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# PORTFOLIO OPTIMIZATION BASED ON GARCH-EVT-VINECOPULA VIA INFORMATION RATIO, SHARPE RATIO AND SORTINO INDEX

**Porto Alegre**
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Dissertação submetida ao Programa de Pós-Graduação em Economia da Faculdade de Ciências Econômicas da UFRGS, como requisito parcial para obtenção do título de Mestre em Economia, com ênfase em Economia Aplicada.

Orientador: Prof. Flávio A. Ziegelmann

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"if you're going to try, go all the way.otherwise, don't even start..." - Bukwoski, Charles.

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**ABSTRACT**

This study employs an ARMA-GARCH-EVT modeling approach to capture marginal features and Vine copula models to understand tail dependence in a portfolio of 60 stocks listed on the Brazilian stock market, specifically the Ibovespa. Throughout this analysis, we focus on portfolio optimization using five key investment performance metrics, including Information Ratio, Sharpe Ratio, and Sortino Ratio. These optimizations were carried out at two rebalancing frequencies, all focusing on risk mitigation and index optimization, and considering market transaction costs. In total, we propose 20 portfolios to achieve a positive Alpha on the Ibovespa Index. Notably, one of the significant findings of this study is the ability to achieve positive excess returns compared to the Ibovespa Index during 2019 and 2020. It is worth noting that this superior performance is particularly noteworthy, as seen, in terms of cumulative returns, in 19 of the 20 portfolios proposed in this dissertation.

Keywords: Portfolio optimization. Vine-Copula. Extreme value theory. Ibovespa.

# RESUMO

Este estudo emprega uma abordagem de modelo ARMA-GARCH-EVT para capturar caracterís- ticas marginais e modelos de cópula Vine para entender a dependência da cauda em uma carteira de 60 ações listadas no mercado de ações brasileiro, especificamente o Ibovespa. Ao longo desta análise, nos concentramos na otimização do portfólio usando cinco métricas principais de desempenho de investimento, incluindo Informatio Ratio, Índice de Sharpe e Índice de Sortino. Essas otimizações foram realizadas em duas frequências de rebalanceamento, todas com foco na mitigação de riscos e na otimização dos índices, e considerando os custos de transação de mercado. No total, propomos 20 carteiras com o objetivo de atingir um Alpha positivo no Índice Ibovespa. Notavelmente, uma das descobertas significativas deste estudo é a capacidade de obter retornos excedentes positivos em comparação ao Índice Ibovespa durante 2019 e 2020. Vale ressaltar que esse desempenho superior é particularmente digno de nota, como visto, em termos de retornos acumulados, em 19 dos 20 portfólios propostos nesta dissertação.

Palavras-chave: Otimização de portfólio. Vine-Copulas. Teoria de valor extremo. Ibovespa.

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# 1 INTRODUCTION

The main goal of portfolio optimization is to find the optimal asset allocation that strikes a balance between maximizing returns and managing risk. In this case, the selection of an optimal portfolio depends on the initial assumptions about the behavior of the assets and the choice of a measure of risk. Markowitz (1952) is a pioneer in the construction of the optimal portfolio considering a trade-off between risk and return and proposing the Mean-Variance approach, which paved the way for the development of The Modern Portfolio Theory. This study proposes an ARMA-GARCH EVT Vinecopula model to capture the marginals, tail probability, and dependence structure of 60 assets, to perform portfolio optimization throughout the years 2019 and 2020. Five optimization criteria, namely Sharpe Ratio, Sharpe VaR, Sharpe CVaR, Sortino, and Information Ratio, are employed in the evaluation process.

However, despite being a pioneering work in portfolio optimization, Markowitz (1952)'s work depends on certain assumptions that may limit its applicability in the real-world context. Firstly, it assumes that returns follow a normal distribution, which may not accurately represent the non-normal characteristics, such as fat tails and skewness, often observed in financial asset returns. Additionally, the approach only captures a linear dependence between financial returns, disregarding the more complex and dynamic nature of correlations in different market conditions. Secondly, the reliance on variance as a measure of risk is a notable limitation. Variance treats positive and negative deviations from the mean symmetrically. In financial markets, investors are typically more concerned about downside risk (losses) than upside volatility (gains). Other risk measures, such as downside deviation or semi-variance, focus specifically on negative deviations and may better capture the risk that investors are concerned about. Furthermore, nonlinear risk measures, such as Value at Risk (VaR) or Conditional Value at Risk (CVaR), may provide a more accurate representation of risk in such situations.

The reasons explained above serve as strong incentives for applying copulas to capture the real dependence structure between assets. The copula model emerges as a more predominant approach and addresses the limitations of conventional linear models. Sklar's theorem (Sklar,1959), a fundamental result that has guided the entire copula theory literature, states that any multivariate joint distribution function can be decomposed into its marginal distribution functions and a copula function that captures the dependence structure between variables. In other words, the theorem provides a way to separate the marginal distributions of variables from their dependence structure, which can be useful for modeling purposes. The copula model accommodates marginal distributions and allows for a more flexible dependence structure, encompassing linearity, nonlinearity, or even tail-only dependence. Thus, the finantial time series literature has used ARMA-GARCH copula models, where the marginal time series are modeled by univariate ARMA-GARCH models, while the dependence structure is explained by copula models.

A notable advancement beyond traditional copulas was achieved with the emergence of pair copulas. The pioneering work of Joe (1997) delved into constructing multivariate copulas adapted to various types of dependency structures. Unfortunately, the options for multivariate copulas are limited compared to the bivariate scenario, where a range of copula types exhibiting flexible and complex dependence patterns is available. The innovative work of Bedford e Cooke (2001) and Bedford e Cooke (2002) introduces the concept of regular vine (R-vine), a convenient graphical model for categorizing pair copula constructions, displaying a hierarchical structure. The authors also propose two distinct structure types of regular vine copula: drawable (D-vine) and canonical (C-vine). Additional insights into the properties and statistical inference of vine copula can be found in the works of Aas et al. (2009), Joe (2014), and Czado (2019).

Dissmann et al. (2013) developed an automated strategy that uses a sequential approach to search for the optimal R-vine tree structure, pair-copula families, and parameter values for each asset. This process starts with identifying the first tree and its corresponding pair-copula families, followed by estimating their parameters. Using transformed variables, the specification of the second tree is based on the choices made in the first tree. They use a maximum spanning tree algorithm to select each tree, with edge weights reflecting large dependencies. Pair-copula are chosen independently using the Akaike information criterion, which performs well in this context. Finally, the sequential estimation approach is used for the corresponding pair-copula parameter estimation for D-vine and C-vine. Overall, this approach allows for a more efficient and accurate analysis of financial data. The authors tested this strategy in 16-dimensional financial data and found that their approach was highly effective. The results indicated that R-vine distributions provide a better fit than both C- and D-vines for this particular data set. This improvement allows us to expand the implementation of our approach to higher dimensions, which is critical for assessing the risk associated with larger financial portfolios.

Patton (2006) extended the concept to model dynamic dependence and applied theSklar’s theorem to conditional copulas. In finance, copulas have been widely used in, mong others, in risk management (Chiou; Tsay, 2008; Kole; Koedijk; Verbeek, 2007), dependence modeling (Genest; MacKay, 1986; Schweizer; Wolff, 1981; Wei; Zhang; Guo, 2004) and portfolio optimization (Wu et al., 2006; Kakouris; Rustem, 2014), among others.

Tófoli, Ziegelmann e Silva Filho (2017) propose a novel method for modeling the dependence in financial return data over time by introducing Markov switching into the copula dynamics. The study investigates the dynamics of dependence and the models' ability to predict capital losses, focusing on tail risk in risk management. They compare models based on Value-at-Risk (VaR) forecasts for portfolios and find that their approach outperforms others in capturing time-varying dependencies, particularly during financial crises. In a subsequent study Tófoli et al. (2019), the authors introduce a dynamic D-vine copula model for analyzing multivariate financial return data, demonstrating its effectiveness in capturing time-varying dependencies and improving VaR forecasts, especially during bear markets. The findings suggest valuable insights for risk management in different market conditions, highlighting the model's superior predictive accuracy compared to static models.

In terms of empirical applications in the area of portfolio management, Zhang et al. (2014) explore the application of C-, D- and R-vine for 10 international stock indices. Based on the structure estimated by each vine copula model, they simulate returns for each series and measure the VaR and ES for the international stock portfolio. This article indicates that the Vine copula model is more accurate and reliable in risk prediction compared with traditional methods, such as: mean-variance (MV), historical simulation (HS) and multi DCC-GARCH (DCC). The authors demonstrate that Vine copula models are able to effectively forecast the VaR of the international stock markets portfolio on the base of VaR measurement, and D-vine copula is superior to other copulas.

Low et al. (2013) investigate the ability of using C-vine copula models to forecast returns for portfolios from 3 to 12 assets could produce superior investments performance compared to traditional methods. The authors assume that investors have no short-sales constraints and a utility function characterized by Conditional Value-at-Risk (CVaR) minimization. They examine the efficient frontiers produced by each model and, as the traditional Mean-Variance (MV) model does not take into account asymmetry in return distributions, there is a clear need for more advanced portfolio management strategies that incorporate asymmetries into the forecasting process and during the optimization of the investor's utility function. Furthermore, the article also shows that as the number of assets in the portfolio increases, modeling the dependence structure across the assets has a greater impact. They conclude that Clayton C-vine copulas are 'worth it' when managing portfolios of high dimensions due to their ability to better capture asymmetries within the dependence structure than either the Clayton Standard Copula or multivariate normal models.

In light of the recent Covid-19 pandemic and increasing volatility at global financial markets, a requirement for good estimation of risk measures, such as VaR and ES, has become even greater for any national institution and asset management. Zhang, Zhang e Lee (2022) explore the impact of the COVID-19 pandemic on the risk spillover structure and comparatively analyze the changes in the risk spillover structure before and after the pandemic. The authors aim to construct the GARCH-Copula-CoVaR model to improve the inadequacy of the quantile regression measure in previous studies and to explore the direct spillover value between important global stock markets before and after the pandemic through a bidirectional spillover matrix. They also analyze the indirect infection path in risk spillover and obtain the impact of the pandemic on an indirect path by measuring the indirect spillover value with the R-vine structure. The empirical results of this paper suggest that the COVID-19 pandemic has a significant impact onthe risk spillover structure of the global stock market. The authors find that the direct spillovervalue between important global stock markets has increased significantly after the pandemic, indicating that the systemic risk of the global stock market has increased. They also find that the indirect spillover path in risk spillover has changed significantly after the pandemic, and the impact of the pandemic on the indirect path is more significant than that on the direct path. The authors identify key nodes for risk prevention and provide policy implications for global financial risk management.

Sommer, Bax e Czado (2023) focus on extending the estimation of unconditional risk measures and introduce a novel algorithm for forecasting conditional risk measures, particularly Expected Shortfall (ES) over April, 2020 and October, 2021. The goal is to measure the risk of large asset portfolios in a conditional setting using a stress factor, providing more accurate estimation of various risk measures for financial portfolios. The application of a flexible class of vine copulas, specifically D-vine copulas, allows for the determination of the conditional distribution of portfolio components given a stress factor. This approach helps model complex dependence structures and adequately incorporates potentially high-dimensional dependence in financial portfolios. Using European and American market indices as stress factors, they find that a portfolio of the largest Spanish stocks could potentially serve as a hedge in the American market, but not in the European market. The sensitivity of the risk measure to the market is noted to be higher for Spanish assets.

In terms to risk management, Han e Li (2020) apply mixture of vine copulas to describe the distribution of assets returns, they use a rolling window to forecast the worst-case CVaR (WCVaR) of multi-assets portfolios. The first portfolio composed of ten industry indices of

Chi- nese equities which includes Energy, Materials, Industrials, Consumer Discretionary, Consumer Staples, Health Care, Financials, Information Technology, Telecommunication Services and Utilities. Their results show that mixture C-vine copula performs best in terms of sharpe ratio, average returns, the expected maximum drawdown, total return and cumulative returns, followed by the mixture R-vine, while mixture D-vine method performs the worst. Meanwhile, the second portfolio composed by five representative of the major asset classes in China: SHIBOR1W (representing cash), Wind full A Index (representing stock market), ChinaBond Composite Total Return Index (Total Value) (representing bond market), SSE Fund Index (representing fund market) and Wind Commodity Index (representing commodity market), the mixture of R-vine copula performs better in terms of sharpe ratio, average returns, total returns and cumulative returns.

A novel approach proposed by Al Janabi, Ferrer e Shahzad (2019) investigate the liquidity-adjusted Value-at-Risk (LVaR)1 optimization of a portfolio consisting of stock market indexes of the G-7 countries along with gold, a global commodity index and the cryptocurrency bitcoin. To validate the framework performance, they compare out-of-sample of optimal allocations derived by the vine copula based LVaR optimization approach under various scenarios (e.g., only long positions, both long and short sales with budget restrictions, and different liquidation periods). The study suggests that the vine copula-based LVaR optimization approach outperforms the traditional MV VaR model, leading to a relevant improvement in optimal portfolio selection under various operational and budget constraints. The out-of-sample analysis further confirms the superior performance of the vine copula-based LVaR approach over other frameworks, including Markowitz's MV method and the equally weighted portfolio strategy.

To address the heavy tails present in return series, Longin e Solnik (2001) introduced the Extreme Value Theory (EVT) as a powerful tool for dissecting financial data characterized by clear deviations from normality. EVT operates as a methodological framework designed to explore extreme values, making it an important tool for capturing rare events that exist in the distribution's tails. This theory has attracted considerable attention from researchers seeking to delineate and comprehend rare occurrences within the field of finance, quantitative risk management, and the insurance market (Tsevas; Panaretos, 1998; Diebold; Schuermann;

[1] According to the CFA (Chartered Financial Analysts) Institute, liquidity risk represents any risk of economic loss because of the need to sell relatively less liquid assets to meet liquidity requirements; the risk that a financial instrument cannot be purchased or sold without a significant concession in price because of the market's potential inability to efficiently accommodate the desired trading size.

Stroughair, 1998; Bensalah, 2000; Embrechts; Meister, 1997; Embrechts; Resnick; Samorodnitsky, 1999; Embrechts, 2000). EVT focuses on modeling the stochastic patterns governing extreme events situated in the tails of probability distributions. The primary objective of EVT lies in forecasting the probabilities associated with infrequent events and drawing meaningful comparisons with past recorded instances.

The application of GARCH-EVT-Copula models to risk management has been the focus of several studies. Wang et al. (2010) measure the risk of a multi-dimensional foreign exchange rates portfolio. Bhatti e Nguyen (2012) suggests the applicability of a conditional EVT and time-varying copula for modeling the tail dependency between stock markets. Huang e Hsu (2015) consider two GARCH EVT-Copula models for simulating future stock market returns, employing a rolling window to compute the optimal weights based on the Min-CvaR allocation for the out-of-sample period. They also manage four rebalancing strategies - daily, weekly, biweekly, and monthly - for each model. Sahamkhadam, Stephan e Östermark (2018), explores portfolio optimization using GARCH-EVT-Copula models and ARMA-GARCH-EVT-Copula models, employing three portfolio optimization techniques (Min-CVaR, GMV, and CET) to calculate the optimal weights of different portfolios with different rebalancing strategies, without transaction costs.

Our work follows the approach of Sahamkhadam, Stephan e Östermark (2018), utilizing an ARMA-GARCH model as a foundational tool in portfolio optimization. This model serves as a basis for capturing essential characteristics in financial series. Our data is composed of 60 assets presented in the Ibovepa Index. To improve the GARCH model specification and provide a more accurate description of the dynamic volatility structures of each asset, we improved the specification of the GARCH model using the AIC criterion. Testing three GARCH families (GARCH, EGARCH, GJR-GARCH) for each asset in every iteration - a novel comparative approach within the existing literature. Subsequently, we employ the Extreme Value Theory (EVT) to explore and model the extreme values present in the tail in every iteration. Moving forward, we adopt the automated strategy proposed by Dissmann et al. (2013) to identify the optimal family copula model for each asset pair. This selection process is based on Kendall's Tau maximization, thus determining the most suitable vine structure for our dataset. Then we incorporate the parameters from the ARMA-GARCH models and dependency structures from the vine copula models. To obtain optimal portfolio allocations, we perform the optimization of five different criteria (Sharpe Ratio, Sharpe Ratio VaR, Sharpe Ratio CVaR, Sortino Ratio, and Information Ratio). Furthermore, the central objective of our study revolves around returns to outperforming benchmarks. We achieve this by combining two different Vine copula models, different portfolio optimization techniques, and two rebalancing frequencies, culminating in the creation of twenty different portfolios.

This work strongly contributes to the existing literature in several dimensions. Firstly, we propose a vine copula approach for 60 assets, a higher dimension than most of the GARCH-EVT-Copula literature. Notably, our best portfolio - weekly rebalancing C-Vine Information Ratio - in terms of annualized returns, gave us an excess of 217.43% over the Ibovespa benchmark and 112.12% over the daily rebalancing Markowitz portfolio. This result takes into account B3's trading fee for local investment funds and clubs, which amounts to 0.023% of the financial value of the transaction. Our empirical investigation uses daily data of 60 Brazilian stock prices presented in the Ibovespa Index during the period 4 February 2014 to 30 December 2020. The primary findings indicate portfolios exhibit increasing risk and reduced diversification over time. However, as our goal is to propose an approach capable of achieving positive returns above the Ibovespa Index, our emphasis is on constructing a highly flexible model devoid of constraints. Subsequent research efforts could explore imposing limitations on asset weights to avoid excessive concentration in only a few. Additional criteria focused exclusively on minimizing Value at Risk (VaR) and minimizing standard deviation may also be considered in future investigations. Secondly, this study is focused on an entirely quantitative methodology, using a rolling window made up of 1,223 daily returns in all calculations, the only fixed results in the out-of-sample are the vines structure. This involves re-estimating all GARCH, EVT, and vines parameters at each iteration. Additionally, we evaluate three different GARCH models for all assets and select the optimal model for each day based on the Akaike Information Criterion (AIC). This approach provides us with a more adaptable model for all out-of-sample data, resulting in a superior fit to our dataset through continuous model refinement.

The rest of the dissertation is organized as follows. Chapter 2 presents the methodology, including the ARMA-GARCH model, Extreme Value Theory, Vine copula model, and opti- mization criteria employed in our study. Moving on to Chapter 3, we provide a comprehensive statistical description of the dataset and a detailed exposition of our empirical findings. Finally, Chapter 4 concludes our analysis.

# 2 METHODOLOGY

This chapter describes the ARMA-GARCH-EVT-Vinecopula and the maximization criteria for the optimal weights for portfolio allocation.

## 2.1 ARMA-GARCH FORECASTING

A range of time series models have been proposed in the literature to explain the dynamics of financial time series and its characteristics. The applications of the Autoregressive Conditional Heteroscedasticity (ARCH) model introduced by Engle (1982) or its extension GARCH by Bollerslev (1986) in finance field have become common. Following previous studies, we adopt the univariate ARMA(1,1)-GARCH(1,1) model for the assets to obtain our marginal distribution, defined as follows:

$$\begin{cases} r_{it} = \mu_{it} + \varphi_i \left(r_{i,t-1} - \mu_{it}\right) + \theta_i \epsilon_{i,t-1} + \epsilon_{it} \\ \epsilon_{it} = z_{id} \sigma_{it} \\ z_{it} \approx (\text{ i.i.d.}) \\ \sigma_{it}^2 = \omega_i + \alpha_i \epsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2 \end{cases}, \tag{2.1}$$

where $r_{i,t}$ is the actual return for asset i = 1, 2, ..., d, zit is the standardized error, and the parameter restrictions are $\omega_i > 0$, $\alpha_i \geq 0$, $\beta_i \geq 0$, $\alpha_i + \beta_i < 1$, $\varphi_i + \theta_i \neq 0$. Following empirical studies, we assume $z_{i,t}$ follows Skew-Student $t$ distribution proposed by Fernandez e Steel (1998).

To attain optimal results, we assessed two additional GARCH models for each asset and compared their performance with the initial model using the Akaike Information Criterion (AIC).

### 2.1.1 EGARCH

Nelson (1991) introduces the Exponential GARCH model to overcome some limitations that need to be addressed in the GARCH (p, q) model. One limitation is its inability to provide distinct responses to returns with different signs but the same absolute value. Additionally, thenon-negativity constraints on the volatility equation parameters can lead to increased volatilitywith any increments in the absolute values of returns, which eliminates random oscillatory behavior.

An exponential generalised autoregressive conditionally heteroscedastic model with order $p = 1$ and $q = 1$, denoted by EGARCH$(1, 1)$, is defined as

$$\log \sigma_{it}^2 = \omega_i + \alpha_i \left(|\epsilon_{t-1}| - \gamma_i \epsilon_{t-1}\right) + \beta_i \log \sigma_{t-1}^2, \tag{2.2}$$

where $\gamma_i$ represents the asymmetry parameter, also known as the leverage effect. Responsible for positive and negative shocks, making it a valuable tool for capturing the dynamics of financial return data.

### 2.1.2 GJR-GARCH

Proposed by Glosten, Jagannathan e Runkle (1993) the GJR-GARCH can capture the asymmetry and fat tails in the distribution of returns. This model includes an additional parameter to capture the impact of negative returns on volatility, as well as a kurtosis parameter to capturethe degree of fat tails ($\gamma_i$). The GJR-GARCH(1,1) can be written as

$$\sigma_{i,t}^2 = \omega_i + \alpha_i \epsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2 + \gamma_i \psi \left[\varepsilon_{i,t-1} < 0\right] \varepsilon_{i,t-1}^2. \tag{2.3}$$

The GJR-GARCH and T-GARCH (Threshold GARCH) models have some similarities, but they differ in one key aspect: the GJR-GARCH includes the skewed Generalized Error Distribution, whereas the T-GARCH model does not account for the shape parameter.

