A Dynamic Extension for L_gV Controllers

Alexandre S. Bazanella, Petar V. Kokotović, and Aguinaldo S. e Silva

Abstract—**A dynamic state feedback control structure is proposed in this paper. The scheme is conceived as an adaptive controller with the equilibrium in an L_gV control law as the uncertain parameter, which allows the implementation of the controller for systems with unknown equilibrium. A new property of L_gV controllers is given, and it is proven that this and other important properties carry on to the proposed scheme. A synchronous machine case study shows that the proposed scheme may give better results than the L_gV controller from which it is derived.**

Index Terms—**Dynamic feedback, passivity, stability domains.**

I. Introduction

L_gV controllers arise in Lyapunov analysis as a means of providing asymptotic stability to a Lyapunov stable system or increasing the damping of an asymptotically stable system. This class of controllers occupies an important role in the study of feedback stabilizability and stabilization of nonlinear systems. Significant results have been presented regarding global asymptotic stabilization of Lyapunov stable systems [2], [13], [15], involving geometric characterizations [6], [9], [11], Lyapunov analysis [12], and passivity concepts [4]. In this paper we study L_gV controllers in a slightly different setting, which allows us to deal with nonglobal stability and stabilization. In fact, our main concern will be to study the effect of the control on the size of the region of attraction—or its estimate.

On the other hand, L_gV controllers, as most state feedback controllers designed to improve the dynamic performance of a system around a given equilibrium point, require the knowledge of this equilibrium. This is not a reasonable assumption in many control problems, such as in power system stabilizers [10] and other applications [1], [16]. We treat the equilibrium as an uncertain parameter in the L_gV control law and design an adaptive mechanism to track it while maintaining the overall system stability. In so doing, knowledge of the equilibrium is not required. The resulting controller presents a dynamic state feedback structure and is referred to as *dynamic L_gV controller*, in opposition to its nonadaptive counterpart, referred to as *static L_gV controller* in this paper. Some important properties of static L_gV controllers carry on to the dynamic L_gV controllers. Moreover, in some examples the dynamic L_gV controller outperforms its static counterpart, as shown by a case study presented in this paper.

The paper is organized as follows. In Section II some concepts of L_gV control are reviewed as we fix the notation and state some mild assumptions on the system and the Lyapunov functions to be made throughout the paper. It is proven that under these conditions an L_gV controller enlarges the estimate of the region of attraction of the equilibrium. The dynamic L_gV control is presented in Section III, and it is proven that it inherits the properties of infinite gain margin and enlargement of the estimate of the region of attraction from the static L_gV controllers. The application of the proposed scheme to the control of a synchronous machine is presented in Section IV. Finally, in Section V the conclusions are given.

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II. L_gV Controllers

We consider nonlinear systems affine in the input, with the usual assumptions for existence and uniqueness of solutions

$$\dot{x} = f(x) + g(x)u \tag{1}$$

with $x \in \mathcal{X} = \Re^n$ and $u \in \mathcal{U} = \Re^m$, and its equilibrium x_e^o at which it is to operate in steady state

$$f(x_e^o) = 0$$

where the superscript o stands for "operating point." It is assumed that x_e^o is an asymptotically stable equilibrium of the open-loop system (2)

$$\dot{x} = f(x). \tag{2}$$

Hence there exists a continuous function $V(x)$ which satisfies, in some neighborhood $\mathcal{D}$ of x_e^o

$$V(x) > 0, \qquad \forall x \in \{\mathcal{D} - x_e^o\} \tag{3}$$

$$V(x_e^o) = 0 \tag{4}$$

$$L_fV(x) < 0 \tag{5}$$

where

$$L_fV(x) = \frac{\partial V(x)}{\partial x} f(x)$$

is the Lie derivative of $V(x)$ along the vector field $f(x)$. The Lyapunov function also satisfies a nondecreasing condition, which we assume to hold all over the set $\mathcal{D}$

$$\mathcal{L}_V(c_1) \supset \mathcal{L}_V(c_2) \quad \text{iff} \quad c_1 > c_2, \qquad \forall c_1\colon \mathcal{L}_V(c_1) \subset \mathcal{D}$$

where $\mathcal{L}_V(c)$ is the interior of the level surface $V(x) = c$ and $\subset$ is used in the strict sense.

