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FACULDADE DE CIÊNCIAS ECONÔMICAS
DEPARTAMENTO DE ECONOMIA E RELAÇÕES INTERNACIONAIS

FELIPE FREITAS CAMBOIM

COMPARATIVE ANALYSIS OF ASSET PRICES: A SIMULATION OF THE LUCAS ASSET PRICING MODEL TO BRAZILIAN AND U.S. MARKETS

Porto Alegre
2024

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Trabalho de conclusão submetido ao Curso de Graduação em Ciências Econômicas da Faculdade de Ciências Econômicas da UFRGS, como requisito parcial para obtenção do título Bacharel em Economia.

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BANCA EXAMINADORA:

Prof. Dr. Nelson Seixas dos Santos - Orientador
UFRGS

Prof. Flávio Tosi Feijó
UFRGS

Prof. Ronald Otto Hillbrecht
UFRGS

RESUMO

Esta pesquisa explora a dinâmica dos mercados financeiros, enfatizando a interação entre preços de ativos e consumo na economia em evolução do Brasil. Integrando teorias clássicas e modernas de crescimento econômico em uma revisão de literatura com o Modelo de Precificação de Ativos de Lucas, examina a influência de tendências macroeconômicas sobre comportamentos de mercado. O estudo contrasta Brasil e EUA, destacando como diferentes estruturas econômicas impactam os mercados financeiros. Os resultados mostram que o poder preditivo do modelo é limitado pelas complexidades do mundo real. Especificamente no Brasil, fatores externos como taxas de câmbio influenciam significativamente o comportamento do mercado, o que não é totalmente encapsulado pelo modelo. Em contraste, nos Estados Unidos, o modelo se alinha mais de perto com as tendências de mercado, sugerindo uma maior utilidade preditiva em ambientes econômicos mais estáveis. Destaca-se a importância de combinar modelos teóricos com dados empíricos para um entendimento abrangente das dinâmicas econômicas em vários contextos, assim como ajudar a formular políticas efetivas.

Palavras-chave: Mercados Financeiros. Preços de Ativos. Consumo. Teorias de Crescimento Econômico. Modelo de Lucas.

ABSTRACT

This research explores financial market dynamics, emphasizing the interplay between asset prices and consumption in Brazil's emerging economy and also the United States' developed economy. Integrating classical and modern economic growth theories in a literature review with Lucas's Asset Pricing Model, it examines the influence of macroeconomic trends on market behaviors. The study contrasts Brazil and the USA, highlighting how different economic structures impact financial markets. Results show that the model's predictive power is limited by real-world complexities. Particularly in Brazil, external factors such as exchange rates significantly influence market behavior, which is not fully encapsulated by the model. Conversely, in the United States, the model aligns more closely with market trends, suggesting a greater predictive utility in more stable economic environments. These outcomes highlight the significance of combining theoretical models with empirical data for a comprehensive understanding of economic dynamics in various contexts, as well to help formulate effective policies.

Keywords: Financial Markets. Asset Prices. Consumption. Economic Growth Theories. Lucas Model.

SUMÁRIO

1 INTRODUÇÃO

The exploration of financial market dynamics in this research is deeply influenced by seminal works in the field. The contribution of Solow (1956) lays the foundation for macroeconomic growth theory, emphasizing the role of capital accumulation, labor growth, and technological advancements in driving economic development. Rubinstein (1976), meanwhile, approaches about decision-making under uncertainty, exploring the concept of risk aversion and its implications for financial markets. The Lucas Asset Pricing Model (Lucas, 1978) is a cornerstone, which integrates rational expectations with intertemporal consumption choices. This model is pivotal in understanding how asset prices are shaped by consumer behavior, particularly in markets undergoing rapid changes like Brazil's. These works collectively offer a comprehensive view of the forces shaping financial markets, from individual decision-making processes to broader economic growth patterns. They guide our investigation into the nuanced interplay between economic growth, consumption patterns, and asset pricing, especially in emerging economies where these dynamics are most pronounced.

This research go through the relation of asset prices and consumption in financial markets, focusing on Brazil's evolving economy. We begin by examining how macroeconomic growth theories, particularly those centered on capital accumulation and technological advancements, shape financial markets. These theories provide a foundational understanding of economic development and their implications on asset pricing.

We then explore how modern economic theories, emphasizing internal drivers like innovation and human capital, contribute to our understanding of financial market dynamics. These theories highlight the significance of internal economic policies and institutional settings in shaping growth and market behaviors.

The study also investigates the role of individual decision-making in financial markets, particularly in the context of risk assessment and consumption choices. This aspect is crucial for understanding how microeconomic behaviors, influenced by broader economic trends, impact asset pricing and market equilibrium.

Finally, by applying these theories to both Brazilian and American markets, the research aims to draw comparative insights into how different economic structures and growth patterns influence financial markets. This comparative approach allows for a deeper understanding of diverse financial systems and their responses to economic changes.

2 ANÁLISE COMPARATIVA DOS PREÇOS DE ATIVOS: UMA SIMULAÇÃO DO MODELO DE PRECIFICAÇÃO DE ATIVOS DE LUCAS PARA OS MERCADOS BRASILEIROS E DOS EUA

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Keywords: Financial Markets. Asset Prices. Consumption. Economic Growth Theories. Lucas Model.

2.1 INTRODUCTION

In this paper, we delve into the dynamic world of financial markets, focusing on the relationship between asset prices and consumption patterns. Our study is particularly relevant in the context of Brazil, a rapidly developing economy that exemplifies the complexities of emerging financial markets. We aim to dissect these complexities using theoretical models and simulations, moving beyond mere data analysis to gain a deeper understanding of the underlying mechanisms.

The review begins by addressing classical and modern economic growth theories, highlighting their significance in financial market analysis. We explore foundational theories, starting from risk allocation and measurement, and gradually move towards more sophisticated concepts like rational expectations and their role in asset pricing models.

Central to our discussion is the Lucas Asset Pricing Model, a key framework in macroeconomic finance that integrates rational decision-making based on available information, risk aversion, and economic and political shocks. This model is instrumental in understanding the fluctuations in asset prices as a response to changing consumer behaviors, especially in the context of Brazil's rapidly changing economy.

By applying the Lucas model to both Brazil and the USA, we provide a comparative analysis of how different economic environments influence financial market dynamics. This comparative approach allows us to draw insights into the unique characteristics of each market, enhancing our understanding of global financial behavior.

2.2 FROM LONG RUN TO SHORT RUN: GROWTH AND TRANSITION DYNAMICS

This literature review intends to connect the dots between classical and contemporary economic growth theories and their application in the realm of financial market analysis. We commence with an examination of the fundamental concepts of risk allocation and measurement, progressing towards an exploration of rational expectations theories and their implications in asset pricing models.

2.2.1 From Solow to modern growth theories

The first formal model to explain post-war economic growth consistent with the stability of economies observed in data is due to Solow (1956). Solow states labor growth and capital accumulation on the basis of technological advancements are the key drivers of economic growth. However, in his assessement, he considers technological progress as an external factor, not directly influenced by the economy itself, which leaves room for modern growth theories, often referred to as endogenous, pivot from this standpoint.