## 2.2 TAIL BEHAVIOR

The tail behavior of the asset returns can be modeled by extreme value theory, specifically through the peak over threshold (POT). When integrated with an ARMA(p,q)-GARCH(1,1) model, this approach allows us to derive the marginal distribution function Gj(zˆj) for ith asset. We achieve this by employing the parametric generalized Pareto distribution (GPD) to model the

$$G_i\left(\widehat{z}_i\right) = \begin{cases} \frac{k_l}{n}\left[\hat{\xi}_j^l \frac{u_i^l - z_i}{\beta_i^l}\right]^{-\frac{1}{\xi_i^l}}, \text{ for } z_i < u_i^l \\ \varphi\left(\hat{z}_i\right), \text{ for } u_i^l < z_i < u_{ji}^r \\ 1 - \frac{k_r}{n}\left[\hat{\xi}_i^r \frac{z_i - u_i^r}{\beta_i^r}\right]^{-\frac{1}{\xi_i^r}}, \text{ for } z_i > u_i^r \end{cases}, \tag{2.4}$$

lower and upper tails, while the middle part is approximated using the Gaussian kernelwhere superscripts l and r denote the left and the right tails, respectively. β is the scale parameter and $\xi$ is the shape parameter. When $\xi < 0$, it indicates that the distribution has a finite tail, if $\xi > 0$ the distribution has a fat tail, and when $\xi = 0$ it indicates that the distribution has a thin tail. $n$ is the number of observations and k is the amount of observations beyond threshold u.

Selecting an appropriate threshold u is the main key of this approach. If the threshold is set too high, little data above the threshold will lead to high variance. On the other hand, if the threshold is set too low some observations may not belong to the extremes, which will lead to estimation bias. Previous studies have recommended different threshold levels or methods for determining the optimal level (McNeil e Frey (2000) and Longin e Solnik (2001) for example).

## 2.3 INTRODUCTION TO COPULAS

A copula is a mathematical function that describes the relationship among multiple random variables while taking into account their individual distributions. It is defined as a cumulative distribution function (CDF) with uniform marginals on the unit interval. This means that the copula function operates on variables that are transformed to have uniform distributions between 0 and 1, see for example, Joe (1996) and Nelsen (2007).

According to the theorem of Sklar (1959), for random continuous variables, if we have the cdf of each individual variable, denoted as $F_i(y_i)$ for $Y_j$, then combining this cdf through a copula function $C(F_1(y_1), \ldots F_d(y_d))$ yields a multivariate distribution for $\boldsymbol{Y} = (Y_1, \ldots, Y_d)$ with the desired marginal distributions $F_i$, $i = 1, \ldots d$. Conversely, if we have a continuous multivariate distribution with individual marginal cdf $F_1, \ldots, F_d$, then there exists a unique copula function $C$ that corresponds to this distribution, such that

$$F(\mathbf{y}) = C\left(F_1\left(y_1\right), \ldots, F_d\left(y_d\right)\right), \quad \forall \mathbf{y} = (y_1, \ldots, y_d). \tag{2.5}$$

The corresponding density is

$$f\left(x_1, \ldots, x_d\right) = \left[\prod_{k=1}^{d} f_k\left(x_k\right)\right] \times c\left(F_1\left(x_1\right), \ldots, F_d\left(x_d\right)\right), \tag{2.6}$$

where $c(u_1, \ldots, u_d)$ is the d-dimensional copula density and $f_i$, $i = 1, \ldots, d$, are the corresponding marginal densities.

Copulas have been combined with time series models for modeling financial series returns, i.e., copula modeling operates with the standardized residuals obtained from ARMA-GARCH models.

In the copula context, vines emerged as a solution to overcome limitations in the flexibility of traditional copula models. In particular, such Archimedean copulas, characterized by typically having only one or two parameters, impose strict dependence properties by assuming ex- changeability and uniformity across all multivariate margins. Elliptical copulas as the Gaussian copula, operate under the assumption of a single global dependence structure that applies uniformly to all pairs of variables. Although the Student-t copula allows for symmetric tail dependence, it suffers from a drawback: it has only a single parameter to govern the tail dependence of all variable pairs. The category of vine copulas adeptly addresses the limitations associated with elliptical and Archimedean copulas, as well as other copulas. Simultaneously, it leverages their advantageous features in the bivariate case.

## 2.4 VINE COPULAS

Introduced by Aas et al. (2009), the vine structure can be illustrated in three dimensions. Assuming that all necessary densities exist, let $\mathbf{Y} = (y_1, y_2, y_3)' \sim F$, then

$$f(y_1, y_2, y_3) = f_1(y_1)\, f(y_2 \mid y_1)\, f(y_3 \mid y_1, y_2). \tag{2.7}$$

Using Sklar (1959) theorem (2.1), it follows

$$\begin{aligned} f(y_2 \mid y_1) &= \frac{f(y_1, y_2)}{f_1(y_1)} \\ &= \frac{c_{1,2}(F_1(y_1), F_2(y_2))\, f_1(y_1)\, f_2(y_2)}{f_1(y_1)} \\ &= c_{1,2}(F_1(y_1), F_2(y_2))\, f_2(y_2), \end{aligned} \tag{2.8}$$

where c1,2 is a copula function that links y1 and y2 and F1, F2 and F3 are the respective marginal distribution functions. Similarly, the second conditional density in Eq.(2.7) can be factorized as:

$$\begin{aligned} f(y_3 \mid y_1, y_2) &= \frac{f(y_2, y_3 \mid y_1)}{f(y_2 \mid y_1)} \\ &= c_{2,3|1}(F(y_2 \mid y_1), F(y_3 \mid y_1))\, c_{1,3}(F_1(y_1), F_3(y_3))\, f_3(y_3), \end{aligned} \tag{2.9}$$

Combining the Equations (2.7)-(2.9), the joint density of the three-dimensional can be obtained as follows:

$$\begin{aligned} f(y_1, y_2, y_3) =& f_1(y_1)\, f_2(y_2)\, f_3(y_3)\, c_{1,2}(F_1(y_1), F_2(y_2)) \\ & c_{1,3}(F_1(y_1), F_3(y_3))\, c_{2,3|1}(F(y_2 \mid y_1), F(y_3 \mid y_1)). \end{aligned} \tag{2.10}$$

The joint density function can be calculated as the product of the marginal densities and three bivariate copulas that collectively form the vine copula. The copulas $c_{1,2}(F_1, F_2)$ and $c_{1,3}(F_1, F_3)$ are unconditional, and the copula $c_{2,3|1}(F_{2|1}, F_{3|1})$ is conditional on $y_1$. Each of these copulas might not belong to the same copula family, indicating that the vine copula provides extensive flexibility for modeling dependency in high-dimensional data. These findings can be extended to the n-dimensional case.

Proposed by Joe (1996), the marginal conditional distributions, $F(y \mid v)$, which are necessary for the pair-copula decomposition of a multivariate density, can be computed as:

$$F(y \mid v) = \frac{\partial C_{yv_j|v_{-j}}\left(F\left(y \mid v_{-j}\right), F\left(v_j \mid v_{-j}\right)\right)}{\partial F\left(v_j \mid v_{-j}\right)} \tag{2.11}$$

where $C_{xv_j|v_j}$ is a bivariate copula and $v_j$ denotes a vector with the $jth$ component $v_j$ removed.

The decomposition in Eq.(2.7) is not unique. $f(y_1, y_2, y_3)$ can be expressed as $f_1, f_2, f_3 c_{1,3} c_{2,3} c_{1,2|3}$ or as $f_1, f_2, f_3 c_{1,2} c_{2,3} c_{1,3|2}$, which implies that the choice of the particular vine copula structure, the organization of the different possibilities of decomposition, becomes a crucial issue.

Vines were introduced by Joe (1996), Bedford e Cooke (2001) and Bedford e Cooke (2002) and described in more detail in Kurowicka e Cooke (2006) and Kurowicka e Joe (2010).

Following the definition of Kurowicka e Cooke (2006), a regular vine (R-vine) on d variables is a sequence nested set of d − 1 trees such that each, where the edges of the (j − 1)th tree. The proximity condition ensures that two nodes in the (j − 1)th tree are connected by an edge only if these nodes share a common node in the jth tree. It's worth noting that the initial tree's set of nodes comprises all indices from 1 to d, while the set of edges consists of d − 1 pairs of these indices. In subsequent trees, the set of nodes includes sets of pairs of indices, and the set of edges is formed by pairs of pairs of indices, and so on.

According to Theorem 4.2 of Kurowicka e Cooke (2006), the R-vine copula density is given by

$$c\left(F_1\left(y_1\right), \ldots, F_d\left(y_d\right)\right) = \prod_{i=1}^{d-1} \prod_{e \in E_i} c_{j(e),k(e)|D(e)}\left(F\left(y_{j(e)} \mid y_{D(e)}\right), F\left(y_{k(e)} \mid y_{D(e)}\right)\right) \tag{2.12}$$

where $c_{j(e)}, k(e) \mid D(e)$ represents a bivariate conditional density copula with $j(e)$ and $k(e)$ as the conditioned nodes, and $D(e)$ as the conditioning set. The parameter $e = j(e), k(e)|D(e)$is an edge that belongs to the edge set $\epsilon = E_1, \ldots, E_{d-1}$. $Y_{D(e)}$ is the vector of variables conditioned by the components of the conditioning set $D(e)$.

A particular case of R-vines, which we considered in this work, is the canonical vines (C-vines). An R-vine is called C-vine if each tree $T_i$ has a unique node with degree $d - 1$, the *root node*. According to Aas et al. (2009), the C-vine density can be write as

$$c\left(F_1\left(y_1\right), \ldots, F_d\left(y_d\right)\right) = \prod_{i=1}^{d-1} \prod_{j=1}^{d-i} c_{i,i+j|1,\ldots,i-1}\left(F\left(y_i \mid y_1, \ldots, y_{i-1}\right), F\left(y_{i+j} \mid y_1, \ldots, y_{i-1}\right)\right) \tag{2.13}$$

The model selection for both C-vine and R-vine copulas structures are determined using the maximum spanning tree approach proposed by Dissmann et al. (2013). The optimization problem for each tree is established based on the Kendall correlation coefficient between pairs of stock markets. The procedure begins with the calculation of the empirical Kendall's Tau $\hat{\tau}_{i,k}$ forall conceivable pairs of variables denoted as $\{i, k\}$, $1 \leq j \leq k \leq d$. Subsequently, the primary objective is to identify the spanning tree that maximizes the aggregate of absolute empirical Kendall's Tau values., i.e.,

$$\max \sum_{\text{edge } e=\{i,k\} \text{ in spanning tree}} |\hat{\tau}_{i,k}| \tag{2.14}$$

for each pair $\{i, k\}$ identified within the chosen spanning tree, the procedure selects a copula and estimates the corresponding parameter(s), for $i = 2, \ldots, d-1$. The next step involves computing the empirical Kendall's Tau $\hat{\tau}_{i,k|D}$ for all conditional variable pairs $\{i, k|D\}$ that can be part of the tree $T_i$. Among these edges, select the spanning tree that maximizes the sum of absolute empirical Kendall's taus, i.e.,

$$\max \sum_{\text{edge } e=\{i,k|D\} \text{ in spanning tree}} \left|\hat{\tau}_{i,k|D}\right| \tag{2.15}$$

Then for each $\{i, k|D\}$ in the select spanning tree, we computed observations $F(y \mid v)$ from the estimated pair copulas using the Eq. (2.11).

### 2.4.1 The portfolio allocation criteria

In this dissertation, we will explore five different criteria for determining the optimal weights for portfolio allocation.

### 2.4.2 Sharpe Ratio

The Markowitz's optimal portfolio theory has two main characteristics: the expected return maximization and the risk measure (variance) minimization. Several studies in recent decades have focused on Markowitz's mean-variance portfolio, and one important extension is the maximization of the Sharpe ratio (SR), proposed by Sharpe (1963), Sharpe (1994). The SR is defined as the average return earned over a risk-free rate per unit of volatility or total risk. This means how much expected return your allocation is accept when adding a unit of risk.

Let's suppose a d-dimensional portfolio with asset returns $r_t = (r_{t1}, r_{t2}, ..., r_{td})$, $i = 1, 2, ..., d$, asset weights $w_t = (w_{t1}, w_{t2}, ..., w_{td})$ and a $d \times d$ covariance matrix $\sum$ at time $t$.

Then the portfolio expected return and variance are, respectively, $\boldsymbol{w_t^T} r_t$ and $\boldsymbol{w_t^T} \sum w_t$. The Sharpe Ratio maximization can be expressed as follows:

$$\begin{array}{lll} \underset{w_t}{\text{maximize}} & \frac{w_t^T rt}{\sqrt{wt^T \Sigma w_t}} & \text{Sharpe ratio} \\ \text{subject to} & w_t^T 1 = 1, & \text{full investment} \\ & \forall i \in \{1, 2, \ldots, d\} : & \\ & w_{t,i} \geq 0, & \text{long positions only} \end{array} \quad . \qquad (2.16)$$

### 2.4.3 Sharpe Ratio VaR

Since investors are primarily concerned with downside risk rather than windfall gains, variance alone may not be an effective tool for reflecting the risk profile of cautious investors. Consequently, Value-at-Risk (VaR) is proposed as a more suitable risk measure in risk management.

The Sharpe Ratio in combination with the Standard Deviation, when faced with a high variance caused by extraordinary gains, can penalize the optimization process, resulting in asub-optimal portfolio allocation. To address this challenge, many studies have been considering VaR as the risk measure in combination with Markowitz's mean-variance portfolio (Consigli, 2002). So, we propose a Sharpe Ratio that incorporates VaR to provide a more accurate and comprehensive assessment of risk-adjusted returns. Following Rockafellar, Uryasev et al. (2000) the probability of f (wt, rt) not exceeding a threshold α is given by

$$VaR_\alpha = \Psi(w_t, \alpha) = \int_{f(w_t, r_t) \geq \alpha} p(r_t) dr_t \qquad (2.17)$$

where $f(w_t, r_t)$ is the loss function for asset weights $w_t$ and $p(r_t)$ is the probability of $r_t$ at time $t$. From the optimization, $w_t$ is the vector of asset weights that maximizes the Ratio. In thiswork, we chose an $\alpha = 5\%$.

By incorporating VaR, we can rewrite equation (2.16) as follows:

$$\begin{array}{lll} \underset{w_t}{\text{maximize}} & \frac{w_t^T rt}{VaR_\alpha} & \text{Sharpe ratio VaR} \\ \text{subject to} & w_t^T 1 = 1, & \text{full investment} \\ & \forall i \in \{1, 2, \ldots, d\} : & \\ & w_{t,i} \geq 0, & \text{long positions only} \end{array} \quad . \qquad (2.18)$$

### 2.4.4 Sharpe Ratio CVaR

Another variation of the Sharpe Ratio is applying the CVaR as an alternative for losses beyond the VaR threshold (Xu et al., 2016). Following Rockafellar and Uryasev (2000, 2002) algorithm, the CVaR's integral is:

$$\text{CVaR}_{\beta}(w_t) = \frac{1}{1-\beta}\int_{f(wt,\mathbf{r}t)\leq\alpha\beta(wt)} f(\mathbf{w}_t,\mathbf{r}_t)\,p(\mathbf{r}_t)\,d\mathbf{r}_t, \tag{2.19}$$

where $\beta$ is a specified confidence level value (here 0.9). It integrates the joint distribution of portfolio returns and weights over a range where the portfolio returns exceed a certain threshold $\alpha\beta(w_t)$. The result represents the expected tail loss of the portfolio, given that it falls in the tail region beyond the threshold ($\alpha$).

The maximization of the Sharpe Ratio CVaR problem can be expressed as:

$$\begin{array}{lll} \underset{w_t}{\text{maximize}} & \frac{w_t^T \mathbf{r}t}{\text{CVaR}_{\beta}(w_t)} & \text{Sharpe ratio CVaR} \\ \text{subject to} & w_t^T 1 = 1, & \text{full investment} \\ & \forall i \in \{1,2,\ldots,d\}: & \\ & w_{t,i} \geq 0, & \text{long positions only} \end{array} . \tag{2.20}$$

### 2.4.5 Sortino Ratio

The Sortino ratio, a variation of the Sharpe ratio, utilizes downside deviation as the risk measure instead of standard deviation. In this approach, only returns that fall below a user-defined target, or required rate of return, are considered risky. This approach provides a more focused assessment of downside risk and can be used to evaluate portfolio performance with greater accuracy.

Sortino and Brian Rom were responsible for coining the term "Sortino ratio" in the field of investment terminology. The first mention of this ratio was in the August 1980 issue of Financial Executive magazine, and its first calculation was published in a series of articles in the Journal of Risk Management in September 1981. These references highlight the historical development of the Sortino ratio and its importance in the field of finance.

The target downside deviation can be mathematically defined as the root-mean-square (RMS) of the deviations between the realized returns and the target return, where all returns above the target return are treated as underperformance of 0. This approach enables more precise measurement of downside risk and is an important component in the calculation of the Sortino ratio. Mathematically,

$$\text{Downside Deviation} = \sqrt{\frac{1}{N}\sum_{i-1}^{N}(\text{Min}(0, w_t^T \mathbf{r}t - rf))^2}, \tag{2.21}$$

where r f is the risk free asset or a target return.

Then,

$$\begin{array}{lll} \underset{w_t}{\text{maximize}} & \frac{w_t^T \mathbf{r}t}{\text{Downside Deviation}} & \text{Sortino Ratio} \\ \text{subject to} & w_t^T 1 = 1, & \text{full investment} \\ & \forall i \in \{1,2,\ldots,d\}: & \\ & w_{t,i} \geq 0, & \text{long positions only} \end{array} . \tag{2.22}$$

### 2.4.6 Information Ratio

The information ratio (IR) is a crucial metric for evaluating portfolio performance. By taking into account the excess returns of a portfolio compared to a chosen benchmark, typically an index representing a specific sector or industry, as well as the volatility of those returns, the IR helps to gauge a portfolio manager's skill in generating excess returns.

To provide a more nuanced picture of performance consistency, the IR incorporates a tracking error component, which measures the degree to which a portfolio's returns "track" the performance of the chosen benchmark. A low tracking error indicates consistent outperformance of the benchmark over time, whereas a high tracking error implies greater volatility and less reliable excess returns relative to the benchmark. By considering both excess returns and tracking error, the IR offers a more comprehensive assessment of portfolio performance than traditional metrics that focus only on returns:

$$\text{Tracking Error} = \sqrt{\frac{1}{N}\sum_{i-1}^{N}(w_t^T \mathbf{r}t - R_b)^2}, \tag{2.23}$$

where $R_b$ is the return or fund used as benchmark.

The maximization problem of the Information Ratio can be expressed as,

$$\begin{array}{lll} \underset{w_t}{\text{maximize}} & \frac{w_t^T \mathbf{r}t - R_b}{\text{Tracking Error}} & \text{Information Ratio} \\ \text{subject to} & w_t^T 1 = 1, & \text{full investment} \\ & \forall i \in \{1,2,\ldots,d\}: & \\ & w_{t,i} \geq 0, & \text{long positions only} \end{array} . \tag{2.24}$$

### 2.4.7 Portfolio Rebalancing

Rebalancing is the process of realigning the weights of an asset's portfolio, which means, buying or selling, periodically, assets in a portfolio to maintain an original or desired level of return or risk. However, the main question for investors is how often a portfolio should be rebalanced and whether the benefits of rebalancing outweigh the costs of doing so. To test the cost transaction in portfolio rebalancing, we test two frequencies - daily and weekly (5 businessdays)- and apply the B3 trading fee, settlement fee, and registration fee to assets funds and localinvestment clubs. The accumulated cost is equivalent to 0.023% of the financial value of the transaction, equivalent to $0.23 for every $1,000 (purchase or sale).

