# Trapping state stabilization in a micromaser with a mixed atomic beam

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A scheme which stabilizes the trapping states in micromasers is proposed and studied. It uses an atomic beam composed of a mixture of two types of atoms. Numerical simulations based on the Monte Carlo wave-function method show that, despite collective events, it is possible to obtain a steady-state photon number distribution limited (''trapped'') to a certain domain of the photon number space. For the one-photon trapping condition this steady state approaches a Fock state in the limit of low cavity losses. [S1050-2947(97)05302-X]

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## I. INTRODUCTION

Generation of sub-Poissonian photon statistics [1], collapses and revivals of the atomic population inversion [2] and the so-called ''trapping states'' [3] are the main quantum features of the micromaser [4,5] which cannot be explained in the viewscope of a classical field theory. While the first two features have already been realized experimentally [6–8], the trapping states remain to be observed. These states have been largely studied theoretically and they constitute a proposal for Fock state generation [3]. Recently, a scheme for producing entangled Fock states and a ''Schrödinger cat'' [9] has been proposed which uses the concept of trapping states [10]. Also, squeezing in a two-mode micromaser pumped by three-level ($\Lambda$-type) atoms running Raman transitions into the cavity has been theoretically predicted under the trapping conditions [11].

Depending on the atom-field coupling constant and interaction time, for certain values of the photon number inside the cavity the atoms will have maximum probability of leaving the cavity in the excited state. If they were initially prepared in the excited state, this corresponds to a situation in which the atoms undergo a number of full Rabi oscillations inside the cavity. Thus, for fields initially in the vacuum state, the photon number distribution will be limited (''trapped'') to a certain domain of the Fock space. Moreover, in the lossless case this process leads to a $\delta$ distribution, i.e., a Fock state [3]. However, due to its marginal stability [1,12], the realization of a trapping state is very sensitive to external sources of noise like thermal fluctuations, stray electric fields inside the cavity (or resonator), atomic velocity dispertion, and collective events in which the field interacts with more than one atom at a time inside the cavity. Lately, special attention has been given to the problem of cooperative effects in the micromaser [13–16]. An analytical and numerical calculation of the trapping states lifetime can be found in Ref. [16].

Among the difficulties in the realization of a trapping state, collective events play a special role. Even though in standard micromaser experiments the atomic beam flux is very small, so that most events involve only one atom, there is a nonzero probability for a collective event of two ore more atoms to occur. After the interaction with a pair of atoms, for instance, the number of photons inside the cavity may be altered and the trapping condition violated. A single atom that follows this event will then have a nonzero probability of adding one more photon, so that a nontrapped field is built up. As demonstrated before [14], too many one-atom events are needed in order to recover the trapping photon number, so that another collective event is very likely to occur in the meantime.

In the present work we propose a scheme which overcomes the difficulty created by the collective events. It is based on the production of an atomic beam composed of two types of atoms with different transition frequencies. One set is prepared in the upper level of the masing transition, and the other in the lower level. With a suitable cavity tuning it is possible to obtain a situation in which both sets of atoms have the same effective Rabi frequency. Under the trapping condition the atoms develop an integer number of Rabi oscillations without changing the trapping photon number. When the trapping condition is violated by a collective event, for example, the excited atoms that follow will have a nonzero probability for emitting a photon but, at the same time, atoms in the lower level will have a nonzero probability for absorbing a photon. By adjusting the fluxes of the two sets of atoms one may obtain a stable trapping state as we will show below. In fact, the role played by the atoms prepared in the lower level is to reduce the time taken by the micromaser to reestablish the trapping condition so that it happens before another collective event takes place.

## II. MICROMASER DYNAMICS IN THE PRESENCE OF A MIXED ATOMIC BEAM

### A. Atom-field interaction

We now turn to a quantitative argument about the considerations above. Assuming that the atoms can be approximated by a two-level system, and adopting the usual electric dipole and rotating-wave approximations the atom-field interaction Hamiltonian in the rotating frame may be written as

