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Electrical fire risk indexing using fuzzy Petri nets

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ABSTRACT

Electrical fires are a significant cause of dwelling fires, but the existing information on electrical fires is often vague and imprecise, making it challenging to develop a comprehensive risk assessment method. To address this problem, this paper proposes a risk assessment method based on Fuzzy Petri nets (FPNs), a modeling tool that is suitable for complex systems under uncertainty. This approach allows for modeling various relationships between risk factors and their respective importance, supplying a flexible and comprehensive way to perform risk assessment. The proposed method involves responding to a yes/no questionnaire based on code recommendations, which allows for the prediction of the failure mode response. Based on the simulation results, it can be evaluated the overall risk, determined the relevance of each electrical issue in a residential distribution system, or assessed the impact of electrical installation improvements and alterations on safety. Despite the complexity of failure modes modeling, this approach offered a practical and straightforward means of identifying potential electrical fire risks.

1. Introduction

When electrical current flows through a circuit, it generates heat, which is safely dissipated in a well-designed and maintained system. However, electrical faults, such as short circuits, series arcs, glowing connections, and earth leakage, among others, can provoke excessive heating and potentially ignite the cable cover or nearby flammable materials [1]. Nonetheless, electrical fires typically result from a complex combination of multiple failure modes and mechanisms, rather than a single factor. Overload, protection device failure, overheating, and insulation breakdown can all contribute to a fire, either simultaneously or in sequence. Due to this complexity, it can be difficult to accurately identify the primary cause of electrical faults, and the cause may be misclassified or remain undetermined [2].

Electrical failures account for a considerable number of home fires on a global scale [3]. According to the National Fire Protection Association (NFPA), electrical fires account for approximately 13% of all home fires in US [4], while in Europe, the Forum for European Electrical Domestic Safety reports a rate of 30% [5]. Despites the lack of statistics, the problems could be even greater in poor and developing countries. For instance, authorities in India report that electrical fires constitute approximately 70% of all fires in the country. A study by the US's Federal Emergency Management Agency (FEMA) shows that poverty is associated with an increased risk of fires [6]. The study highlights that the poor-quality electrical installations, the use of substandard electrical components and inadequate maintenance practices can increase the likelihood of hazardous defects in electrical systems, which increase the risk of fire outbreaks.

Electrical codes and standards, such as BS 7671 (British Standard for Electrical Installations) and NFPA 70 (National Electrical Code), provide guidelines and requirements to ensure the safety of electrical installations. Despite implementing the best design and installation practices, electrical faults can still occur due to factors such as environmental conditions, aging components, and human error. Regular inspections and testing of electrical wiring are recommended by codes to reduce the likelihood electrical issues [7,8]. These practices can help identify potential hazards and failures in the system before they escalate and lead to significant problems. Inspectors typically use checklists based on codes to assess the safety of an installation, considering adherence to code items. However, this approach could not be the best way of risk management, especially when fully complain with the codes are not a feasible option. These checklists comprise valuable tools to identify fire

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risk factors, but they do not distinguish among the importance of these factors [9]. A more advanced risk assessment can help determining which updates and repairs are priority, with the aim to reduce risk to an acceptable level. However, it is worth noting that there is currently no comprehensive method available for accurately assessing the risk of electrical fires.

Probabilistic risk analysis (PRA), that produce quantitative values, are the most informative approach to fire risk assessment. PRA approaches include, but they are not limited to: the Fault Tree Analysis (FTA); Failure Mode and Effects Analysis (FMEA) and Event Tree Analysis (ETA). In conventional PRA, failure data is required for the purposes of quantitative analysis [10]. Due to the complexity involved in electrical fires, the available data are usually vague and imprecise. According to the NFPA report [11], it was not possible to identify the specific electrical failure responsible for the majority of registered fires in the United States. In situations where there is uncertainty or imprecision in the available information, it can be challenging to make accurate predictions or assessments about the behavior of complex systems. To handle such situations, fuzzy set theory has been successfully used in novel PRA approaches for safety and reliability evaluation under conditions of uncertainty, such as Fuzzy Petri nets (FPN) [10].

Developed by Carl Adam Petri in the 1960s, a Petri net is a graphical tool used to model and analye systems that involve discrete events. In contrast, a Fuzzy Petri net utilizes fuzzy logic principles to represent the imprecise and uncertain information often associated with real-world systems. It is widely used in risk assessment and fault diagnoses to simulate the behavior of a system under different conditions and evaluate the likelihood of failure [12]. A comprehensive risk assessment framework based on FPNs has been proposed by Y. Chang et al. [13] for deep-water drilling riser based on analyses of accidents and identification of risk factors. Wan S. [14] used FPN to represent the generation and propagation of faults in electrical equipment to evaluate a smart electrical substation's reliability. A combination of Fault Tree Analysis and fuzzy Petri net was used by He L. et al. [15] to develop a novel method for fire risk assessment of cables in utility tunnels [15].