### 2.4.8 Steps

In this chapter we present step by step each procedure of this dissertation. First, we present the procedure for obtaining the weight matrix ($W^m$) for each $m$ portfolio and then we present the steps simulating the portfolio manager's trading.

a) We employ three distinct models: ARMA(1,1)-GARCH(1,1) (Eq.2.1), ARMA(1,1)-eGARCH(1,1) (Eq.2.2), and ARMA(1,1)-gjrGARCH(1,1) (Eq.2.3). To select the optimal model for each asset, we utilize the Akaike Information Criterion (AIC). Subsequently, we estimate the model parameters using maximum likelihood estimation (MLE) and obtain the standardized residuals:

$$\hat{z}_t = (\hat{z}_{1t}, ...., \hat{z}_{dt}).$$

b) We use estimated standardized residuals zt from step 1 to estimate the center of the distribution as Gaussian kernel and the upper/lower tails as a Generalized Pareto Distribution (GPD) (Eq. 2.4):

$$\hat{v}_{it} = G_j\left(\hat{z}_{it}\right), \in [1, d], v_{it} \sim U(0,1).$$

c) We introduce the estimated uniforms $v_{it}$ into R-Vine (Eq. 2.12). Following the approach outlined in Dissmann et al. (2013), we begin by drawing the vine structure based on the Kendall correlation coefficient between pairs (Eq. 2.15). Subsequently, we proceed to estimate the parameters of each tree:

$$F(y \mid v) = \frac{\partial C_{yv_j|v_{-j}}\left(F\left(y \mid v_{-j}\right), F\left(v_j \mid v_{-j}\right)\right)}{\partial F\left(v_j \mid v_{-j}\right)}$$

d) Generate 1000 sets of uniform random numbers $(x_{n1}, \ldots x_{nd})$, with $n = 1, \ldots, 1000$,for each series $i = 1, \ldots, d$. These uniform random numbers are then inserted into the estimated empirical vine copula structure obtained in step 3. As a result, we obtain 1000sets of uniform random variables that inherit the estimated dependency structure from theempirical vine copula:

$$\hat{\mathbf{u}}_i = \hat{C}\left(\hat{F}_1\left(x_{i1}\right), \ldots, \hat{F}_d\left(x_{id}\right)\right) = \left(\hat{u}_{i1}, \ldots, \hat{u}_{id}\right), \hat{u}_{in} \sim U(0,1).$$

e) We incorporate the simulated uniform $\hat{\mathbf{u}}_i$ using the estimated dependency structure into the inverse of the estimated cumulative distribution function from step 2. This enables usto estimate new standardized residuals, which are then utilized in portfolio optimization at time t.

$$\hat{Z} = (\hat{F}_1^{-1}(\hat{u}_{n1}), \ldots, \hat{F}_d^{-1}(\hat{u}_{nd})) = (\hat{\zeta}_1^1, \ldots, \hat{\zeta}_d^n)$$

f) We compute the one-step ahead returns ($\mu_{it}$) and the one-step ahead conditional volatility ($\sigma_{it}$) for each asset using the specified ARMA-GARCH models identified in Step 1. Then

we multiply the estimated conditional volatility by the vector of estimated standardized residuals, which added to the expected return for one step forward gives us 1000 simulated returns for each asset:

$$\begin{aligned} \hat{r}_{it}^n &= (\hat{\mu}_{it} + \hat{\sigma}_{it} \times \hat{\zeta}_i^n) \\ &= (\hat{r}_t^1, \ldots, \hat{r}_t^n) \end{aligned}$$

g) We utilize the one-step-ahead simulated returns$\wedge r_t^n$ obtained in the previous step and substitute them into the optimization criteria described in Chapter (2.5). This allows us todetermine the optimal weights for each $m$ portfolio, enabling us to construct the optimalportfolios based on the forecasted returns. In Appendix C we present the weights matrixfor each portfolio.

$$\begin{aligned} \mathbf{W}^m &= \begin{bmatrix} w_t^m & w_{t+1}^m & \ldots & w_T^m \end{bmatrix} \\ &= \begin{bmatrix} w_{1t}^m & w_{2t}^m & \ldots & w_{dt}^m \\ w_{1t+1}^m & w_{2t+1}^m & \ldots & w_{dt+1}^m \\ \ldots & \ldots & \ldots & \ldots \\ w_{1T}^m & w_{2T}^m & \ldots & w_{dT}^m \end{bmatrix} \end{aligned}$$

The next step is to simulate the portfolio manager's trading. We start with an amount of \$1,000,000 and the weight vector $w_t^m$. The first trading happen in t, with the simulated returns for $t + 1$:

$$Quantities_{it} = Round\left(\frac{w_{it}^{m} \times 1,000,000}{p_{it}/100}, 0\right) \times 100$$

$$Cost_t = \sum_{i=1}^{d}(Quantities_{it} \times p_{it} \times 0.023\%)$$

where Round (. . . , 0) is a function, as the name says, that rounds the number with 0 decimals. This function provides the portfolio manager with the quantities, in multiples of 100, that he needs to buy in t, this function avoids purchases or sales of quantities below 100, which would implicitly take him to the fractional market.[2]

For $t + 1$:

$$Portfolio_t^m = \sum_{i=1}^{d}(Quantities_{it} \times p_{it}) - Cost_t$$

$$Quantities_{it+1} = Round\left(\frac{|w_{it}^{m} - w_{it+1}^{m}| \times Portfolio_t^m}{p_{it+1}/100}, 0\right) \times 100$$

$$Cost_{t+1} = \sum_{i=1}^{d}(|Quantities_{it} - Quantities_{it+1}| \times p_{it+1} \times 0.023\%)$$

For $[t + 2, \ldots, T]$:

$$Portfolio_{t+1}^m = \sum_{i=1}^{d}(Quantities_{it+1} \times p_{it+1} \times (1 + r_{it+1})) - Cost_{t+1}$$

$$Quantities_{it+2} = Round\left(\frac{|w_{it+1}^{m} - w_{it+2}^{m}| \times Portfolio_{t+1}^m}{p_{it+2}/100}, 0\right) \times 100$$

$$Cost_{t+2} = \sum_{i=1}^{d}(|Quantities_{it+1} - Quantities_{it+2}| \times p_{it+2} \times 0.023\%)$$

In simpler terms, the portfolio manager initiates asset purchases based on closing prices at time t. In the subsequent period, t + 1, they take into account the costs associated with the value of their portfolio from time t and adjust asset weights by buying or selling the difference between the weight vectors at t and t + 1. When new information emerges (rt+1) at t + 2, the portfolio manager considers the returns of Port f oliom between the trades at t and t + 1, incorporating this return into their portfolio value.

[2] The Stock Exchange's fractional market is an environment that allows the purchase and sale of shares in small quantities – ranging from 1 to 99 units. However, the liquidity is usually lower compared to the full market, as there is a lower supply and purchase volume.

# 3 EMPIRICAL ANALYSIS

In this chapter we describe the database and present the results.

## 3.1 DATA

The dataset used in our analysis consists of daily adjusted prices for 60 stocks that are included in the Ibovespa Index, obtained from the Bloomberg terminal. We use the logarithmic returns, defined as $log(p_{t-1}/p_t)$, where $p_t$ denote the price of an asset at time $t$. The data spans from February 4, 2014, to December 30, 2020, giving a total of 1723 observations.We implemented a rolling window of 1223 observations and left the years 2019 and 2020 as out-of-sample.

According to B3 (2023), the Ibovespa is the primary performance indicator of the Brazilian stock market and is constructed through a set of rules that include several criteria for selecting assets and determining their weighting. These criteria are designed to ensure that the index is representative of the market and is rebalanced periodically to maintain its relevance. Overall, the Ibovespa methodology is widely recognized and respected by investors and market participants.

The methodology construction follows the rules outlined below:

- Selection Criteria: To be among the assets that represent 85% in descending order of Negotiability Index (NI) (buffer 90%); 95% presence in trading sessions; 0.1% of financial volume in the spot market (standard lot); and not be a penny stock.
- Weighting: Free float market value/cap 20% per company/cap 2x NI
- Rebalancing: Every four months (1st Monday of January, May, and September)

The weights and assets present in the Ibovespa index in our out-of-sample are present in the 6.

Since we started forecasting on the first Monday of 2019, we have chosen to exclude assets that were added to the Ibovespa in mid-2019 and 2020. Additionally, companies such as LOGG3, BRDT3, and RAIL3, which were included in the Ibovespa index in January 2019, had their initial public offering (IPO) after February 2014 and were therefore not included in our forecast model. We were also unable to locate historical prices for SUZB3 before 10/11/2017, likely due to the company's IPO on the B3 stock exchange in 2012 and subsequent transition to the *Novo Mercado* segment five years later. Lastly, VVAR3 is experiencing some convergence problems, leading us to prefer to exclude this asset from our sample.

Our dataset consists of 60 assets, which collectively represent varying percentages of the total companies in the Ibovespa Index. Specifically, they represent 96.5%, 94.6%, 90.6%, 85.7%, 83.7%, and 90.6% of the Ibovespa portfolio for the months of January 2019, May 2019, September 2019, January 2020, May 2020 and September 2020, respectively. These percentages correspond to the six rebalancing of the Ibovespa index during 2019 and 2020. In Appendix A we present the daily returns of each asset.

We acknowledge the presence of survival bias in our analysis. In our context, assets that were removed from the Ibovespa Index mid-sample should be excluded from the analysis, and new assets added to the index should be integrated into the data set. Despite this consideration,we made a deliberate choice to maintain a constant set of 60 assets throughout the analysis. Thisdecision aligns with our objective of working with a substantial number of assets. Introducing new assets could potentially lead to a reduction in the dataset size, either due to convergence issues (KS test or ARMA-GARCH model orders) or a scarcity of observations.

For instance, companies such as Azul SA (AZUL4) and Banco BTG Pactual (BPAC11) entered the stock market in mid-2017. However, we opted not to include these companies in our analysis because, as of their inclusion in the Ibovespa Index in May 2019 (AZUL4) and September 2019 (BPAC11), for each there were only approximately 500 historical observations available. In contrast, we utilized a rolling window of size 1,223 to estimate the model parameters, ensuring a more robust analysis despite the limited historical data for these specific companies.

Table 1 provides descriptive statistics for the returns of the 60 assets. The Jarque- Bera test results indicate that the null hypothesis of normality is rejected for all series. This suggests that the returns of the assets do not follow a normal distribution. Specifically, only nineassets have a positive skewness (BRKM5, BTOW3, CIEL3, ELET3, ELET6, GOLL4, HYPE3,MGLU3, SANB11, USIM5). The presence of positive kurtosis in all series means that each series exhibits a sharper peak than a standard normal distribution. Furthermore, the AugmentedDickey-Fuller (ADF) suggests strong evidence to reject the null hypothesis of a unit root and conclude that all-time series are stationary.

As a benchmark to evaluate and validate the effectiveness of our model, we chose an equally weighted portfolio, the global minimum variance (GMV) portfolio, and the Markowitz portfolio. All of these portfolios are rebalanced at the same frequency as our portfolios and are constructed using the identical set of 60 assets from our sample. GMV and Markowitz are also performed using a moving window of 1,223 observations.

## 3.2 RESULTS

We utilize an ARMA-GARCH-EVT model to establish the marginal distribution for the innovations. In this model, we assume that the conditional distribution for the residuals follows a Skew-Student t distribution. To determine the best GARCH specification for each asset, we employ the Akaike Information Criterion (AIC) as a model selection criterion. As the GARCH specification evolves during the out-of-sample period, accompanied by changes in its

Table 1 – Descriptive statistics for the daily log returns

| TICKER | Mean | Std. dev. | Skewness | Kurtosis | JB | ADF |
|---|---|---|---|---|---|---|
| ABEV3 | 0,0133 | 0,0171 | -0,73 | 11,0763 | 8987,0357*** | -12,329*** |
| B3SA3 | 0,1251 | 0,024 | -0,1129 | 5,4949 | 2179,1148*** | -11,7809*** |
| BBAS3 | 0,0611 | 0,0301 | -0,4538 | 7,5447 | 4159,0202*** | -11,9916*** |
| BBDC3 | 0,0529 | 0,0233 | -0,0502 | 5,2675 | 1999,9955*** | -11,3488*** |
| BBDC4 | 0,0653 | 0,0236 | -0,0613 | 5,698 | 2340,2593*** | -11,3171*** |
| BBSE3 | 0,0469 | 0,0202 | -0,1834 | 3,4104 | 848,3304*** | -12,6689*** |
| BRAP4 | 0,0865 | 0,0302 | -0,3764 | 6,5125 | 3095,9382*** | -11,0233*** |
| BRFS3 | -0,0333 | 0,0246 | -0,7981 | 12,862 | 12093,7655*** | -11,2941*** |
| BRKM5 | 0,0326 | 0,0316 | 0,0934 | 19,5064 | 27392,6076*** | -11,6888*** |
| BRML3 | 0,0079 | 0,0263 | -0,7388 | 10,7098 | 8416,1063*** | -12,2231*** |
| BTOW3 | 0,0696 | 0,0364 | 0,2802 | 3,4672 | 889,3591*** | -12,5239*** |
| CCRO3 | 0,012 | 0,0262 | -0,0314 | 8,378 | 5055,2753*** | -12,3817*** |
| CIEL3 | -0,0721 | 0,0272 | 0,4046 | 10,4219 | 7868,1966*** | -11,0147*** |
| CMIG4 | 0,0388 | 0,0296 | -0,6015 | 7,43 | 4080,3115*** | -10,7553*** |
| CSAN3 | 0,0646 | 0,0239 | -0,4793 | 5,4491 | 2205,4742*** | -12,1292*** |
| CSNA3 | 0,0739 | 0,04 | -0,0287 | 4,4492 | 1426,9446*** | -10,3805*** |
| CVCB3 | 0,0352 | 0,034 | -2,1557 | 35,4131 | 91597,0514*** | -10,5352*** |
| CYRE3 | 0,0621 | 0,0281 | -1,1538 | 14,3746 | 15258,8769*** | -10,6014*** |
| ECOR3 | 0,0244 | 0,0282 | -0,2868 | 6,7243 | 3280,7797*** | -11,0119*** |
| EGIE3 | 0,0508 | 0,0164 | -0,0267 | 3,3162 | 793,1931*** | -11,4117*** |
| ELET3 | 0,1225 | 0,0368 | 0,6299 | 11,8575 | 10237,4686*** | -11,4094*** |
| ELET6 | 0,0991 | 0,0323 | 0,2315 | 7,3985 | 3957,9361*** | -11,7516*** |
| EMBR3 | -0,039 | 0,0266 | -0,8836 | 18,8529 | 25810,5285*** | -11,3025*** |
| ENBR3 | 0,0628 | 0,0202 | -0,0259 | 5,1308 | 1897,0461*** | -12,0655*** |
| EQTL3 | 0,1044 | 0,0173 | -0,5072 | 4,4184 | 1481,0317*** | -10,8799*** |
| ESTC3 | 0,0488 | 0,033 | -0,2145 | 8,0295 | 4656,5821*** | -10,8661*** |
| FLRY3 | 0,0809 | 0,0208 | -0,5592 | 6,2462 | 2900,5376*** | -11,3834*** |
| GGBR4 | 0,0295 | 0,031 | -0,1828 | 3,5785 | 932,8625*** | -10,5293*** |
| GOAU4 | -0,028 | 0,0348 | -0,4004 | 5,0699 | 1898,2911*** | -10,59*** |
| GOLL4 | 0,0573 | 0,0485 | 0,0962 | 13,5902 | 13299,8338*** | -11,4361*** |
| HYPE3 | 0,0604 | 0,0206 | 0,1674 | 12,8215 | 11843,8937*** | -12,6663*** |
| IGTA3 | 0,0427 | 0,0228 | -0,9433 | 13,5034 | 13383,8478*** | -12,5339*** |
| ITSA4 | 0,0699 | 0,0204 | -0,1904 | 2,9664 | 645,1054*** | -11,2813*** |
| ITUB4 | 0,0643 | 0,0212 | -0,0344 | 3,0694 | 679,8228*** | -11,4025*** |
| JBSS3 | 0,0682 | 0,0325 | -0,3853 | 15,4903 | 17316,8138*** | -13,0089*** |
| KLBN11 | 0,0591 | 0,0193 | -0,3561 | 6,9356 | 3501,3384*** | -12,919*** |
| KROT3 | -0,0307 | 0,0329 | -0,6739 | 7,1454 | 3808,1881*** | -11,3166*** |
| LAME4 | 0,0609 | 0,0249 | -0,5683 | 9,1884 | 6172,6641*** | -11,5574*** |
| LREN3 | 0,0983 | 0,0234 | -0,5842 | 11,2997 | 9291,8129*** | -12,2136*** |
| MGLU3 | 0,2743 | 0,0391 | 0,659 | 9,3885 | 6472,3851*** | -11,2217*** |
| MRFG3 | 0,0738 | 0,0302 | -0,3089 | 10,7879 | 8407,2608*** | -12,4253*** |
| MRVE3 | 0,0739 | 0,026 | -0,6125 | 8,6653 | 5515,447*** | -11,5318*** |
| MULT3 | 0,0371 | 0,0231 | -0,5654 | 14,4025 | 15025,6987*** | -12,7264*** |
| NATU3 | 0,0665 | 0,0275 | -0,5672 | 12,3828 | 11132,4719*** | -12,3647*** |
| PCAR4 | -0,1065 | 0,0457 | -28,0983 | 1022,7121 | 75491980,4322*** | -11,5556*** |
| PETR3 | 0,05 | 0,034 | -0,8544 | 11,121 | 9115,0827*** | -11,3292*** |
| PETR4 | 0,0495 | 0,0342 | -0,8495 | 10,4802 | 8116,349*** | -11,1437*** |
| QUAL3 | 0,0595 | 0,0291 | -0,8144 | 22,8879 | 37898,3384*** | -11,245*** |
| RADL3 | 0,1309 | 0,0197 | -0,0875 | 2,9428 | 626,8546*** | -12,9934*** |
| RENT3 | 0,1238 | 0,0268 | -0,2557 | 13,1954 | 12554,8241*** | -10,9006*** |
| SANB11 | 0,1029 | 0,0239 | 0,0275 | 4,6434 | 1554,0812*** | -10,9845*** |
| SBSP3 | 0,0543 | 0,0251 | -0,3757 | 6,2075 | 2816,5026*** | -11,1323*** |
| SMLS3 | 0,016 | 0,0342 | -3,7897 | 52,3654 | 201477,6285*** | -11,6463*** |
| TAEE11 | 0,0827 | 0,0157 | -0,2663 | 2,0829 | 333,6314*** | -12,085*** |
| TIMP3 | 0,0191 | 0,0222 | -0,1148 | 4,657 | 1566,705*** | -11,2989*** |
| UGPA3 | 0,0045 | 0,0243 | -0,5929 | 18,7445 | 25393,6638*** | -10,5563*** |
| USIM5 | 0,0164 | 0,0393 | 0,2715 | 5,2829 | 2032,1765*** | -10,7143*** |
| VALE3 | 0,0739 | 0,0302 | -0,3544 | 7,8142 | 4433,9814*** | -11,5262*** |
| VIVT4 | 0,0211 | 0,017 | -0,1795 | 7,1347 | 3675,8426*** | -12,5993*** |
| WEGE3 | 0,1377 | 0,021 | -0,8307 | 13,7618 | 13833,4444*** | -11,4051*** |

Fonte: Elaborated by the author.
Note: The total number of observations is 1723 for each stock. The mean is expressed as a percentage.
*** Denotes statistical significance at the 1% level.

parameters, Table 3 in Appendix b displays the outcomes of tests that exclusively compare the GARCH models using in-sample data and their associated AIC values.

Additionally, just for the in-sample data Table 2 offers the estimated parameters and GARCH family classification for each asset within our training sample. Specifically, we present these details exclusively for the period spanning from February 4, 2014, to January 4, 2019, which is immediately before our forecast period. It's important to note that during each iteration within the rolling window, we reestimate the parameters and evaluate the three GARCH models mentioned above using the AIC criterion to determine the optimal estimate. These estimated parameters reveal the distinctive characteristics and behavioral characteristics inherent to the model, providing valuable information about the specific dynamics of each asset.

In summary, the key distinction among EGARCH (Eq. 2.2), GJR-GARCH (Eq. 2.3) and the usual GARCH (Eq. 2.1) models, are their treatments of volatility dynamics and asymmetry, as exemplified by the parameter $\gamma$ in Table 2. EGARCH allows for flexible modeling of asymmetry through the use of logarithmic transformations. In contrast, GJR-GARCH explicitly introduces an asymmetry parameter to capture different effects of positive and negative shocks. Meanwhile, the basic GARCH model assumes symmetric effects of shocks on volatility.

The initial 45 assets are modeled by an EGARCH, 9 with GARCH, and 5 with GJR-GARCH. Only the assets BRFS3, CCRO3, CIEL3, CSNA3, GGBR4, GOAU4, MRFG3, NATU3, and PCAR4 had an estimated constant term ($\mu i$) equal or lower than 0, indicating a negative average return. In general, the coefficients ($\alpha$ and $\beta$) in the variance equation are significant for most series, implying that the GARCH models fit the volatilities of the return series reasonably well. Interestingly, the higher values in the $\beta$ parameter suggest a high degree of persistence in volatility over time, in other words, large changes in volatility from the past are expected to have a lasting impact on volatility in the current period.

We utilize extreme value theory to fit the Generalized Pareto Distribution (GPD) to the lower and upper tails of the standardized residuals. We apply the peak-over-threshold (POT) method, following the approach outlined by Wang et al. (2010). Specifically, we choose the 10th percentile of the standardized residuals as the threshold. The estimated parameters, including u (threshold), $\xi$ (scale), and $\beta$ (location), for the upper and lower tails are presented in Table 3, again, the parameters are for the in-sample database only.

As it was mentioned earlier, after fitting the GPD to the upper and lower tails of the standardized residuals and obtaining the marginal distribution, we use the R- and C-Vine structure obtained from the Eq. 2.14 to estimate the dependency structure between the assets. This incorporates the interaction or dependence of assets on each other within the portfolio. Following that, we generate 60,000 simulated uniform numbers, which are then transformed using the inverse function of the previously obtained marginal distribution for each series. This allows us to create 60,000 synthetic one-day-ahead returns. Using these simulated returns, we proceed to calculate the performance of different portfolios. Such as Information Ratio (IR),