Under these conditions, a control law of the form $u = -k(L_gV(x))^T$, $k > 0$ is called an L_gV *controller* and $V(x)$ is called an L_gV *control Lyapunov function* [7], [15]. The closed-loop system is then described by

$$\dot{x} = f(x) - kg(x)(L_gV(x))^T. \tag{6}$$

In a slightly different setting, in which the equilibrium of (1) is globally Lyapunov stable, but not asymptotically, such L_gV controllers have been used to prove some important conditions for global asymptotic stabilizability [12]. It has also been noted that the L_gV control can be interpreted as a unit gain negative output feedback imposed on the passive system defined choosing for (1) the output map

$$y = k(L_gV(x))^T. \tag{7}$$

Then the above-mentioned result can be seen as a consequence of the passivity of the plant (1), (7) and the strictly positive real (SPR) property of the unit gain feedback [4]. This paper is concerned with the effect of the control on the size of the region of attraction of the stable equilibrium when it does not encompass the whole state space. Accordingly, nonglobal asymptotic stability of the open-loop system is assumed so that we can talk about the size of the region of attraction of its stable equilibrium. This does not represent loss of generality on the class of systems under study with respect to the above-mentioned publications, since the asymptotically stable system can always be thought of as a Lyapunov stable system that satisfies the stabilizability conditions to which a previous stabilizing control has been applied.

A well-known property of L_gV controllers is that they guarantee infinite gain margin. Also, an L_gV controller does not shift the position of the equilibrium, since

$$\left.\frac{\partial V(x)}{\partial x}\right|_{x=x_e^o} = 0$$

which implies that the control vanishes at x_e^o. We now prove a useful property of L_gV controllers: that they enlarge the estimate of the region of attraction of the stable equilibrium obtained with the same Lyapunov function. First, let us define some notation and state a few facts needed in the proof.

Let $\mathcal{N}_o$ be the largest connected set containing the equilibrium such that $L_fV(x) < 0$. Then an estimate of the region of attraction of x_e^o can be obtained as

$$\hat{\mathcal{R}}_o = \mathcal{L}_V(\overline{a}_o)$$

where $\overline{a}_o \triangleq \max a$: $\mathcal{L}_V(a) \subseteq \mathcal{N}_o$. Similarly, an estimate of the region of attraction of x_e^o in closed loop can be obtained as

$$\hat{\mathcal{R}}_c = \mathcal{L}_V(\overline{a}_c)$$

where $\overline{a}_c \triangleq \max a$: $\mathcal{L}_V(a) \subseteq \mathcal{N}_c$ and $\mathcal{N}_c$ is the largest connected set containing the equilibrium such that $L_fV(x) - k(L_gV(x))(L_gV(x))^T < 0$. Note that $\mathcal{L}_V(\overline{a}_o)$, $\mathcal{L}_V(\overline{a}_c)$, $\mathcal{N}_o$, and $\mathcal{N}_c$ are open sets.

Let ∂ denote the boundary of a set. The following facts come directly from the definitions above and the smoothness of the Lyapunov function.

Fact 1:

$$L_fV(x) = 0, \qquad \forall x \in \partial\mathcal{N}_o$$

and

$$L_fV(x) - k(L_gV(x))(L_gV(x))^T = 0, \qquad \forall x \in \partial\mathcal{N}_c.$$

□

Fact 2:

$$\partial\hat{\mathcal{R}}_o \cap \partial\mathcal{N}_o \neq \emptyset$$

and

$$\partial\hat{\mathcal{R}}_c \cap \partial\mathcal{N}_c \neq \emptyset.$$

□

Fact 3: If $\mathcal{N}_c \supseteq \mathcal{N}_o \supset \hat{\mathcal{R}}_o$, then $\partial\hat{\mathcal{R}}_o \cap \partial\mathcal{N}_o \supseteq \partial\hat{\mathcal{R}}_o \cap \partial\mathcal{N}_c$. □

Theorem 4: Let $V(x)$ be an L_gV control Lyapunov function for the open-loop system (2), satisfying the continuity and smoothness conditions in some domain $\mathcal{D} \supseteq \hat{\mathcal{R}}_c$, and consider the closed-loop system (6). Then $\hat{\mathcal{R}}_c \supseteq \hat{\mathcal{R}}_o$. If it is further assumed that $L_gV(x) \neq 0 \forall x \in \partial\mathcal{N}_o \cap \partial\hat{\mathcal{R}}_o$, then $\hat{\mathcal{R}}_c \supset \hat{\mathcal{R}}_o$. □

Proof: The time derivative $\dot{V}_c(x)$ of the Lyapunov function $V(x)$ in closed loop is

$$\dot{V}_c(x) = L_fV(x) - k(L_gV(x))(L_gV(x))^T \leq L_fV(x)$$

which implies that $\mathcal{N}_c \supseteq \mathcal{N}_o$. But then $\mathcal{L}_V(\overline{a}_o) \subseteq \mathcal{N}_c$, and therefore $\mathcal{L}_V(\overline{a}_c) \supseteq \mathcal{L}_V(\overline{a}_o)$, that is, $\hat{\mathcal{R}}_c \supseteq \hat{\mathcal{R}}_o$.