These theories, as the one presented in Romer (1990), propose that economic growth is fueled by factors internal to the economy. Emphasizing innovation, knowledge spillovers, and human capital development, these models argue that such elements are not only vital for sustained economic growth but are also significantly shaped by internal economic policies and institutional frameworks. This shift marks a critical development in understanding the mechanisms driving economic growth, linking internal dynamics to long-term economic outcomes.

Particularly, the endogenous models emphasizing innovation and human capital, interplay with the dynamics of financial markets. This connection is explained in the Lucas Asset Pricing Model (Lucas, 1978), where consumption decisions of a representative agent determines the

price of capital stocks. But, to explain that, we need an excursion on the theory of finance and that is what we going to do now.

2.2.2 Agents' utility and efficient risk allocation

The concept of expected utility, introduced in Bernoulli (1954), which was translated the original work published in 1738 in latin language, marked a significant advancement in how decision-making under uncertainty is modeled. This concept is foundational in analyzing how investors' preferences for utility shape their decision-making, especially when facing uncertain future outcomes. Central to financial theory is understanding the relationship between agents' utility and the efficient allocation of risk. Markowitz's groundbreaking work (Markowitz, 1952) laid the base for modern portfolio theory. He underscored the importance of considering not only asset returns but also their interrelationships, particularly covariances. He advocated for portfolio diversification as a means to reduce overall risk, introducing the concept of the "efficient frontier" to depict the optimal balance between risk and return.

Markowitz's contributions further extended to quantifying diversification by differentiating between "non-systematic risk" (diversifiable) and "systematic risk" (market risk). His introduction of the "minimum variance" formula for optimal asset allocation in a portfolio, based on their covariances, significantly influenced the development of modern risk management and portfolio selection strategies. His methodologies have become integral in constructing diversified and efficient portfolios in a variety of financial contexts.

Translated from the original paper in 1953, Arrow (1964) expanded upon risk-related discussions in finance and explored achieving optimal resource allocation in competitive environments by examining agents' risk behavior. He posits that a general equilibrium in financial markets under uncertainty is attainable, particularly when agents are risk-averse.

Furthering this framework, Sharpe developed the Capital Asset Pricing Model (CAPM) (Sharpe, 1964), which has played a crucial role in elucidating the relationship between investors' risk perceptions and asset prices. CAPM integrates rational expectations into a model that correlates asset prices with their associated risks, thereby offering a nuanced understanding of asset price formation in financial markets.

Rubinstein (1976) built on the assumption that investors display risk aversion, aiming to establish the present value of a series of cash flows obtained at multiple future dates consistent with agents' risk-averse behavior and the hypothesis of market equilibrium. The significance of this discussion lies in the need to evaluate uncertain income flows over time, rather than just in a single period. Rubinstein developed a formula to assess these future cash flows, taking into account the uncertainties of future incomes, market financial equilibrium, and investors' rational behavior. His formula is used to derive sufficient restrictions on economic variables to ensure that the market portfolio return rate adheres to the conditions of a random walk. This methodology can be applied to the pricing of securities, investment return rates, capital allocation projects, and other assets.

Focusing on the optimization of asset combinations to enhance expected returns at a given risk level, Elton *et. al* (2014) further extends the scope of portfolio theory. Their work demonstrates how investors can incorporate utility expectations within broader strategies for asset allocation, taking into account diversification and risk exposure.

2.2.3 Rational expectations and asset pricing models

The theory of rational expectations, a cornerstone of modern financial economics, saw significant advancements through the works of Muth (1961) and Mossin (1966). This theory contends that economic agents form their expectations based on all accessible information, aligning these expectations with actual market conditions. This concept has been pivotal in comprehending how economic agents assimilate information and its subsequent impact on their financial decision-making.

Fama's exploration of the efficient market hypothesis (Fama, 1970) furthered these concepts. His research illuminates both the usual efficiency of markets in reflecting all accessible information and the occurrence of short-term anomalies and inefficiencies. This work underscored the intricate nature of information processing in financial markets.

Lucas's Asset Pricing Model (Lucas, 1978) significantly advanced asset pricing theory by focusing on determining the equilibrium price of assets. This model considers agents' rational expectations and their decisions regarding intertemporal consumption, as well as productivity variations. Lucas emphasized the importance of macroeconomic variables, like consumption patterns, in financial market analysis.

Quantecon (2024) offers a unique platform for understanding and analyzing economic models, including the Lucas Asset Pricing Model. While Quantecon does not directly simulate Lucas's model, it provides valuable resources, including theoretical explanations and coding tools. This makes it an essential resource for researchers and students seeking to explore Lucas's model and understand the complex interplay between rational expectations, consumption choices, and market equilibrium. Quantecon thus stands as a bridge between theoretical economic concepts and practical analytical tools, contributing significantly to the field of financial economics.

2.3 LUCAS ASSET PRICING MODEL

The Lucas Asset Pricing Model (Lucas, 1978) builds upon these concepts by integrating the individual decision-making processes concerning consumption and investment. It offers a nuanced view of how agents' utility preferences, shaped by their expectations of future economic conditions, directly influence market dynamics. This model effectively bridges the gap between macroeconomic growth theories and microeconomic behavior, highlighting the importance of individual actions in the broader economic landscape.

The model goes into the dynamics of a pure exchange economy, characterized by a singular, representative consumer or household. This framework, presented and adapted from Quantecon (2024) simplifies the computation of competitive equilibrium prices by assuming that all endowments are given and that consumers share identical resources and preferences, thereby nullifying the need for trade. Central to the model is a "productive unit" or "Lucas tree," a metaphor for an asset that generates a sequence of consumption goods, denoted as $\{y_t\}_{t=0}^{\infty}$. This sequence represents the economy's consumption endowment, assumed to follow a Markov process described by $y_{t+1} = (y_t, \xi_{t+1})$, where $\{\xi_t\}$ is an independently and identically distributed (IID) shock with a known distribution ϕ, ensuring $y_t \geq 0$. The representative consumer's preferences are expressed as:

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{2.1}$$

Here, $\beta \in (0, 1)$ represents the discount factor, derived from the marginal utility of consumption, indicating the consumer's preference for current over future consumption.

The utility function, u, expressed in the CRRA (Constant Relative Risk Aversion) format, is strictly increasing, concave, and continuously differentiable, signifying diminishing marginal utility from consumption. It is essential for capturing the consumer's valuation of consumption across different times. The function is given by $u(c_t)$, where c_t represents consumption at time t. The CRRA utility function is typically formulated as:

$$u(c_t) = c_t^{1-\gamma}/(1-\gamma) \tag{2.2}$$

In this equation, γ is the coefficient of relative risk aversion, quantifying the consumer's aversion to fluctuations in consumption. In Lucas's theoretical model, the representative consumer maximizes utility by balancing current consumption against expected future utility. This decision-making process, influenced by forecasts of future economic conditions, is constrained by a budget linking current consumption, asset holdings, and income.

The crux of the Lucas Model is to ascertain the fair price for a claim on the consumption endowment. An ex-dividend claim implies the buyer acquires future dividends, not the current period's yield, by paying a price p_t today. The consumer's challenge is to optimize utility by deciding on the share π_t of the claim to hold, subject to their budget constraint:

$$c_t + \pi_{t+1} p_t \leq \pi_t y_t + \pi_t p_t \tag{2.3}$$

This sets the stage for a dynamic programming problem, with the consumer's behavior and the equilibrium prices being interdependent.