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$$H(t)=\sum_i \hbar\kappa_i(a\sigma_i^+ + a^\dagger\sigma_i^-)-\hbar\Delta_i\sigma_i^+\sigma_i^- , \qquad (1)$$

where $\kappa_i$ and $\Delta_i$ are, respectively, the atom-field coupling constant and the detuning $(\omega_{cav}-\omega_i)$ corresponding to atom $i$, $a$ and $a^\dagger$ are the creation and annihilation operators for the cavity mode, and $\sigma_i^+$ and $\sigma_i^-$ are the Pauli spin flip matrices corresponding to atom $i$. Let us first consider one-atom events only so that the summation appearing in the Hamiltonian is irrelevant for the moment. In addition, let us suppose that the atomic beam is composed of two sets of atoms. One set is prepared in the upper level before entering the cavity and detuned by $\Delta_{emt}$ $[\Delta_{emt}=(\omega_{cav}-\omega_{emt})]$ from the cavity frequency. From now on we will refer to these atoms as *emitters*, where $\omega_{emt}$ is their atomic transition frequency. The other set is prepared in the lower level and detuned by $\Delta_{abs}$ $[\Delta_{abs}=(\omega_{cav}-\omega_{abs})]$. We will call them *absorbers*, and $\omega_{abs}$ is their atomic transition frequency. From the Hamiltonian above, the transition probability for both sets of atoms may be calculated in terms of the interaction time $t_{int}$ and the photon number $n$ inside the cavity. We then find the well-known results describing the Rabi oscillations

$$P^g_{emt}(n)=\frac{4\kappa^2_{emt}(n+1)}{\Delta^2_{emt}+4\kappa^2_{emt}(n+1)}\sin^2[\Omega_{emt}(n)t_{int}] \qquad (2)$$

and

$$P^e_{abs}(n)=\frac{4\kappa^2_{abs}n}{\Delta^2_{abs}+4\kappa^2_{abs}n}\sin^2[\Omega_{abs}(n)t_{int}], \qquad (3)$$

where $P^{g(e)}_{emt(abs)}(n)$ is the probability that an emitter (absorber) leaves the cavity in the lower (upper) level, that is, the emission (absorption) probability when there are $n$ photons inside the cavity. $\Omega_{emt}(n)$ and $\Omega_{abs}(n)$ are the $n$-photon Rabi frequencies for the corresponding sets of atoms, given by

$$\Omega_{emt}(n)=\left[\left(\frac{\Delta_{emt}}{2}\right)^2+\kappa^2_{emt}(n+1)\right]^{1/2} \qquad (4)$$

and

$$\Omega_{abs}(n)=\left[\left(\frac{\Delta_{abs}}{2}\right)^2+\kappa^2_{abs}n\right]^{1/2}. \qquad (5)$$

The factor $(n+1)$ in Eqs. (2) and (4) accounts for the spontaneous Rabi oscillation performed by an emitter in the vacuum field.

In the usual micromaser experiments [5] the atomic beam is prepared in the excited state and the cavity is tuned to resonance. In this case, for one-atom operation, it is predicted [3] that a trapped photon number distribution is obtained when the atom-field interaction time is such that the emission probability vanishes for some value $n_0$ of the photon number. This corresponds to a situation where the atoms develop an integer number of full Rabi oscillations before leaving the cavity. From the equations above we see that, in principle, one can conceive a situation in which both sets of atoms perform an integer number of Rabi oscillations for a given number of photons $n_0$ in the cavity and for a given

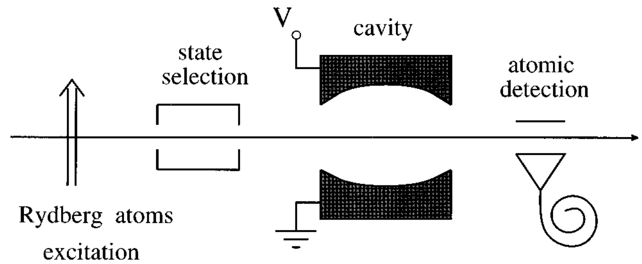


FIG. 1. A proposed setup for production of a mixed atomic beam composed by emitters and absorbers. The alternating between the two sets of atoms is achieved by periodically switching on and off the state selection and the dc electric field between the cavity mirrors in a synchronized way. The dc field provides the suitable tuning of the absorbers in order to cancel the absorption when the trapping condition is fulfilled.

interaction time. This situation is achieved, for example, when $\Omega_{emt}(n_0)t_{int}=\Omega_{abs}(n_0)t_{int}=q\pi$, with $q$ an integer. A simple and straightforward calculation shows that this condition can be fulfilled provided the interaction time is set to the suitable value and the cavity is tuned to the frequency

$$\omega_{cav}=\frac{\omega_{abs}+\omega_{emt}}{2}+\frac{2\kappa^2_{abs}n_0-2\kappa^2_{emt}(n_0+1)}{\omega_{abs}-\omega_{emt}}. \qquad (6)$$

We can see from Eqs. (2) and (3) that both $P^g_{emt}$ and $P^e_{abs}$ vanish when the trapping condition is satisfied. Furthermore, if an extra photon is added to the cavity field, so that the trapping condition is no longer fulfilled, both probabilities will have a nonzero value and the trapping condition may be reestablished due to the absorption of a photon by an absorber. As we will see in Sec. IV, in this case one can obtain a trapped photon number distribution despite the presence of collective events.