Table 2 – GARCH parameters for daily log returns of assets in the training sample database

| TICKER | GARCH model | μ | φ | θ | ω | α | β | γ |
|---|---|---|---|---|---|---|---|---|
| ABEV3 | eGARCH | 0.0004** | 0.7858*** | -0.8177*** | -0.365*** | -0.0741*** | 0.9581*** | 0.1304*** |
| B3SA3 | GARCH | 0.0014*** | 0.7879*** | -0.8115*** | 0*** | 0.0608*** | 0.9159*** | |
| BBAS3 | eGARCH | 0.0018** | 0.5248*** | -0.5097*** | -0.2392*** | -0.027 | 0.9668*** | 0.1856*** |
| BBDC3 | eGARCH | 0.001* | -0.2193*** | 0.1741*** | -0.2249*** | -0.0071 | 0.971*** | 0.1231*** |
| BBDC4 | eGARCH | 0.0012** | 0.4125*** | -0.4574*** | -0.132*** | -0.0224 | 0.983*** | 0.101*** |
| BBSE3 | GARCH | 0.0007* | 0.7915*** | -0.8273*** | 0 | 0.078*** | 0.904*** | |
| BRAP4 | GARCH | 0.0006 | -0.099 | 0.128 | 0 | 0.0531*** | 0.9454*** | |
| BRFS3 | eGARCH | -0.0002 | 0.2595 | -0.2183 | -0.1342*** | -0.0734*** | 0.983*** | 0.0428*** |
| BRKM5 | gjrGARCH | 0.0007 | 0.5965 | -0.5784 | 0*** | 0.0035 | 0.9664*** | 0.0406*** |
| BRML3 | eGARCH | 0.0003 | 0.4659*** | -0.5047*** | -0.2464*** | -0.0295 | 0.9677*** | 0.1219*** |
| BTOW3 | eGARCH | 0.0008 | -0.1359*** | 0.2043*** | -0.8462** | -0.014 | 0.8748*** | 0.2881*** |
| CCRO3 | eGARCH | -0.0002 | 0.5666*** | -0.6375*** | -0.2007*** | -0.0358** | 0.9737*** | 0.124*** |
| CIEL3 | eGARCH | -0.0001 | -0.6658*** | 0.6941*** | -0.137*** | -0.0664*** | 0.9826*** | 0.0696*** |
| CMIG4 | GARCH | 0.0004 | 0.4699 | -0.5033* | 0.0003*** | 0.1912*** | 0.436*** | |
| CSAN3 | GARCH | 0.0004 | -0.2662 | 0.244 | 0*** | 0.0617*** | 0.9089*** | |
| CSNA3 | eGARCH | -0.0002 | -0.1151*** | 0.15*** | -0.0978*** | 0.0069 | 0.9852*** | 0.127*** |
| CVCB3 | gjrGARCH | 0.0013** | -0.7423* | 0.7567** | 0** | 0.0605** | 0.8683*** | 0.047*** |
| CYRE3 | GARCH | 0.0005 | 0.556 | -0.5725 | 0** | 0.0882*** | 0.8489*** | |
| ECOR3 | eGARCH | 0.0004 | 0.091*** | -0.2107*** | -0.1726*** | -0.0581*** | 0.9769*** | 0.119*** |
| EGIE3 | eGARCH | 0.0004 | 0.4916*** | -0.5807*** | -0.0378*** | -0.0008 | 0.9956*** | 0.0512*** |
| ELET3 | eGARCH | 0.0008 | -0.7929*** | 0.8176*** | -0.523 | 0.0314 | 0.9228*** | 0.148*** |
| ELET6 | eGARCH | 0.0011** | -0.6335*** | 0.6894*** | -0.1854*** | -0.0221 | 0.9734*** | 0.135*** |
| EMBR3 | eGARCH | 0.0003 | 0.0388 | -0.1116 | -1.3319 | -0.0115 | 0.8292*** | 0.1475*** |
| ENBR3 | eGARCH | 0.0005 | 0.2193** | -0.3883*** | -0.0855*** | -0.0299** | 0.9893*** | 0.0749*** |
| EQTL3 | eGARCH | 0.001*** | -0.4387*** | 0.4135*** | -0.6855*** | -0.1256*** | 0.9194*** | 0.1496*** |
| ESTC3 | eGARCH | 0.0003 | 0.5723*** | -0.5548*** | -0.1071*** | -0.0488*** | 0.9847*** | 0.0571*** |
| FLRY3 | eGARCH | 0.0007** | 0.4893*** | -0.5161*** | -0.76** | -0.0209 | 0.9037*** | 0.1961*** |
| GGBR4 | GARCH | -0.0003 | 0.5986 | -0.6036 | 0 | 0.0579*** | 0.9328*** | |
| GOAU4 | eGARCH | -0.0025*** | 0.998*** | -0.9907*** | -0.0684*** | -0.0061 | 0.9902*** | 0.137*** |
| GOLL4 | eGARCH | 0.0047*** | 0.9962*** | -0.9844*** | -0.8087*** | 0.0018 | 0.873*** | 0.3601*** |
| HYPE3 | eGARCH | 0.0006 | 0.0352 | 0.001 | -0.1352*** | -0.0774*** | 0.9837*** | 0.0429*** |
| IGTA3 | eGARCH | 0.001** | -0.1949*** | 0.0922 | -0.2869*** | -0.0241 | 0.9643*** | 0.1309*** |
| ITSA4 | eGARCH | 0.0012** | -0.286** | 0.2603** | -0.2004*** | -0.029 | 0.9747*** | 0.1259*** |
| ITUB4 | eGARCH | 0.0011** | 0.5551*** | -0.5987*** | -0.1575*** | -0.0237 | 0.98*** | 0.1099*** |
| JBSS3 | eGARCH | 0.0008 | 0.4708*** | -0.5888*** | -0.387*** | -0.0799*** | 0.9456*** | 0.1335*** |
| KLBN11 | GARCH | 0.0006 | -0.0093 | 0.0343 | 0* | 0.0853*** | 0.8526*** | |
| KROT3 | eGARCH | 0.0006 | 0.796*** | -0.8072*** | -0.1193*** | -0.0546*** | 0.9835*** | 0.0958*** |
| LAME4 | eGARCH | 0.0005 | -0.7956*** | 0.8442*** | -0.2827*** | -0.0671*** | 0.9633*** | 0.0917*** |
| LREN3 | eGARCH | 0.0013** | 0.4944*** | -0.5638*** | -0.0849*** | -0.0388*** | 0.9892*** | 0.0373*** |
| MGLU3 | eGARCH | 0.0019*** | -0.7463*** | 0.8043*** | -0.7658*** | -0.0493 | 0.8839*** | 0.3969*** |
| MRFG3 | eGARCH | -0.0001 | 0.4154*** | -0.4594*** | -0.4628*** | -0.05* | 0.9367*** | 0.2454*** |
| MRVE3 | eGARCH | 0.0009** | -0.4515*** | 0.3862*** | -0.3531*** | -0.0175 | 0.9545*** | 0.1475*** |
| MULT3 | gjrGARCH | 0.0007* | 0.5848*** | -0.6604*** | 0** | 0.0618** | 0.8392*** | 0.0718*** |
| NATU3 | eGARCH | -0.0002 | 0.5236*** | -0.4971*** | -2.9957** | 0.0679 | 0.6015*** | 0.2117*** |
| PCAR4 | eGARCH | 0 | -0.1387*** | 0.206*** | -0.1409*** | -0.042** | 0.982*** | 0.1145*** |
| PETR3 | eGARCH | 0.0014* | -0.336*** | 0.2673*** | -0.074*** | -0.0129 | 0.9893*** | 0.1361*** |
| PETR4 | eGARCH | 0.0012* | -0.2717*** | 0.19*** | -0.1351*** | -0.0336** | 0.9804*** | 0.1459*** |
| QUAL3 | GARCH | 0.0002 | 0.1995 | -0.2575 | 0.0002*** | 0.1295*** | 0.5405*** | |
| RADL3 | eGARCH | 0.0011** | -0.0617 | 0.0765 | -0.0599*** | -0.0178** | 0.9925*** | 0.0435*** |
| RENT3 | gjrGARCH | 0.0011** | 0.5761*** | -0.6296*** | 0 | 0.0328** | 0.9439*** | 0.03*** |
| SANB11 | eGARCH | 0.0013*** | 0.4795*** | -0.5546*** | -0.136*** | -0.0305** | 0.9823*** | 0.0719*** |
| SBSP3 | eGARCH | 0.0003 | 0.3881 | -0.4634 | -0.0418*** | -0.0338*** | 0.9946*** | 0.0295*** |
| SMLS3 | eGARCH | 0.0014** | -0.4596*** | 0.436*** | -0.4391*** | -0.0135 | 0.9406*** | 0.2572*** |
| TAEE11 | eGARCH | 0.0006* | 0.5121*** | -0.5979*** | -0.9475*** | 0.0446 | 0.8855*** | 0.251*** |
| TIMP3 | eGARCH | 0.0002 | 0.2165** | -0.318*** | -0.0793*** | -0.0458*** | 0.9898*** | 0.0431*** |
| UGPA3 | eGARCH | 0.0001 | 0.3729*** | -0.4336*** | -0.0307*** | -0.054*** | 0.9963*** | 0.0406*** |
| USIM5 | eGARCH | 0.0005 | -0.2446** | 0.3065*** | -0.1417*** | -0.0058 | 0.9784*** | 0.177*** |
| VALE3 | gjrGARCH | 0.0008 | -0.3778 | 0.4271 | 0 | 0.0649*** | 0.9458*** | -0.0286*** |
| VIVT4 | eGARCH | 0.0004 | 0.613*** | -0.6705*** | -0.148*** | -0.0039 | 0.9822*** | 0.1068*** |
| WEGE3 | eGARCH | 0.0009** | 0.5075*** | -0.5523*** | -0.1232*** | -0.0292** | 0.985*** | 0.0862*** |

Fonte: Elaborated by the author.
Note: The total number of observations is 1223 for each stock, considering only the database in the training sample.

Table 3 – Estimates of EVT parameters for standardized residuals in the training sample database

| TICKER | $u^r$ | $\xi^r$ | $\beta^r$ | $u^l$ | $\xi^l$ | $\beta^l$ |
|---|---|---|---|---|---|---|
| ABEV3 | 1.1859 | 0.1049 | 0.4976 | -1.2511 | 0.1459 | 0.5085 |
| B3SA3 | 1.2421 | 0.0277 | 0.5471 | -1.1956 | 0.002 | 0.5812 |
| BBAS3 | 1.1714 | -0.0877 | 0.6895 | -1.1854 | 0.1449 | 0.4919 |
| BBDC3 | 1.2069 | -0.1269 | 0.6929 | -1.1797 | 0.1241 | 0.4957 |
| BBDC4 | 1.2591 | 0.0886 | 0.4591 | -1.1604 | 0.1242 | 0.5385 |
| BBSE3 | 1.2994 | -0.2728 | 0.6267 | -1.2655 | 0.1119 | 0.4613 |
| BRAP4 | 1.2505 | 0.0431 | 0.4976 | -1.2467 | -0.1413 | 0.6014 |
| BRFS3 | 1.1396 | 0.0409 | 0.5893 | -1.1516 | 0.1991 | 0.5016 |
| BRKM5 | 1.141 | 0.24 | 0.5706 | -1.0674 | 0.2602 | 0.427 |
| BRML3 | 1.2797 | -0.039 | 0.6113 | -1.1916 | 0.1758 | 0.4412 |
| BTOW3 | 1.1543 | -0.0148 | 0.6755 | -1.1805 | 0.1427 | 0.4691 |
| CCRO3 | 1.2248 | -0.0036 | 0.5014 | -1.1984 | 0.1642 | 0.5277 |
| CIEL3 | 1.1916 | -0.065 | 0.5681 | -1.1827 | 0.0703 | 0.6075 |
| CMIG4 | 1.1533 | 0.0006 | 0.6401 | -1.2035 | 0.1612 | 0.4753 |
| CSAN3 | 1.2486 | -0.1308 | 0.6011 | -1.212 | 0.0867 | 0.5176 |
| CSNA3 | 1.2305 | -0.121 | 0.6848 | -1.1933 | 0.0583 | 0.4821 |
| CVCB3 | 1.2888 | -0.2379 | 0.6295 | -1.1986 | 0.1435 | 0.4774 |
| CYRE3 | 1.2303 | -0.049 | 0.5523 | -1.1637 | 0.2098 | 0.503 |
| ECOR3 | 1.2637 | -0.0996 | 0.5389 | -1.191 | 0.115 | 0.5496 |
| EGIE3 | 1.2213 | 0.0733 | 0.5588 | -1.2094 | 0.0885 | 0.506 |
| ELET3 | 1.1832 | 0.2313 | 0.5494 | -1.123 | 0.0726 | 0.5148 |
| ELET6 | 1.1267 | 0.1181 | 0.6626 | -1.1718 | 0.0401 | 0.5029 |
| EMBR3 | 1.1051 | 0.1498 | 0.5445 | -1.1582 | 0.2435 | 0.5047 |
| ENBR3 | 1.2134 | 0.0842 | 0.5612 | -1.1899 | 0.0373 | 0.5746 |
| EQTL3 | 1.179 | -0.1351 | 0.587 | -1.2376 | -0.0983 | 0.6857 |
| ESTC3 | 1.1425 | 0.3422 | 0.3546 | -1.1742 | 0.0146 | 0.6653 |
| FLRY3 | 1.2396 | -0.0184 | 0.5939 | -1.1847 | 0.2042 | 0.4419 |
| GGBR4 | 1.2141 | -0.1618 | 0.6674 | -1.1782 | 0.0617 | 0.5225 |
| GOAU4 | 1.2335 | -0.125 | 0.5725 | -1.2472 | 0.0941 | 0.489 |
| GOLL4 | 1.1323 | 0.232 | 0.5932 | -1.1012 | 0.0889 | 0.4586 |
| HYPE3 | 1.181 | 0.0935 | 0.5507 | -1.1848 | 0.2691 | 0.429 |
| IGTA3 | 1.225 | -0.1374 | 0.6734 | -1.1914 | 0.1936 | 0.441 |
| ITSA4 | 1.2599 | -0.0423 | 0.5604 | -1.1749 | 0.151 | 0.5077 |
| ITUB4 | 1.2069 | -0.0672 | 0.6416 | -1.217 | 0.1872 | 0.4396 |
| JBSS3 | 1.1279 | 0.1382 | 0.545 | -1.0552 | 0.2881 | 0.513 |
| KLBN11 | 1.2286 | -0.1354 | 0.6534 | -1.2898 | -0.0169 | 0.4505 |
| KROT3 | 1.2278 | -0.0617 | 0.5753 | -1.2235 | 0.1175 | 0.4874 |
| LAME4 | 1.1665 | -0.0538 | 0.5978 | -1.1779 | 0.1053 | 0.582 |
| LREN3 | 1.2433 | 0.0966 | 0.51 | -1.1773 | 0.0038 | 0.5156 |
| MGLU3 | 1.0724 | 0.1865 | 0.6595 | -1.0889 | 0.1778 | 0.4557 |
| MRFG3 | 1.2085 | 0.0629 | 0.6704 | -1.1212 | -0.0842 | 0.6014 |
| MRVE3 | 1.2189 | -0.0005 | 0.5316 | -1.2191 | 0.138 | 0.4877 |
| MULT3 | 1.2035 | -0.1163 | 0.7339 | -1.2038 | 0.2131 | 0.3851 |
| NATU3 | 1.1993 | 0.2089 | 0.4951 | -1.169 | 0.1414 | 0.4637 |
| PCAR4 | 1.2249 | 0.0486 | 0.5376 | -1.2177 | 0.1375 | 0.4728 |
| PETR3 | 1.1956 | 0.0012 | 0.5854 | -1.2118 | 0.1717 | 0.4529 |
| PETR4 | 1.2019 | -0.0909 | 0.622 | -1.1883 | 0.1224 | 0.5299 |
| QUAL3 | 1.1508 | -0.1115 | 0.5781 | -1.1817 | 0.3478 | 0.467 |
| RADL3 | 1.237 | -0.0154 | 0.5853 | -1.2331 | -0.2091 | 0.6127 |
| RENT3 | 1.2064 | -0.2222 | 0.7158 | -1.2217 | 0.0208 | 0.5296 |
| SANB11 | 1.1718 | 0.1536 | 0.5195 | -1.1547 | 0.002 | 0.6281 |
| SBSP3 | 1.1504 | 0.0034 | 0.5586 | -1.1592 | 0.1608 | 0.5966 |
| SMLS3 | 1.1664 | -0.1113 | 0.554 | -1.184 | 0.2987 | 0.509 |
| TAEE11 | 1.2207 | -0.1116 | 0.5621 | -1.2032 | -0.183 | 0.743 |
| TIMP3 | 1.1976 | 0.1749 | 0.4765 | -1.1999 | 0.1065 | 0.4999 |
| UGPA3 | 1.1687 | -0.1585 | 0.6293 | -1.2023 | 0.1029 | 0.5722 |
| USIM5 | 1.2164 | -0.0095 | 0.6657 | -1.1939 | 0.0164 | 0.5099 |
| VALE3 | 1.2388 | -0.09 | 0.5838 | -1.251 | -0.2446 | 0.6721 |
| VIVT4 | 1.2004 | 0.2208 | 0.4445 | -1.1927 | 0.1084 | 0.5231 |
| WEGE3 | 1.2795 | -0.2171 | 0.6194 | -1.2283 | -0.0136 | 0.5591 |

Fonte: Elaborated by the author.
Note: The total number of observations is 1223 for each stock, considering only the database in the training sample.

Sharpe Ratio (SR), Sharpe Ratio with Value at Risk (SR-VaR), Sharpe Ratio with Conditional Value at Risk (SR-CVaR), and Sortino Ratio (SoR). We repeat the entire procedure for each day in our out-of-sample period and perform ten portfolio allocations. For the weekly rebalancing, the same procedure is repeated, but forecasts are made five days ahead, which gives us ten more portfolio allocations. Figure 1 illustrates the portfolio's accumulation of wealth with an initial investment of $ 1,000,000. The dashed red vertical line represents March 11, 2020 - a pivotal date in global affairs. On this day, the World Health Organization (WHO) officially declared the novel coronavirus (COVID-19) outbreak as a global pandemic. The placement of this marker highlights the relevant impact of this event on the financial scenario.

As highlighted earlier, our primary objective is to secure a positive alpha on the Ibovespa Index. Consequently, we begin by presenting the portfolio returns in Table 4, comparing them to the benchmarks. In general, in daily rebalancing, R-vine performs better, and in weekly rebalancing, C-vine achieves better results. Following this, in Table 5, we conduct an in-depth analysis of the performance and risk metrics associated with these returns.

In terms of cumulative returns, 19 out of 20 portfolios exhibited positive returns over the Ibovespa Index. However, when compared to the Markowitz portfolio, only 10 out of 20 demonstrated comparable performance against this benchmark. Our focused analysis centers on these particular cases. Within the Daily Rebalancing strategy, only 4 out of 10 portfolios showcased an annualized return superior to the Markowitz daily rebalancing. Notably, the R-vine's Sharpe CVaR and Information Ratio, with 23.17% and 23.22% excess returns over the Markowitz, outperformed in this category. Similarly, the C-vine's Sharpe VaR and Sharpe CVaR exhibited excess returns of 16.87% and 7.49%, respectively, over the Markowitz.

Shifting to the Weekly Rebalancing strategy, 6 out of 10 portfolios outperformed the Markowitz weekly rebalancing, with returns ranging from a minimum of 12.80% over the benchmark to a maximum of 50.26%. This contrast can be attributed to the higher costs associated with the daily strategy.

When we analyze returns before and after the Covid-19 pandemic, it becomes clear that portfolios rebalanced daily suffered a more pronounced disadvantage compared to their counterparts that employed weekly rebalancing. Despite this, daily rebalancing strategies, for the most part, have demonstrated a remarkable ability to quickly recover lost wealth.

In the pursuit of identifying the optimal portfolio among various tested performance options, the portfolio manager carefully considers risks and other crucial metrics. Table 5 reveals seven metrics for each portfolio.

Beta, a metric measuring the portfolio's sensitivity to changes in the overall market, is a valuable tool for assessing systematic or market risk. It quantifies the portfolio's relative volatility in comparison to a benchmark index, in this case, the Ibovespa Index. Interestingly, all ten daily portfolios exhibit a Beta close to zero but negative, suggesting that these strategies

Figure 1 – Portfolio Allocations accumulation wealth for an initial investment of $1,000,000


Fonte: Elaborated by the author.

tend to move in the opposite direction to the Ibovespa Index. In contrast, the weekly strategies display a positive Beta, indicating that in bull markets, these strategies yield higher returns compared to the market. This difference can be explained by the higher returns daily volatility that occurs in a higher frequency rebalancing.

Alpha, a key concept, provides insight into whether a portfolio has outperformed or underperformed its expected return based on its level of risk, as compared to the benchmark. Remarkably, all strategies, including the benchmark portfolios, achieve a positive Alpha over the Ibovespa Index. Comparatively, under the daily rebalancing, we observe that 4 out of 10 alphas surpass the Markowitz portfolio, while under weekly rebalancing, 7 out of 10 portfolios exhibit higher alphas than the Markowitz portfolio.

Analyzing the annualized volatility, it becomes evident that the two daily Information Ratio portfolios exhibit the highest values within this strategy, surpassing the Ibovespa Index by a factor of two. A similar trend is observed in the weekly rebalancing, where these portfolios demonstrate volatility three times that of the Ibovespa Index. This helps elucidate the returns presented in Table 4 this criterion involves assuming a greater risk than its peers. As detailed in Appendix C, a closer look reveals that this strategy is more concentrated than the other portfolios.

Table 4 – Portfolio Returns

| Vine-Copula | Allocation criteria | | Retu (%) | | |
|---|---|---|---|---|---|
| Daily Rebalancing | | Cumulative | Pre Covid | Post Covid | Annualized |
| R-vine | Sharpe | 115.45 | 37.18 | 57.06 | 47.69 |
| R-vine | Sharpe VaR | 59.58 | -5.94 | 69.67 | 26.8 |
| R-vine | Sharpe CVaR | 216.21 | 29.55 | 143.97 | 79.43 |
| R-vine | Information Ratio | 216.21 | -23.15 | 311.47 | 79.48 |
| R-vine | Sortino | 126.88 | 24.94 | 81.59 | 51.63 |
| C-vine | Sharpe | 70.39 | 28.07 | 33.04 | 31.1 |
| C-vine | Sharpe VaR | 194.56 | 27.1 | 131.76 | 73.13 |
| C-vine | Sharpe CVaR | 163.98 | 22.8 | 114.97 | 63.75 |
| C-vine | Information Ratio | 30.97 | -10.04 | 45.59 | 14.69 |
| C-vine | Sortino | 98.78 | 4.49 | 90.24 | 41.77 |
| Weekly Rebalancing | | | | | |
| R-vine | Sharpe | 93.47 | 43.24 | 35.07 | 42.45 |
| R-vine | Sharpe VaR | 95.7 | 30.93 | 49.48 | 43.33 |
| R-vine | Sharpe CVaR | 74.83 | 7.76 | 62.24 | 34.91 |
| R-vine | Information Ratio | 42.75 | 18.15 | 20.82 | 21.02 |
| R-vine | Sortino | 137.88 | 42.35 | 67.11 | 59.13 |
| C-vine | Sharpe | 172.61 | 71.25 | 59.19 | 71.19 |
| C-vine | Sharpe VaR | 202.13 | 83.15 | 64.96 | 80.89 |
| C-vine | Sharpe CVaR | 142.56 | 44.08 | 68.35 | 60.8 |
| C-vine | Information Ratio | 252.85 | 97.19 | 78.94 | 96.59 |
| C-vine | Sortino | 195.56 | 55.92 | 89.55 | 78.77 |
| Benchmarks | | | | | |
| | Ibovespa Index | 35.42 | 11.5 | 21.45 | 16.55 |
| | Markowitz Daily | 140.73 | 46.84 | 63.95 | 56.26 |
| | GMV Daily | 29.4 | 22.89 | 5.3 | 13.99 |
| | Equal Weights Daily | 58.69 | -5.35 | 67.67 | 26.44 |
| | Markowitz Weekly | 103.44 | 56.42 | 30.06 | 46.33 |
| | GMV Weekly | 15.3 | 21.43 | -5.05 | 7.93 |
| | Equal Weights Weekly | 25.49 | 11.96 | 12.08 | 12.94 |

Fonte: Elaborated by the author.

It's crucial to highlight that this criterion also boasts the least favorable Maximum Drawdown metric under both rebalancing frequencies. This metric, indicating the maximum loss from peak to trough, underscores the downside risk associated with this strategy.

In summary, while the high returns of Information Ratio under R-vine daily and C-vine weekly suggest the potential for outperformance, the portfolio manager should carefully consider this against the backdrop of increased risk, concentration, and the less favorable Maximum Drawdown metric associated with this criterion.