The equilibrium price function emerges from the first-order condition of the consumer's optimization problem, yielding a consumption-based asset pricing equation:

$$p(y) = \beta \int \frac{u'[G(y,z)]}{u'(y)} [G(y,z) + p(G(y,z))] \phi(dz) \tag{2.4}$$

In a sequential notation:

$$p_t = \mathbb{E}_t \left[\beta \frac{u'(c_{t+1})}{u'(c_t)} (y_{t+1} + p_{t+1}) \right] \tag{2.5}$$

Here, p_t represents the asset price at time t, $\mathbb{E}_t$ denotes the expectation based on information available at t, β is the fixed intertemporal discount factor, which represents the agents' willingness to value the present more than the future, and y_{t+1} is the consumption endowment at time $t + 1$. The consumption path in the Lucas model is assumed to follow a Markov process, reflecting the random nature of economic shocks and their impact on consumption choices.

Lucas's model portrays agents as rational entities who make informed decisions based on rational expectations. They face complex choices regarding consumption and investment, with the aim of maximizing utility. Market equilibrium is attained when these choices align with prevailing asset prices, indicating that asset price fluctuations mirror changes in available information and agents' future expectations.

While Lucas (1978) presented the theoretical aspects, the QuantEcon (2024) platform provides a detailed mathematical formulation of the model, offering practical applications of Lucas's theoretical concepts. This mathematical representation facilitates a deeper comprehension of the model's real-world economic implications.

2.3.1 Applying the model to Brazil and United States' contexts

The application of Robert Lucas's Asset Pricing Model to both the Brazilian and United States contexts provides a valuable perspective in understanding the interactions between asset prices and consumption in varying economic environments.

Lucas's model (Lucas, 1978) sets the groundwork for examining how asset prices and consumption patterns are deeply interconnected. This model acknowledges the significance of investors' intertemporal preferences, highlighting that consumption and investment decisions are shaped by long-term expectations. Adapting this model to accommodate the distinct economic, political, and social intricacies of Brazil and the United States is a crucial step in this exploration.

In Brazil, a nation characterized by its dynamic economic landscape and unique market conditions, applying the model involves careful consideration of local factors. These include varying interest rates, inflation trends, and specific governmental policies that influence economic behavior. Similarly, in the United States, with its own set of economic indicators and market dynamics, the model must be adjusted to reflect these differing conditions. This comparative approach allows for a broader understanding of how asset prices and consumption patterns interact under different economic systems.

The use of advanced analytical tools and economic platforms, such as Quantecon (2024), facilitates this complex analysis. Although Quantecon does not directly simulate Lucas's model, it offers theoretical insights and coding resources that are instrumental in studying these economic dynamics. By leveraging such tools, researchers can go through the variations of the model, examining the influence of macroeconomic factors on asset prices and consumption in both Brazil and the United States.

This comparative study is particularly insightful, as it not only sheds light on the individual economic characteristics of each country but also reveals broader patterns and principles that govern asset price and consumption dynamics globally. Through this analysis, researchers can draw more comprehensive conclusions about the interplay between economic variables, enhancing our understanding of global economic behavior and decision-making processes.

2.4 DATA AND METHODS

In this section, we present the data and methodologies used to investigate the economic behaviors of Brazil and the United States through the Lucas Asset Pricing Model. Our approach utilizes comprehensive GDP data from reliable sources, combined with theoretical model simulations. This strategy is aimed at dissecting the interactions between economic variables and their impact on market dynamics, shedding light on the contrasting economic environments of a developing and a developed country.

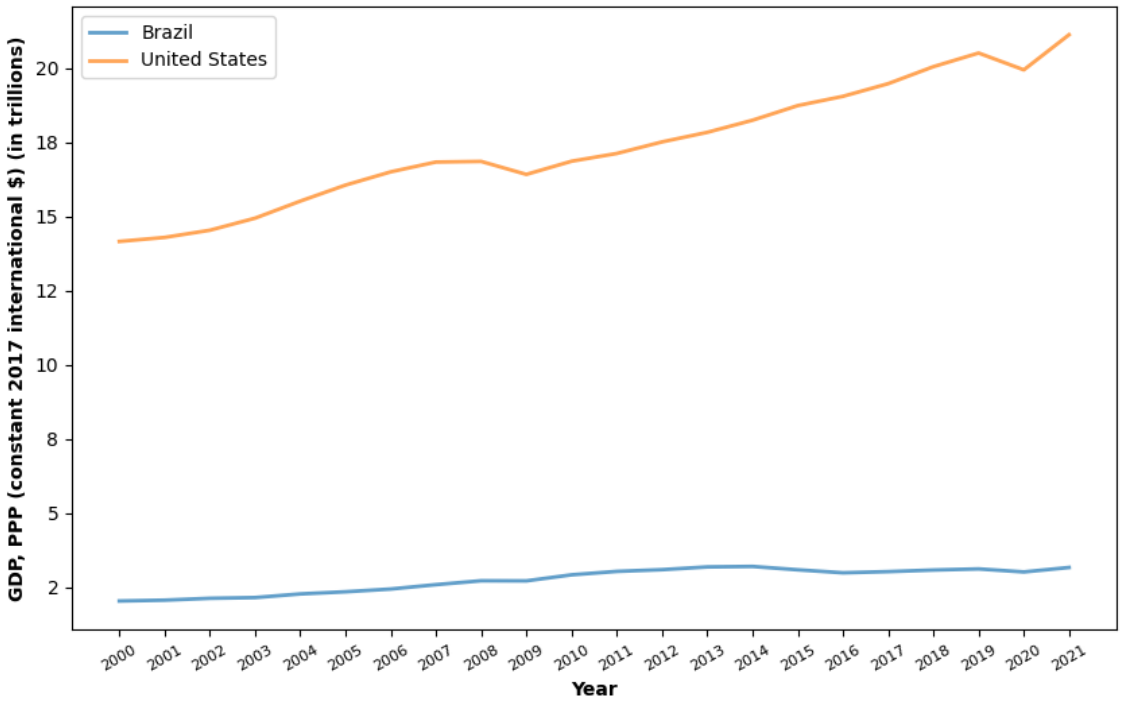
2.4.1 Data

The raw data were extracted from the World Bank (2024a) database, focusing on GDP figures adjusted for Purchasing Power Parity (PPP) in constant 2017 US dollars, spanning from 2000 to 2021. This approach involved transforming the GDP values into their logarithmic form to linearize the growth rates, thereby facilitating a more nuanced interpretation of economic dynamics. Subsequently, the annual percentage change was calculated by taking the difference between the logarithmic values of consecutive years. This transformation process not only

standardizes the data for comparative analysis but also underscores the relative growth and contraction phases within and across the economies of the United States and Brazil. This dataset serves as the foundation for the examination, which goes into the economic trajectories and resilience of these nations through the lens of their annual growth rates, revealing patterns of stability, volatility, and recovery in the face of global economic challenges.