In what concerns the system parameters the considerations above are quite general and we may investigate their consequences in a simpler context. For example, let us set $\Delta_{emt}=0$ and $\kappa_{emt}=\kappa_{abs}\equiv\kappa$. In this case, Eq. (6) implies $\Delta_{abs}=\pm 2\kappa$. Since the Rabi frequency depends on the square of the detuning, the absorbers may be detuned either above or below the cavity frequency. The hypothesis of equal coupling constants is in general not verified if the two sets of atoms are from different atomic species since the electric dipole moments may be appreciably different. However, if both sets are from the same specie their coupling constants will be the same.

Let us consider, for instance, the situation sketched in Fig. 1. The atomic beam crosses a region where the atoms are excited to the Rydberg state corresponding to the upper masing level, which means that all atoms are initially prepared as emitters. The preparation of the absorbers may be achieved by means of a microwave $\pi$ pulse, represented in the figure by the state selection box, which flips the atoms to the lower masing level. This atomic flipping must be synchronized with the application of a constant electric field inside the cavity that will shift the masing levels via the Stark effect and provide the detuning of the atomic frequency for the absorbers. By switching the state selection and the dc field on and off periodically, one obtains the alternating between emitters and absorbers in the beam.

### B. Trapping state recovering time

The main feature behind the trapping state stabilization is the reduction of the number of one-atom events required for the micromaser to recover the trapping condition after its violation due to a collective event. In order to check this argument let us first consider one-atom events only. In the presence of a mixed atomic beam satisfying a Poissonian injection statistics, the micromaser dynamics is described by the following master equation:

$$\begin{aligned}\dot{\pi}_n =& f_{\mathrm{emt}} r[-P^g_{\mathrm{emt}}(n)\pi_n + P^g_{\mathrm{emt}}(n-1)\pi_{n-1}]\\ &+ f_{\mathrm{abs}} r[-P^e_{\mathrm{abs}}(n)\pi_n + P^e_{\mathrm{abs}}(n+1)\pi_{n+1}]\\ &+ \gamma_{\mathrm{cav}}(n_{\mathrm{th}}+1)[(n+1)\pi_{n+1} - n\pi_n]\\ &- \gamma_{\mathrm{cav}} n_{\mathrm{th}}[(n+1)\pi_n - n\pi_{n-1}],\end{aligned} \tag{7}$$