While the Sharpe Ratio provides an overall assessment of performance, Sortino places specific emphasis on downside risk, aiming to assess the portfolio's ability to mitigate losses. Thus, compared to the Sharpe Ratio criteria, the Sortino results for the daily R-vine strategy show a relatively low negative value for Beta, suggesting a negative correlation with the benchmark, and a smaller Maximum Drawdown compared to the strategy by Sharpe. This indicates that the R-vine strategy, although potentially underperforming in absolute terms, has a superior ability to protect against adverse market movements and limit downside risk. Likewise, Sortino's results from the C-vine strategy mirror those from the R-vine approach. The negative Beta and lower Maximum Drawdown imply that the C-vine strategy is effective in managing downside risk, contributing to a favorable Sortino metric.

Focusing on weekly rebalancing, Sortino's results for the R-vine and C-vine strategies continue to demonstrate their resilience in mitigating downside risk. Positive values indicate overall positive risk-adjusted performance, with an emphasis on minimizing the impact of negative price movements. Here, both vine achieve a lower maximum drawdown than Markowitz Weekly, but greater volatility.

By concentrating exclusively on downward volatility, the Sortino ratio is more sensitive to the portfolio's ability to manage and minimize losses. This can lead to variations in the performance assessment compared to the Sharpe ratio. We can see this in the Maximum Drawdown, both daily Sortino portfolios had a Maximum Drawdown lower than the Sharpe Ratios, as well as in the weekly C-vine about the Sharpe Ratios portfolios.

Additionally, following DeMiguel et al. (2009) we present the Turnover Ratio:

$$\text{Turnover } = \frac{1}{T-1} \sum_{t=1}^{T-1} \sum_{i=1}^{d} \left( \left| \mathrm{w}_{i,t}^{m} - \mathrm{w}_{i,t+1}^{m} \right| \right) . \tag{3.1}$$

In simpler terms, the equation measures the average percentage of the portfolio that is bought or sold during each period, offering insights into the trading activity and potential transaction costs associated with managing the portfolio. A higher turnover indicates more frequent changes in the portfolio, which may result in higher transaction costs and could impact the overall performance of the portfolio. On the other hand, lower turnover implies a more stable portfolio composition over time. For example, in the R-vine Sharpe strategy with daily rebalancing, the turnover is 2.63%. This means that, on average, about 2.63% of the total portfolio value is bought or sold each day to maintain the specified allocation criteria.

As expected, all daily rebalanced portfolios had a higher turnover rate when compared to their weekly rebalanced pair, except for the GMV and Markowitz Benchmarks, and the R-Vine IR portfolio.

Table 5 – Performance and Risk Analysis

| Vine-Copula | Allocation criteria | Financial Metrics | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Daily Rebalancing | | Beta | Alpha | Sharpe Ratio | Max Drawdown (%) | Volatility ann(%) | Daily VaR (%) | Turnover (%) |
| R-vine | Sharpe | -0.1 | 0.47 | 1.28 | -38.61 | 35.42 | -3.49 | 2.63 |
| R-vine | Sharpe VaR | -0.06 | 0.36 | 0.74 | -47.71 | 47.64 | -4.8 | 2.79 |
| R-vine | Sharpe CVaR | -0.05 | 0.72 | 1.4 | -42.17 | 30.51 | -4.95 | 2.89 |
| R-vine | Information Ratio | -0.03 | 0.82 | 1.21 | -63.26 | 67.72 | -6.69 | 1.4 |
| R-vine | Sortino | -0.02 | 0.49 | 1.3 | -37.98 | 37.47 | -3.69 | 2.71 |
| | | | | | | | | |
| C-vine | Sharpe | -0.09 | 0.35 | 0.94 | -41.54 | 35.55 | -3.55 | 2.66 |
| C-vine | Sharpe VaR | -0.11 | 0.69 | 1.35 | -41.54 | 49.8 | -4.89 | 2.85 |
| C-vine | Sharpe CVaR | -0.08 | 0.64 | 1.2 | -41.68 | 52.13 | -5.15 | 2.89 |
| C-vine | Information Ratio | -0.09 | 0.37 | 0.54 | -59.79 | 64.79 | -6.57 | 1.4 |
| C-vine | Sortino | -0.09 | 0.44 | 1.09 | -37.46 | 38.9 | -3.86 | 2.74 |
| | | | | | | | | |
| Weekly Rebalancing | | | | | | | | |
| R-vine | Sharpe | 0.38 | 1.56 | 3.17 | -31.63 | 60.09 | -5.47 | 2.37 |
| R-vine | Sharpe VaR | 0.42 | 1.8 | 2.38 | -44.27 | 91.83 | -8.65 | 2.19 |
| R-vine | Sharpe CVaR | 0.3 | 1.59 | 2.06 | -39.1 | 90.22 | -8.61 | 2.35 |
| R-vine | Information Ratio | 0.37 | 1.17 | 1.43 | -53.77 | 104.69 | -10.25 | 1.57 |
| R-vine | Sortino | 0.27 | 2.3 | 3.37 | -32.7 | 75.31 | -6.8 | 2.55 |
| | | | | | | | | |
| C-vine | Sharpe | 0.38 | 2.48 | 4.47 | -32.29 | 63.13 | -5.42 | 2.37 |
| C-vine | Sharpe VaR | 0.34 | 2.86 | 4.26 | -32.78 | 74.22 | -6.44 | 2.26 |
| C-vine | Sharpe CVaR | 0.33 | 2.33 | 3.33 | -23.22 | 78.66 | -7.11 | 2.3 |
| C-vine | Information Ratio | 0.46 | 3.43 | 3.66 | -39.91 | 104.83 | -9.34 | 1.36 |
| C-vine | Sortino | 0.33 | 2.78 | 4.39 | -31.17 | 70.09 | -6.04 | 2.53 |
| | | | | | | | | |
| Benchmarks | | | | | | | | |
| | Ibovespa Index | - | - | 0.58 | -46.82 | 33.94 | -3.44 | - |
| | Markowitz Daily | -0.1 | 0.52 | 1.58 | -31.93 | 31.42 | -3.06 | 0.09 |
| | GMV Daily | -0.06 | 0.17 | 0.67 | -32.09 | 24.02 | -2.42 | 0.02 |
| | Equal Weights Daily | -0.08 | 0.32 | 0.81 | -46.16 | 37.7 | -3.78 | 0 |
| | Markowitz Weekly | 0.46 | 1.63 | 3.37 | -34.35 | 60.54 | -5.46 | 0.20 |
| | GMV Weekly | 0.5 | 0.04 | 1.02 | -36.24 | 48.32 | -4.81 | 0.04 |
| | Equal Weights Weekly | 0.76 | 0.18 | 1.21 | -49.71 | 71.29 | -7.05 | 0 |

Fonte: Elaborated by the author.

# 4 CONCLUSIONS

This study employs ARMA-GARCH-EVT-Vinecopula models to explore the potential advantages of portfolio optimization. We aim to provide portfolio managers with tools to seek superior performance in market indices, with a specific focus on the Brazilian stock market under two rebalancing frequencies.

In terms of wealth accumulation, our findings are surprising. For the years 2019 and 2020, our years of analysis, all 20 proposed portfolios managed to achieve a positive Alpha over the Ibovespa Index. Surprisingly, 12 out of 20 achieve more than 100% cumulative returns out of the sample, considering B3's transaction costs. Furthermore, 17 of the 20 portfolios also had a Sharpe ratio above 1.00.

Furthermore, our work encompasses a comprehensive quantitative approach. Not only allowing greater flexibility in adapting the EVT and GARCH parameters but also allowing adjustments to the GARCH specification throughout the out-of-sample period. This innovation increases the accuracy of tuning each asset in each interaction. Notably, we incorporate Vinecop- ulas to capture the tail dependence between data points, a departure from the traditional approach found in the literature, which primarily uses copulas.

Looking ahead, future research should strive to further refine the EVT methodology, techniques such as the mean excess function (MEF) and Hill plots could be explored to increase the accuracy of tail risk estimation. Furthermore, expanding the procedure to cover portfolios with lower risks, more diversified, and other rebalancing frequencies can incorporate better efficiency gains in terms of transaction costs and risk exposure. Allocation criteria such as GMV, Min VaR, and Min CVaR can be considered.

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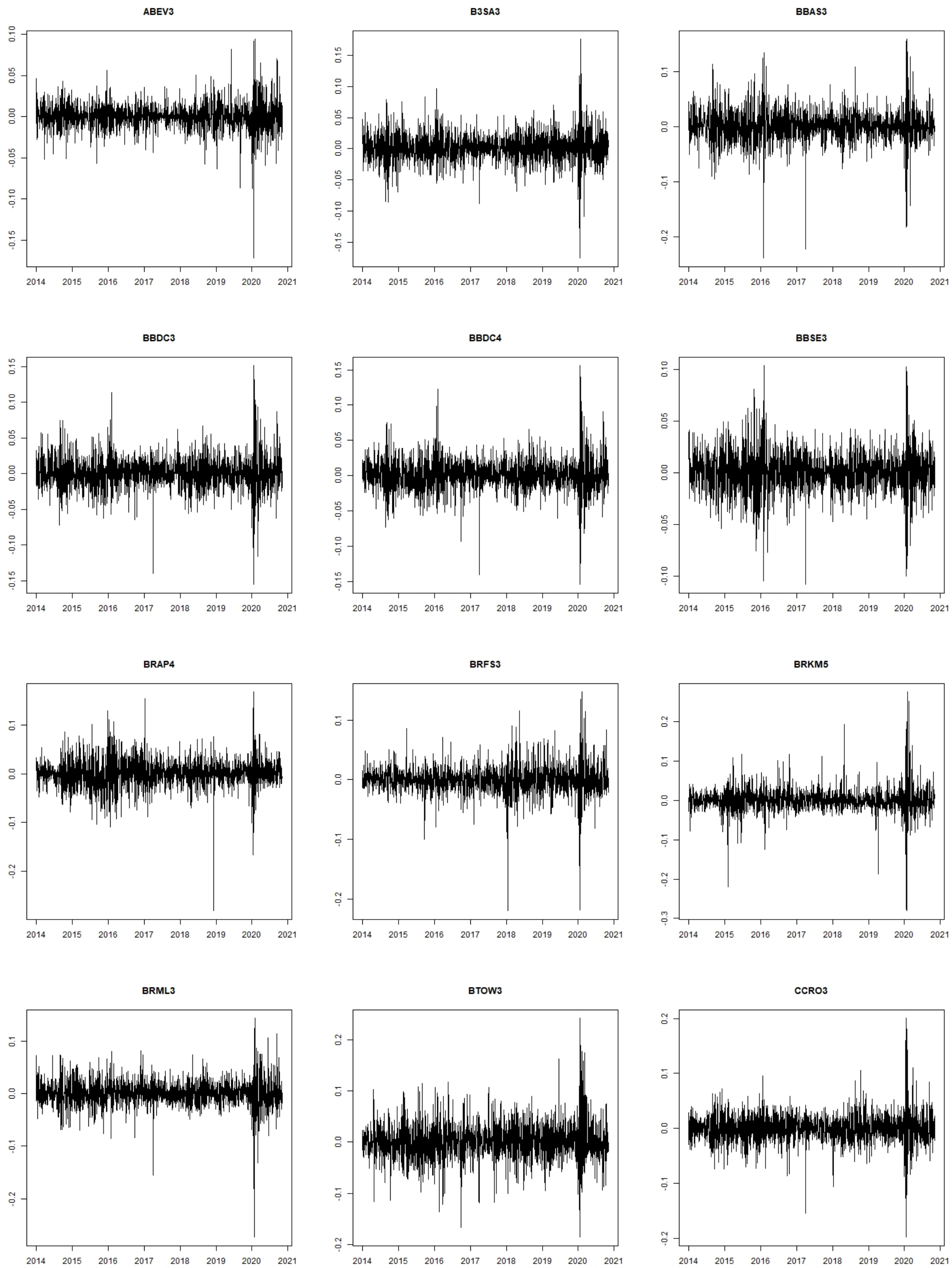
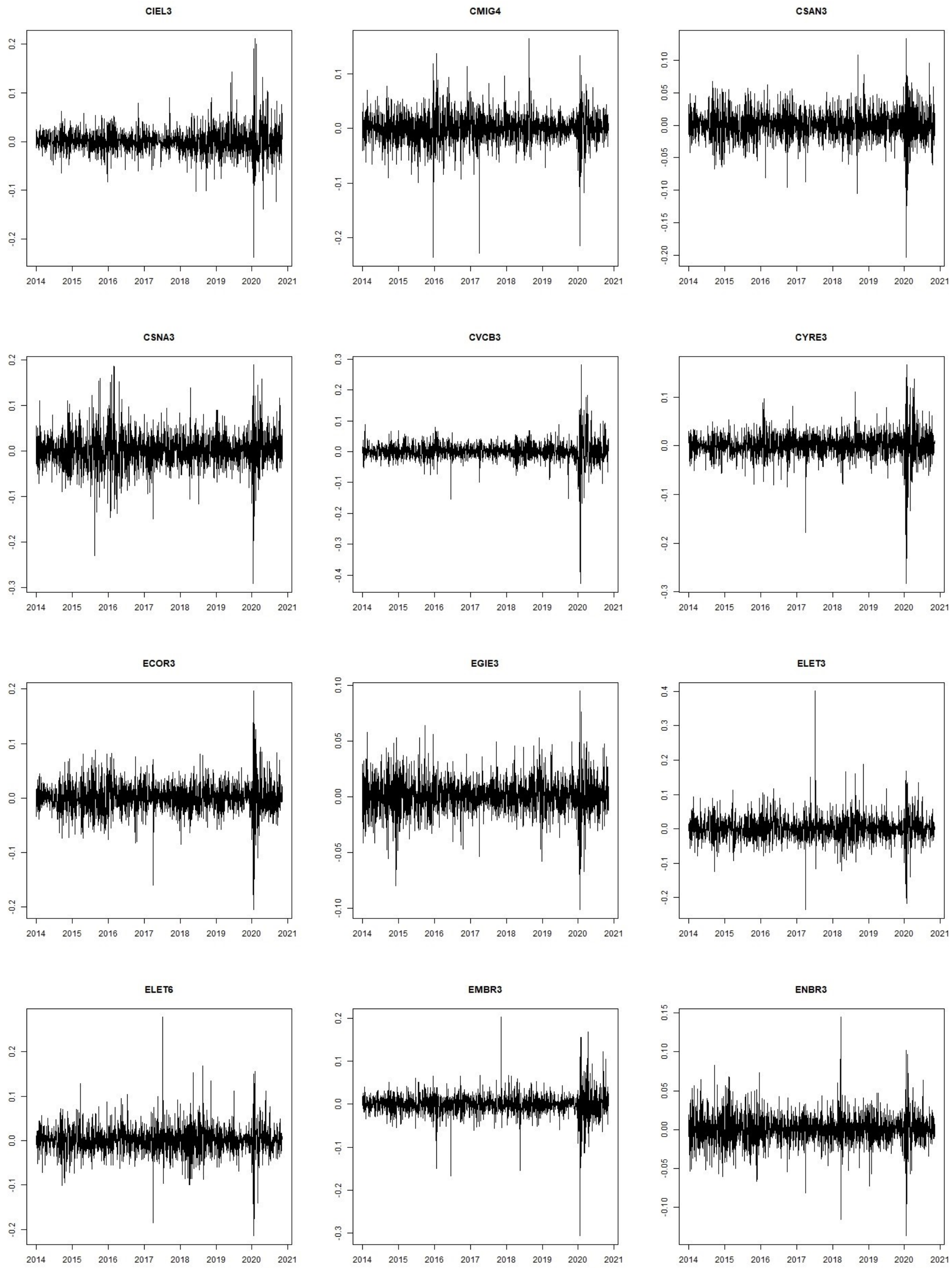
# APPENDIX A - ASSETS RETURNS

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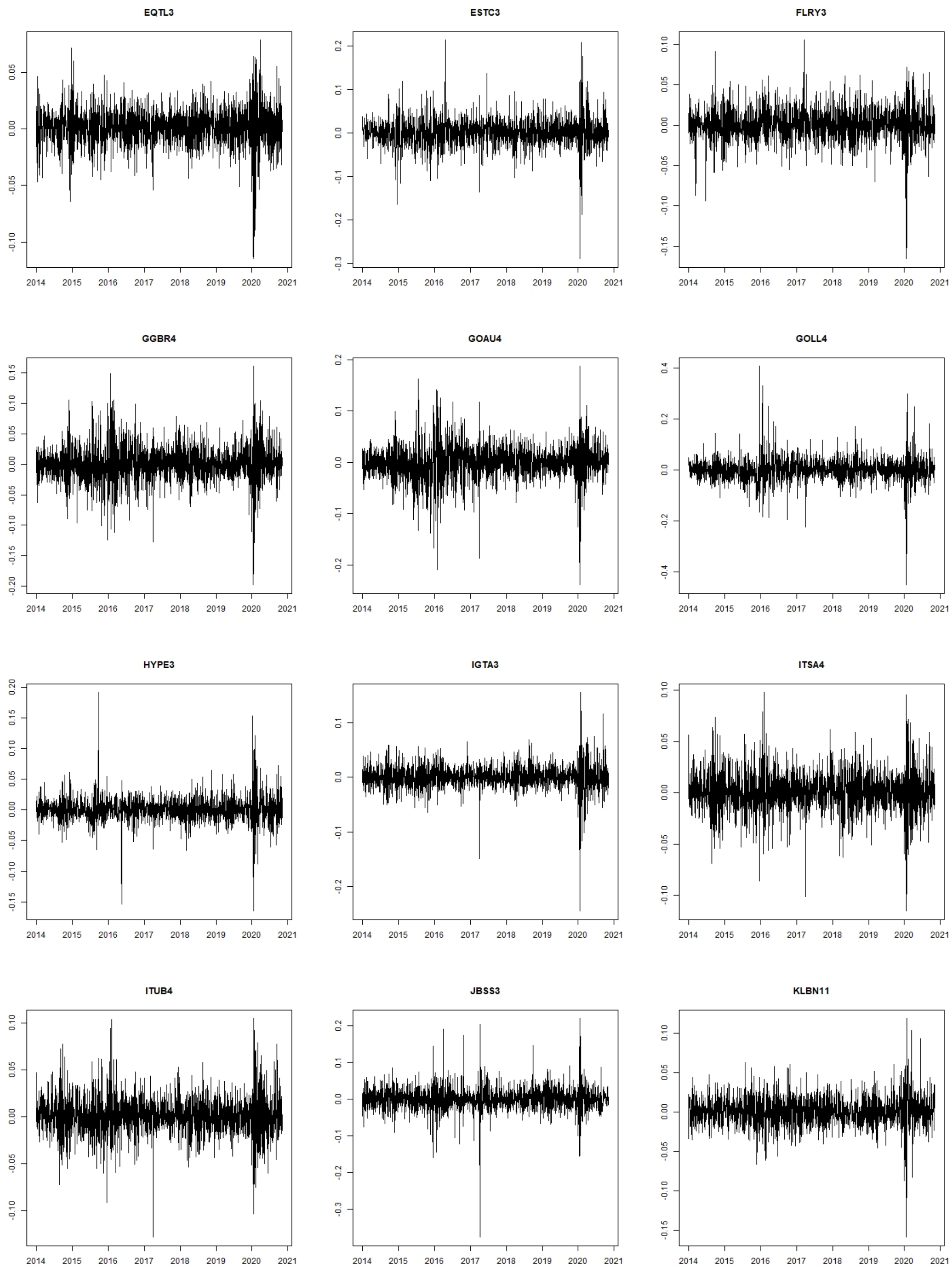
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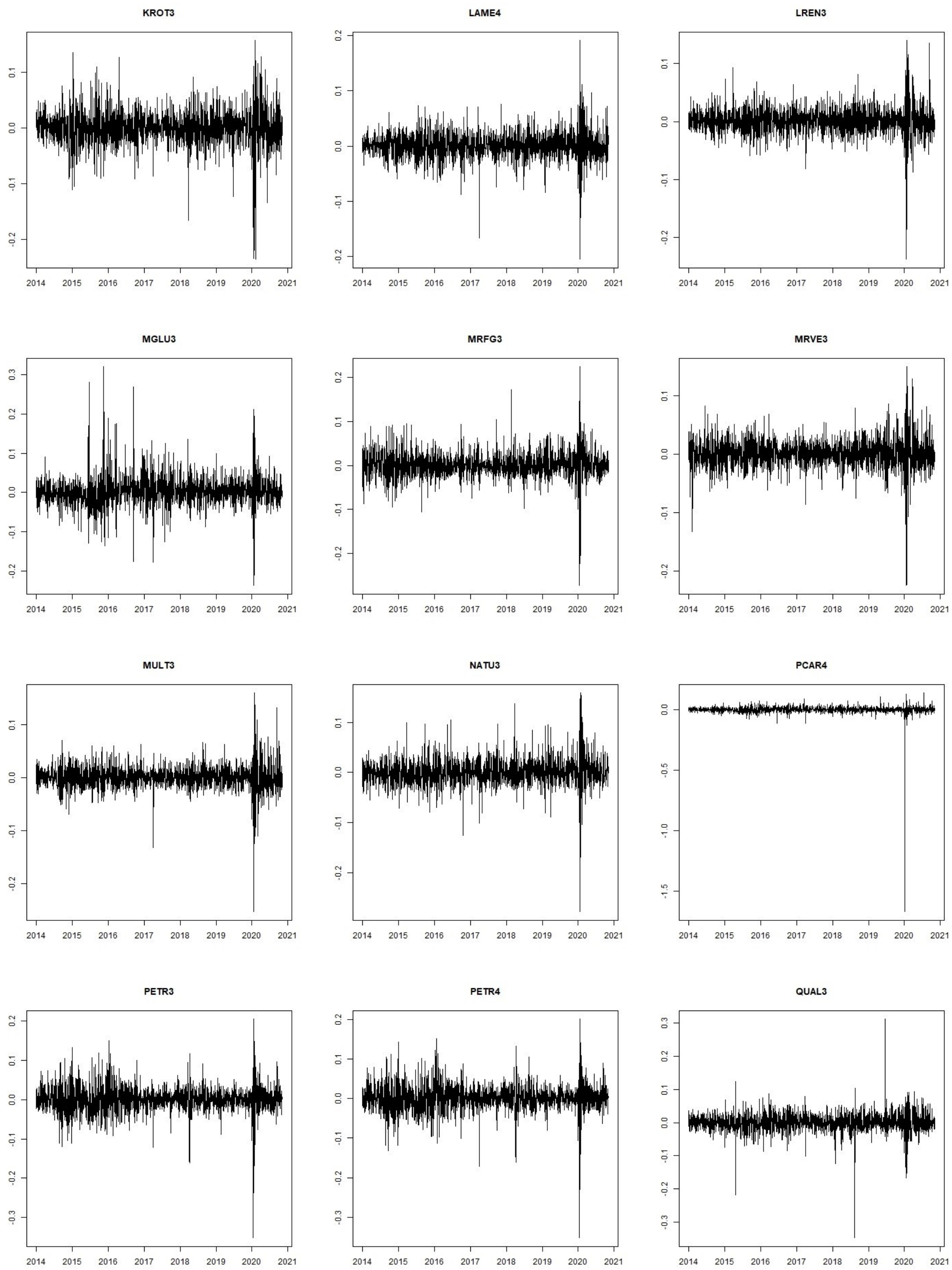
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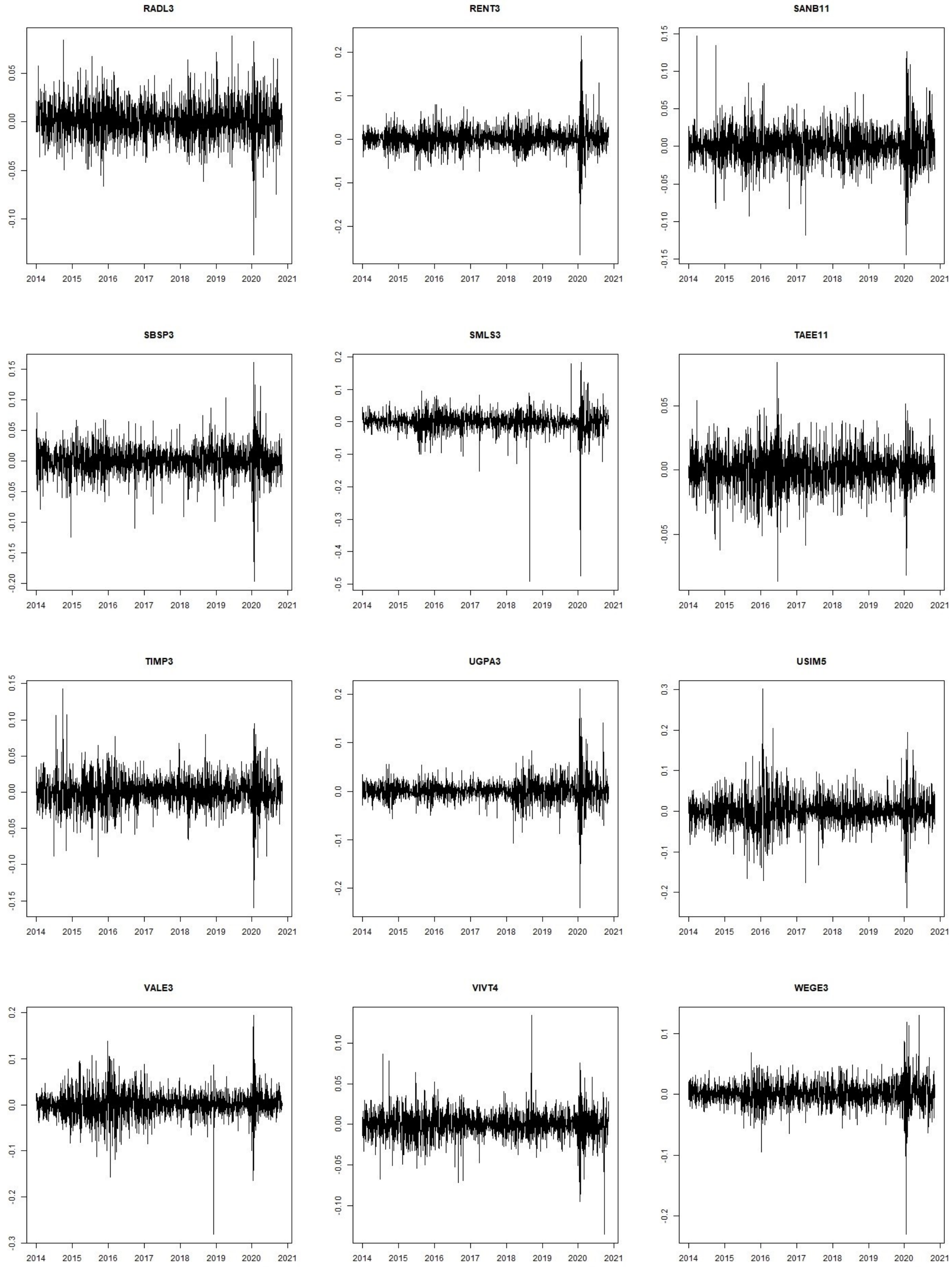
PETR4
QUAL3