For the United States, a pattern of steady growth is punctuated by significant downturns in 2009 and 2020, correlating with the global financial crisis and the COVID-19 pandemic, respectively. These years stand out with negative growth rates, marking periods of economic contraction. However, the resilience of the US economy is evident in its rapid recovery post-recession, particularly in 2021, when it experienced a remarkable upswing of 0.19%, indicating a strong rebound from the pandemic-induced downturn. The United States exhibits an average annual growth rate of approximately 0.06%, with a standard deviation of also 0.06%, indicating a relatively stable yet varied economic expansion over the period. In contrast, Brazil's economic journey through these years is marked with periods of vigorous expansion followed by sharp contractions. The years 2004, 2007, and 2010 are notable for significant growth. However, the Brazilian economy faced deep recessions, particularly during 2015-2016, and again in 2020, the latter due to the global impact of the COVID-19 pandemic. These downturns were followed by periods of recovery, with 2021 showing a substantial growth rate of 0.17%, although not as pronounced as the rebound observed in the US economy. Brazil's mean growth rate stands at 0.07%, suggesting a marginally higher average growth rate compared to the United States. However, Brazil also shows a higher standard deviation of 0.10%, reflecting greater volatility and fluctuations in its economic growth.

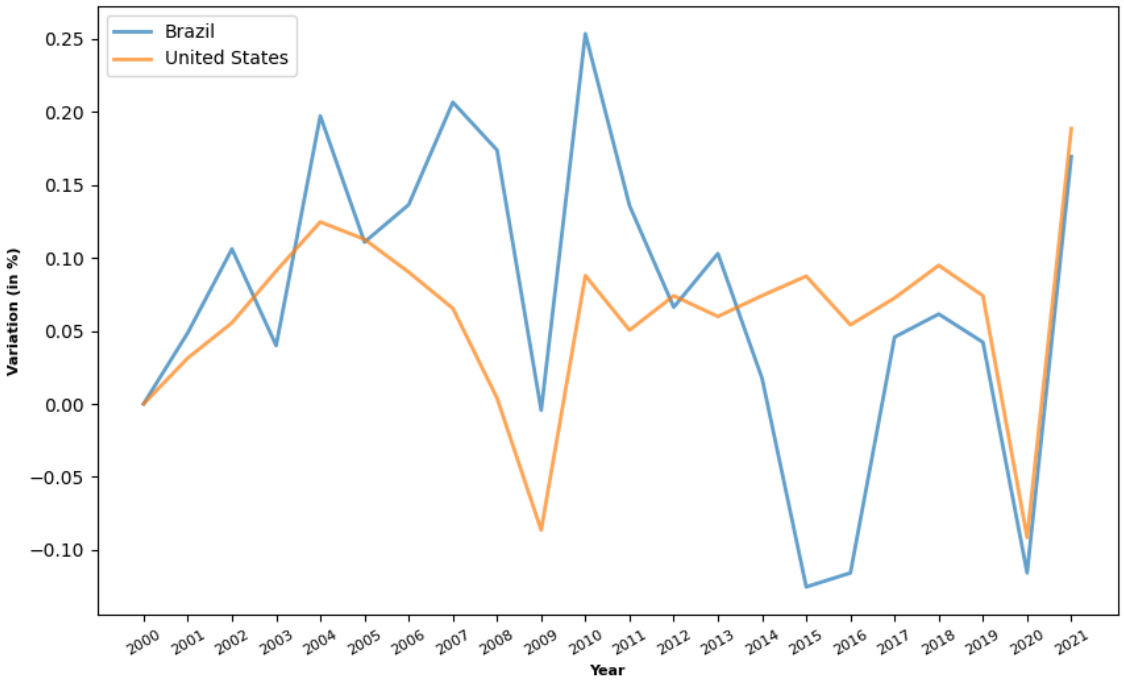
Figure 1 - GDP (PPP) in constant 2017 international $ - Brazil and United States



Source: Data from World Bank (2024a). Elaborated by the author (2024).

The PPP-adjusted household consumption data (also includes the expenditures of nonprofit institutions serving households) for Brazil and the United States (World Bank, 2024b), covering the period from 2000 to 2021, underwent a logarithmic transformation before the annual

Figure 2 - Log GDP (PPP) Variations (Annual %) - Brazil and United States



Source: Data from World Bank (2024a). Elaborated by the author (2024).

Table 1 - Log GDP (PPP) Variations - Descriptive Statistics

Statistic	Brazil	United States
Mean	0.070599	0.059762
Standard Deviation	0.103565	0.062116
Minimum	-0.125387	-0.091571
Maximum	0.253549	0.188592

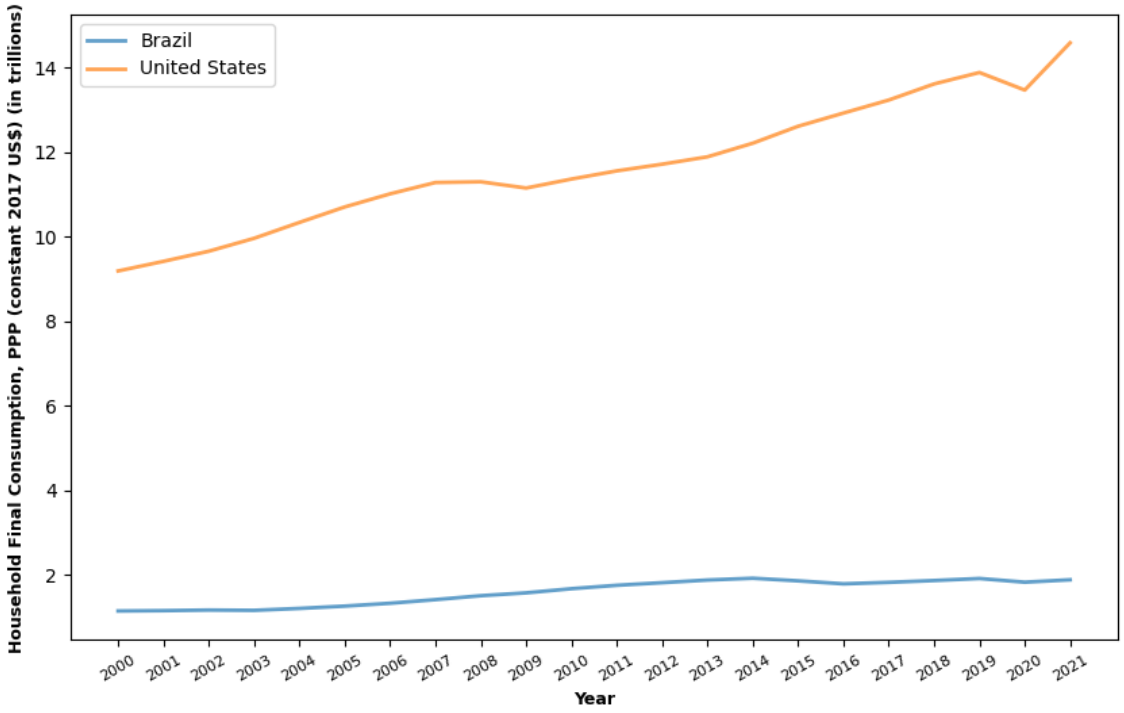
Source: Elaborated by the author (2024).

percentage changes were calculated.

