrone fits in Figs 3–10 and Figs 14–16. Other CMDs are presented online or can be provided on request.

We present the obtained  $[\text{Fe}/\text{H}]$ , distances, reddenings, and ages for the most important CMDs in Table 4. For comparison, we convert the obtained reddenings into  $E(B - V)$ , given in parentheses, by use of extinction coefficients from Casagrande & Vandenberg (2014), Casagrande & Vandenberg (2018a), Casagrande & Vandenberg (2018b), or CCM89 with  $R_V = 3.1$ .<sup>24</sup>

Table 4 provides the empirical systematic uncertainty of obtained reddening after its value. This systematic uncertainty is defined as the maximal deviation of the best-fitting isochrone from the bulk of

the stars along the reddening vector (i.e. nearly along the colour). The systematic uncertainty never drops below 0.03 mag and it is usually larger than the predicted statistic uncertainty. The latter is described in the balance of uncertainties (see appendix A of Paper II). The largest values in such pairs of the systematic and statistic uncertainties are shown by the extinction error bars in Figs 17 and 18 with resulting empirical extinction laws.

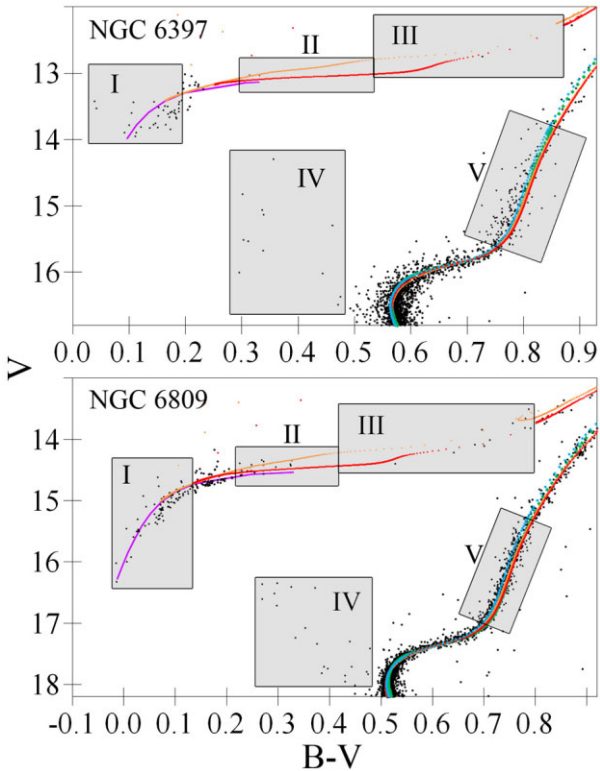
### 5.1 Issues

The *VISTA* photometry for the brightest stars is biased, as discussed in Paper IV. This is seen in Fig. 14. However, accurate parameters of the clusters can be obtained by use of the remaining *VISTA* stars.

In Paper IV, we discussed drawbacks of UV, UV–optical, optical–IR, and IR–IR CMDs with respect to (w.r.t.) a typical optical CMD (with filters within  $430 < \lambda_{\text{eff}} < 1000$  nm). Especially, the UV CMDs are highly affected by the multiple population chemical patterns (see Sbordone et al. 2011 and Cassisi et al. 2013 for the first results, as well as a recent discussion by Vandenberg, Casagrande & Edvardsson 2022). However, some of these CMDs can give reliable  $[\text{Fe}/\text{H}]$ , age, reddening, and distance estimates (see Table 4).

Fig. 15 shows an example UV CMD with the *HST*/WFC3  $F336W - F438W$  colour, where almost all domains are fitted by both the BaSTI and DSED isochrones with reasonable residuals. The maximal colour offset of the best-fitting isochrones from these data is 0.05 mag and it is the same for both the models and both the clusters. Moreover, the maximal colour offsets are  $< 0.05$  mag for the UV CMDs with the  $u - v$  Strömgren colour from the GCL99 and AT2000 data sets, as well as for the CMDs with the  $U - B$  colour from the SPZ19 data sets. For comparison, such offsets were 0.08–0.10 mag for the UV CMD with *HST*/WFC3  $F336W - F438W$  colour for NGC 6362 and

<sup>24</sup>Extinction-to-reddening ratio  $R_V \equiv A_V/E(B - V) = 3.1$  is defined for early-type MS stars, while the observed ratio  $A_V/E(B - V)$  depends on intrinsic spectral energy distribution of stars under consideration (Casagrande & Vandenberg 2014). For rather cool and metal-poor stars of the GCs under consideration the observed reddening is calculated as  $E(B - V) = A_V/3.48$ , while the extinction coefficients are calculated for the median effective temperature 6400 K of the cluster members.



**Figure 11.** A central part of the  $B - V$  versus  $V$  CMDs with the *Gaia* DR3 cluster members from the SPZ19 data sets (i.e. the data from Fig. 10). The isochrones from BaSTI (red) and DSED (green) for  $Y = 0.25$ , the BaSTI HB for  $Y = 0.25$  (magenta), and isochrones from BaSTI (orange) and DSED (blue) for  $Y = 0.267$  are calculated with the best-fitting parameters from Table 4. The grey areas are the CMD domains of the (i) blue HB, (ii) blue AGB, (iii) RR Lyrae, (iv) blue stragglers, and (v) faint RGB, which are discussed in the text.

NGC 6723 in Paper IV. Thus, it seems that the BaSTI and DSED UV isochrones better fit UV CMDs for low metallicity GCs (NGC 6397 and NGC 6809) than for those with a higher metallicity (NGC 6362 and NGC 6723). However, Table 4 shows that some distance and age estimates derived from the UV CMDs are unreliable. Thus, the results from UV, UV–optical, optical–IR, and IR–IR pairs (including those in Table 4) are not used for our final estimates.

We do not use the SPZ19 photometry in the  $R$  filter for our final estimates due to its lower precision than in the  $BVI$  filters.

Similar to Paper IV, our cross-identification of data sets, which use the same or similar filters, reveals some systematic differences up to 0.04 mag in magnitudes and colours for some data sets. They are common and expected (SPZ19). Our DR corrections reduce these differences and, hence, confirm that they are mostly due to some systematic errors of the data sets (see Section 4.5). We do not take into account the residual systematics after the DR correction, since we find little, if any, influence of these systematics to the derived parameters, as seen in Table 4 and in Figs 17 and 18. In particular, similar to Paper IV and in contrast to Paper III, we do not adjust the data sets. The adjustment would slightly decrease the scatter of the derived parameters. However, without the adjustment, we better see the real influence of the data set systematics. Yet, we eliminate the brightest stars of the Kaluzny et al. (2010) and Narloch et al. (2017) data sets for NGC 6809 due to their exceptionally large systematic deviation from any reasonable isochrone, as seen in Fig. 16 in comparison with Fig. 10 for the SPZ19 data set.