In this context, the paper proposes a methodology to assess the risk of electrical fires that utilizes a Fuzzy Petri net to simulate potential failure modes and their responses in various scenarios. The goal of this methodology is to support decision-making and risk assessment by considering non-compliance issues identified through a verification script that is based on recommendations outlined in electrical codes. This approach could aid in effective risk management and improve safety.

To offer readers a more comprehensive understanding of the proposed method, the paper delves in the Fuzzy Petri nets fundamentals in section 2 and in the electrical fire failure modes in section 3.

The developed methodology is shown in section 4, which comprises three parts: electrical fires Fuzzy Petri net, in which the graphical representation and production rules are presented as a response model to behavior system. The model was built based on the review present in section 3 and specialists' opinions. Based on the code's recommendations, in the second part it is presented the verification script, aims at evaluating the current installation conditions. Finally, it is presented an algorithm to automatic execution of the model.

In section 5, the method was demonstrated using Matlab software to simulate three case studies. The first case evaluated the response of each hazard identified in the verification script individually, revealing that the presence of an oxidation spot on electrical connections alone greatly increases the probability of a fire outbreak. The second case compared the results of the proposed method with a conventional indexing method, showing significant differences due to the use of fuzzy arithmetic in calculating risk in the Fuzzy Petri nets. In the third case, the introduction of an Arc Fault Circuit Interrupter (AFDD) was simulated to assess its impact on safety and the reduction of arc fault occurrences. While the device significantly reduced the likelihood of arc faults, the average risk reduction was only 14%, as concluded in section 6.

2. Fuzzy Petri net fundamentals

The Fuzzy Petri net (FPN) is a powerful modeling tool that combines the strengths of Petri nets and fuzzy logic. Petri nets are a graphical tool used to model systems with concurrency, synchronization, and resource sharing, while fuzzy logic is a mathematical framework used to deal with uncertainty and imprecision. FPNs allow the modeling of complex systems that involve both exact and uncertain information, making them well-suited for modeling electrical systems with multiple interacting components and uncertain operating conditions [16].

In the context of electrical safety assessment, the FPN methodology provides a way to represent and analyze the behavior of electrical systems under different operating conditions, including malfunction and failure. By identifying the characteristics, risk factors, and potential defects of the system, the FPN can help identifying the need for corrective maintenance and estimate the safety impacts on installation modifications [17].

The utilization of expert knowledge is a vital component of the proposed methodology. Expert systems are employed to gather and archive domain experts' knowledge in fire safety, enabling individuals without expertise in the field to conduct comprehensive risk assessments. Fuzzy logic is commonly used to represent uncertain knowledge, while Petri nets are well-suited to represent precise knowledge. By merging these two frameworks, the proposed methodology provides a robust tool to represent and resolve intricate issues in electrical safety assessment [18].

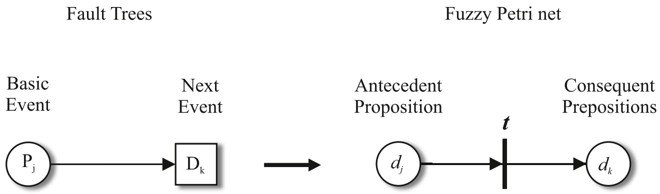
2.1. Definition

Fuzzy Petri nets (FPNs) can model complex systems that have both exact and uncertain information, making them a powerful tool to model real-world systems with multiple interacting components and uncertain operating conditions [19].

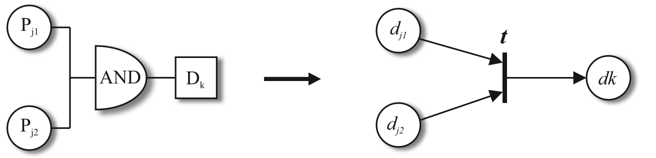
A Petry net (PN) may be identified as a particular kind of bipartite directed graph populated by three types of objects. These objects are places (represented by a circle), transitions (represented by a bar), and directed arcs (represented by an arrow) connecting places to transitions and transitions to places. Places represent the system's state, and transitions represent the events that can occur. The arcs represent the connections between places and transitions, and the weights assigned to the arcs indicate the degree of influence that one component has on another [20]. The transitions from an antecedent to a consequent place, are defined firing rules, regulated by the “IF-THEN” and the systems interactions by the “OR”/“AND” logical operators [21] The same logical operators are used in the Fault Analysis Tree (FTA), but its graphical representation are different [10], as shown in Fig. 1.

The Fuzzy Petri net (FPN) is a powerful tool to model complex systems due to its capability to reason and understand representation. In order to define the FPN model, it is necessary to consider several parameters, including the Certainty Factor (μ), the threshold value (τ), and the weight (w) in Weight Fuzzy Petri nets (WFPNs) [22], as shown Fig. 2.