For Brazil, the early 2000s show a gradual increase in consumption growth, peaking in the late 2000s, particularly in 2008 with a 0.23% increase, reflecting a period of robust economic expansion and increased consumer confidence. However, the data also reveal periods of contraction, most significantly in 2015 and 2016, where consumption decreased by 0.12% and 0.14%, respectively. The subsequent recovery in 2017 and continued growth until 2019 highlight the resilience and variability of consumer behavior in Brazil, before facing another significant downturn in 2020 with a decrease of 0.16%, followed by a recovery in 2021. In contrast, the United States exhibits a more consistent pattern of consumption growth over the same period, starting with modest growth in the early 2000s and experiencing a significant increase from 2003 onwards, with the exception of a downturn in 2009 reflecting the impact of the global financial crisis on consumer spending. The growth in consumption is particularly strong in the mid-2000s and again in 2021, where there is an extraordinary increase of 0.26%, reflecting the rebound from the COVID-19 pandemic's impact on consumption in 2020. The US data generally show less volatility compared to Brazil, indicating a more stable consumer spending environment, albeit not without its own periods of contraction and recovery.

For analyzing the price data of Brazil and the United States, specific Exchange-Traded Funds (ETFs) were selected to represent the performance of each country's stock market. The

Figure 3 - Household Consumption (PPP) in constant 2017 international $ - Brazil and United States



Source: Data from World Bank (2024b). Elaborated by the author (2024).

Table 2 - Log Household Consumption (PPP) Variations - Descriptive Statistics

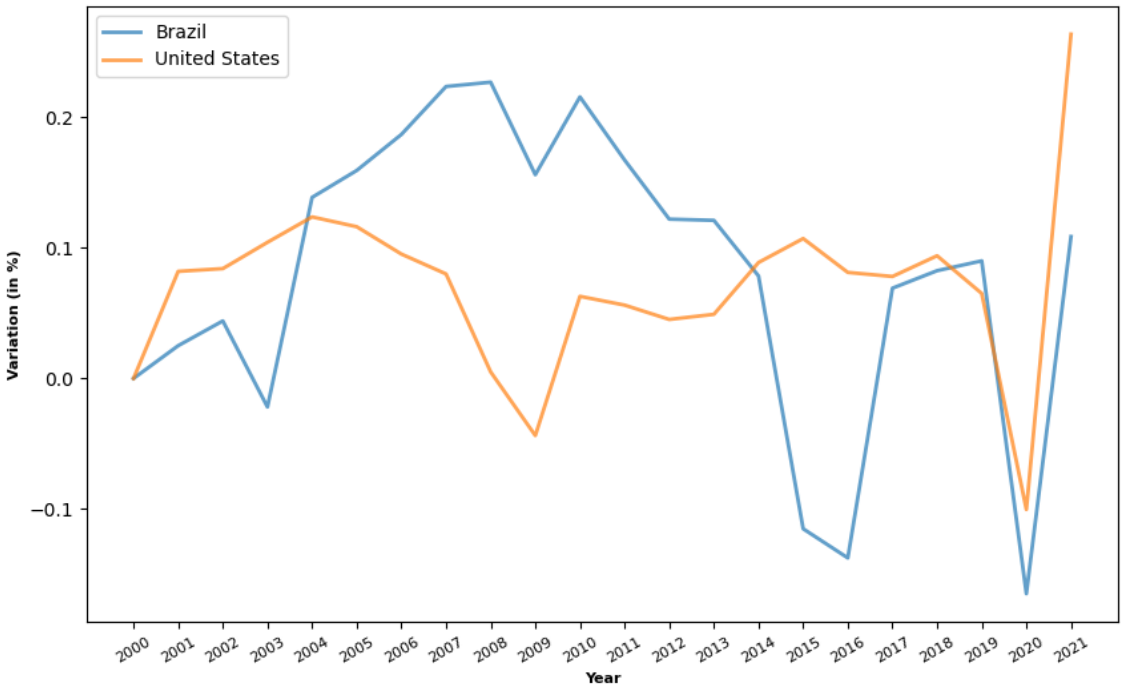
Statistic	Brazil	United States
Mean	0.080678	0.069852
Standard Deviation	0.112500	0.068859
Minimum	-0.164782	-0.100323
Maximum	0.226774	0.263505

Source: Elaborated by the author (2024).

data for these ETFs were extracted from *Yahoo! Finance* (2024a, 2024b), from 2009 to 2021 and each year with a unique daily price from the first of december. From Brazil, the iShares Ibovespa Index Fund (BOVA11.SA) was chosen, aiming to replicate the performance of the Ibovespa Index, which is a benchmark for the Brazilian stock market, encapsulating the performance of the most traded stocks on the B3 exchange. For the United States, the SPDR S&P 500 ETF Trust (SPY) was selected, designed to mirror the performance of the S&P 500 Index, an American stock market index based on the market capitalizations of 500 large companies having common stock listed on the NYSE or NASDAQ. Both ETFs serve as proxies for the broader market performance in their respective countries, providing a tangible means for investors to gauge market trends. To transform the prices from the BOVA11.SA to USD, it was used the daily exchange rate from Banco Central do Brasil (2024), also with the values from december first for each year.

The logarithmic variations in the prices of the iShares Ibovespa Index Fund (BOVA11.SA) for Brazil and the SPDR SP 500 ETF Trust (SPY) for the United States reveal significant fluctuations over the observed periods, reflecting the dynamic nature of stock market movements and investor sentiment. In the case of Brazil, the data show a wide range of changes, from a substantial decrease of 18.88% in 2015 to a steep incline of 18.60% in 2017, indicating periods of high volatility and rapid shifts in market conditions. Conversely, the USA's ETF prices also exhibit

Figure 4 - Log Household Consumption (PPP) Variations (Annual %) - Brazil and United States



Source: Data from World Bank (2024b). Elaborated by the author (2024).

fluctuations, though with a distinct pattern characterized by a major drop of -9.88% in 2017, suggesting a significant market correction or downturn. However, the overall trend includes periods of recovery and growth, such as an increase of 5.62% in the next year, showcasing the resilience and potential for rebound within the US stock market.