## 5.2 Metallicity

We obtain  $[\text{Fe}/\text{H}] = -1.86$  and  $-1.82$  for NGC 6397 from DSED and BaSTI, respectively, as the average from 16 independent optical CMDs with the well-populated bright RGB or faint MS. Their mean  $[\text{Fe}/\text{H}] = -1.84 \pm 0.02 \pm 0.1$  is adopted as our final  $[\text{Fe}/\text{H}]$  estimate for NGC 6397 and used for other CMDs. The statistic uncertainty  $\pm 0.02$  is calculated as the standard deviation of one estimate divided by the square root of the number of the estimates, while the systematic uncertainty is the uncertainty of the iron scale 0.1 dex, discussed in Section 2, which is larger than the DSED–BaSTI systematic difference of 0.04 dex. Similarly, using 10 CMDs for NGC 6809, we obtain  $[\text{Fe}/\text{H}] = -1.82$  (DSED) and  $-1.74$  (BaSTI) and their mean  $[\text{Fe}/\text{H}] = -1.78 \pm 0.02 \pm 0.1$  as our final  $[\text{Fe}/\text{H}]$  estimate.

Comparing our  $[\text{Fe}/\text{H}]$  estimates with those from spectroscopy, mentioned in Section 2, we conclude that our estimates support the higher estimates from Mészáros et al. (2020) ( $[\text{Fe}/\text{H}] = -1.89 \pm 0.09$  and  $-1.76 \pm 0.07$  for NGC 6397 and NGC 6809, respectively) and Wang et al. (2017) ( $[\text{Fe}/\text{H}] = -1.86 \pm 0.06$  for NGC 6809), but not the lower ones from Carretta et al. (2009) ( $[\text{Fe}/\text{H}] = -1.99 \pm 0.02$  and  $-1.93 \pm 0.02$  for NGC 6397 and NGC 6809, respectively) and Rain et al. (2019) ( $[\text{Fe}/\text{H}] = -2.01 \pm 0.02$  for NGC 6809). Accordingly, following the discussion of Mészáros et al. (2020), our estimates support the reference solar abundance mixture from Grevesse, Asplund & Sauval (2007), but not from Gratton et al. (2003).

## 5.3 Reddening and extinction

We verify the agreement of reddening estimates from all CMDs with each other and with an extinction law by combining all derived reddening estimates into empirical extinction laws. These laws are presented in Figs 17 and 18.

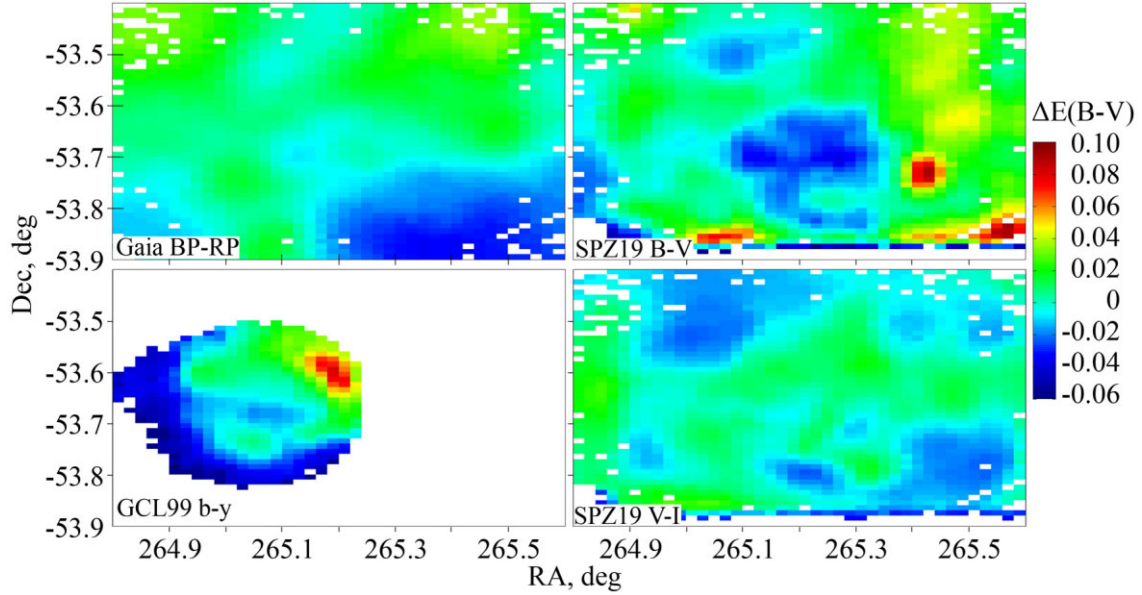
Similar to our previous papers, in order to draw these laws, we cross-identify all possible data sets with the 2MASS, VISTA, and unWISE data sets and calculate extinctions in all filters from the derived reddenings and IR extinctions. For example,

$$A_V = (A_V - A_{W1}) + A_{W1} = E(V - W1) + A_{W1}, \quad (1)$$

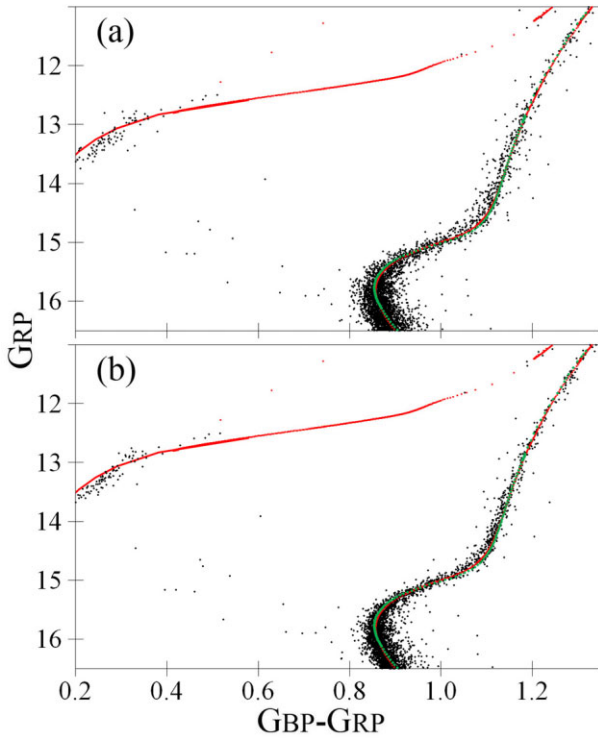
where  $E(V - W1)$  is obtained from a CMD, while very low extinction  $A_{W1}$  in the  $W1$  filter is slightly upgraded iteratively with upgrade of extinction law.