Because $\mathcal{N}_c \supseteq \mathcal{N}_o \supset \hat{\mathcal{R}}_o$, we have $\partial\hat{\mathcal{R}}_o \cap \partial\mathcal{N}_c \subseteq \partial\hat{\mathcal{R}}_o \cap \partial\mathcal{N}_o$. But, by assumption, $L_gV(x) \neq 0 \forall x \in \partial\hat{\mathcal{R}}_o \cap \partial\mathcal{N}_o$, which implies $\dot{V}_c(x) < 0 \forall x \in \partial\hat{\mathcal{R}}_o \cap \partial\mathcal{N}_o$, so that (from Fact 1) no point $x \in \partial\hat{\mathcal{R}}_o \cap \partial\mathcal{N}_o$ belongs to $\partial\mathcal{N}_c$. We then conclude that $\partial\hat{\mathcal{R}}_o \cap \partial\mathcal{N}_c = \emptyset$, so that, from Fact 2, $\hat{\mathcal{R}}_c \neq \hat{\mathcal{R}}_o$ and thus

$$\hat{\mathcal{R}}_c \supset \hat{\mathcal{R}}_o.$$

□

We work with negative definite derivative of the Lyapunov function and base our results on Lyapunov's theorem for the sake of simplicity. For a semidefinite Lyapunov derivative the above result—as well as Theorem 6—can be proven based on LaSalle's invariance principle, under the additional assumption that the operating point is the only invariant set inside the set $\{x \in \mathcal{L}_V(\overline{a}_c)\colon L_fV(x) = 0\}$.

III. The Dynamic Extension

It is assumed in the following that the control law is of the form:

$$u = -k(L_gV(x))^T = \varphi(x) - \varphi(x_e^o). \tag{8}$$

Instead of implementing the control law like in (8), the equilibrium value of the function $\varphi(x)$ can be thought of as an uncertain parameter $\theta \triangleq \varphi(x_e^o)$. Then a certainty equivalence controller with an adaptation mechanism for this uncertain parameter can be applied. The proposed control structure is

$$\dot{x} = f(x) + g(x)(\varphi(x) - \hat{\theta}) \tag{9}$$

$$\dot{\hat{\theta}} = A(\varphi(x) - \hat{\theta}) \tag{10}$$

with $A \in \Re^{m\times m}$, $A = A^T > 0$. This control structure will be referred to as *dynamic L_gV controller,* in opposition to the original control law (8), referred to as *static L_gV controller.*

Although the Lyapunov function depends on the equilibrium, its knowledge usually does not require the knowledge of the equilibrium, since the Lyapunov function can be parameterized in terms of a generic equilibrium. This point is made clearer in the example. The knowledge of the equilibrium is required for the L_gV controller only at the point of implementation of (8). On the other hand, the equilibrium does not appear in (9) and (10) so that the implementation of this control does not require its knowledge. Thus, the dynamic L_gV controller does not require the knowledge of the operating point x_e^o, which allows its direct implementation in systems with unknown operating point. Moreover, the operating point is invariant under this feedback, as shown below.

Fact 5: To each equilibrium x_e of the open-loop system (2) there corresponds one and only one equilibrium of the closed-loop system (9), (10), and this equilibrium is $[x_e^T \;\; \varphi^T(x_e)]^T$. □

Proof: A given point $[x_0^T \;\; \hat{\theta}_0^T]^T$ is an equilibrium of (9), (10) if and only if both equations as follows are satisfied:

$$f(x_0) + g(x_0)(\varphi(x_0) - \hat{\theta}_0) = 0 \tag{11}$$

$$A(\varphi(x_0) - \hat{\theta}_0) = 0. \tag{12}$$

Since $A > 0$, (12) is equivalent to $\varphi(x_0) = \hat{\theta}_0$, so that (11) and (12) are equivalent to

$$f(x_0) = 0 \tag{13}$$

$$\varphi(x_0) = \hat{\theta}_0. \tag{14}$$

Hence $[x_0^T \;\; \hat{\theta}_0^T]^T$ is an equilibrium of (9), (10) if and only if x_0 satisfies the open-loop equilibrium equation (13) and $\hat{\theta}_0 = \varphi(x_0)$. □

The positions of the original equilibria are maintained in a robust way, in the sense that this is a structural property of the control scheme and therefore does not depend on the parameters of the controller. On the other hand, all the equilibria of the original system are maintained, not only the operating point x_e^o.

We shall prove below that the dynamic controller (9), (10) preserves the stability and the infinite gain margin of the static L_gV

controller. The issue of the size of the region of attraction deserves more attention because the dynamic controller increases the dimension of the state-space of the system. What is to be compared with the open-loop region of attraction is the size of the closed-loop region of attraction in the x-directions. It is clear that

$$\mathcal{X} = \left\{ \begin{bmatrix} x \\ \hat{\theta} \end{bmatrix} : \hat{\theta} = \theta \right\}$$

and that the operating point of the closed-loop system (9), (10) is given by

$$\begin{bmatrix} x \\ \hat{\theta} \end{bmatrix} = \begin{bmatrix} x_e^o \\ \varphi(x_e^o) \end{bmatrix}.$$

Let $\hat{\mathcal{R}}_{cd}$ be the region of attraction of this operating point. Then we are interested in the size of the set $\hat{\mathcal{R}}_{cd} \cap \mathcal{X}$. In other words, we want to prove that for any given $x_1 \in \hat{\mathcal{R}}_o$, it follows that $[x_1^T \hat{\theta}^T] \in \hat{\mathcal{R}}_{cd} \forall \hat{\theta}$.