Table 3 - Log ETFs' Prices Variations - Descriptive Statistics

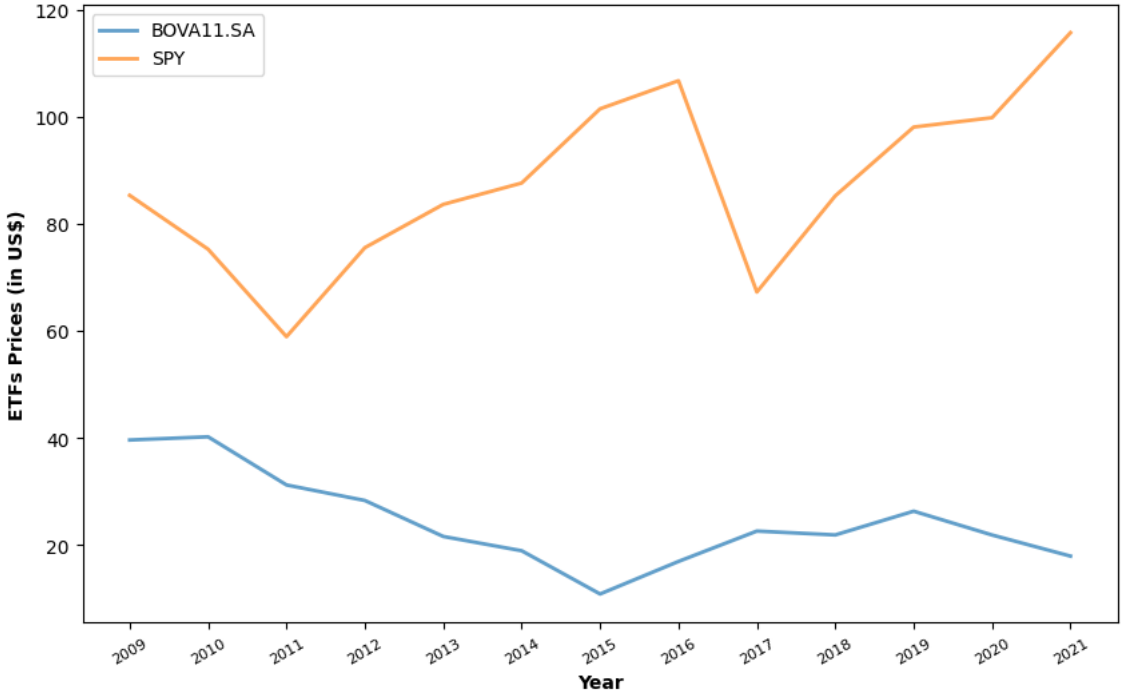
Statistic	Brazil	United States
Mean	-1.452525	0.605979
Standard Deviation	9.254477	4.478511
Minimum	-18.872892	-9.883538
Maximum	18.599741	6.095460

Source: Elaborated by the author (2024).

In the model, the correlation (α) and volatility (σ) coefficients play pivotal roles in capturing the economic dynamics. The alpha parameter, representing the serial correlation of GDP, was derived from applying the Augmented Dickey-Fuller (ADF) test to the logarithmized GDP series, whereby the coefficients from this test were designated as alpha. This parameter would only be adopted if it was statistically significant; otherwise, it was set to zero. For Brazil, alpha was determined to be zero, indicating no significant serial correlation in the GDP's logarithmic series over the period analyzed. On the other hand, the sigma parameter was obtained from the standard deviation of the logarithmic series' variations, reflecting the volatility or the inherent economic uncertainty captured through the fluctuations in the GDP data.

Additionally, the fixed intertemporal discount factor (β) and CRRA utility parameter (γ), crucial for our model's simulation, are derived from Issler and Piqueira (2000). This methodological approach ensures that the parameters accurately reflect the underlying economic conditions and the degree of predictability or volatility in the GDP growth rates.

Figure 5 - ETFs' Prices in USD - Brazil and United States



Source: Data from Yahoo! Finance (2024a, 2024b). Elaborated by the author (2024).

Table 4 - Parameters used in the model for each country

Parameters	Brazil	United States
Discount Factor (β)	0.890	0.995
CRRA Utility Parameter (γ)	4.890	2.250
Correlation Coefficient (α)	0.000	-0.551
Volatility Coefficient (σ)	0.103	0.062

Source: Adapted from Issler and Piqueira (2000, p. 231).

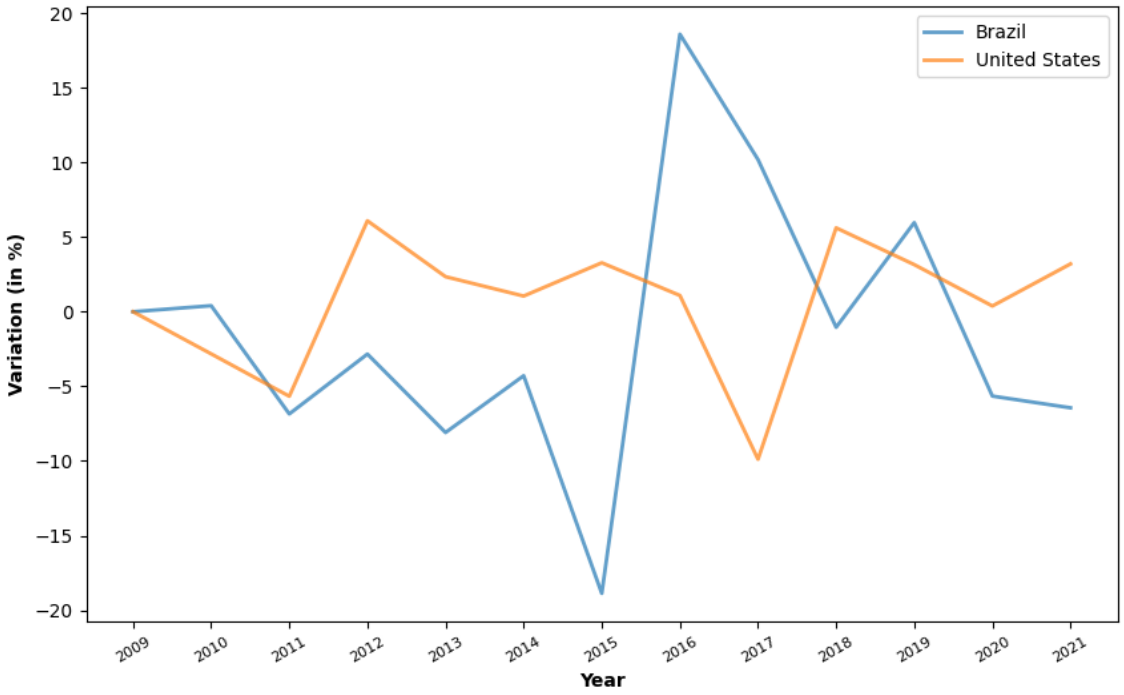
2.4.2 Model methodology

In our empirical approach for simulating the Lucas Asset Pricing Model, we begin by defining the Lucas Tree class, which encapsulates key parameters of the model. These include the CRRA utility parameter (γ), the discount factor (β), the correlation coefficient (α) and the volatility coefficient (σ). The model studies asset pricing in a pure exchange economy, with those parameters reflecting each country's economic reality.

The simulation, following methodologies from Quantecon (2024), utilizes Python for computational purposes. The Lucas operator will be used to compute and update the function on points in an interval which contains the stationary distribution of the consumption endowment, employing Monte Carlo integration. The model will iterate the Lucas operator to find the equilibrium price function associated with the Lucas tree, determining equilibrium prices under various scenarios.

This process is visualized through a plot that represents the relationship between the consumption endowment and its corresponding price, providing a graphical representation of the model's results. This approach not only showcases the practical application of the Lucas model (Lucas, 1978) but also links theoretical concepts with real-world economic scenarios and data interpretation. The code used for this work is available at GitHub (ECOMPFIN, 2024).

Figure 6 - Log ETFs' Prices Variations Variations (Annual %) - Brazil and United States



Source: Data from Yahoo! Finance (2024a, 2024b). Elaborated by the author (2024).

2.5 RESULTS AND DISCUSSION

In this section, we will analyze the collected data and simulations. We'll look at the Gross Domestic Product (GDP) of Brazil and the United States and compare these real-world figures with predictions from the Lucas Asset Pricing Model. Our aim is to understand how this theoretical model aligns with the economic patterns observed in these two countries, highlighting similarities and discrepancies between theory and practice in financial markets.