Note that some data set pairs cannot be cross-identified. The main reasons are a very small common field or common magnitude range of such data sets. Namely, the NLP18 data set for NGC 6397 contains only rather faint stars in a small field and, hence, has only a few common stars with 2MASS. Other such pairs: the NLP18 data set for NGC 6809 versus VISTA and unWISE and Kaluzny et al. (2010) data set for NGC 6809 versus unWISE. The data sets of MFR, Piotto et al. (2002), Richer et al. (2008), SBA18, and Ahumada et al. (2021) are not cross-identified with any IR data set. The extinctions for the MFR, Piotto et al. (2002), and Ahumada et al. (2021) data sets are calculated by adopting the CCM89 extinction law with our best-fitting  $R_V$  (described later) for their  $I$ ,  $HST/WFPC2 F555W$ , and  $I$  filters, respectively.

The common  $HST/ACS F814W$  filter for the NLP18, Richer et al. (2008), SBA18, and Libralato et al. (2022) data sets allows us to process their reddenings together by adopting the same extinction  $A_{F814W}$ . Extinctions derived for all  $HST$  filters, detectors, and data sets are shown together in Figs 17 and 18 by the red diamonds. These extinction estimates agree with each other following the same smooth extinction laws without outliers. This is a robust confirmation



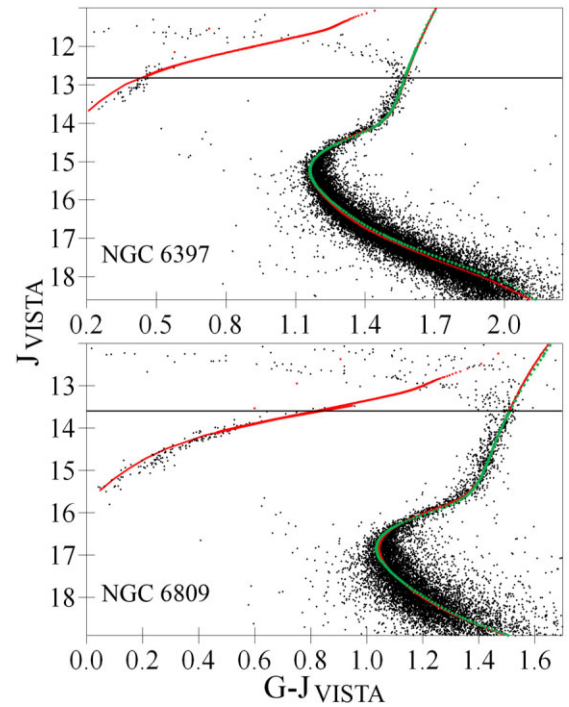
**Figure 12.** The DR maps for the same NGC 6397 field obtained by use of four CMDs.



**Figure 13.** A central part of the *Gaia* DR3  $G_{BP} - G_{RP}$  versus  $G_{RP}$  CMD for NGC 6397 from Fig. 3 (a) before and (b) after DR correction.

of the systematic accuracy of our reddening and extinction estimates at the level of a few hundredths of a magnitude. Thus, we use all the *HST* results for its optical filters ( $\lambda_{\text{eff}} > 430$  nm) for our final estimates of  $[\text{Fe}/\text{H}]$ , age, reddening, and distance.

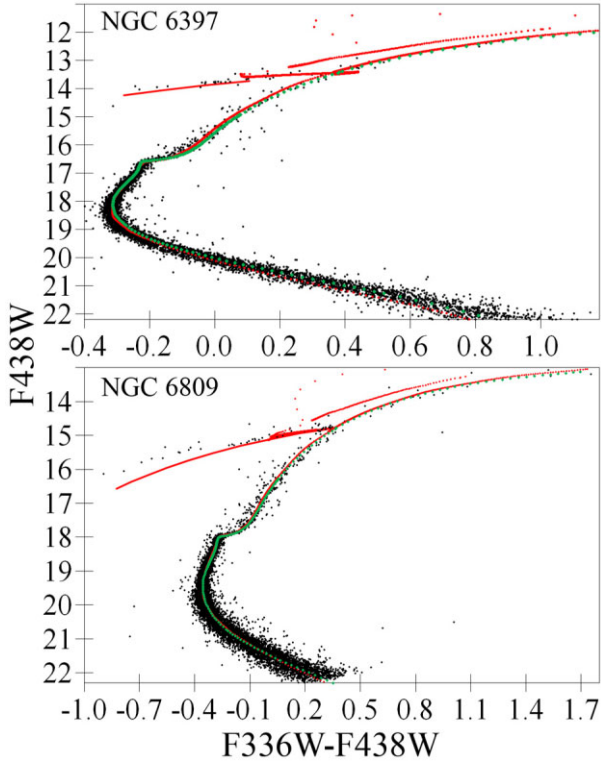
2MASS versus unWISE and *VISTA* versus unWISE are cross-identified via common *Gaia* DR3 cluster members. The *VISTA*-unWISE CMDs represent a very short-wavelength baseline and, hence, provide uncertain age, distance, and  $[\text{Fe}/\text{H}]$ . Therefore, fixing



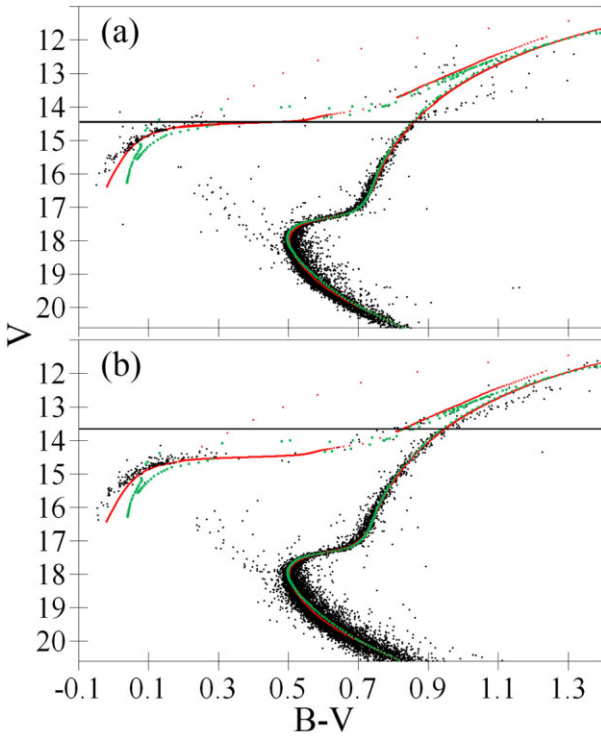
**Figure 14.** The same as Fig. 3 but for the *Gaia* – *VISTA*  $G - J_{\text{VISTA}}$  colour. The black horizontal line shows the cut of bright stars with systematically erroneous photometry.

these parameters for the *VISTA*-unWISE pair, we derive only the reddening  $E(J_{\text{VISTA}} - W1)$  as the average difference w.r.t.  $G_{\text{RP}}$  and  $V$  from SPZ19:  $E(J_{\text{VISTA}} - W1) = [E(G_{\text{RP}} - W1) - E(G_{\text{RP}} - J_{\text{VISTA}}) + E(V - W1) - E(V - J_{\text{VISTA}})]/2$ .