RADL3
RENT3
SANB11
SBSP3
SMLS3
TAEE11
TIMP3
UGPA3
USIM5
VALE3
VIVT4
WEGE3

# APPENDIX B - IBOVESPA COMPOSITION ANDGARCH INITIAL TEST

Table 1 – Ibovespa composition (%) in 2019 and 2020

| TICKER | 1st Monday of Jan,2019 | 1st Monday of May,2019 | 1st Monday of Set,2019 | 1st Monday of Jan, 2020 | 1st Monday of May,2020 | 1st Monday of Set,2020 |
|---|---|---|---|---|---|---|
| ABEV3 | 4.8429 | 4.8909 | 4.6512 | 3.9575 | 3.3953 | 4.6512 |
| AZUL4 | - | 0.718 | 0.854 | 0.8662 | 0.3798 | 0.854 |
| B3SA3 | 3.8457 | 4.3772 | 5.175 | 4.3129 | 5.4053 | 5.175 |
| BBAS3 | 4.3736 | 4.1909 | 3.5251 | 3.6629 | 2.7798 | 3.5251 |
| BBDC3 | 1.6034 | 1.8037 | 1.9449 | 1.8307 | 1.5131 | 1.9449 |
| BBDC4 | 8.5709 | 8.7545 | 7.3002 | 6.9859 | 5.6118 | 7.3002 |
| BBSE3 | 1.2815 | 1.183 | 1.2549 | 1.2167 | 1.2261 | 1.2549 |
| BEEF3 | - | - | - | - | 0.2186 | - |
| BPAC11 | - | - | 0.6503 | 0.7186 | 0.5849 | 0.6503 |
| BRAP4 | 0.4772 | 0.4381 | 0.3611 | 0.4118 | 0.456 | 0.3611 |
| BRDT3 | 0.6019 | 0.4801 | 1.1994 | 1.0698 | 0.9778 | 1.1994 |
| BRFS3 | 1.1711 | 1.566 | 1.77 | 1.4021 | 1.0819 | 1.77 |
| BRKM5 | 0.856 | 0.7665 | 0.4271 | 0.4039 | 0.3847 | 0.4271 |
| BRML3 | 0.7585 | 0.6678 | 0.6476 | 0.7528 | 0.5822 | 0.6476 |
| BTOW3 | 0.4742 | 0.4019 | 0.5176 | 0.6015 | 0.9812 | 0.5176 |
| CCRO3 | 0.9199 | 0.84 | 1.0369 | 1.0079 | 0.9475 | 1.0369 |
| CIEL3 | 0.7647 | 0.5187 | 0.4957 | 0.4549 | 0.3122 | 0.4957 |
| CMIG4 | 0.9046 | 0.8544 | 0.8186 | 0.6401 | 0.6324 | 0.8186 |
| COGN3 | - | - | - | 0.8753 | 0.6969 | - |
| CPFE3 | - | - | - | - | 0.3399 | - |
| CRFB3 | - | - | - | 0.4543 | 0.5293 | - |
| CSAN3 | 0.3778 | 0.4424 | 0.4443 | 0.5392 | 0.6366 | 0.4443 |
| CSNA3 | 0.4135 | 0.5663 | 0.5276 | 0.4503 | 0.3958 | 0.5276 |
| CVCB3 | 0.5849 | 0.5255 | 0.4474 | 0.3084 | 0.1382 | 0.4474 |
| CYRE3 | 0.2809 | 0.2752 | 0.3797 | 0.4156 | 0.3076 | 0.3797 |
| ECOR3 | 0.1307 | 0.1011 | 0.1213 | 0.133 | 0.1234 | 0.1213 |
| EGIE3 | 0.6092 | 0.6817 | 0.6555 | 0.6407 | 0.6855 | 0.6555 |
| ELET3 | 0.5541 | 0.5457 | 0.7085 | 0.5872 | 0.597 | 0.7085 |
| ELET6 | 0.5165 | 0.4868 | 0.6067 | 0.4188 | 0.4595 | 0.6067 |
| EMBR3 | 1.0271 | 0.9023 | 0.7554 | 0.7221 | 0.4379 | 0.7554 |
| ENBR3 | 0.2958 | 0.3253 | 0.3396 | 0.3131 | 0.3453 | 0.3396 |
| ENGI11 | - | - | - | - | 0.7534 | - |
| EQTL3 | 1.0403 | 0.998 | 1.0856 | 1.1302 | 1.2733 | 1.0856 |
| ESTC3 | 0.5092 | 0.4987 | - | - | - | - |
| EZTC3 | - | - | - | - | - | - |
| FLRY3 | 0.435 | 0.3981 | 0.4187 | 0.44 | 0.4784 | 0.4187 |
| GGBR4 | 1.0307 | 0.8836 | 0.7554 | 1.0111 | 0.8325 | 0.7554 |
| GNDI3 | - | - | 1.1704 | 1.6088 | 1.7276 | 1.1704 |
| GOAU4 | 0.2823 | 0.2454 | 0.2134 | 0.309 | 0.2347 | 0.2134 |
| GOLL4 | 0.2239 | 0.1981 | 0.2556 | 0.2323 | 0.1148 | 0.2556 |
| HAPV3 | - | - | - | 0.7212 | 0.7827 | - |
| HGTX3 | - | - | - | 0.2124 | 0.1299 | - |
| HYPE3 | 0.8403 | 0.7005 | 0.7498 | 0.7198 | 0.8149 | 0.7498 |
| IGTA3 | 0.2436 | 0.2085 | 0.2286 | 0.228 | 0.1967 | 0.2286 |
| IRBR3 | - | 0.8477 | 1.9263 | 1.7721 | 0.6489 | 1.9263 |

Table 2 – (Continued) Ibovespa composition (%) in 2019 and 2020

| TICKER | 1st Monday of Jan,2019 | 1st Monday of May,2019 | 1st Monday of Set,2019 | 1st Monday of Jan, 2020 | 1st Monday of May,2020 | 1st Monday of Set,2020 |
|---|---|---|---|---|---|---|
| ITSA4 | 3.8466 | 3.3235 | 3.1539 | 3.0552 | 2.7812 | 3.1539 |
| ITUB4 | 10.8017 | 9.9977 | 9.1946 | 8.5731 | 7.4144 | 9.1946 |
| JBSS3 | 1.3543 | 2.0676 | 2.7474 | 2.1404 | 2.6567 | 2.7474 |
| KLBN11 | 0.6762 | 0.6177 | 0.5493 | 0.5945 | 0.7811 | 0.5493 |
| KROT3 | 0.9272 | 0.8723 | 0.8638 | - | - | 0.8638 |
| LAME4 | 0.892 | 0.6776 | 0.7371 | 0.8986 | 1.1911 | 0.7371 |
| LOGG3 | 0.0237 | - | - | - | - | - |
| LREN3 | 1.9686 | 2.0195 | 2.2591 | 2.1451 | 2.0731 | 2.2591 |
| MGLU3 | 0.8117 | 0.8337 | 1.1502 | 1.5557 | 2.2618 | 1.1502 |
| MRFG3 | 0.1539 | 0.1799 | 0.1843 | 0.2091 | 0.3736 | 0.1843 |
| MRVE3 | 0.2447 | 0.2654 | 0.33 | 0.3093 | 0.307 | 0.33 |
| MULT3 | 0.4427 | 0.4063 | 0.4017 | 0.4359 | 0.3882 | 0.4017 |
| NATU3 | 0.5464 | 0.5419 | 0.6605 | - | - | 0.6605 |
| NTCO3 | - | - | - | 0.6878 | 1.6394 | - |
| PCAR3 | - | - | - | - | 0.7067 | - |
| PCAR4 | 0.8681 | 0.9316 | 0.7832 | 0.6746 | - | 0.7832 |
| PETR3 | 5.0156 | 5.0323 | 4.7697 | 4.2012 | 3.783 | 4.7697 |
| PETR4 | 7.2084 | 7.3107 | 6.5927 | 6.6176 | 5.6103 | 6.5927 |
| QUAL3 | 0.2157 | 0.2578 | 0.3659 | 0.54 | 0.5018 | 0.3659 |
| RADL3 | 0.8157 | 0.9194 | 1.1242 | 1.1527 | 1.4945 | 1.1242 |
| RAIL3 | 1.291 | 1.237 | 1.3319 | 1.2985 | 1.434 | 1.3319 |
| RENT3 | 1.0165 | 1.2637 | 1.5089 | 1.3185 | 1.3947 | 1.5089 |
| SANB11 | 1.1821 | 1.0493 | 0.8825 | 0.8512 | 0.6725 | 0.8825 |
| SBSP3 | 0.7782 | 0.8963 | 1.0053 | 0.9727 | 0.937 | 1.0053 |
| SMLS3 | 0.1736 | 0.1775 | 0.1245 | 0.1124 | - | 0.1245 |
| SULA11 | - | - | - | 0.8691 | 0.8577 | - |
| SUZB3 | 1.457 | 1.9841 | 1.2073 | 1.4186 | 1.967 | 1.2073 |
| TAEE11 | 0.3713 | 0.3576 | 0.3512 | 0.3254 | 0.4133 | 0.3512 |
| TIMP3 | 0.6513 | 0.5827 | 0.5682 | 0.6135 | 0.7087 | 0.5682 |
| TOTS3 | - | - | - | 0.4565 | 0.5469 | - |
| UGPA3 | 1.927 | 1.4465 | 1.0191 | 1.32 | 1.0832 | 1.0191 |
| USIM5 | 0.3492 | 0.2863 | 0.232 | 0.2371 | 0.1696 | 0.232 |
| VALE3 | 10.7745 | 9.9707 | 8.2044 | 8.1892 | 10.1549 | 8.2044 |
| VIVT4 | 1.3272 | 1.198 | 1.2743 | 1.152 | 1.3031 | 1.2743 |
| VVAR3 | 0.1161 | 0.1419 | 0.5072 | 0.6337 | 0.7238 | 0.5072 |
| WEGE3 | 0.927 | 0.8679 | 0.9532 | 1.2389 | 1.893 | 0.9532 |
| YDUQ3 | - | - | 0.5392 | 0.6594 | 0.6254 | 0.5392 |

Fonte: UP2DATA B3.

Table 3 – AIC and BIC results

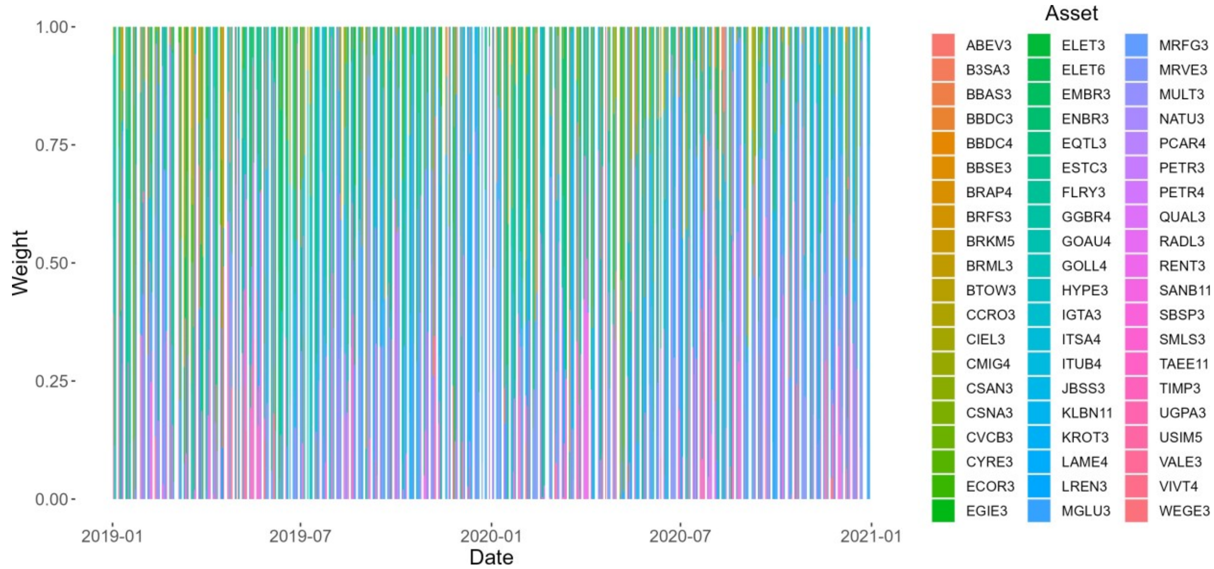
| | AIC | | | BIC | | |
|---|---|---|---|---|---|---|
| TICKER | sGARCH | eGARCH | gjrGARCH | sGARCH | eGARCH | gjrGARCH |
| ABEV3 | -5.8984 | -5.9099 | -5.9026 | -5.865 | -5.8723 | -5.865 |
| B3SA3 | -4.9464 | -4.9434 | -4.9448 | -4.913 | -4.9058 | -4.9072 |
| BBAS3 | -4.3758 | -4.3817 | -4.3752 | -4.3424 | -4.3441 | -4.3376 |
| BBDC3 | -4.9676 | -4.9678 | -4.966 | -4.9342 | -4.9302 | -4.9284 |
| BBDC4 | -4.9404 | -4.9456 | -4.9394 | -4.907 | -4.908 | -4.9018 |
| BBSE3 | -5.1323 | -5.1313 | -5.1307 | -5.0989 | -5.0937 | -5.0931 |
| BRAP4 | -4.3096 | -4.306 | -4.3081 | -4.2762 | -4.2684 | -4.2705 |
| BRFS3 | -5.1526 | -5.1724 | -5.1639 | -5.1192 | -5.1348 | -5.1263 |
| BRKM5 | -4.7763 | -4.7823 | -4.7855 | -4.7429 | -4.7447 | -4.7479 |
| BRML3 | -4.8301 | -4.8316 | -4.8302 | -4.7967 | -4.794 | -4.7926 |
| BTOW3 | -3.9812 | -3.9835 | -3.9797 | -3.9478 | -3.9459 | -3.9421 |
| CCRO3 | -4.7918 | -4.7983 | -4.7922 | -4.7584 | -4.7608 | -4.7546 |
| CIEL3 | -5.0746 | -5.0874 | -5.083 | -5.0412 | -5.0498 | -5.0454 |
| CMIG4 | -4.302 | -4.2978 | -4.3017 | -4.2686 | -4.2602 | -4.2641 |
| CSAN3 | -4.8913 | -4.8902 | -4.8902 | -4.8579 | -4.8526 | -4.8526 |
| CSNA3 | -3.7716 | -3.7727 | -3.77 | -3.7382 | -3.7351 | -3.7324 |
| CVCB3 | -4.8628 | -4.8612 | -4.863 | -4.8294 | -4.8236 | -4.8254 |
| CYRE3 | -4.9012 | -4.9004 | -4.9002 | -4.8678 | -4.8628 | -4.8626 |
| ECOR3 | -4.6039 | -4.6139 | -4.6096 | -4.5704 | -4.5763 | -4.572 |
| EGIE3 | -5.6614 | -5.662 | -5.6606 | -5.628 | -5.6244 | -5.623 |
| ELET3 | -4.003 | -4.0082 | -4.0019 | -3.9696 | -3.9706 | -3.9643 |
| ELET6 | -4.2048 | -4.2133 | -4.2067 | -4.1714 | -4.1757 | -4.1691 |
| EMBR3 | -5.046 | -5.0506 | -5.0444 | -5.0126 | -5.013 | -5.0068 |
| ENBR3 | -5.1522 | -5.1575 | -5.1517 | -5.1187 | -5.1199 | -5.1141 |
| EQTL3 | -5.6436 | -5.6608 | -5.6569 | -5.6102 | -5.6232 | -5.6193 |
| ESTC3 | -4.2742 | -4.2823 | -4.282 | -4.2408 | -4.2447 | -4.2444 |
| FLRY3 | -5.1162 | -5.1211 | -5.0988 | -5.0828 | -5.0835 | -5.0612 |
| GGBR4 | -4.2907 | -4.2876 | -4.2891 | -4.2573 | -4.25 | -4.2515 |
| GOAU4 | -4.0615 | -4.062 | -4.0603 | -4.0281 | -4.0244 | -4.0227 |
| GOLL4 | -3.6514 | -3.6531 | -3.6498 | -3.618 | -3.6155 | -3.6122 |
| HYPE3 | -5.4489 | -5.4841 | -5.4636 | -5.4154 | -5.4465 | -5.426 |
| IGTA3 | -5.2324 | -5.2336 | -5.2308 | -5.199 | -5.196 | -5.1932 |
| ITSA4 | -5.1391 | -5.1405 | -5.1384 | -5.1057 | -5.1029 | -5.1008 |
| ITUB4 | -5.1035 | -5.1088 | -5.1027 | -5.0701 | -5.0712 | -5.0651 |
| JBSS3 | -4.3535 | -4.3657 | -4.3641 | -4.3201 | -4.3281 | -4.3265 |
| KLBN11 | -5.3531 | -5.3505 | -5.3527 | -5.3197 | -5.3129 | -5.3151 |
| KROT3 | -4.3926 | -4.4048 | -4.3985 | -4.3592 | -4.3672 | -4.3609 |
| LAME4 | -4.9212 | -4.9328 | -4.9258 | -4.8878 | -4.8952 | -4.8882 |
| LREN3 | -5.0672 | -5.0769 | -5.0742 | -5.0337 | -5.0393 | -5.0366 |
| MGLU3 | -3.9241 | -3.9301 | -3.9255 | -3.8907 | -3.8925 | -3.8879 |
| MRFG3 | -4.5366 | -4.5472 | -4.541 | -4.5032 | -4.5096 | -4.5034 |
| MRVE3 | -4.9094 | -4.9146 | -4.9077 | -4.876 | -4.877 | -4.8702 |
| MULT3 | -5.2804 | -5.279 | -5.2819 | -5.247 | -5.2414 | -5.2443 |
| NATU3 | -4.7149 | -4.7158 | -4.7147 | -4.6815 | -4.6782 | -4.6771 |
| PCAR4 | -5.0449 | -5.0558 | -5.0466 | -5.0115 | -5.0182 | -5.009 |
| PETR3 | -4.1667 | -4.172 | -4.1652 | -4.1333 | -4.1344 | -4.1276 |
| PETR4 | -4.0928 | -4.1008 | -4.0941 | -4.0594 | -4.0632 | -4.0565 |
| QUAL3 | -4.6299 | -4.6282 | -4.6288 | -4.5965 | -4.5906 | -4.5912 |
| RADL3 | -5.1989 | -5.2012 | -5.2006 | -5.1655 | -5.1636 | -5.163 |
| RENT3 | -4.8615 | -4.8619 | -4.8621 | -4.8281 | -4.8243 | -4.8245 |
| SANB11 | -4.9016 | -4.9118 | -4.9041 | -4.8682 | -4.8742 | -4.8665 |
| SBSP3 | -4.8682 | -4.8748 | -4.8662 | -4.8347 | -4.8372 | -4.8286 |
| SMLS3 | -4.5961 | -4.6026 | -4.5947 | -4.5627 | -4.565 | -4.5571 |
| TAEE11 | -5.4547 | -5.457 | -5.4544 | -5.4213 | -5.4194 | -5.4168 |
| TIMP3 | -4.9704 | -4.979 | -4.9782 | -4.937 | -4.9414 | -4.9406 |
| UGPA3 | -5.4773 | -5.485 | -5.4811 | -5.4439 | -5.4475 | -5.4435 |
| USIM5 | -3.7901 | -3.7938 | -3.7885 | -3.7567 | -3.7562 | -3.7509 |
| VALE3 | -4.3853 | -4.3828 | -4.3858 | -4.3519 | -4.3452 | -4.3482 |
| VIVT4 | -5.5299 | -5.5353 | -5.5283 | -5.4965 | -5.4977 | -5.4907 |
| WEGE3 | -5.3617 | -5.362 | -5.3618 | -5.3283 | -5.3244 | -5.3242 |

Note: The total number of observations is 1223 for each stock, considering only the database in the training sample.