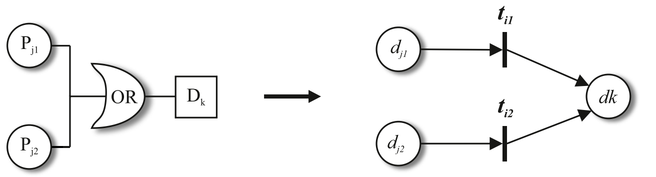
The Certainty Factor (μ) is the degree of belief in the rule, as determined by domain experts. This factor indicates the level of confidence in the rule and its potential impact on the system. The Certainty Factor can range from 0 to 1, with 0 indicating complete uncertainty and 1 indicating complete certainty. The threshold value (τ) is associated with transitions in the FPN and represents the minimum requirements of actual support needed for the transition to occur. The threshold value is used to determine whether the transition can fire or not, based on the current state of the system. If the input to the transition is below the threshold value, the transition will not fire. The Weight Fuzzy Petri nets (WFPNs) introduce an additional parameter, weight (w), which represents the importance of the place for the transition. The weight is used to determine the degree of influence that a particular place has on the transition. A higher weight indicates a greater influence on the transition, while a lower weight indicates a lesser influence. The use of these



a) Simple relationship



b) AND gate



c) OR gate

Fig. 1. Fault Tree and Fuzzy Petri net graphical representation comparison.

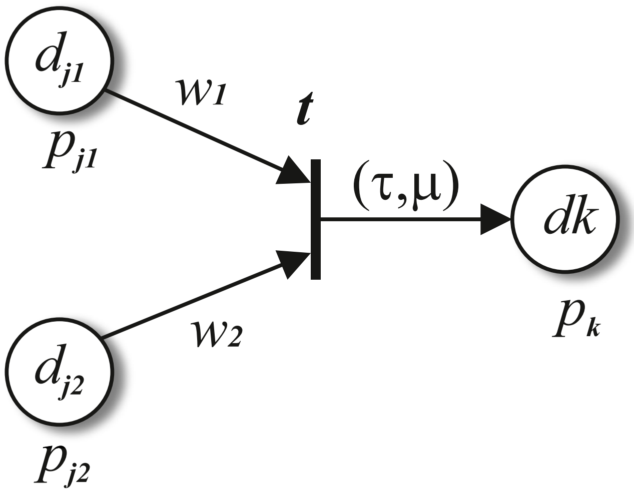


Fig. 2. Weight fuzzy Petri net complete graphical representation.

parameters in FPNs and WFPNs allows for a more precise representation of the system and its behavior. By adjusting the Certainty Factor, threshold value, and weight, the behavior of the system can be fine-tuned to better reflect real-world conditions. This allows for the identification of potential problems or risks and the selection of the best course of action [23].

2.2. Fuzzy production rules

Fuzzy production rules (FPRs) supply an alternative method to represent a system model using Fuzzy Petri net (FPN) by a set of rules rather than a graphical representation. FPRs consist of a set of rules that describe the relationships between the input variables and the output variables. To represent these relationships, FPRs use fuzzy operators, which allow for the representation of linguistic (fuzzy) variables. The fuzzy simulation theory adopts "MIN" operator to manage "AND" problems and "Max" operator to manage "OR" problems. [13], as shown Table 1.

The minimum operator is used to represent the logical AND operator in fuzzy logic. It is used to determine the degree of support for a rule based on the weakest premise. In FPRs, the minimum operator is used to calculate the degree of membership of a fuzzy set. The maximum operator is used to represent the logical OR operator in fuzzy logic. It is used to determine the degree of support for a rule based on the strongest premise. In FPRs, the maximum operator is used to calculate the degree of membership of a fuzzy set [24].

2.3. Matrix representation

The matrix representation of WFPNs provides an effective way to perform computational simulations of the system. By representing the system as a two-dimensional matrix, it is possible to perform various operations on the system, such as fire sequence determination of transitions and the behavior analysis of the system under different conditions.

One advantage of the matrix representation is that it allows for efficient computation of the system's behavior. The matrix can be updated based on the firing of transitions, allowing for real-time simulations, leading for rapid prototyping and testing of complex systems before their implemented in the real-life [25].

In addition, the matrix representation of WFPNs allows for the use of various mathematical and computational tools to analyze the system. For example: it is possible to use linear algebra techniques to analyze the matrix and determine the steady-state behavior of the system.

One of these approaches is presented in Liu H. et al. [26], where the authors introduce a matrix representation that enables computational simulation of complex systems. The Dynamic Adaptative Fuzzy Petri net (DAFPN) can be defined as an 11-tuple [26].

$\mathrm{DAFPN} = (\mathrm{P};\ \mathrm{T};\ \mathrm{I};\ \mathrm{O};\ \mathrm{D};\ \alpha;\ \beta;\ \mathrm{W};\ \mathrm{U};\ \mathrm{Th};\ \mathrm{M})$, where:

$\mathrm{P} = \{p_1, p_2, \ldots, p_m\}$ denotes a finite nonempty set of places; $\mathrm{T} = \{t_1, t_2, \ldots, t_n\}$ denotes a finite nonempty set of transitions.

$\mathrm{I} : \mathrm{P} \times \mathrm{T} \rightarrow \{0, 1\}$ is an $m \times n$ input incidence matrix defining the directed arcs from place to transitions.