The extinctions in Figs 17 and 18 show a low scatter of a few hundredths of a magnitude around the CCM89 extinction law with best-fitting  $R_V = 3.3$  and 2.9 for NGC 6397 and NGC 6809, respectively.



**Figure 15.** The same as Fig. 3 but for *HST*  $F336W - F438W$  versus  $F438W$  CMDs (NLP18).



**Figure 16.** The same as Fig. 5 but for the  $B - V$  colour from the (a) Kaluzny et al. (2010) and (b) Narloch et al. (2017) data sets for NGC 6809. The black horizontal lines show the cut of bright stars with their biased colour.

We derive our final reddening and extinction estimates for the clusters through the use of all 19 and 14 independent optical CMDs for NGC 6397 and NGC 6809, respectively. Table 8 presents the final reddening estimates. The DSED estimates are systematically higher than those from BaSTI by about  $\Delta E(B - V) = 0.01$  mag. This is related to the systematically lower  $[\text{Fe}/\text{H}]$  of the DSED best-fitting isochrones.

We calculate our final  $A_V$  estimates as the averages of all its direct measurements (six for NGC 6397 and four for NGC 6809) using equation (1) and its counterparts for other IR filters and the Strömgren  $y$  filter, which is very close to the  $V$  one. We obtain  $A_V = 0.59 \pm 0.01 \pm 0.02$  and  $0.37 \pm 0.01 \pm 0.04$  mag (statistical and model-to-model uncertainties) for NGC 6397 and NGC 6809, respectively. Accordingly, the ratio of these  $A_V$  and  $E(B - V)$  estimates,  $R_V = 3.32^{+0.32}_{-0.28}$  and  $3.16^{+0.66}_{-0.56}$  (total uncertainty) for NGC 6397 and NGC 6809, respectively.

The systematic uncertainty 0.1 dex for  $[\text{Fe}/\text{H}]$  (see Section 5.2) is the dominant contribution to systematic uncertainty of all our reddening and extinction results, which is equivalent to  $\sigma_{E(B - V)} = 0.01$  and  $\sigma_{A_V} = 0.03$ .

Our  $E(B - V)$  estimates agree with those in Table 2 for both the clusters, except very high estimate by Meisner & Finkbeiner (2015) for NGC 6397 and very low estimate by Harris (1996) for NGC 6809. We find no reason for these outliers. Also, our  $E(B - V)$  estimates agree with those from isochrone fitting in Table 1, except very low  $E(B - V)$  by Martinazzi et al. (2014) for NGC 6397 and by Valcin et al. (2020) for NGC 6809.

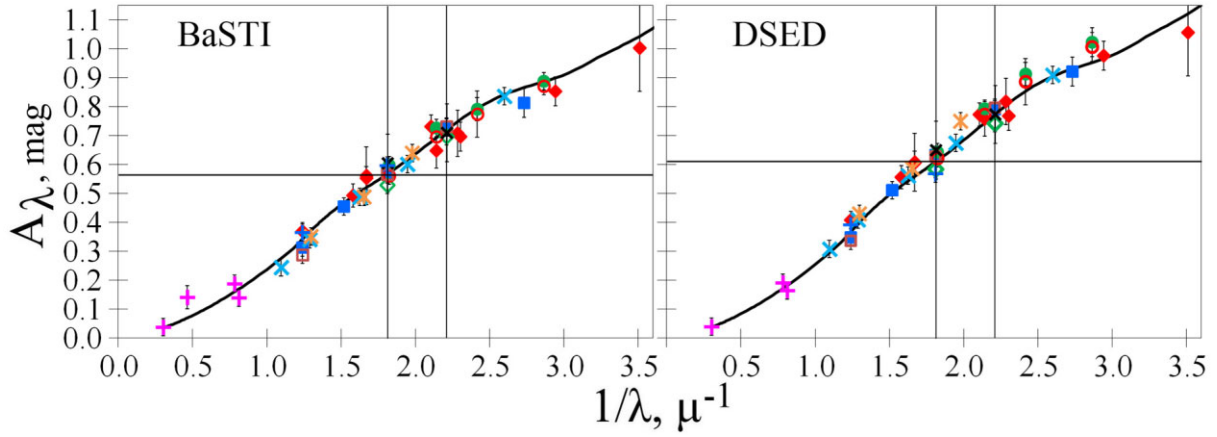
Reddening estimates calculated through alternative methods are: Olech et al. (1999) found  $E(B - V) = 0.11 \pm 0.03$  for RR Lyrae variables of NGC 6809; AT2000 found  $E(B - V) = 0.179 \pm 0.003 \pm 0.011$  (statistic and systematic uncertainties) from the Strömgren photometric data of the TO stars of NGC 6397; Hansen et al. (2007) found  $E(F606W - F814W) = 0.20 \pm 0.03 \approx E(B - V)$  from the white dwarf cooling sequence of NGC 6397; and Pych et al. (2001) with a correction from McNamara (2011) derived  $E(B - V) = 0.135 \pm 0.005$  from the data for SX Phe variables in NGC 6809. All these estimates perfectly agree with ours.