Consider the following Lyapunov function candidate for the closed-loop system (9), (10):

$$U(x, \hat{\theta}) = V(x) + \frac{1}{2k} (\theta - \hat{\theta})^T A^{-1} (\theta - \hat{\theta}). \tag{15}$$

An estimate for the region of attraction of the closed-loop system is given by $\mathcal{L}_U(\overline{a}_{cd})$, where $\overline{a}_{cd} \triangleq \max a$: $\mathcal{L}_U(a) \subseteq \mathcal{N}_{cd}$ and $\mathcal{N}_{cd}$ is the largest connected set containing the equilibrium such that $\dot{U}(x, \hat{\theta}) < 0$. We are now ready to state our main result.

Theorem 6: Let $V(x)$ be an L_gV control Lyapunov function for the system (1) and $u = \varphi(x) - \varphi(x_e^o) = -k(L_gV(x))^T$. Then $[x_e^{oT} \quad \varphi^T(x_e^o)]^T$ is an asymptotically stable equilibrium of the closed-loop system (9), (10) and the control has infinite gain margin. Furthermore, $\hat{\mathcal{R}}_{cd} \cap \mathcal{X} \supseteq \hat{\mathcal{R}}_o$. Finally, if it is further assumed that $L_gV(x) \neq 0 \, \forall x \in \partial \hat{\mathcal{R}}_o \cap \partial \mathcal{N}_o$, then $\hat{\mathcal{R}}_{cd} \cap \mathcal{X} \supset \hat{\mathcal{R}}_o$. □

Proof: Let

$$u = \varphi(x) - \hat{\theta} = -k.(L_gV(x))^T + \theta - \hat{\theta}$$

and consider the Lyapunov function candidate (15). Then the Lyapunov derivative is

$$\begin{aligned}\dot{U}(x, \hat{\theta}) &= L_fV(x) + L_gV(x)(\varphi(x) - \hat{\theta}) \\ &\quad + \frac{1}{k} (\theta - \hat{\theta})^T A^{-1} (-A(\varphi(x) - \hat{\theta})) \\ &= L_fV(x) - \frac{1}{k} (\varphi(x) - \hat{\theta})^T (\varphi(x) - \hat{\theta}) < 0 \end{aligned} \tag{16}$$

which is zero only at the equilibrium $[x_e^{oT} \quad \varphi^T(x_e^o)]^T$, therefore establishing its asymptotic stability. That this control has infinite gain margin is clear from (16), since $\dot{U}$ is negative definite for all $k > 0$.

Take a point $[x_1^T \quad \hat{\theta}_1^T]^T \in \partial \mathcal{L}_U(\overline{a}_o)$, that is, $U(x_1, \hat{\theta}_1) = \overline{a}_o$; then

$$V(x_1) = \overline{a}_o - \frac{1}{2k} (\theta - \hat{\theta}_1)^T A^{-1} (\theta - \hat{\theta}_1) < \overline{a}_o$$

and, because $V(\cdot)$ is continuous and nondecreasing, $x_1 \in \mathcal{L}_V(\overline{a}_o)$. But $\mathcal{L}_V(\overline{a}_o) \subseteq \mathcal{N}_o$ and therefore

$$\begin{bmatrix} x_1 \\ \hat{\theta}_1 \end{bmatrix} \in \mathcal{L}_U(\overline{a}_o) \rightarrow x_1 \in \mathcal{N}_o.$$

Now, (16) also implies that $\dot{U}(x, \hat{\theta}) < 0 \; \forall [x^T \hat{\theta}^T]^T$: $x \in \mathcal{N}_o$, so that $x \in \mathcal{N}_o \rightarrow [x^T \quad \hat{\theta}^T]^T \in \mathcal{N}_{cd} \; \forall \hat{\theta}$.

Putting the pieces together, we have

$$\begin{bmatrix} x \\ \hat{\theta} \end{bmatrix} \in \mathcal{L}_U(\overline{a}_o) \rightarrow \begin{bmatrix} x \\ \hat{\theta} \end{bmatrix} \in \mathcal{N}_{cd}$$

or, in other words

$$\mathcal{N}_{cd} \supseteq \mathcal{L}_U(\overline{a}_o)$$

which implies that $\overline{a}_{cd} \geq \overline{a}_o$, and therefore $\mathcal{L}_V(\overline{a}_{cd}) \supseteq \mathcal{L}_V(\overline{a}_o)$. But $\mathcal{L}_U(\overline{a}_{cd}) \cap \mathcal{X} = \mathcal{L}_V(\overline{a}_{cd})$ and thus

$$\mathcal{L}_U(\overline{a}_{cd}) \cap \mathcal{X} \supseteq \mathcal{L}_V(\overline{a}_o)$$

which is the same as

$$\hat{\mathcal{R}}_{cd} \cap \mathcal{X} \supseteq \hat{\mathcal{R}}_o. \tag{17}$$