where $f_{\mathrm{emt}}$ ($f_{\mathrm{abs}} = 1 - f_{\mathrm{emt}}$) is the fraction of emitters (absorbers) in the beam, $r$ is the total atomic flux, $\gamma_{\mathrm{cav}}$ is the cavity decay rate, $n_{\mathrm{th}}$ is the average number of thermal photons in the mode (given by Planck's distribution), and $\pi_n \equiv \langle n|\rho|n\rangle$ is the density-matrix diagonal element in the Fock state basis. One can easily verify that in the absence of losses ($\gamma_{\mathrm{cav}}=0$) a Fock state $|n_0\rangle$, corresponding to a trapping condition $\Omega_{\mathrm{emt}}(n_0)t_{\mathrm{int}} = \Omega_{\mathrm{abs}}(n_0)t_{\mathrm{int}} = q\pi$, is a steady-state solution of the above master equation. In the presence of a zero temperature reservoir the steady-state photon number distribution will be limited to the $\{|0\rangle, \ldots, |n_0\rangle\}$ subspace. Let us consider, for example, the one photon trapping state ($\kappa t_{\mathrm{int}}\sqrt{2} = \pi$). In this case both the emitters and absorbers will develop a full Rabi oscillation when there is one photon in the resonator. The steady state $\pi_n$ will then vanish for $n \neq 0,1$. When a collective event takes place there is a finite probability for the atoms to add an extra photon to the cavity field so that the subsequent atoms will no longer develop a full Rabi oscillation. In the absence of the absorbers the cavity losses take a very long time to reestablish the trapping condition, so that another collective event is very likely to occur in the meantime [14]. When the absorbers are present in the beam they provide an extra loss mechanism which is triggered when the trapping condition is violated. This extra loss significantly reduces the time taken by the micromaser to reestablish the trapped photon number distribution, and at the same time are ineffective under the trapping condition due to the coherent nature of the atom-field interaction. In order to give a quantitative meaning to this argument we have numerically solved the master equation (7) taking the Fock state $|2\rangle$ as the initial condition for the cavity field. Of course, other initial conditions may be considered, but they do not add much physical insight. We then compute the time, in units of the average time interval $1/r$ between two consecutive atoms, taken by the photon number distribution to become trapped in the $\{|0\rangle,|1\rangle\}$ subspace. To do so we had to establish a numerical criterion for the trapping condition so that we actually calculated the time $t_{99}$ for which $\pi_0(t_{99}) + \pi_1(t_{99})$ reaches the value 0.99. In Fig. 2(a) we show the time $t_{99}$ in units of $1/r$ as a function of $f_{\mathrm{abs}}$. Under the usual micromaser condition ($f_{\mathrm{abs}}=0$), about 616 atoms are required for the system to recover the trapping condition. As $f_{\mathrm{abs}}$ increases, $t_{99}$ decreases very fast and after about 33 atoms the photon number distribution is already trapped when $f_{\mathrm{abs}} = f_{\mathrm{emt}} = 0.5$. The time evolution scale increases with the ratio between the cavity damping rate $\gamma_{\mathrm{cav}}$ and the atomic flux $r$. In Fig. 2(a) we assumed a rather low value ($N_{\mathrm{ex}} \equiv r/\gamma_{\mathrm{cav}} = 10$) in order to save computation time, but the result may be easily extended to higher values.

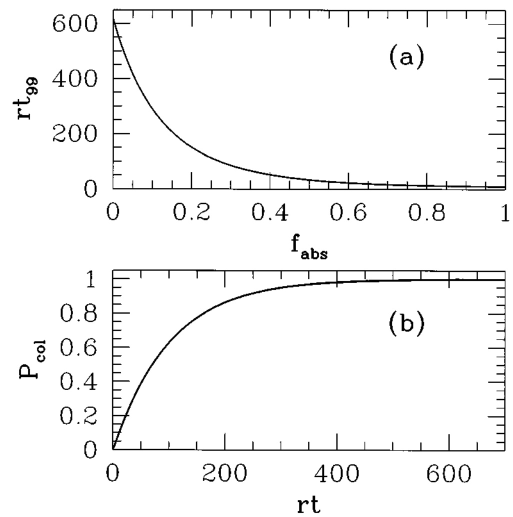


FIG. 2. In (a) the trapping state recovering time $t_{99}$ vs the fraction $f_{\mathrm{abs}}$ of absorbers in the atomic beam ($N_{\mathrm{ex}}=10$) is shown. In (b) the probability $P_{\mathrm{col}}$ that at least one collective event takes place during the time interval $t$ is given as a function of $t$ ($N_{\mathrm{at}}=0.01$). In both figures, the time is expressed in units of the average time interval between two consecutive atoms.

We may develop some intuition about the competition between the evolution toward the trapping condition and the collective events by calculating the probability $P_{\mathrm{col}}(t)$ that at least one collective event takes place during the time interval $t$. An approximate expression for $P_{\mathrm{col}}$ as a function of time is obtained in the Appendix and the result is

$$P_{\mathrm{col}}(t) = 1 - \frac{e^{-rt}}{p}(e^{prt} + p - 1), \tag{8}$$

where $p \equiv e^{-rt_{\mathrm{int}}}$ is the probability that the time interval between two consecutive atoms exceeds $t_{\mathrm{int}}$ for a beam obeying Poisson's distribution function. Notice that a relevant parameter in what concerns the collective events is the average number of atoms in the resonator $N_{\mathrm{at}} \equiv rt_{\mathrm{int}}$. In Fig. 2(b) we show the probability $P_{\mathrm{col}}(t)$ as a function of time for $N_{\mathrm{at}}=0.01$, which is a typical value in micromaser experiments [6]. For the time $t_{99}=616/r$, taken by the usual micromaser scheme to recover the trapping condition, the probability $P_{\mathrm{col}}$ exceeds 0.997, while for $t_{99}=33/r$, corresponding to $f_{\mathrm{abs}} = f_{\mathrm{emt}} = 0.5$, it is smaller than 0.28. Of course, higher values of $f_{\mathrm{abs}}$ may be considered, but in this case collective events involving absorbers will populate the vacuum state. In Sec. IV we will see that a trapped photon number distribution is obtained when $f_{\mathrm{abs}} = f_{\mathrm{emt}} = 0.5$.