#### 5.4 Distance and age

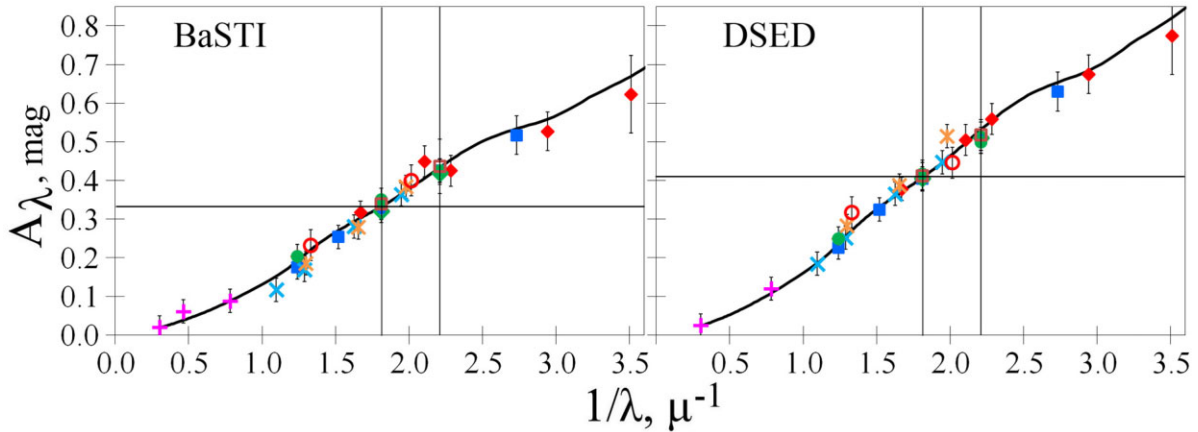
We average our distance and age estimates from all 19 and 14 optical CMDs for NGC 6397 and NGC 6809, respectively. Table 9 with the final results shows the consistent standard deviations for the models and for the mean values and, hence, a good agreement between BaSTI and DSED in their distance and age estimates. Our final estimates for NGC 6397 and NGC 6809, respectively, are as follows:

- (i) Age is  $12.9 \pm 0.1 \pm 0.8$  and  $13.0 \pm 0.1 \pm 0.8$  Gyr (statistic and systematic uncertainties),
- (ii) Distance is  $2.45 \pm 0.02 \pm 0.06$  and  $5.24 \pm 0.02 \pm 0.18$  kpc,
- (iii) Distance modulus ( $m - M$ )<sub>0</sub> =  $11.95 \pm 0.01 \pm 0.05$  and  $13.60 \pm 0.01 \pm 0.07$  mag, and
- (iv) Apparent  $V$ -band distance modulus ( $m - M$ )<sub>V</sub> =  $12.54 \pm 0.03 \pm 0.06$  and  $13.97 \pm 0.03 \pm 0.08$  mag.

Despite the rather sparse HB populations in NGC 6397 and NGC 6809, the statistic uncertainty of their derived distances is lower than the systematic uncertainty. The latter can be estimated from the scatter of the previous estimates of the distance moduli or distances presented in the compilation of all GC distance determinations by



**Figure 17.** The empirical extinction laws for NGC 6397 from the isochrone fitting drawn by the different models. The data sets are: *HST* WFC3 and ACS by Richer et al. (2008), *NLP18*, *SBA18*, Libralato et al. (2022) – red diamonds; *Gaia* – yellow snowflakes; *SPZ19* – blue squares; SMSS – blue inclined crosses; *HST* WFC2 by Piotto et al. (2002) – black inclined crosses; Nardiello et al. (2015) – open brown squares; Kaluzny (1997) – open green diamonds; *GCL99* – green circles; *AT2000* – open red circles; Ahumada et al. (2021) – blue upright crosses; and IR data sets from 2MASS, *VISTA*, and unWISE – purple upright crosses. The vertical lines denote the effective wavelengths of the *B* and *V* filters. The black curve shows the extinction law of CCM89 with  $R_V = 3.3$  tied to the obtained  $A_V$ , which is shown by the horizontal line.



**Figure 18.** The same as Fig. 17 but for NGC 6809. The data sets are: *HST* WFC3 and ACS by *NLP18*, *SBA18* – red diamonds; *Gaia* – yellow snowflakes; *SPZ19* – blue squares; SMSS – blue inclined crosses; *MFR* – green circles; Kaluzny et al. (2010) – open green diamonds; Narloch et al. (2017) – open brown squares; PS1 – open red circles; and IR data sets by *VISTA* and unWISE – purple upright crosses. The lines and curves are the same as in Fig. 17, but for  $R_V = 2.9$ .

**Table 8.** The  $E(B - V)$  values found through the various models. The model estimates are average values for all optical CMDs, provided with the standard deviations of the mean values. The final values are the mean values of the models with their uncertainties as half the differences between the model estimates.

	NGC 6397	NGC 6809
BaSTI	$0.172 \pm 0.006$	$0.112 \pm 0.004$
DSED	$0.184 \pm 0.006$	$0.125 \pm 0.004$
Final value	$0.178 \pm 0.006$	$0.118 \pm 0.006$

Baumgardt & Vasiliev (2021).<sup>25</sup> All recent (since Dotter et al. 2010) estimates of the distance modulus for NGC 6397 by isochrone-to-CMD fitting, except outlying 12.12 from Valcin et al. (2020), are

<sup>25</sup>This compilation is so comprehensive that our distance estimates do not need a comparison with individual estimates from the literature.

**Table 9.** Our NGC 6397 and NGC 6809 age (Gyr) and distance (kpc) estimates from optical CMDs. All the uncertainties are standard deviations of one measurement.

	DSED	BaSTI	Mean value
	NGC 6397		
Mean distance	$2.46 \pm 0.08$	$2.44 \pm 0.08$	$2.45 \pm 0.08$
Mean age	$12.95 \pm 0.62$	$12.89 \pm 0.59$	$12.92 \pm 0.60$
	NGC 6809		
Mean distance	$5.26 \pm 0.12$	$5.22 \pm 0.10$	$5.24 \pm 0.11$
Mean age	$12.96 \pm 0.60$	$12.96 \pm 0.54$	$12.96 \pm 0.56$

within 11.95–12.05 (including our own 11.95). Similarly, those for NGC 6809, except outlying 13.70 from Valcin et al. (2020), are within 13.53–13.67 (including our own 13.60). Assuming this scatter is due to some systematics, we accept a systematic uncertainty of distance moduli as  $\pm 0.05$  and  $\pm 0.07$  for NGC 6397 and NGC 6809, respectively, which converts to  $\pm 62$  and  $\pm 178$  pc distance systematic

**Table 10.** The relative estimates for the obtained [Fe/H] (dex), age (Gyr), distance (kpc), and  $E(B - V)$  (mag) in the sense ‘NGC 6809 minus NGC 6397’. All the values are shown with uncertainty of one measurement. The right column presents the differences of the parameters in the sense ‘NGC 6809 minus NGC 6397’ derived from all optical CMDs.