Now, suppose $\hat{\mathcal{R}}_{cd} \cap \mathcal{X} = \hat{\mathcal{R}}_o$. Then, since $\hat{\mathcal{R}}_{cd} = \mathcal{L}_U(\overline{a}_{cd})$, $\hat{\mathcal{R}}_o = \mathcal{L}_V(\overline{a}_o)$, and $\mathcal{L}_U(\overline{a}_{cd}) \cap \mathcal{X} = \mathcal{L}_V(\overline{a}_o)$, we have $\overline{a}_{cd} = \overline{a}_o$. Since $\mathcal{L}_U(\overline{a}_o)$ is the closed-loop region of attraction, $\partial \mathcal{L}_U(\overline{a}_o) \cap \partial \mathcal{N}_{cd} \neq \emptyset$. It is clear from (15) and (16) that $\partial \mathcal{L}_U(\overline{a}_o) \cap \partial \mathcal{N}_{cd} \subset \mathcal{X}$, for if $\partial \mathcal{L}_U(\overline{a}_o)$ does not intersect $\partial \mathcal{N}_{cd}$ for $\hat{\theta} = \theta$ then this intersection does not happen for any other $\hat{\theta}$ either. Therefore, $\partial \mathcal{L}_U(\overline{a}_o) \cap \partial \mathcal{N}_{cd} = \partial \mathcal{L}_V(\overline{a}_o) \cap \partial \mathcal{N}_o$. Now, because $\mathcal{N}_{cd} \supset \mathcal{N}_o$, $\partial \mathcal{L}_V(\overline{a}_o) \cap \partial \mathcal{N}_o \neq \emptyset$. Then $\exists x \in \partial \mathcal{L}_V(\overline{a}_o)$: $\dot{U}(x, \theta) = 0$. But since $L_gV(x) \neq 0 \forall x \in \partial \mathcal{L}_V(\overline{a}_o) \cap \partial \mathcal{N}_o$, $\dot{U}(x, \theta) < 0 \forall x \in \partial \mathcal{L}_V(\overline{a}_o) \cap \partial \mathcal{N}_o$ and we have a contradiction. We thus conclude that $\hat{\mathcal{R}}_{cd} \cap \mathcal{X} \neq \hat{\mathcal{R}}_o$, which together with (17) gives

$$\hat{\mathcal{R}}_{cd} \cap \mathcal{X} \supset \hat{\mathcal{R}}_o.$$

□

Again we can think of the control as an output feedback for the plant (1), (7). Then (16) and the resulting properties of asymptotic stability and infinite gain margin are a direct consequence of the passivity of the plant (1), (7) and the fact that the feedback presents an SPR property.

IV. Application to Synchronous Machines

Consider the model of a synchronous machine

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -b_1 x_3 \sin x_1 - D x_2 + P \\ \dot{x}_3 &= b_3 \cos x_1 - b_4 x_3 + E + u\end{aligned}$$

where x_1 is the load angle, x_2 is the shaft speed deviation, x_3 is the internal voltage, u is the control input, and b_1, b_2, b_4, P, and E are positive parameters.

The open-loop ($u \equiv 0$) system has multiple equilibria, and the operating point is the equilibrium $x_e^o = [x_{1e}^o \quad 0 \quad x_{3e}^o]^T$ with $x_{1e}^o \in (0, \pi/2)$. The operating point is asymptotically stable with a Lyapunov function given by

$$\begin{aligned}V(x) &= \tfrac{1}{2} x_2^2 + b_1 x_3 (\cos x_{1e}^o - \cos x_1) - P(x_1 - x_{1e}^o) \\ &\quad + \frac{b_1}{2} \frac{b_4}{b_3} (x_3 - x_{3e}^o)^2\end{aligned}$$

whose time derivative in open loop is

$$\dot{V}(x) = -\frac{b_1}{b_3} \{[\phi(x) - \phi(x_e^o)]^2 - D x_2^2\}$$

where

$$\phi(x) \triangleq b_3 \cos x_1 - b_4 x_3.$$

The Lyapunov function is locally positive definite and nondecreasing in a region $\mathcal{D}$ around the equilibrium, while its time derivative is globally negative semidefinite which, together with LaSalle's invariance principle, establishes the asymptotic stability of x_e^o. An estimate of the region of attraction of x_e^o is given by the region $\mathcal{D}$ [14]. Under these conditions no controller can improve the estimate for the region of attraction obtained with this Lyapunov function, although an L_gV controller—whether static or dynamic—is guaranteed not to reduce this estimate according to Theorems 4 and 6. Moreover, the *actual* region of attraction can be changed by an L_gV controller, as will be seen in the sequel. It is also worth noticing that the Lyapunov