$\mathrm{I}_{ij} = 1$, if there is a directed arc from pi to t_j, and $\mathrm{I}_{ij} = 0$, I_{ij} if there is no directed arcs from pi to t_j, for p_i to $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

$\mathrm{O} : \mathrm{T} \times \mathrm{P} \rightarrow 0, 1$ is an $m \times n$ output incidence matrix defining the directed arcs from transitions to places. $\mathrm{O}_{ij} = 1$, if there is a directed arc from p_i to t_j, and $\mathrm{O}_{ij} = 0$ if there is no directed arcs from p_i to t_j, for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$, $\mathrm{D} = \{d_1, d_2, \ldots, d_m\}$ denotes a finite set of propositions.

$\mathrm{P} \cap \mathrm{T} \cap \mathrm{D} = \varnothing$, $|\mathrm{P}| = |\mathrm{D}|$. $\alpha : \mathrm{P} \rightarrow [0, 1]$ is an association function which maps from places to real values between 0 and 1.

$\beta : \mathrm{P} \rightarrow \mathrm{D}$ is an association function representing a bijective mapping from places to propositions.

$\mathrm{W} : \mathrm{I} \rightarrow [0, 1]$ is an input function and it can be expressed as a $m \times n$-dimensional matrix. The value of an element in W, $w_{ij} \in [0, 1]$, is the weight of the input place, which indicates how much the place p_i impacts its following transition t_j connected by I_{ij}.

$\mathrm{U} : \mathrm{O} \rightarrow [0, 1]$ is an output function and can be expressed as a $m \times n$-dimensional matrix. The value of an element in U, $\mu_{ij} \in [0, 1]$, is the value of certainty factor, which indicates how much a transition t_j impacts its output places p_i, if the transition fires.

$\mathrm{Th} : \mathrm{O} \rightarrow [0, 1]$ is an output function which assigns a certainty value

Table 1
Fuzzy production rule fuzzy operators.

Relationship	Production Rule	Fuzzy Operator
AND	**If** $\alpha(p_{i1})$ **AND** $\alpha(p_{i2})$ **THEN** $\beta(p_i)$	*min* $(\alpha(p_{i1}), \alpha(p_{i2}))$
OR	**If** $\alpha(p_{i1})$ OR $\alpha(p_{i2})$ **THEN** $\beta(p_i)$	*max* $(\alpha(p_{i1}), \alpha(p_{i2}))$

between 0 and 1 to each output place of a transition, $\mathrm{Th} = (\tau_{ij})_{mxn}, i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$, denoting the output threshold of this place. $\tau_{ij} \in [0, 1]$, if there is a direct arc from t_j to pi, and $\tau_{ij} = +\infty$, if there are no direct arcs from t_j to p_i, for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$. M denotes a marking of the Petri Net, $M = (\alpha(p_1), \alpha(p_2), \ldots, \alpha(p_m),)^T$, where $\alpha(p_1)$ is the truth value of place p_j. The initial marking is denoted by M_0. Its determination should be based on an actual status. Hence, its value could be understood as the dynamic input and directly influence the dynamic behavior of DAFPN.

The evolution of failures chain depends on the execution of production rules, related to a fire of transitions mechanisms.

The notation $I(t) = \{p_{I1}, p_{I2}, \ldots, p_m,\}$ represents the input with corresponding weights $w_{I1}, w_{I2}, \ldots, w_{Im}$, Similarly, $O(t) = \{p_{O1}, p_{O2}, \ldots, p_n,\}$ represents the output with corresponding output thresholds $\tau_{O1}, \tau_{O2}, \ldots, \tau_{On}$ and certainty factors $\mu_{O1}, \mu_{O2}, \ldots, \mu_{On}$. The enabling and firing rules are specified as follows.

Enabling rule: $\forall t \in T$, t is enabled and fired if $\forall p_{Ij} \in I(t)$,

$$\{\alpha(p_{ij}) > 0\} \wedge \{\mu(t) \geq \min(\tau_{OK})\}, j = 1, 2, \ldots, m; k = 1, 2, \ldots, n. \tag{1}$$

Where $\alpha(p_{ij})$, $\alpha(p_{ij}) \in [0, 1]$, is the fuzzy truth value in place $p_{Ij}, p_{Ij} \in I(t)$, wich indicates the truth degree of proposition d_{Ij}, if $\beta(p_{Ij}) = d_{Ij}$; $\mu(t) = \sum_{j=1}^{m} \alpha(p_{ij}) w_{Ij}, j = 1, 2, \ldots, m$ is the equivalente fuzzy truth of input places at transitions when t is enabled.

After t is fired, the tokens in input places are copied, and tokens with fuzzy truth are put into each output place whose output thresholds are lesser than the equivalent fuzzy truth. The new fuzzy truth values of output places are defined as:

$$\alpha(p_{Oi}) = \begin{cases} \mu_{Oi}\mu(t), \mu(t) \geq \tau_{Oi} \\ 0, \mu(t) < \tau_{Oi} \end{cases} i = 1, 2, \ldots, n. \tag{2}$$

If a place has more than one input transition and more than one of its input transitions fires, then the transition produces the new fuzzy truth value of the output place with the maximum fuzzy truth.