	DSED	BaSTI	Mean value	All optical CMDs
[Fe/H]	$0.04 \pm 0.16$	$0.09 \pm 0.12$	$0.06 \pm 0.14$	0.06
Age	$0.00 \pm 0.54$	$0.13 \pm 0.44$	$0.06 \pm 0.48$	0.04
Distance	$2.76 \pm 0.09$	$2.73 \pm 0.05$	$2.75 \pm 0.08$	2.79
$E(B - V)$	$-0.055 \pm 0.017$	$-0.059 \pm 0.13$	$-0.057 \pm 0.015$	-0.060

uncertainty. Such large systematics may be due to contamination of the HB and SGB of both the clusters by the MS of Sagittarius dwarf galaxy, as seen in Fig. 10. Note that our distance estimates agree with the most probable distance estimates of Baumgardt & Vasiliev (2021) presented in Table 2: within 31 and 108 pc, i.e.  $0.8\sigma$  and  $1.5\sigma$  of their stated statistical uncertainties for NGC 6397 and NGC 6809, respectively, and well inside the systematic uncertainties. Both ours and Baumgardt & Vasiliev (2021)’s estimates for both the clusters differ considerably from those of Harris (1996) (see Table 2) and, hence, can update them.

We convert our distance estimates into parallaxes to compare them in Table 6 with our parallax estimates from the *Gaia* DR3 astrometry (see Section 4.2) and with those from VB21. A good agreement between the parallaxes within the stated uncertainties is seen. Note that for such nearby GCs, the parallax estimates from the *Gaia* DR3 astrometry have nearly the same precision as those from our isochrone fitting, unlike more distant GCs in our previous studies, whose isochrone fitting parallaxes are more precise.

The systematic uncertainty of age was discussed and estimated in Section 3.1 of Paper IV. This should take into account the scatter of the previous age estimates in Table 1 and others [e.g. NGC 6397’s age of  $12.8 \pm 0.50 \pm 0.75$  Gyr (statistic and systematic uncertainties) from a population synthesis study of the white dwarf population by Torres et al. (2015) and  $13.4 \pm 0.7 \pm 1.2$  from the  $V$  luminosity at the TO by Brown et al. (2018)]. Taking into account this scatter and the discussion of age uncertainty by VandenBerg & Denissenkov (2018) and Valcin et al. (2020), we assign 0.8 Gyr as the systematic uncertainty of our derived ages.

## 5.5 Eclipsing binary

NGC 6809 contains the detached eclipsing binary V54, whose component masses and radii were measured accurately by Kaluzny et al. (2014). Hence, we can verify the cluster parameters by fitting isochrones to the precise V54 mass–radius relation.

Kaluzny et al. (2014) estimated the age, distance modulus, and helium abundance of V54 as 13.3–14.7 Gyr,  $(m - M)_V = 13.94 \pm 0.05$  mag, and  $Y = 0.25$ , respectively, adopting  $E(B - V) = 0.115 \pm 0.010$  for the effective temperatures of the primary component. They found that their fitting by the VR and DSED isochrones provides similar results. These estimates of the cluster parameters agree with ours.

The mass–radius relation for V54 is fitted with the VR isochrones by VandenBerg & Denissenkov (2018) and discussed by VandenBerg, Casagrande & Edvardsson (2022). Analysing also RR Lyrae variables in agreement with the distance obtained from MS fits to local subdwarfs and modelling the cluster HB populations, VandenBerg, Casagrande & Edvardsson (2022) conclude that NGC 6809 has  $(m - M)_V = 13.95 \pm 0.05$ ,  $[\text{Fe}/\text{H}] = -1.85 \pm 0.1$ ,  $[\text{O}/\text{Fe}] = 0.5 \pm 0.1$ ,  $0.25 < Y < 0.27$ , and an age of about  $12.9 \pm 0.8$  Gyr. All of these estimates agree with ours.

Fig. 19 presents our fitting of V54 with the DSED and BaSTI isochrones for various [Fe/H],  $Y$ , and age. A better fit is seen for higher age or lower [Fe/H] or higher  $Y$ . The latter, about  $Y = 0.26$ , is most fruitful to obtain the best fit.

## 5.6 Relative estimates

Similar to Paper IV, we consider the relative estimates for the cluster parameters separately derived for each model. Systematic errors of the models must be cancelled out in such relative estimates. Therefore, the relative estimates may be more accurate than the absolute ones.

We use eight independent CMDs of five twin data sets with accurate photometry in optical filters in order to derive relative estimates for the cluster parameters: (i)  $F438W - F606W$  and (ii)  $F606W - F814W$  from NLP18, (iii)  $F475W - F814W$  from SBA18, (iv)  $B - V$  and (v)  $V - I$  from SPZ19, (vi)  $G_{\text{BP}} - G_{\text{RP}}$  from *Gaia* DR3, (vii)  $g_{\text{SMSS}} - r_{\text{SMSS}}$ , and (viii)  $r_{\text{SMSS}} - z_{\text{SMSS}}$  from SMSS. Table 5.6 presents the relative estimates. The models are consistent in them, i.e. the distribution of the combined sample of the DSED and BaSTI relative estimates for each parameter is nearly Gaussian and each uncertainty of the combined sample agrees with those of the models. This confirms that, indeed, systematic errors of the models are cancelled out in the relative estimates.