TABLE I
PARAMETER VALUES FOR THE CASE STUDY

Parameter	Value (pu)
b_1	34.29
b_3	0.1490
b_4	0.3341
P	28.22
E	0.2394

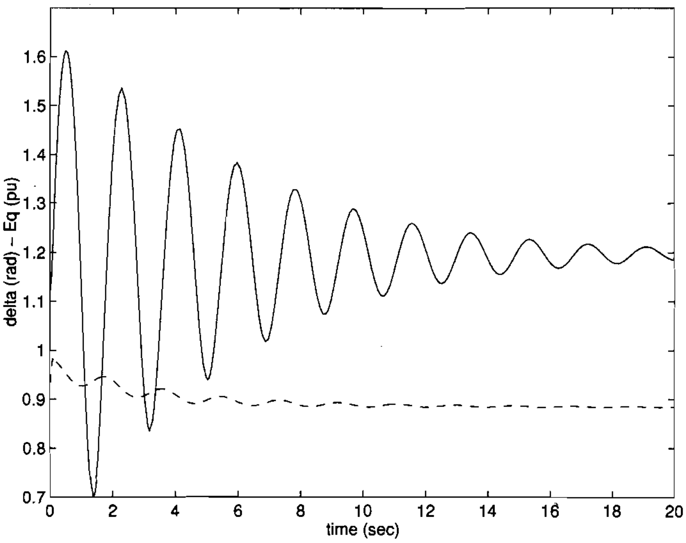

Fig. 1. Load angle (x_1, solid line) and internal voltage (x_3, dashed line) behavior in open loop for $t_{cl} = 90$ ms.

function is parameterized in the *unknown* equilibrium x_e^o, so that we do not need to know this equilibrium to obtain its analytical expression.

Symmetric short circuits at the machine's terminal are considered the most important disturbances in a power system. It is assumed that the short circuit is removed after a *clearing time* t_{cl}. An important security measure of a power system is the *critical clearing time* t_{cr}, which is the maximum clearing time after which the system will still return to the operating point in a stable manner [14]. Since t_{cr} is directly related to the size of the region of attraction of x_e^o, the latter is of major importance in power systems operation. On the other hand, the dynamic performance of the machine following a major disturbance is also of great concern. A controller that provides both improved damping and increased critical clearing time is thus highly desirable.

Consider a case study, with the system parameters given in Table I. Then the operating point is

$$x_e^o = \begin{bmatrix} 1.12\,\text{rad} \\ 0 \\ 0.914\,\text{pu} \end{bmatrix}.$$

By simulating short circuits with increasing clearing times the critical clearing time for the open-loop system is found to be 90 ms. Fig. 1 presents the response of the open-loop system to a short circuit at the machine's terminal with exactly this clearing time.

The region of attraction of x_e^o can be visualized by means of a trajectory on its boundary, which can be obtained by the procedure briefly described below. The boundary of the region of attraction of x_e^o in the synchronous machine case is the stable manifold of the closest unstable equilibrium—which we denote x_e^u. If we linearize the system around x_e^u then the eigenvectors associated to the stable eigenvalues of this linearization define a vector space which is tangent to the stable manifold of x_e^u. Then points arbitrarily close to the stable manifold of x_e^u can be obtained as a small perturbation from this equilibrium along some direction inside this tangent space. If the system is simulated in reverse time with such a point as the initial condition, then the resulting trajectory will remain arbitrarily close to the boundary of the stable manifold of x_e^u. Details of this trajectory reversing approach are given in [5] and [8]. This procedure has been applied to obtain trajectories arbitrarily close to the boundary of the region of attraction for our case study in open-loop and with the controllers proposed. The trajectory obtained for the open-loop system is shown in Fig. 2.

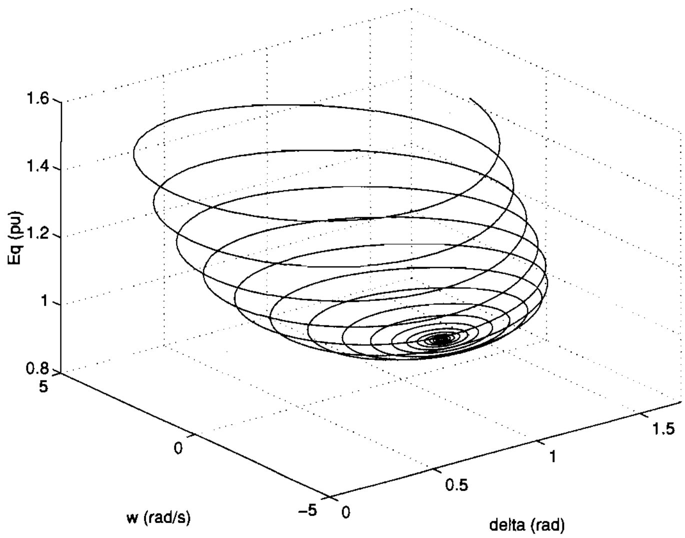

Fig. 2. Trajectory on the boundary of the region of attraction in open loop.