3. Electrical fires

An electrical fire is a dangerous and potentially catastrophic event that occurs when electrical energy ignites a flammable material. Electrical fires can occur because of a wide range of electrical faults, including short-circuits, overloaded circuits, damaged wiring or equipment, and other electrical malfunctions. These faults can generate heat, sparks, or arcs, which can ignite flammable materials such as insulation, wood, paper, or fabric. [3,27,28]

3.1. Electrical fault in wiring systems

Electrical faults, such as short-circuits, earth leakage, glowing connections, series arcs, and overload, can generate excessive heat and energy that can potentially ignite surrounding materials, leading to a fire outbreak. The heating mechanisms involved in these faults include: ohmic heating, which occurs when current flows through a resistance and arcing heating, which occurs when a discharge is present between conductors. Fig. 3 [29] illustrates the wiring faults.

Short-circuits occur when a low resistance path is created between two conductors, causing excessive current to flow, leading to overheating. High current levels are usual in short-circuits, manly in bolted short-circuits; However, under normal conditions, overcurrent protective devices (OCPD) should interrupt the power supply avoiding excessive heating. Some circumstances can lead the failure of a circuit breaker to open under fault conditions due to an improper branch circuit design or lack of maintenance [30].

Overload results when the electrical system is subjected to excessive load, which can lead to cable overheating and, in extreme cases, to fire outbreak. However, experts consider overloads a rare cause of fires in branch-circuit wiring, as they would require around seven times the rated load for the wiring to ignite. Such a gross overloading of wiring is not common, even if the OCPD fails, making this fault relatively less likely to result in cable insulation ignition [31].

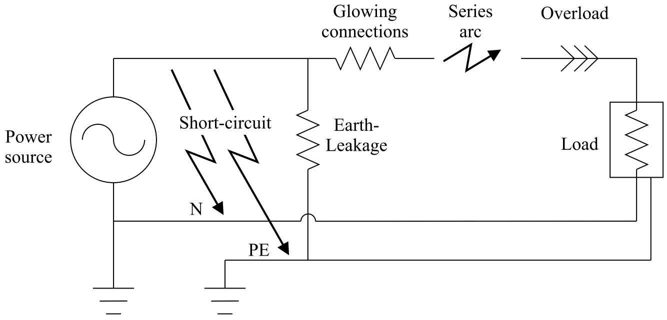


Fig. 3. Electrical faults in low-voltage wiring.

Glowing connections are a significant hazard to electrical safety due to their potential to reach high temperatures and ignite surrounding materials. This fault is often caused by loose, corroded, or poorly connected wires, and its temperature is influenced by several factors, including the current magnitude, resistance, and thermal properties of surrounding materials [1]. Typically, glowing connections can reach temperatures of several hundred degrees Celsius, which can ignite common household materials such as paper, wood, and fabric [32]. It is a dangerous fault because poor connections are not visible, often hidden behind walls and inside junction boxes. Despite standard recommendations, connections are rarely verified periodically, further increasing the risk of fires caused by glowed connections.

Series arcs are closely related to glowing connections and can also lead to fire outbreaks. An arc is a continuous luminous discharge of electricity across an insulating medium, accompanied by a partial volatilization of the electrodes [33]. An arc fault is an unintentional arcing condition in an electrical circuit that can pose a risk of fire ignition under certain conditions if the arcing persists. In dwelling units, AC arc faults can occur due to damaged wires, worn electrical insulation, overheated, or stressed electrical cords and wires, or misapplied electrical components [34]. Temperature of polymeric insulation used in wiring [35]. A loose or broken wire connection that undergoes an intermittent make/break condition under load creates a series of arcing between the conductors. The arc current flows through a plasma column with a temperature of at least several thousand Kelvin. The intense heat generated by this arcing can lead to an overheated resistive joint that can eventually form a molten bridge of copper and copper oxide at a temperature of up to 1230 °C, which is well above the melting and vaporization temperature of polymeric insulation used in wiring [35].

Series arc faults cannot be detected by conventional protection devices because they have a limited influence on the load current. During a series arc fault, there is no leakage to the ground, so RCDs cannot detect such a fault. Additionally, the load current remains unchanged or becomes lower during a series arc fault, and the over-current protections like MCBs or fuses do not trip as expected. To improve protection against arc faults, arc fault circuit interrupters (AFDDs) are available as protection devices [36]. However, their use is still restricted to some countries.

Earth leakage is the term used to describe the passage of electrical current through unintended paths caused by insulation failure or inadequate grounding [31]. This can be caused by various factors such as damage to the insulation layer, contact with conductive materials or the ground, and other electrical faults. This fault can potentially cause overheating and ignition of the surrounding materials, leading to a fire outbreak. When an electrical current leaks to the ground, it can cause excessive heating and result in the degradation of the insulation layer, resulting to further arcing and sparking. This arcing and sparking can continue, conducting to the ignition of the surrounding materials and potentially causing a fire [37]. The stray currents occurrence is also a low dielectric strength in exposed connection. The live parts are exposed to moisture and pollutants, in which wet tracking can occur. The insulation material surrounding this area is gradually heated till carbonization, which forms a carbonized path, a way to current leaks [38]. Despite the existence of proper protection devices, such as ground fault interrupters (GFCI) and residual current circuit breakers (RCCB), most cases aren't present in all branches. Beyond that, to work properly, certain requisites, such as proper installation and regular maintenance, are required.