### III. MONTE CARLO WAVE-FUNCTION APPROACH

In order to study the micromaser dynamics including many features of its operation, we have chosen a Monte Carlo wave-function (MCWF) approach [17] for simulating the master equation. This method has been successfully applied before to study collective effects in the micromaser [15,16], and its results are in very good agreement with the density-matrix approach. The MCWF simulations involve two steps [17]. In the first one, the Schrödinger equation is numerically integrated from $t$ to $t+\delta t$, using the second-order Runge-Kutta method with the effective non-Hermitian Hamiltonian

$$H_{\rm eff}=H-\frac{i\hbar}{2}\sum_m C_m^\dagger C_m, \tag{9}$$

where $H$ is the interaction Hamiltonian given by Eq. (1). The number of atoms, as well as their types (emitter or absorber), included in the Hamiltonian for each time interval $\delta t$ is determined by random choice: before the integration of the Schrödinger equation for each realization, the arrival times of the successive atoms are drafted according to the distribution for time intervals corresponding to a Poissonian pumping [$P(T)=re^{-rT}$, where $1/r$ is the average time interval between successive atoms]. In the present work, we consider up to two atoms inside the cavity, since three-atom events are very rare for the small atomic fluxes considered here. The operators $C_m$ are obtained from the master equation for the reduced density matrix $\rho$ corresponding to the subsystem atom-field mode (obtained by tracing out the reservoir variables for both the atoms and the field), written in Lindblad's form [18]:

$$\dot{\rho}=\frac{i}{\hbar}[\rho,H]+\sum_m [C_m\rho C_m^\dagger-\tfrac{1}{2}C_m^\dagger C_m\rho-\tfrac{1}{2}\rho C_m^\dagger C_m]. \tag{10}$$

The interaction of the field in the cavity with the reservoir is taken into account by the operators $C_1=[\gamma_{\rm cav}(1+n_{\rm th})]^{1/2}a$ and $C_2=[\gamma_{\rm cav}n_{\rm th}]^{1/2}a^\dagger$. Note that if we set $|\tilde{\Psi}(t+\delta t)\rangle=(1-iH_{\rm eff}\delta t/\hbar)|\psi(t)\rangle$, then since $H_{\rm eff}$ is non-Hermitian the state $|\tilde{\Psi}(t+\delta t)\rangle$ is not normalized, the square of its norm being given by

$$\langle\tilde{\Psi}(t+\delta t)|\tilde{\Psi}(t+\delta t)\rangle=1-\delta\mathcal{P}, \tag{11}$$

where

$$\delta\mathcal{P}=\delta t\frac{i}{\hbar}\langle\Psi(t)|H_{\rm eff}-H_{\rm eff}^\dagger|\Psi(t)\rangle=\sum_m \delta p_m, \tag{12}$$

with

$$\delta p_m=\delta t\langle\Psi(t)|C_m^\dagger C_m|\Psi(t)\rangle. \tag{13}$$

The quantity $\delta\mathcal{P}$ is the probability that the mode exchanges a photon with the reservoir between $t$ and $t+\delta t$, while $\delta p_m$ is the probability that the cavity field loses (if $m=1$) or gains (if $m=2$) a photon during the same time interval.