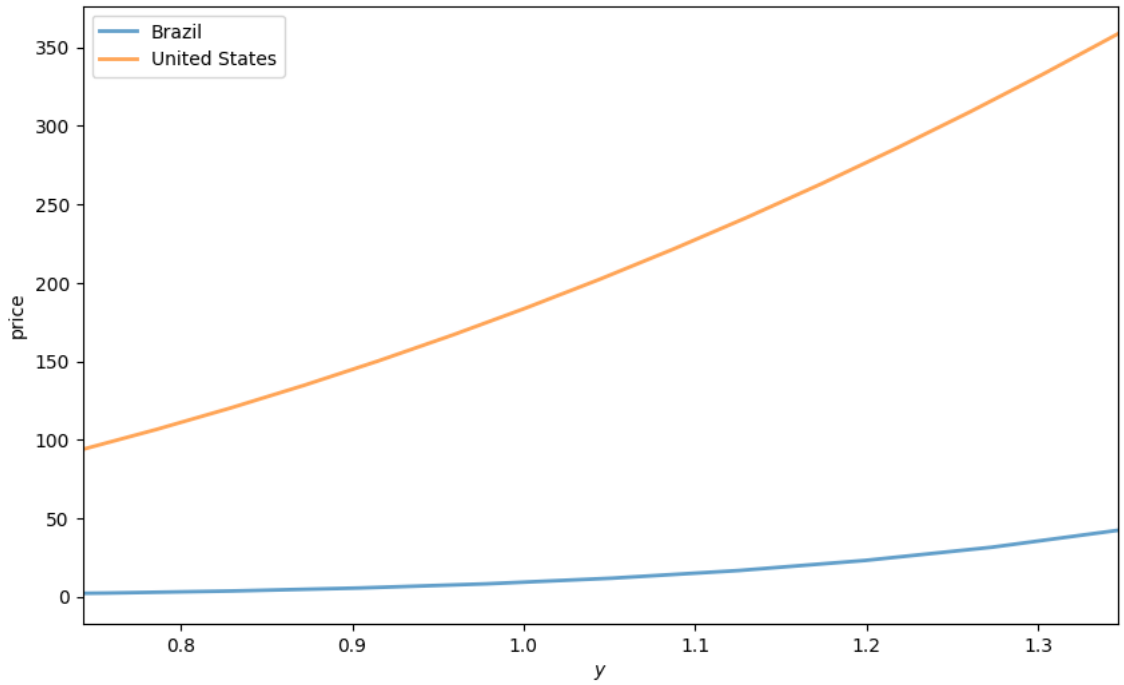
2.5.1 Gross Domestic Product from Brazil and United States

When examined over time, GDP trends offer insights into the patterns of economic growth, cycles of expansion and contraction, and the overall economic resilience of a nation. The period leading up to 2008 witnessed varying rates of GDP growth globally, as economies expanded at different paces. This financial crisis of 2008 marked a significant inflection point, reflecting in a noticeable dip in the GDP trends for both countries.

Moving through the decade following the 2008 crisis, the economies saw recovery and growth, although at uneven rates. However, this trajectory was disrupted during the 2015-2016 period, where Brazil faced another challenging phase marked by economic recession. These years were characterized by declining GDP, which can be attributed to a combination of internal structural weaknesses, political instability, and external economic pressures (ARESTIS; TERRA, 2015). In contrast, the United States displayed more resilience during this period, although not entirely immune to global headwinds.

The advent of the COVID-19 pandemic in 2020 introduced unprecedented challenges, leading to a significant and widespread economic downturn as countries implemented lockdowns and faced disruptions in trade, manufacturing, and services. The pandemic's impact on GDP was immediate and severe, with contractions across both advanced and emerging markets (FER-

Figure 7 - Lucas' model simulation to Brazil and United States



Source: Elaborated by the author (2024).

NANDES, 2020). The recovery paths for both countries have been similar. The GDP data during this period not only reflect the direct economic impact of the pandemic but also the effectiveness of policy measures and the underlying economic resilience of each country.

2.5.2 Lucas' model simulation to Brazil and United States

In the theoretical landscape of the Lucas asset pricing model, the price functions for Brazil and the United States serve as a fundamental baseline for understanding asset valuation influenced by economic fluctuations. Brazil's function, characterized by a relatively flat curve, signifies subdued sensitivity to economic variability, possibly a consequence of the zero alpha parameter. This indicates an absence of significant serial correlation in the nation's consumption growth rates, as determined by the Augmented Dickey-Fuller test. In stark contrast, the United States' negative alpha parameter and steeper price function suggest that its asset prices are more susceptible to shifts in consumption and imply a mean-reverting dynamic in GDP growth rates.

Furthermore, these theoretical price functions derived from the model, though smooth and idealized, starkly contrast with the actual price movements observed in the ETFs BOVA11.SA and SPY. The ETFs embody real-time market valuations that encapsulate not only the core economic indicators like consumption and GDP, reflected in the variations depicted in graphs three and four, but also the multifaceted nature of market sentiment. This is particularly evident in the erratic behavior of ETF prices over time, influenced by an array of factors beyond the scope of the Lucas model's fundamental assumptions. These factors include immediate market forces, geopolitical events, policy changes, and most notably for Brazil, the impact of exchange rate fluctuations.

The ETF price variations, especially in the case of Brazil, are further compounded by the currency valuation effects due to the pricing in USD. Such factors can significantly skew the relationship between economic fundamentals and market prices, resulting in ETF price trajec-

tories that deviate from the predictions of economic models. It's important to note that while the Lucas model assumes a market of rational expectations and frictionless trades, the actual market is replete with behavioral biases, transaction costs, and regulatory constraints that can distort price signals.

The trends in consumption and GDP data, while showing congruent patterns of expansion and contraction across both nations, indicate the underlying rhythm of economic health. Yet, the translation of these rhythms into the language of market prices, as demonstrated by the ETF price variations, involves a complex translation process. Market-specific forces, such as trading behaviors, investor risk profiles, and liquidity constraints, can dramatically alter this translation, often resulting in a narrative that diverges from economic forecasts. In the short term, these forces can dominate price action, while in the long run, the fundamental economic indicators may reassert their influence.

In conclusion, the comparison between the Lucas model's price functions and the real-world ETF prices highlights the inherent limitations of theoretical models when applied to the real world, characterized by complex, adaptive systems. The multifaceted nature of financial markets demands a holistic approach that considers economic fundamentals alongside the myriad influences that shape investor behavior and market outcomes. Understanding this intricate web of factors is crucial for interpreting market signals and making informed investment decisions.

2.6 CONCLUSION

In concluding this examination of asset prices and consumption patterns using the Lucas Asset Pricing Model, we've uncovered intricate connections between theoretical constructs and the realities of the financial markets in Brazil and the United States. Our study has highlighted the nuanced behaviors of economic variables and their implications within different market contexts, offering a bridge between abstract theory and tangible market activities.

This research has contributed to a deeper understanding of financial dynamics, emphasizing the importance of considering varied economic environments when applying theoretical models. It underscores the complexity of financial markets, inviting a more comprehensive approach that encompasses economic, political, and social dimensions.