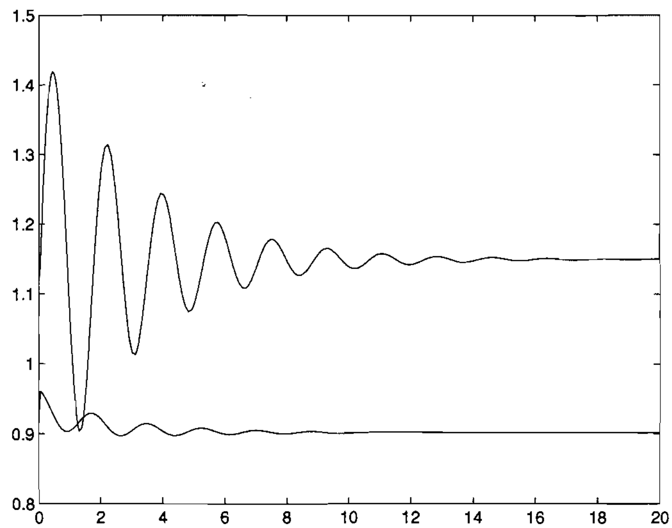

Fig. 3. Load angle and internal voltage behavior with the static L_gV controller; $k = 1$, $t_{cl} = 60$ ms.

Consider now a static L_gV controller

$$u = -kL_gV(x) = k[\phi(x) - \phi(x_e^o)]; \qquad k > 0$$

which is of the form (8) with $\varphi(x) \triangleq k\phi(x)$. The Lyapunov derivative under this control law becomes

$$\dot{V}(x) = -(k+1)\,\frac{b_1}{b_3}\,[\phi(x) - \phi(x_e^o)]^2 - Dx_2^2 \le 0.$$

It can be seen in Fig. 3 that this static L_gV controller provides the system with better damping. However, the region of attraction is not enlarged, as shown in Fig. 4.

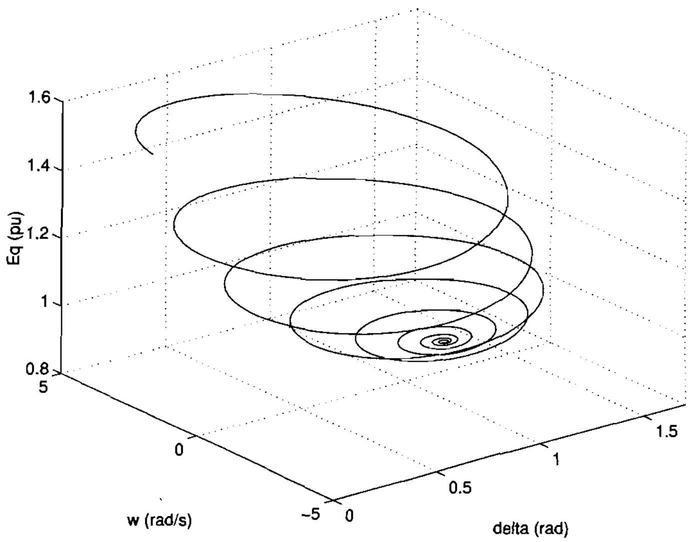

Fig. 4. Trajectory on the boundary of the region of attraction with the static L_gV controller; $k = 1$.

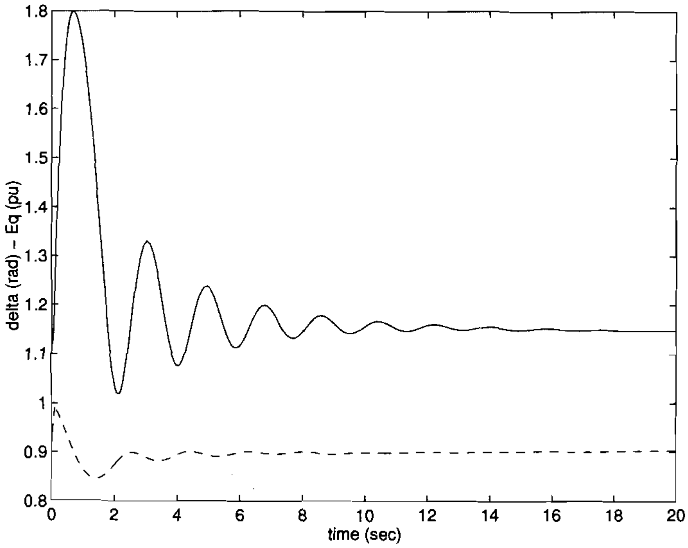

Fig. 5. Load angle and internal voltage behavior with the dynamic L_gV controller; $k = 1$, $\alpha = 1$, $t_{cl} = 108$ ms.