In the second step, the subsystem is subjected to quantum jumps [17] in each interval $\delta t$, according to the probability $\delta\mathcal{P}$. The randomness of this jump is mimicked by the choice of a pseudorandom number [19] $\epsilon$ uniformly distributed between 0 and 1. If $\epsilon>\delta\mathcal{P}$, there is no jump, and we have only to normalize the wave function $|\tilde{\Psi}(t+\delta t)\rangle$, since the time evolution with $H_{\rm eff}$ is not unitary: $|\Psi(t+\delta t)\rangle=|\tilde{\Psi}(t+\delta t)\rangle/\sqrt{1-\delta\mathcal{P}}$. If $\epsilon<\delta\mathcal{P}$, a quantum jump occurs between $t$ and $t+\delta t$. The wave function is projected according to $|\Psi(t+\delta t)\rangle=C_m|\Psi(t)\rangle/(\delta p_m/\delta t)^{1/2}$. The operator $C_m$ to be used in this equation is chosen according to the probability $\delta p_m/\delta\mathcal{P}$. This procedure is repeated $t_{\rm max}/\delta t$ times from $t=0$ to $t=t_{\rm max}$. The expectation value of any operator may be calculated for a single realization at each time interval $\delta t$, while the mean value over an ensemble is obtained by making an average over many realizations. Special attention must be paid to the value of $\delta t$. It must be small enough because $\delta\mathcal{P}$ in Eq. (11) was calculated up to first order in $\delta t$ and also because at most one quantum jump should occur in this interval. Also, it should not be too small, because of the limited precision of the computer-generated pseudorandom numbers $\epsilon$, and also because of the Markovian approximation implicit in the effective Hamiltonian, which implies that $\delta t$ should be larger than the reservoir correlation time (of the order of one optical period). In the following simulations $\delta t$ was chosen so that $\delta\mathcal{P}$ stays in the range from $10^{-3}$ to $10^{-5}$.

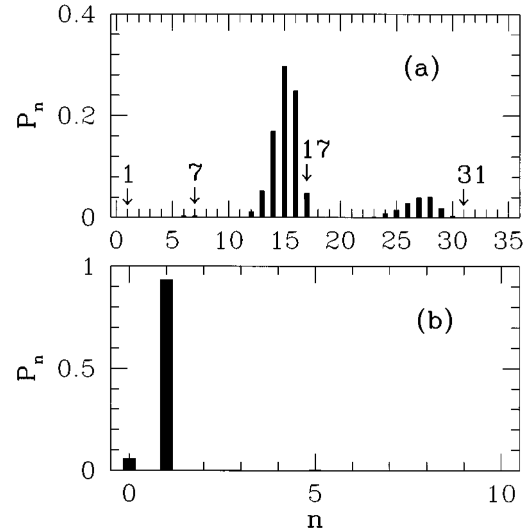


FIG. 3. Steady-state photon number distribution for $N_{\rm ex}=50$, with (a) $f_{\rm abs}=0$ and (b) $f_{\rm abs}=0.5$. The collective events build up a nontrapped photon number distribution in the first case while a trapped steady state is obtained when one-half of the atoms in the beam are absorbers. In the last case the field approaches a one-photon Fock state with decreasing losses.

### IV. NUMERICAL RESULTS

We now present the results obtained with the method described above. Our simulations include collective events involving two emitters, two absorbers, or one of each. We have taken $\kappa_{\rm emt}=\kappa_{\rm abs}\equiv\kappa$, $\Delta_{\rm emt}=0$, and $\Delta_{\rm abs}=+2\kappa$.

Figure 3 shows the steady-state photon number distribution $\pi_n$ for a micromaser operating in the one photon trapping state condition ($\kappa t_{\rm int}\sqrt{2}=\pi$) for $f_{\rm abs}=0$ and 0.5. One can easily see that in the absence of the absorbers [see Fig. 3(a)] the distribution leaks beyond the trapping photon number $n=1$. The situation is quite different when half of the atoms in the beam are absorbers [see Fig. 3(b)], the distribution remains trapped despite the collective events. When an extra photon is added to the cavity field by a collective event an additional loss mechanism is triggered due to the presence of the absorbers, and the distribution is pulled back to the trapping condition. The peak at $n=1$ is approximately 0.94. Large values of $f_{\rm abs}$ will tend to destroy the trapped photon and increase the population of the vacuum state due to the increment of the number of collective events involving absorbers.

We have checked that the differences increase with $N_{\rm ex}$, i.e., with decreasing losses. Once the extra photons left by collective events live longer, the distribution becomes highly spread out when $f_{\rm abs}=0$. On the other hand, for $f_{\rm abs}=0.5$ the distribution remains trapped and the peak at $n=1$ increases since extra photons are removed by the absorbers and the trapped photon lives longer. A one-photon Fock state preparation using the principle of trapping states requires the complete absence of losses. While in the usual micromaser setup the effects of the collective events are stressed in the low loss regime, with the scheme proposed here it is possible to satisfy the compromise between low losses and trapping condition, despite the action of collective events.