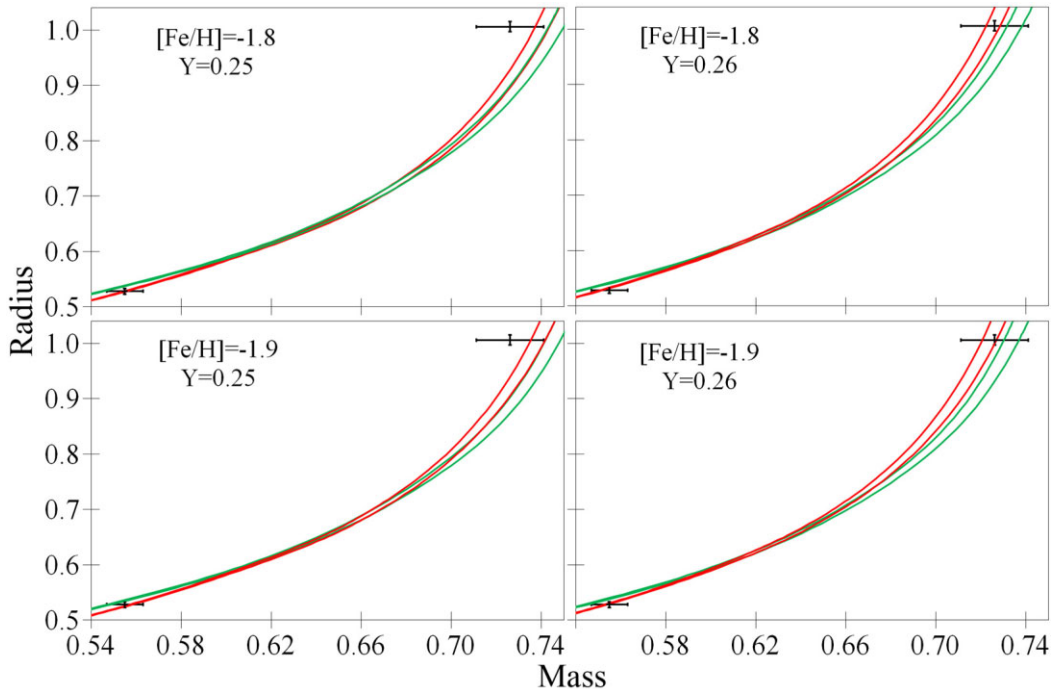
The final uncertainties of the relative estimates are the standard deviations from Table 5.6 divided by the square root of the number of the CMDs and models used (eight CMDs by two models). The relative estimates show that NGC 6809 is  $2.75 \pm 0.02$  kpc further,  $\Delta E(B - V) = 0.057 \pm 0.004$  less reddened,  $0.06 \pm 0.12$  Gyr older (i.e. of nearly the same age), and with  $0.06 \pm 0.03$  dex higher [Fe/H] (i.e. of nearly the same metallicity) than NGC 6397. For comparison, the right column of Table 5.6 presents the absolute differences of the parameters in the sense ‘NGC 6809 minus NGC 6397’ derived from optical CMDs, as described in Sections 5.2–5.4. A good agreement between the relative estimates and absolute differences is evident.

## 5.7 HB morphology difference

NGC 6397 and NGC 6809 show a considerable HB morphology difference (e.g. see Figs 3, 7, 10, 11, 14, and 15) despite nearly the same metallicity and age, which are usually considered as the first and second parameters to explain such a difference (see Paper IV and references therein). Moreover, these clusters have similar low helium enrichments. Therefore, we should describe this HB difference by another parameter, other than metallicity, age, or helium enrichment.

The HB morphology can be presented as the HB types (see Lee, Demarque & Zinn 1994 for definition) of these clusters (0.98 for NGC 6397 and 0.87 for NGC 6809 from Mackey & van den Bergh 2005 with an uncertainty about  $\pm 0.1$  from Torelli et al. 2019), or as their median colour difference between the HB and RGB  $[\Delta(V - I)] = 0.944 \pm 0.012$  for NGC 6397 and  $0.906 \pm 0.021$  for NGC 6809





**Figure 19.** The mass–radius relation (in solar mass and radius) for the eclipsing binary V54 of NGC 6809 (black symbols with error bars) fitted by the BaSTI (red) and DSED (green) isochrones for various  $[\text{Fe}/\text{H}]$ ,  $Y$ , and age (the left and right isochrone in each model pair is for 13.5 and 13 Gyr, respectively).

from Dotter et al. 2010], or an alternative HB morphology index,  $\tau_{\text{HB}} = 8.29 \pm 0.17$  for NGC 6397 versus  $6.59 \pm 0.21$  for NGC 6809, which is introduced by Torelli et al. (2019).<sup>26</sup> All these characteristics represent the HBs of NGC 6397 and NGC 6809 as rather blue and similar. Hence, these indices seem to be an incomplete description of the observed significant HB morphology difference of these clusters.

Given  $-2.0 < [\text{Fe}/\text{H}] < -1.7$ , age about 12.5–13.5 Gyr, and  $Y \approx 0.25$  for both NGC 6397 and NGC 6809, and taking into account the stochastic nature of the mass-loss before the HB, e.g. described by the BaSTI ZAHB predictions (see Section 3), one obtains a realistic *possible* scatter of the HB stars within a wide range of masses, at least  $0.50\text{--}0.78 M_{\odot}$  for our GCs. This *possible* scatter should be compared to the *observed* scatter. The latter is estimated from our fitting of the observed colour distribution of the HB and AGB stars by the BaSTI isochrones:  $0.63\text{--}0.67$  and  $0.58\text{--}0.68 M_{\odot}$  for NGC 6397 and NGC 6809, respectively.<sup>27</sup> These estimates are obtained consistently, within  $\pm 0.02 M_{\odot}$ , for all CMDs with many HB stars. These estimates show a good agreement with those modelled by Gratton et al. (2010):  $0.625\text{--}0.661$  and  $0.645\text{--}0.682$  from *HST* and ground-based observations of NGC 6397, respectively, while  $0.61\text{--}0.70$  from ground-based observations of NGC 6809. Moreover, our mass estimates for the HB stars of NGC 6397 agree with the estimate  $0.64\text{--}0.66 M_{\odot}$  obtained by Ahumada et al. (2021) from their modelling of the HB blue tail with mass-loss at the RGB.

Thus, the main HB morphology difference between these clusters seems to be a narrower HB mass range of NGC 6397 w.r.t. NGC 6809. Note that the wider mass range of the HB stars in NGC 6809 means a

larger amount of stars evolved from them, e.g. of RR Lyrae variables and red AGB stars. The latter is seen in our CMDs: NGC 6397 has no RR Lyrae, while there are several in NGC 6809; NGC 6397 contains twice fewer number of the AGB stars on the red side of the RR Lyrae gap. Yet, the difference in the highest mass is rather small ( $0.67$  versus  $0.68 M_{\odot}$ ). Hence, first of all, the desired HB parameter must explain the existence of the bluest HB stars of  $0.58\text{--}0.63 M_{\odot}$  in NGC 6809, but not in NGC 6397. Note that these stars make up the HB blue tail (i.e. the bluest part of the HB on the blue side of the HB knee), which is observed for NGC 6809, but not for NGC 6397. Hence, a natural explanation for the HB morphology difference between these clusters is that either NGC 6397 has lost or NGC 6809 has acquired the bluest HB stars. The desired parameter may be related to a peculiar evolution of these stars, i.e. with an extreme mass-loss, segregation of stellar masses due to cluster evolution, or other peculiar processes.