Looking forward, the study paves the way for further exploration. Future research could extend the application of the Lucas Model to other emerging markets, offering a more global perspective. Investigating the impact of specific economic policies on model outcomes would deepen our comprehension of the intricate connections between policy decisions, consumption behaviors, and asset pricing. Such integrative research could lead to more effective policy formulations, enhancing global financial stability and growth.

2.7 REFERENCES

ARESTIS, P.; TERRA, F. Economic Policies to Improve the Current State of the Brazilian Economy. **Challenge**, v. 58, p. 532 - 549. Available at: https://doi.org/10.1080/05775132.2015.1105679. Acessed on: 15 Feb. 2024.

ARROW, K. J. The role of securities in the optimal allocation of risk-bearing. **The Review of Economic Studies**, United Kingdom, v. 31, n. 2, p. 91–96, 1964. Available at: http://www.jstor.org/stable/2296188. Accessed on: 29 Nov. 2023.

BANCO CENTRAL DO BRASIL. **Sistema Gerenciador de Séries Temporais**: Taxa de

Câmbio. 2024. Available at: https://www3.bcb.gov.br/sgspub/localizarseries/localizarSeries.do?method=prepararTelaLocalizarSeries. Accessed on: 15 Feb. 2024.

BERNOULLI, D. Exposition of a new theory on the measurement of risk. **Econometrica**, United States, v. 22, n. 1, p. 23–36, 1954. Available at: http://www.jstor.org/stable/1909829. Accessed on: 20 Nov. 2023.

ECOMPFIN (Núcleo de Pesquisa e Extensão em Ciência de Dados e Computacional em Economia e Finanças da Faculdade de Ciências Econômicas da UFRGS). stockprices_consumption_machine_learning. 2024. **GitHub**. Available at: https://github.com/ecompfin-ufrgs/stockprices_consumption_machine_learning. Accessed on: 17 Feb. 2024.

ELTON, E. J.; GRUBER, MARTIN J.; BROWN, STEPHEN J.; GOETZMANN, W. N. **Modern portfolio theory and investment analysis**. 9th ed. United States of America: John Wiley, 2014.

FAMA, E. F. Efficient capital markets: A review of theory and empirical work. **The Journal of Finance**, United States, v. 25, n. 2, p. 383–417, 1970. Available at: http://www.jstor.org/stable/2325486. Accessed on: 9 Dec. 2023.

FERNANDES, N. Economic Effects of Coronavirus Outbreak (COVID-19) on the World Economy. **IESE Business School Working Paper Series**, Spain. https://doi.org/10.2139/ssrn.3557504.

ISSLER, J. V.; PIQUEIRA, N. S. Estimating relative risk aversion, the discount rate, and the intertemporal elasticity of substitution in consumption for brazil using three types of utility function. **Brazilian Review of Econometrics**, Brazil, v. 20, n. 2, p. 201–239, 2000. Available at: https://doi.org/10.12660/bre.v20n22000.2758. Accessed on: 19 Dec. 2023.

LUCAS, R. E. Asset prices in an exchange economy. **Econometrica**, United States, v. 46, n. 6, p. 1429–1445, 1978. Available at: http://www.jstor.org/stable/1913837. Accessed on: 27 Oct. 2023.

MARKOWITZ, H. Portfolio selection. **The Journal of Finance**, United States, v. 7, n. 1, p. 77–91, 1952. Available at: https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1540-6261.1952.tb01525. x. Accessed on: 13 Nov. 2023.

MOSSIN, J. Equilibrium in a capital asset market. **Econometrica**, United States, v. 34, n. 4, p. 768–783, 1966. Available at: http://www.jstor.org/stable/1910098. Accessed on: 10 Dec. 2023.

MUTH, J. F. Rational expectations and the theory of price movements. **Econometrica**, United States, v. 29, n. 3, p. 315–335, 1961. Available at: http://www.jstor.org/stable/1909635. Accessed on: 25 Nov. 2023.

QUANTECON. **Advanced Quantitative Economics with Python**: Asset Pricing II: The Lucas Asset Pricing Model. 2024. Available at:

https://python-advanced.quantecon.org/lucas$_m odel.html. Accessed on : 17 Jan. 2024.

ROMER, P. M. Endogenous technological change. **Journal of Political Economy**, United States, v. 98, n. 5, p. S71–S102, 1990. ISSN 00223808, 1537534X. Available at: http://www.jstor.org/stable/2937632. Accessed on: 15 Jan. 2024.

RUBINSTEIN, M. The valuation of uncertain income streams and the pricing of options. **The Bell Journal of Economics**, United States, v. 7, n. 2, p. 407–425, 1976. Available at: http://www.jstor.org/stable/3003264 Accessed on: 10 Dec. 2023.

SHARPE, W. F. Capital asset prices: A theory of market equilibrium under conditions of risk*. **The Journal of Finance**, United States, v. 19, n. 3, p. 425–442, 1964. Available at: https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1540-6261.1964.tb02865.x. Accessed: 7 Dec. 2023.

SOLOW, R. M. A contribution to the theory of economic growth. **The Quarterly Journal of Economics**, United States, v. 70, n. 1, p. 65–94, 1956. Available at: http://www.jstor.org/stable/1884513. Accessed: 13 Jan. 2024.

WORLD BANK. **GDP, PPP (constant 2017 international $)**. 2024a. Available at: https://databank.worldbank.org/reports.aspx?source=2&series=NY.GDP.MKTP.PP.KD&country=BRA,USA. Accessed on: 17 Feb. 2024.

WORLD BANK. **Households and NPISHs Final consumption expenditure, PPP (constant 2017 international $)**. 2024b. Available at: https://databank.worldbank.org/reports.aspx?source=2&series=NE.CON.PRVT.PP.KD&country=BRA,USA. Accessed on: 17 Feb. 2024.

YAHOO FINANCE. **iShares Ibovespa Index Fund (BOVA11.SA)**. 2024a. Available at: https://finance.yahoo.com/quote/BOVA11.SA/history. Accessed on: 17 Feb. 2024.

YAHOO FINANCE. **SPDR SP 500 ETF Trust (SPY)**. 2024b. Available at: https://finance.yahoo.com/quote/SPY/history. Accessed on: 17 Feb. 2024.

3 CONSIDERAÇÕES FINAIS

This study's comprehensive approach, blending theoretical models with empirical data, offers a multifaceted view of financial market dynamics. The findings provide a deeper understanding of the complex interactions between economic variables in different market environments, highlighting both the practical applications of economic theories and the intricacies of market behaviors in Brazil and the United States. The article concludes with a reflection on the implications of these findings and their contribution to the broader field of economic study.

REFERÊNCIAS

LUCAS, R. E. Asset prices in an exchange economy. **Econometrica**, United States, v. 46, n. 6, p. 1429–1445, 1978. Available at: http://www.jstor.org/stable/1913837. Accessed on: 27 Oct. 2023.

RUBINSTEIN, M. The valuation of uncertain income streams and the pricing of options. **The Bell Journal of Economics**, United States, v. 7, n. 2, p. 407–425, 1976. Available at: http://www.jstor.org/stable/3003264 Accessed on: 10 Dec. 2023.

SOLOW, R. M. A contribution to the theory of economic growth. **The Quarterly Journal of Economics**, United States, v. 70, n. 1, p. 65–94, 1956. Available at: http://www.jstor.org/stable/1884513. Accessed: 13 Jan. 2024.