## V. CONCLUSIONS

We have presented an interesting feature of micromaser operation related to the interaction of a mixed atomic beam with the intracavity field. The scheme presented was theoretically demonstrated to produce a stable trapping state, which constitutes an additional tool for its implementation. We have shown that trapped photon number distributions may be obtained despite the action of collective events. This is achieved by the presence of the absorbers in the atomic beam. The absorbers are ineffective under the trapping condition and constitute an extra loss mechanism when this condition is violated. While in the standard micromaser scheme the reduction of losses makes the effects of collective events even more important, with the mixed beam setup it is possible to approach the one-photon Fock state by reducing losses despite the action of collective events.

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## APPENDIX

In the following we calculate the probability $P_{\rm col}(t)$ that at least one collective event takes place in the time interval $(0,t)$. We start by assuming a Poissonian atomic beam with flux $r$ so that the number $N$ of atoms that cross the cavity during the time interval $t$ obeys the following distribution:

$$P(N)=e^{-\langle N\rangle}\frac{\langle N\rangle^N}{N!}, \tag{A1}$$

where $\langle N\rangle=rt$. As a consequence the probability distribution for the time interval $T$ between two consecutive atoms is

$$P(T)=re^{-rT}. \tag{A2}$$

The probability $p\equiv e^{-rt_{\rm int}}$ that two consecutive atoms are separated by a time interval larger than the interaction time $t_{\rm int}$ is obtained by direct integration of Eq. (A2) from $t_{\rm int}$ to infinity.

If $N$ atoms have crossed the cavity entrance during the time $t$ (considering the entrance of the first atom of the pair at time zero), the condition that no collective events occur requires that all consecutive atoms are separated by a time interval larger than $t_{\rm int}$. Of course, in order to be consistent with the fact that $N$ atoms have crossed the cavity during $t$ the sum of the $N-1$ time intervals between the consecutive atoms must be smaller than $t$. For the values of $N$ such that $(N-1)t_{\rm int}>t$, it will be impossible not to have a collective event. On the other hand, if $N=0,1$ there will certainly be no collective events. Let us define the probability $p_N(t_{\rm int},t)$ that no collective events have occurred during the time interval $t$, given that $N$ atoms have entered the cavity during this time. Taking the considerations above, and with the help of Eq. (A2), we may write it as

$$p_N(t_{\rm int},t)=\begin{cases}1, & N=0,1\\ A, & 2\leqslant N\leqslant \dfrac{t}{t_{\rm int}}+1\\ 0, & N>\dfrac{t}{t_{\rm int}}+1,\end{cases} \tag{A3}$$

where $A=r^{N-1}\int_{\mathcal{V}(t_{\rm int},t)}dt_1\cdots dt_{N-1}e^{-r\Sigma_{i=1}^{N-1}t_i}$, $\mathcal{V}(t_{\rm int},t)$ is the hypervolume limited by the hyperplanes $t_1=t_{\rm int},\ldots,t_{N-1}=t_{\rm int}$ and the hyperplane $S_{N-1}$ over which $t_1+\cdots+t_{N-1}=t$. For the values of $N$ such that $(N-1)t_{\rm int}\ll t$ the integrand in $p_N(t_{\rm int},t)$ is negligible over and beyond $S_{N-1}$. In this case we may extend the integral over $\mathcal{V}(t_{\rm int},t)$ to infinity, so that $p_N(t_{\rm int},t)\approx p^{N-1}$.

The probability that no collective events occur during $t$, regardless of the number of atoms that crossed the cavity, is the sum over $N$ of $p_N(t_{\rm int},t)$ weighed by $P(N)$. For a diluted atomic beam such that $rt_{\rm int}\ll 1$ we have $\langle N\rangle\ll t/t_{\rm int}$, so that the probability $P(N)$ associated with the values of $N$ for which $(N-1)t_{\rm int}\gtrsim t$ is very small. Thus the probability that at least one collective event takes place during $t$ is given by

$$P_{\rm col}(t)=1-\sum_{N=0}^{\infty}P(N)p_N(t_{\rm int},t)$$

$$\approx 1-e^{-\langle N\rangle}\left[1+\sum_{N=1}^{\infty}\frac{\langle N\rangle^N}{N!}p^{N-1}\right]. \tag{A4}$$

A straightforward algebra yields

$$P_{\rm col}(t)\approx 1-\frac{e^{-rt}}{p}(e^{prt}+p-1), \tag{A5}$$

which is the formula presented in the text.

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