This loss or acquisition of low-mass HB stars may relate to the fact that NGC 6397 is a core-collapse cluster, i.e. the one with a highly compact, bright core, with a surface brightness constantly increasing towards the cluster centre. In contrast, NGC 6809, without core collapse, has a very low central concentration of stars and a roughly flat surface brightness of the cluster core. The core collapse relates to the mass segregation during cluster evolution, when massive stars tend to clump in the cluster centre, while less massive ones populate the outskirts, sometimes escaping the cluster (Martinazzi et al. 2014), as well as to the increase of dynamical interactions among stars in the dense core of post-core-collapse cluster (Meylan & Heggie 1997). The current mass of NGC 6397 is just 10 per cent of its initial mass (Dieball et al. 2017). The HB stars are the least centrally concentrated population and absent in the central area of the core of NGC 6397 (Dieball et al. 2017). Hence, most low-mass HB stars may be lost in the dynamical evolution and mass segregation of NGC 6397. Yet, the additional parameter (after metallicity, age, and helium enrichment) is still an issue. Our study may provide sufficient input data to solve it.

<sup>26</sup> $\tau_{\text{HB}}$  is calculated from cumulative number distributions along the HB in the  $I$  magnitude and  $V - I$  colour. This index varies between 0 and 14 for the most red and blue HB.

<sup>27</sup>All stars in the middle part of the NGC 6809’s HB far from the AGB appear RR Lyrae variables after our star-by-star inspection.

## 6. CONCLUSIONS

This study continues the series of [Paper I](#), [Paper II](#), [Paper III](#), and [Paper IV](#) in the estimation of key parameters of Galactic GCs via fitting theoretical isochrones to observed multiband photometry. We have analysed the low-metallicity pair NGC 6397 and NGC 6809 (Messier 55) with similar metallicity, age, helium enrichment, and extinction. The cluster members have been carefully selected through *HST* and *Gaia* DR3 PMs and parallaxes. Accordingly, we provided the lists of reliable members of the clusters, their median parallax ( $0.416 \pm 0.010$  and  $0.203 \pm 0.010$  mas for NGC 6397 and NGC 6809, respectively), and systemic PMs with their total (systematic plus statistic) uncertainties in  $\text{mas yr}^{-1}$ :

$$\begin{aligned}\mu_\alpha \cos(\delta) &= 3.26 \pm 0.02, \quad \mu_\delta = -17.66 \pm 0.02 \\ \mu_\alpha \cos(\delta) &= -3.43 \pm 0.02, \quad \mu_\delta = -9.31 \pm 0.02\end{aligned}$$

for NGC 6397 and NGC 6809, respectively.

We employed the photometry in 32 and 23 filters for NGC 6397 and NGC 6809, respectively, from the *HST*, *Gaia* DR3, SMSS DR3, 2MASS, *VISTA* VHS DR5, unWISE, and other data sets. These filters span a wide wavelength range from the UV to mid-IR, namely from about 230 to 4060 nm. As in our previous studies, we cross-identified some data sets with each other. As a result, we could (i) estimate systematic differences between the data sets and (ii) use the 2MASS, *VISTA*, and unWISE photometry with a very low extinction for determination of the extinction in all other filters and drawing of empirical extinction laws.

We fitted the data by the DSED and BaSTI theoretical models of stellar evolution for  $[\alpha/\text{Fe}] = 0.4$  with nearly primordial helium abundance  $Y \approx 0.25$ . As a result, we obtained  $[\text{Fe}/\text{H}]$ , reddening, age, and distance as the parameters. BaSTI provides metallicity  $\Delta[\text{Fe}/\text{H}] \approx 0.06$  dex systematically higher than DSED and reddening  $\Delta E(B - V) \approx 0.01$  mag systematically lower than DSED.

An important result of this study is the agreed parameters of NGC 6397 and NGC 6809 derived from successful fitting of two recent isochrone sets to all recent photometric data sets, most of which have never been fitted before. To derive reddening, age, and distance, we use 19 and 14 independent CMDs, while 16 and 10 ones to derive  $[\text{Fe}/\text{H}]$  for NGC 6397 and NGC 6809, respectively.

The following estimates were obtained for NGC 6397 and NGC 6809, respectively: metallicities  $[\text{Fe}/\text{H}] = -1.84 \pm 0.02 \pm 0.1$  and  $-1.78 \pm 0.02 \pm 0.1$  (statistic and systematic uncertainties); distances  $2.45 \pm 0.02 \pm 0.06$  and  $5.24 \pm 0.02 \pm 0.18$  kpc; distance moduli  $(m - M)_0 = 11.95 \pm 0.01 \pm 0.05$  and  $13.60 \pm 0.01 \pm 0.07$  mag; apparent *V*-band distance moduli  $(m - M)_V = 12.54 \pm 0.03 \pm 0.06$  and  $13.97 \pm 0.03 \pm 0.08$  mag; ages  $12.9 \pm 0.1 \pm 0.8$  and  $13.0 \pm 0.1 \pm 0.8$  Gyr; reddenings  $E(B - V) = 0.178 \pm 0.006 \pm 0.01$  and  $0.118 \pm 0.004 \pm 0.01$  mag; extinctions  $A_V = 0.59 \pm 0.01 \pm 0.02$  and  $0.37 \pm 0.01 \pm 0.04$  mag; and extinction-to-reddening ratio  $R_V = 3.32^{+0.32}_{-0.28}$  and  $3.16^{+0.66}_{-0.56}$ . These estimates agree with most estimates from the literature, while disapprove other estimates. For example, after our  $[\text{Fe}/\text{H}]$  estimates, higher  $[\text{Fe}/\text{H}]$  estimates by Mészáros et al. (2020) seem to be preferred over the lower ones by Carretta et al. (2009).

There are pairs of similar data sets for the clusters, which are obtained with the same telescope and/or processed within the same pipeline. We used these data sets to derive very precise relative estimates for the parameters. NGC 6809 appears  $2.75 \pm 0.02$  kpc further,  $\Delta E(B - V) = 0.057 \pm 0.004$  less reddened,  $0.06 \pm 0.12$  Gyr older (i.e. of the same age), and with  $0.06 \pm 0.03$  dex higher  $[\text{Fe}/\text{H}]$  (i.e. of the same metallicity) than NGC 6397.

Despite nearly the same metallicity, age, and helium enrichment, these clusters show a considerable HB morphology difference, which must therefore be described by another parameter. Primarily, this parameter must explain the existence of the least massive HB stars of the blue tail (0.58–0.63 solar mass) only in NGC 6809. Probably such stars have been lost by the core-collapse cluster NGC 6397 in its dynamical evolution and mass segregation, unlike NGC 6809, which has a very low central concentration.

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## DATA AVAILABILITY

The data underlying this article will be shared on reasonable request to the corresponding author.

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## SUPPORTING INFORMATION

Supplementary data are available at *MNRAS* online.

**Table 4.** The results of our isochrone fitting for two models and some key CMDs for both the clusters.

**Table 7.** The list of the *Gaia* DR3 members of NGC 6397 and NGC 6809.

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