3.2. Electrical failure causes overview

Failures of electrical systems and components are related to the three items that can be measured. There are three ways that an electrical system fails – insulation loss, connections, and transients. [28]

Insulation loss - Cable overheating is one of the primary causes of electrical cable insulation degradation, typically associated with overload and short-circuits. However, there are other factors that can contribute to cable overheating, compromising its insulation and increasing the likelihood of electrical failures. However, several factors can contribute to the degradation of insulation, including mechanical damages, aging, exposure to elevated temperatures, mechanical stress, chemical exposure, moisture, and overvoltage [39]. These factors include the presence of harmonic distortion in the electrical system, which can increase the cable's ohmic resistance through the skin effect, reducing the effective cross-sectional area for current flow and causing more significant heating [1]. Additionally, the third harmonic ratio distortion, greater than 33%, can origin a neutral overload as an unbalanced load, in a grounded four-wire system [7]. It is particularly dangerous because most neutral conductors are not protected by Over Current Protection Devices (OCPD). Branch circuits passing through thermal insulation can also result in localized overheating and insulation damage. Thermal insulation impedes heat transfer, forcing the cable to retain more heat than it would in a non-insulated environment. This retained heat can cause the cable's operating temperature to rise above safe levels, leading to insulation damage and potential electrical faults [32]. Inadequate ventilation can also contribute to excessive temperature in cables. When cables are not adequately ventilated, heat can accumulate in the cable's surroundings, reducing the heat dissipation capacity and leading to the rise in the operating temperature. External heating sources can also cause excessive temperature in cables. Cables located near heat sources, such as boilers or industrial equipment, can absorb heat, also causing the rising in the operating temperature.

Connections - Poor connections are a quite common electrical issue and underestimated hazard. It is a consensus that poor connections (overheating or glowing connections) are the single most important causes of electrical fires [3]. The loosening of electrical connections is a natural trend, because of the mechanical stress caused by thermal elongation and contraction of the operation cycling [1]. The current flowing through a contact, characterized by an electrical resistance increase, caused by a reduced effective contact area in loose connections, provokes a joint temperature elevation. Even though tinny resistance addition and low power dissipation, the heating generated is concentrated in a small area enabling great temperature rising, creating an oxidation breeding process. The feedback of the process, increases the temperature, even more makes it glow, consequently the heating generated can ignite surrounding materials [40].

Transients - Lightning strikes can induce overvoltage in electrical power systems, generating a phenomenon called Lightning Electromagnetic Pulse (LEMP). This overvoltage can result in wiring electrical breakdown, causing insulation damage and electrical failures [39]. The use of Surge Protection Devices (SPDs) is an effective measure to mitigate the risks associated with LEMP in electrical power systems. These devices divert the surge current to ground, avoiding that a pulse reaching the connected equipment, reducing the risk of damage due to overvoltage. However, to ensure proper operation of SPDs, it is crucial to consider certain installation requirements. Firstly, SPDs must be appropriately selected for the specific application, considering factors such as the type of equipment being protected, the expected surge current magnitude and the system's operating voltage. Secondly, SPDs must be installed at the correct location within the electrical system. For example, the device should be installed as close as possible to the protected equipment and the length of the connecting cables, between the SPD and the equipment, should be minimized to reduce the risk of voltage drop and inductance. Furthermore, the grounding of SPDs is critical to their proper operation. The device must be connected to a l ow-impedance grounding system to ensure effective surge current diversion to ground. The grounding system should comply with applicable standards and regulations, such as the National Electrical Code (NEC) in US [7].

4. Methodology

The development of the proposed methodology consists of three main stages: the first stage is the modeling of possible failures in electrical installations; the second one is the development of a verification script to evaluate the current conditions of the installation. Finally, an algorithm was implemented to simulate the responses of the system.

The first stage involves the identification and modeling of potential failures and malfunctions in electrical systems. This was achieved through a systematic analysis of the system's characteristics, design, and risk factors, among other. The information gathered during this stage was used to develop a Fuzzy Petri net (FPN) model that could help identifying the need for corrective maintenance and estimate the safety impact of installation modifications.

In the second stage, a verification script was developed to evaluate the current conditions of the electrical installation. The script was designed to assess the general condition of the system, including its electrical components and connections, in order to predict the system's behavior and estimate the possibility of electrical issues causing ignition.

Finally, an algorithm was idealized to simulate the responses of the system under different conditions. The algorithm was developed based on the FPN model and the verification script, The complete methodology was implemented using MATLAB software. The algorithm allowed for the simulation of the system's behavior under different scenarios, which could help identify potential problems and inform decisions related to maintenance and modification.