The dynamic extension of the above L_gV controller is given by

$$u = k[\phi(x) - \hat{\theta}]$$
$$\dot{\hat{\theta}} = \alpha[\phi(x) - \hat{\theta}]$$

with $\alpha > 0$. Fig. 5 shows that the dynamic performance of the system with this controller is better than that obtained with the static L_gV controller. Moreover, Fig. 6 shows that the region of attraction is considerably larger than in open loop and with the static controller (notice the different scales in the plots). Indeed, a critical clearing time of 108 ms is obtained with this controller, which is 20% larger than in open-loop.

V. Conclusion

A dynamic state feedback scheme has been proposed for the control of nonlinear systems. This dynamic L_gV controller derives from a static L_gV controller by treating the equilibrium of the open-loop system as an uncertain parameter and applying an adaptation mechanism to estimate this parameter. It has been shown that the proposed control structure preserves the properties of infinite gain margin and enlargement of the estimate of region of attraction present in L_gV controllers. The dynamic L_gV controller does not require the knowledge of the operating point, which is invariant under this feedback. Furthermore, the proposed scheme may yield a better performance than the original L_gV controller from which it is derived. This is indeed the case for the excitation control of synchronous machines, as shown by the case study presented.

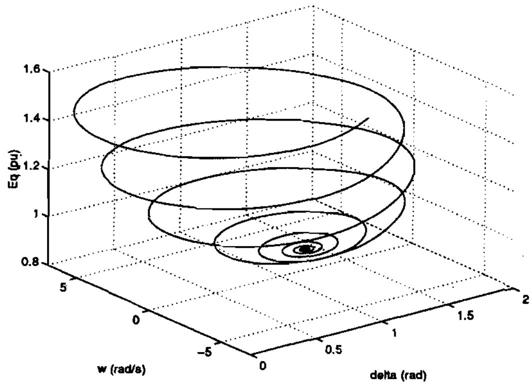

Fig. 6. Trajectory on the boundary of the region of attraction with the dynamic L_gV controller; $k = 1$, $\alpha = 1$.

References

[1] J. Ackerman, "Yaw disturbance attenuation by robust decoupling of car steering," in *Proc. 13th IFAC World Congr.*, San Francisco, CA, 1996, vol. Q, pp. 1–6.

[2] A. Baccioti, *Local Stabilizability of Nonlinear Control Systems*. New York: World Scientific, 1991.

[3] A. S. Bazanella, P. V. Kokotovic, and A. S. e Silva, "On the control of dynamic systems with unknown operating point," in *Proc. 4th European Control Conf.*, Brussels, Belgium, 1997.

[4] C. I. Byrnes, A. Isidori, and J. C. Willems, "Passivity, feedback equivalence, and the global stabilization of minimum phase nonlinear systems," *IEEE Trans. Automat. Contr.*, vol. 36, pp. 1228–1240, 1991.

[5] H. D. Chiang, M. W. Hirsch, and F. F. Wu, "Stability regions of nonlinear autonomous dynamical systems," *IEEE Trans. Automat. Contr.*, vol. 33, pp. 16–26, 1988.

[6] J. M. Coron, "Linearized control systems and applications to smooth stabilization," *SIAM J. Control and Optimization*, vol. 32, pp. 358–386, 1994.

[7] R. A. Freeman and P. V. Kokotovic, *Robust Nonlinear Control Design: State-Space and Lyapunov Techniques*. Boston, MA: Birkhauser, 1996.

[8] R. Genesio, M. Tartaglia, and A. Vicino, "On the estimation of asymptotic stability regions: State of the art and new proposals," *IEEE Trans. Automat. Contr.*, vol. 30, pp. 747–755, 1985.

[9] V. Jurdjevic and J. P. Quinn, "Controllability and stability," *J. Differential Equations*, vol. 28, pp. 381–389, 1978.

[10] P. Kundur, *Power System Stability and Control*. New York: McGraw-Hill/EPRI, 1994.

[11] K. K. Lee and A. Arapostathis, "Remarks on smooth feedback stabilization of nonlinear systems," *Syst. Contr. Lett.*, vol. 10, pp. 41–44, 1988.

[12] W. Lin, "Input saturation and global stabilization of nonlinear systems via state and output feedback," *IEEE Trans. Automat. Contr.*, vol. 40, pp. 776–782, 1995.

[13] H. Nijmeijer and A. J. Van der Schaft, *Nonlinear Dynamic Control Systems*. New York: Springer-Verlag, 1990.

[14] M. A. Pai, *Energy Function Analysis for Power System Stability*. New York: Kluwer, 1989.

[15] R. Sepulchre, M. Janković, and P. V. Kokotović, *Constructive Nonlinear Control*. New York: Springer-Verlag, 1996.

[16] H. O. Wang and E. H. Abed, "Bifurcation control of a chaotic system," *Automatica*, vol. 31, pp. 1213–1226, 1995.