# APPENDIX C - PORTFOLIOS WEIGHT HISTORICAL

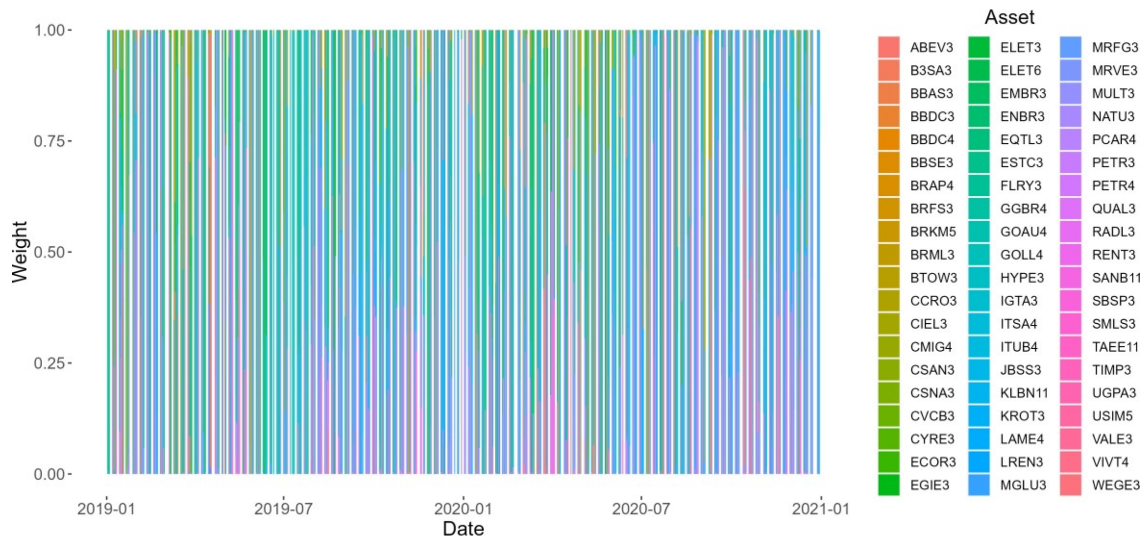
## 3.2 Daily Rebalancing

Figure 2 – Rvine - Sharpe VaR



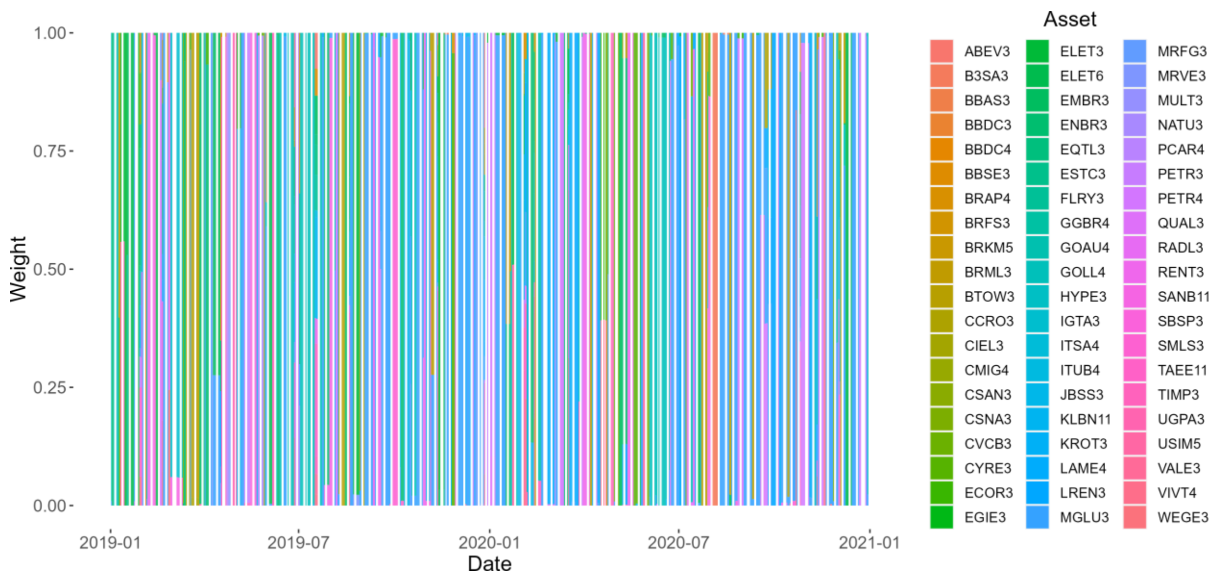

Fonte: Elaborated by the author.

Figure 3 – Rvine - Sharpe CVaR



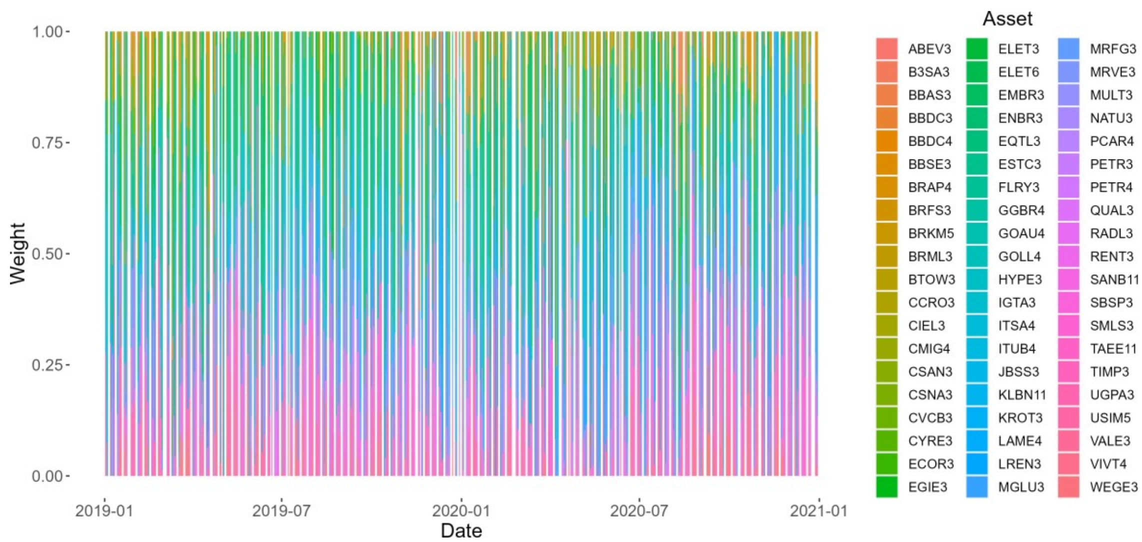

Fonte: Elaborated by the author.

Figure 4 – Rvine - Information Ratio



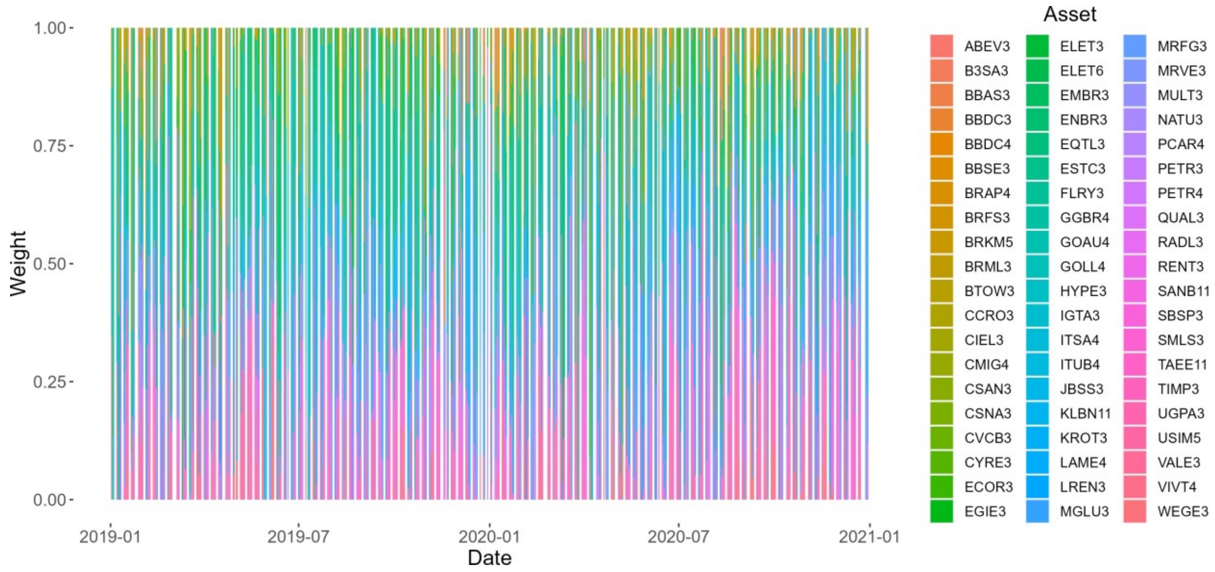

Fonte: Elaborated by the author.

Figure 5 – Rvine - Sharpe Ratio



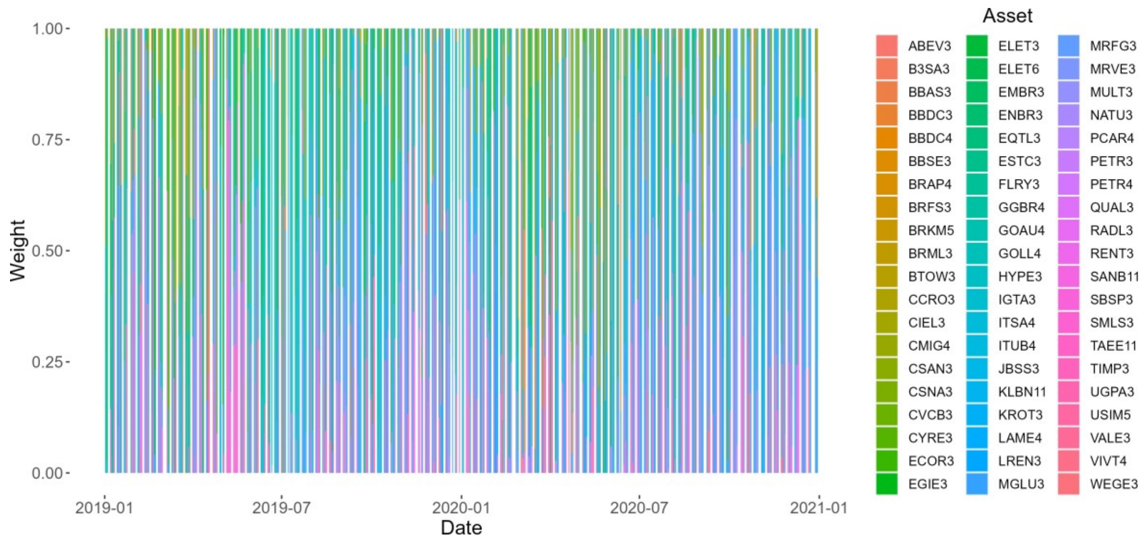

Fonte: Elaborated by the author.

Figure 6 – Rvine - Sortino



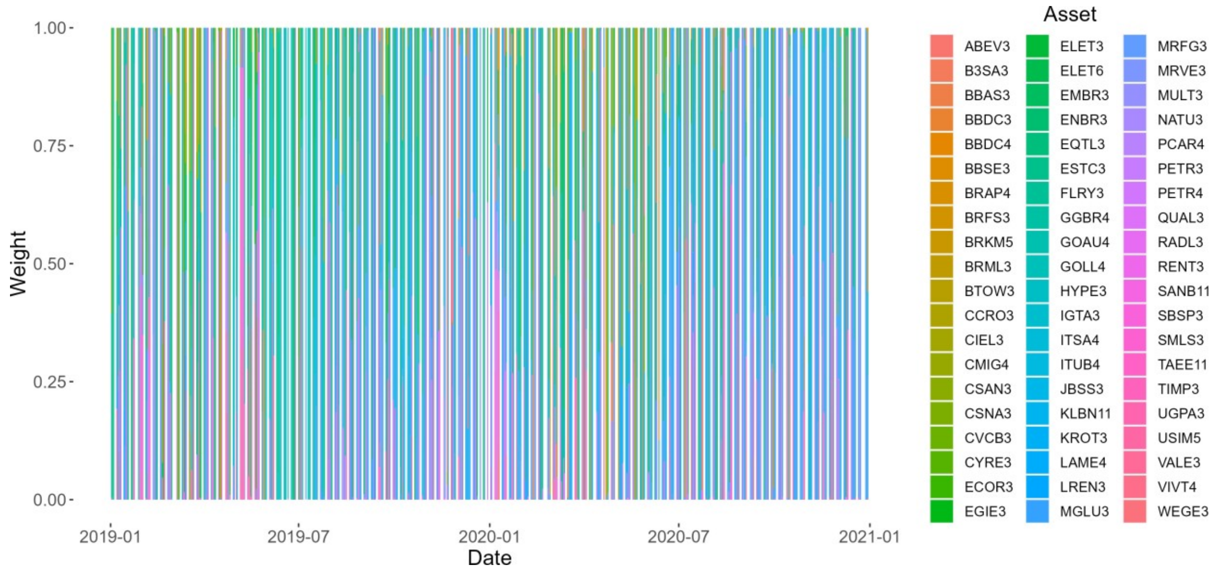

Fonte: Elaborated by the author.

Figure 7 – Cvine - Sharpe VaR



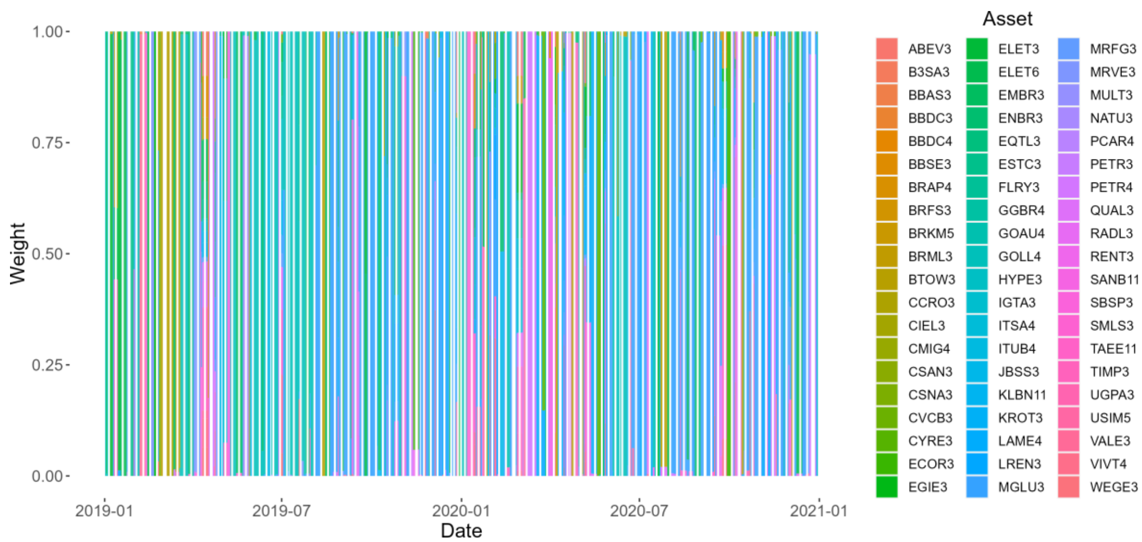

Fonte: Elaborated by the author.

Figure 8 – Cvine - Sharpe CVaR



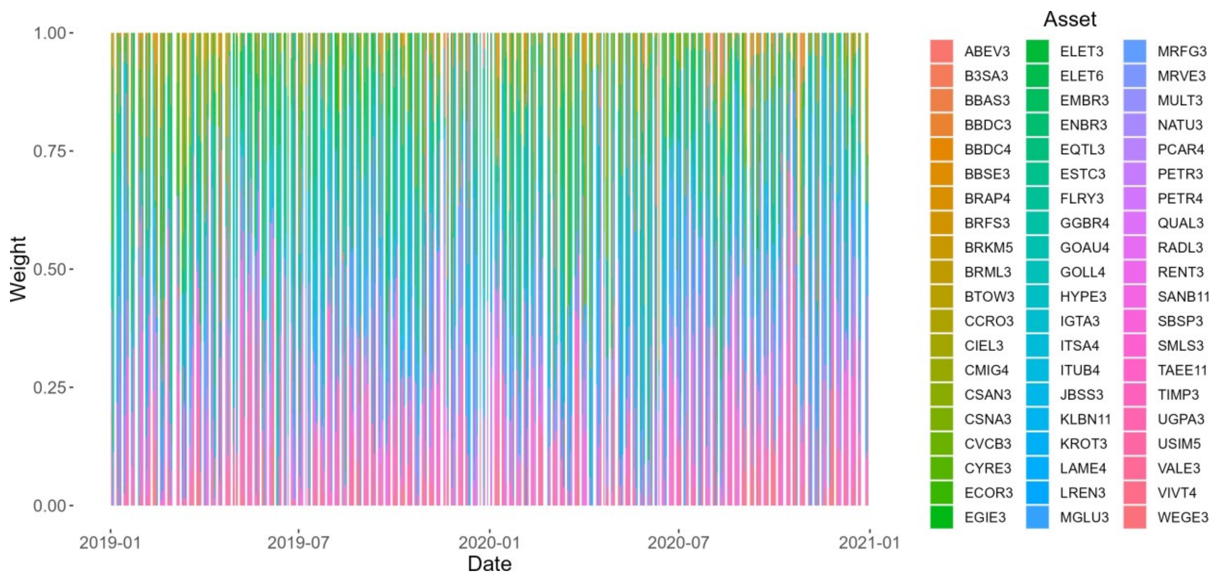

Fonte: Elaborated by the author.

Figure 9 – Cvine - Information Ratio



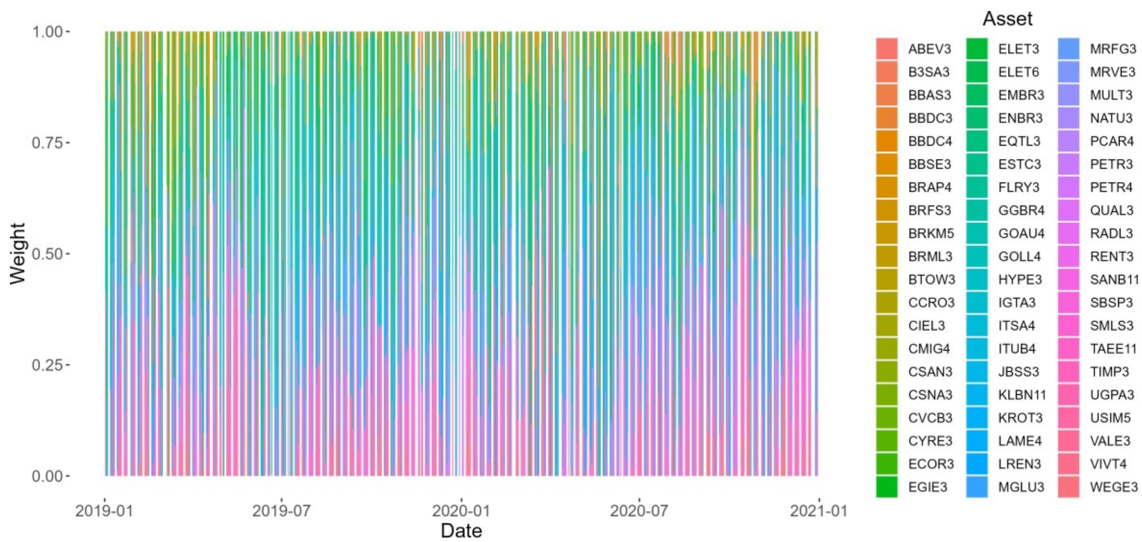

Fonte: Elaborated by the author.

Figure 10 – Cvine - Sharpe Ratio




Fonte: Elaborated by the author.