4.1. Electrical fires FPN

The occurrence of electrical fires is a complex phenomenon that involves various failure modes, mechanisms, and risk factors. In the previous section, a literature review was presented to provide an overview of these factors. However, due to the intrinsic complexity, additional information was necessary to understand all the possibilities of factors and the involved interactions. To address this complexity, interviews with 64 specialists were conducted, in order to fill the gaps in information and provide a more comprehensive understanding of the factors and interactions that can contribute to electrical fires.

The interviews with specialists provided valuable insights into the various failure modes and mechanisms that can lead to electrical fires. One of the key factors highlighted by the specialists was the age of electrical installations. The likelihood of failure in electrical installations tends to increase with age, as they become more vulnerable to wear and tear, corrosion, and various other factors. The periodicity of inspections was also identified as an essential factor in preventing electrical fires. Regular inspections can help identifying potential hazards and allow for maintenance and repairs to be performed before they escalate into serious issues that could lead to electrical fires. Moreover, the interviews revealed that human error, such as improper installation or maintenance of electrical systems and equipment, can also contribute to electrical fires. By utilizing Petri net notations, a comprehensive model of the different electrical faults, failures, physical and chemical processes and risk factors that can contribute to electrical fires was created. This model required a considerable effort in its development, but it provides a powerful tool to represent the interactions between these factors. Using this model, it becomes possible to predict the behavior of the electrical system and evaluate the likelihood of fire outbreak. By analyzing the interactions between different components of the system, the model can identify potential hazards and provide insights into the factors that contribute to the occurrence of electrical fires. The graphical representation of FPN is presented Fig. 4.

To proceed a computational simulation, the system is represented by a linguistic rules, where the rest of parameters: weight (w); threshold (τ) and certainty (μ) must be defined by the specialist using Table 2.

Some factors are direct cause and effect, such as some of the physical processes represented in the model. In these cases, the value CF = 1 is used. In others, the possibility of an event depends on other unknown

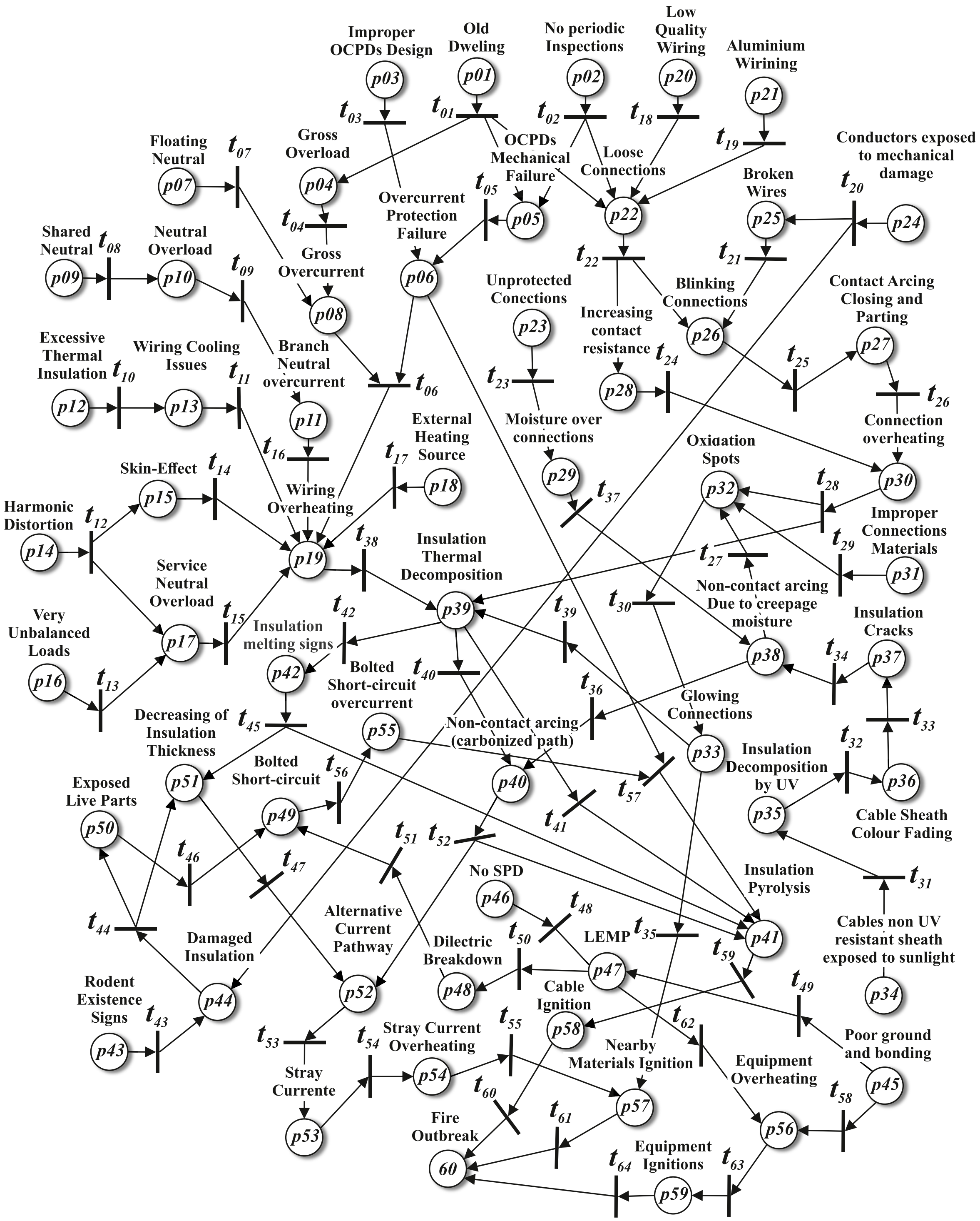


Fig. 4. Electrical fire fuzzy Petri net.