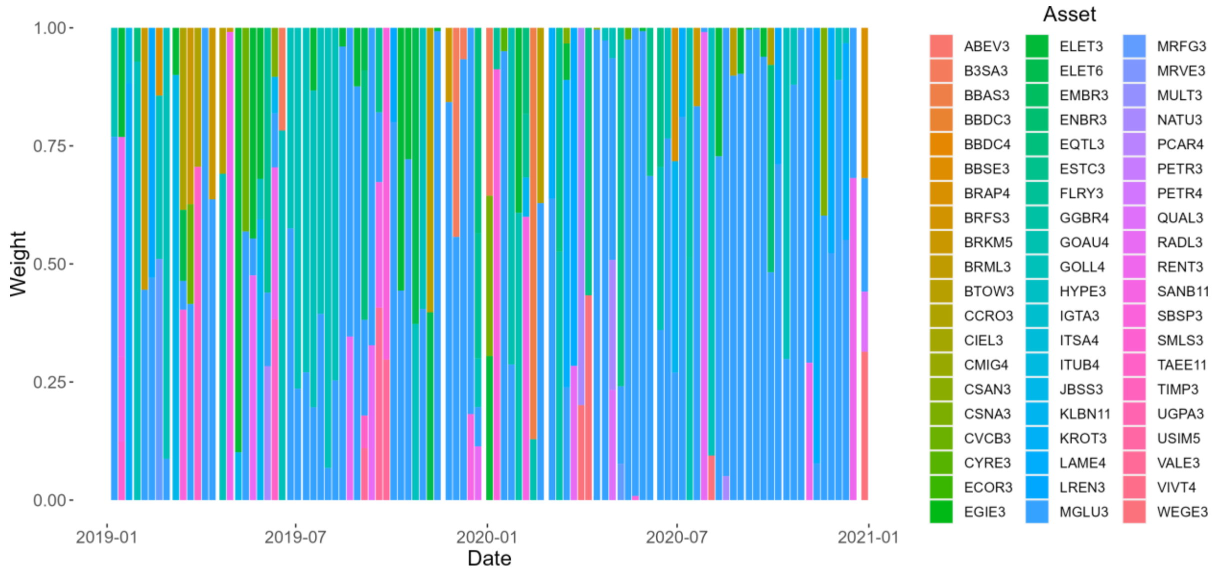
Figure 11 – Cvine - Sortino




Fonte: Elaborated by the author.

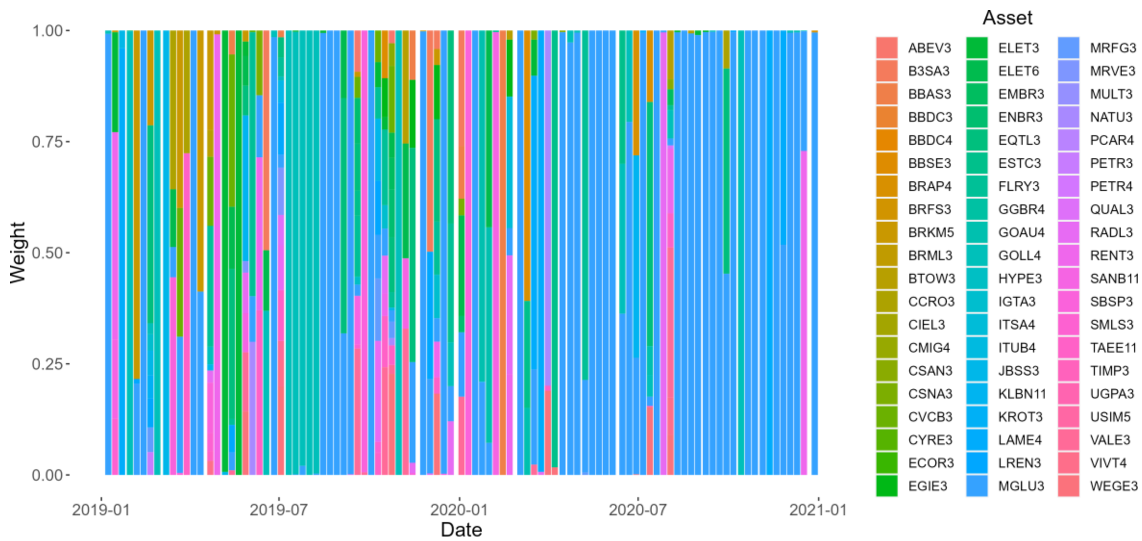
## 3.3 Weekly Rebalancing

Figure 12 – Rvine - Sharpe VaR



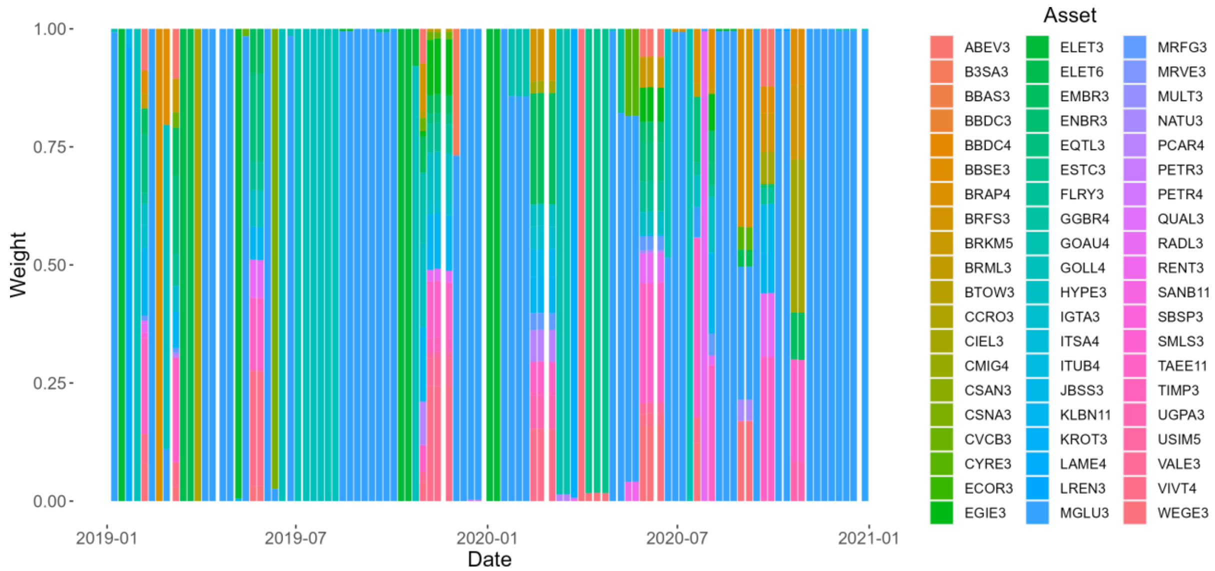

Fonte: Elaborated by the author.

Figure 13 – Rvine - Sharpe CVaR



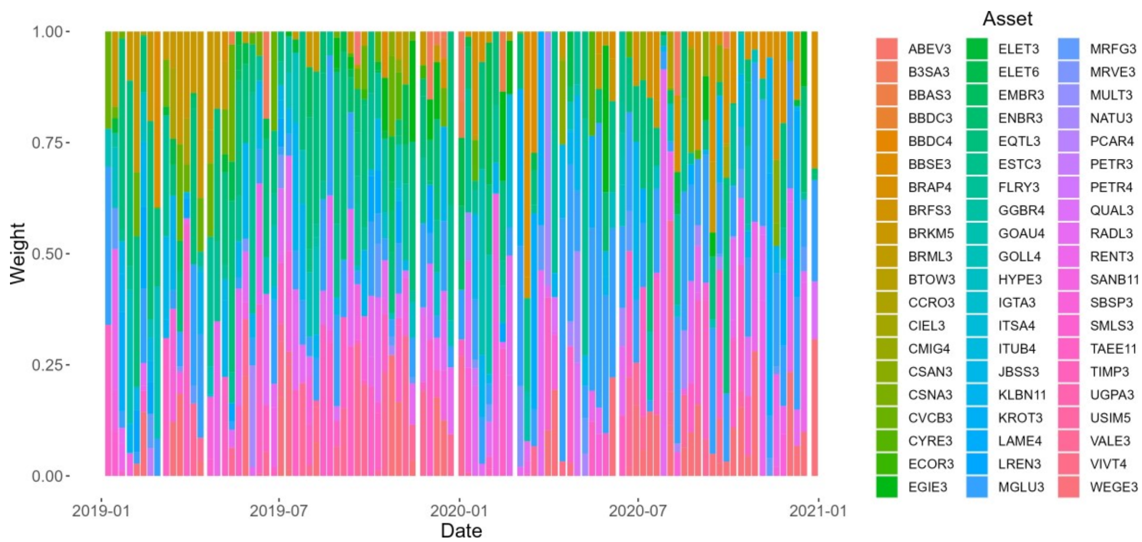

Fonte: Elaborated by the author.

Figure 14 – Rvine - Information Ratio



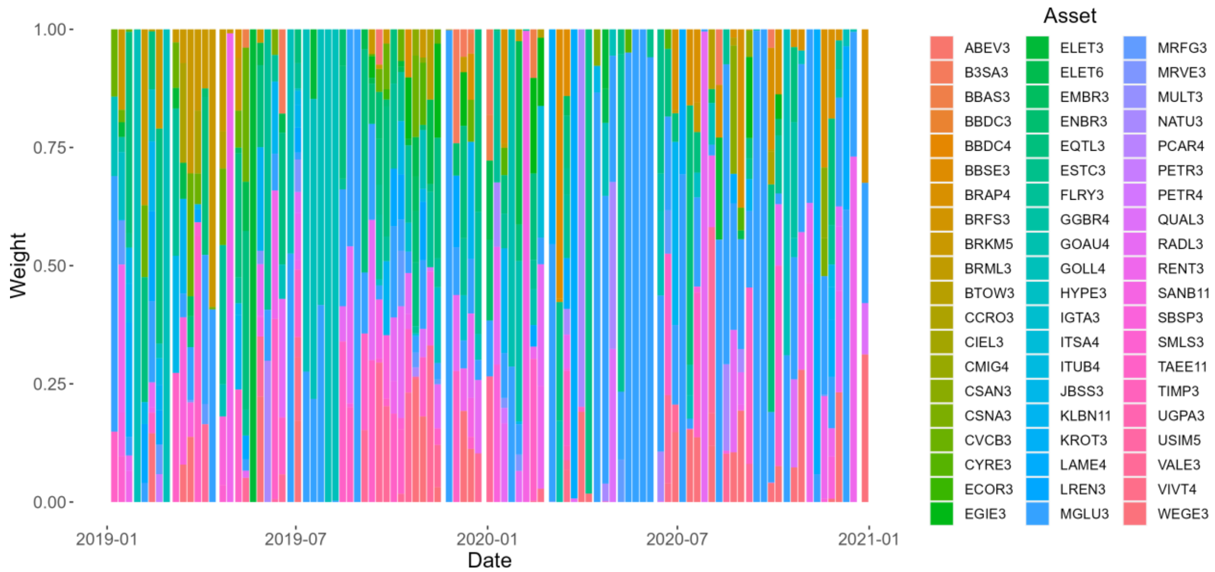

Fonte: Elaborated by the author.

Figure 15 – Rvine - Sharpe Ratio



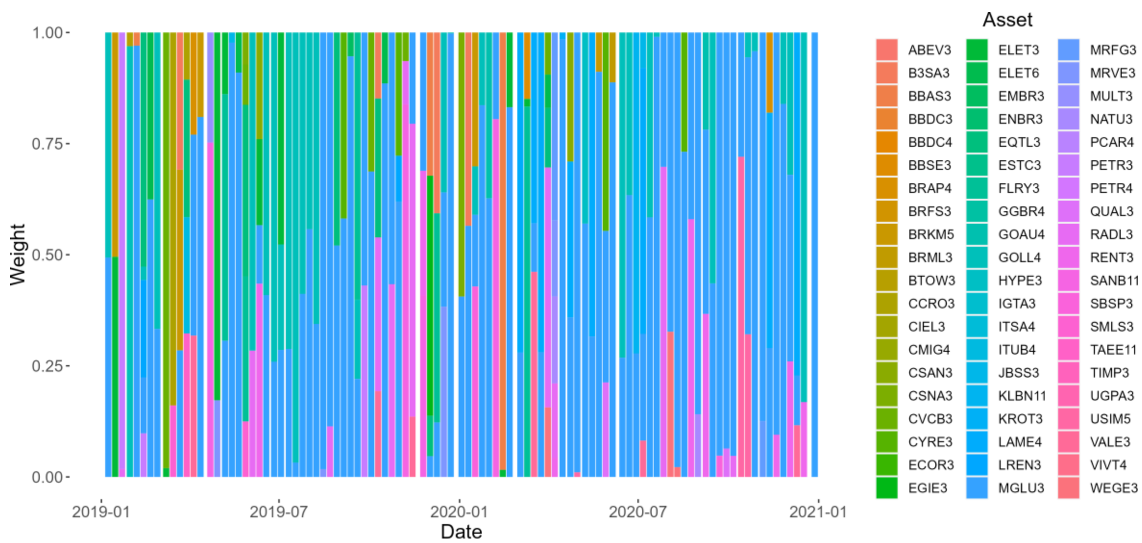

Fonte: Elaborated by the author.

Figure 16 – Rvine - Sortino



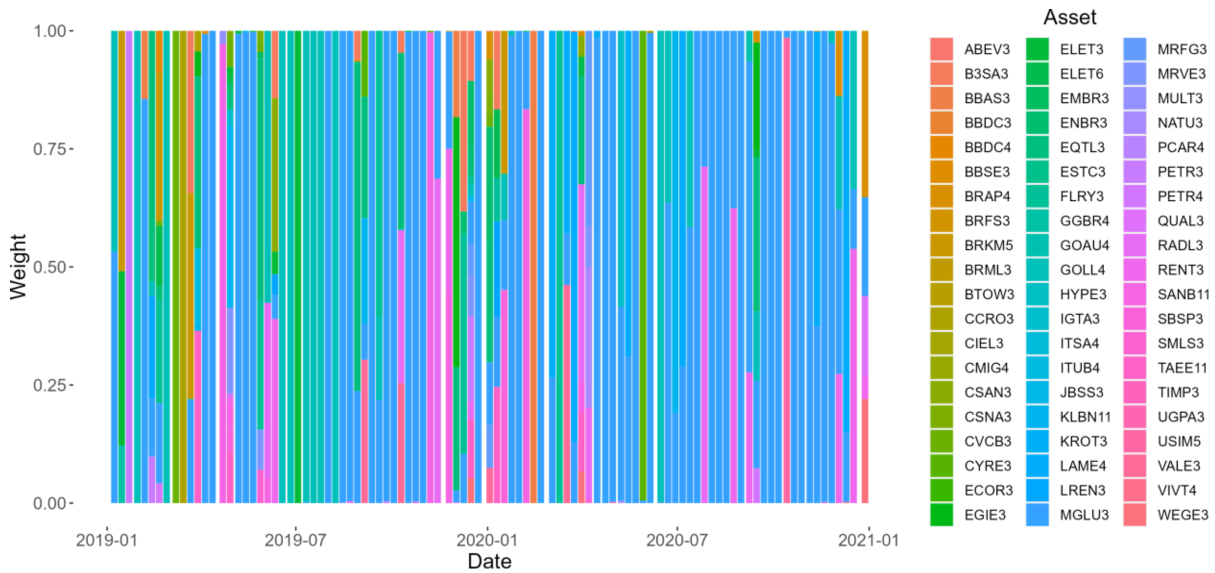

Fonte: Elaborated by the author.

Figure 17 – Cvine - Sharpe VaR



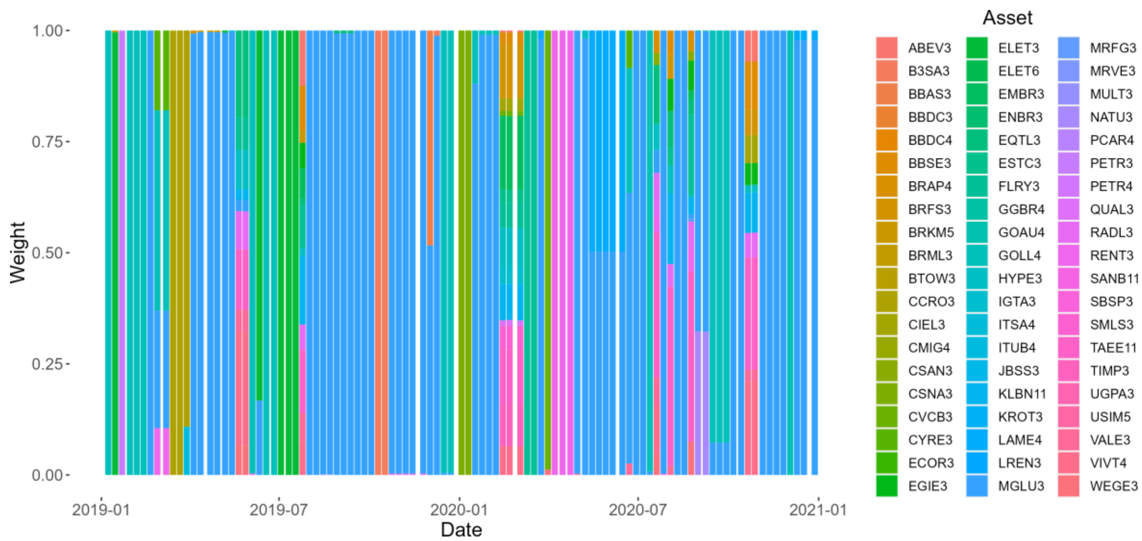

Fonte: Elaborated by the author.

Figure 18 – Cvine - Sharpe CVaR



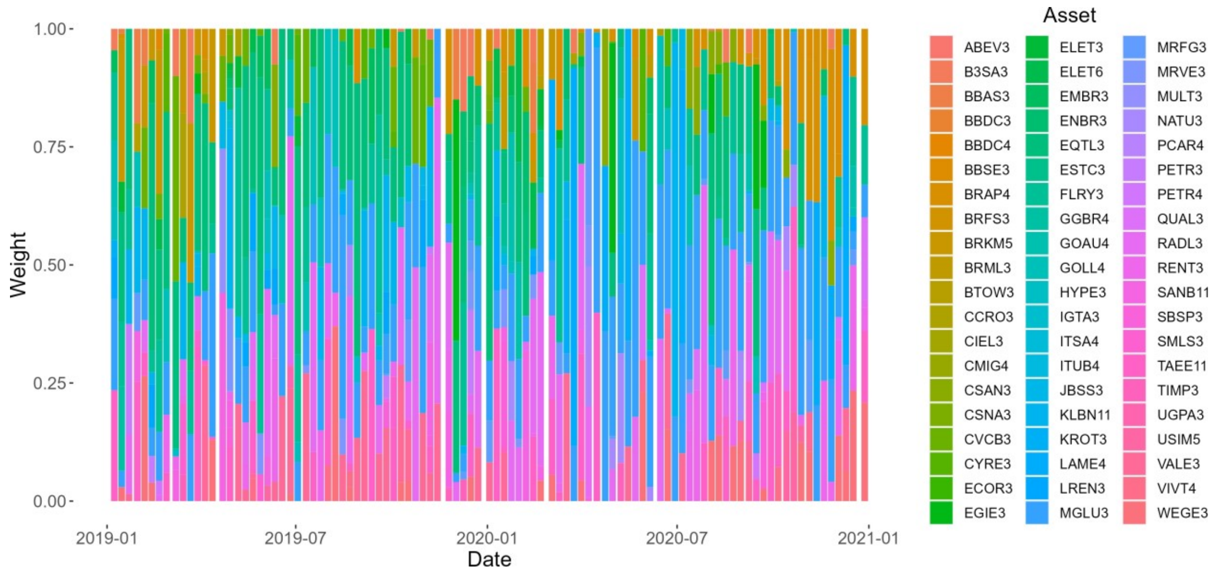

Fonte: Elaborated by the author.

Figure 19 – Cvine - Information Ratio



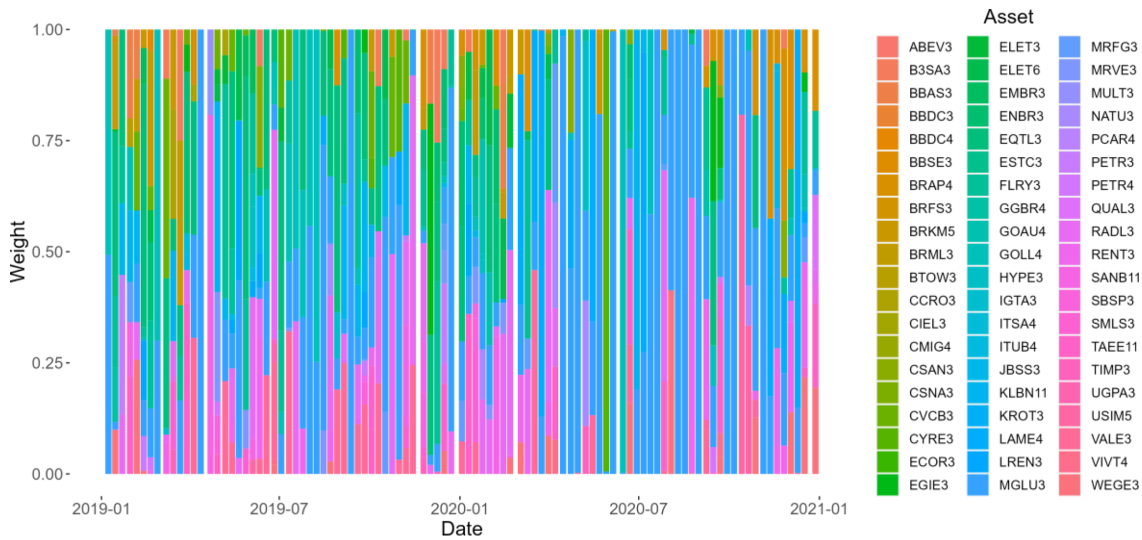

Fonte: Elaborated by the author.

Figure 20 – Cvine - Sharpe Ratio




Fonte: Elaborated by the author.

Figure 21 – Cvine - Sortino




Fonte: Elaborated by the author.