Table 2
Proposed scale for the model.

CF (μ)	Possibility	W	Weight	τ	Threshold
0.25	Possible	*0.5*	Used for Risk Factors	*0.25*	Likelihood
0.5	Likely	*1.0*	Used for failure/malfunction		
0.75	Very Likely				
1.0	Imminent				

conditions, such as the temperature of the cable, the percentage of its current carrying ability exceeded, etc. For these cases, CF values were used: 0.25, 0.5, and 0.75, depending on the situation. For the weight prepositions, W, was adopted 0.5 for a risk factor and 1.0 for failures or malfunctions founds. The threshold value was initially set to 0.25 for the entire Petri net. All its parameters were defined by the specialist's judgment because the lack of failure rates makes it impossible to probabilistically define the occurrence chances of each event. Based on these statements and on the expert judgment, the fuzzy production rules were defined, as can be seen in Table 3.

4.2. *Current condition evaluation*

To determine the initial state matrix M_0 for the proposed model, an inspection script was developed to evaluate the current conditions of electrical installations in a simple and straightforward way. The script consists of a yes/no checklist that enables electrical professionals, who may not have fire safety expertise, to proceed with a fire risk assessment. The checkpoints were defined based on verifications suggested in international standards [7,8], but some aspects were added as risk factors based on a survey with specialists, such as the age of the installation.

The user fills the questionnaire shown in Table 4, indicating whether a particular aspect is present in the installation by checking 'yes' or 'no'. If the question is checked, the truth value of the corresponding preposition is 1; otherwise, it is zero. The values obtained from the questionnaire responses are then used to compose a (1 × 60) matrix that serves as input to the algorithm.

$$M_0 = \begin{bmatrix} M_{1-1} \\ \dots \\ M_{60-1} \end{bmatrix}_{60\times 1} \tag{3}$$

The checklist covers various aspects of the electrical installation, including potential problems like overloads (p04) caused by excessive electrical equipment, unbalanced phases (p16), stray currents (p53), and bolted short circuits (p49), which are always possible and have a minimum value of 0.25 (possible) regardless of the questionnaire response. For other aspects, the value starts as 0, and it is determined by the questionnaire responses. The inspection script is a useful tool for assessing the initial state of the installation and determining the risk factors that need to be addressed to improve fire safety.

Although some risk factors, such as overloads (p04) caused by excess electrical equipment, unbalanced phases (p16), stray currents (p53), and bolted short circuits (p49), are always possible, the minimum value for these prepositions is 0.25, indicating a possibility of occurrence regardless of the questionnaire results. For other factors, the initial value is set as 0, representing no possibility of occurrence until identified during the inspection.

Verifying all the aspects listed in the electrical system verification process can be a challenging task, but it is crucial to ensure electrical safety. While it may not be practical to check every electrical connection and junction, it is recommended to inspect as many as possible since loose connections can be the most dangerous issue for an installation.

During an inspection, if a hazardous situation is found, it is expected that the professional would immediately interrupt the verification process. However, such situations may require more than just electrical expertise. The proposed tool can assist professionals in identifying areas that require intervention through simulation.

Table 3
Fuzzy linguistic rules.

Ri	Rule	Ri	Rule	Ri	Rule
R1	IF d_{j01} THEN $d_{k04}, d_{k05}, d_{k22}$ (0.5; 0.25, 0.25, 0.25; 0.5,0.25,0.5)	R23	IF d_{j23} THEN d_{k29} (0.5; 0.25; 1.0)	R46	IF d_{j50} THEN d_{k49} (1.0; 0.25; 0.25)
R2	IF d_{j02} THEN d_{k05}, d_{k22} (0.5; 0.25, 0.25; 0.25, 0.5)	R24	IF d_{j28} THEN d_{k30} (1.0; 0.25; 0.25)	R47	IF d_{j51} THEN d_{k52} (1.0; 0.25; 1.0)
R3	IF d_{j03} THEN d_{k06} (0.5; 0.25; 1.0)	R25	IF d_{j26} THEN d_{k27} (1.0; 0.25; 1.0)	R48	IF d_{j46} THEN d_{k47} (0.5; 0.25; 0.25)
R4	IF d_{j04} THEN d_{k08} (1.0; 0.25; 1.0)	R26	IF d_{j27} THEN d_{k30} (1.0; 0.25; 1.0)	R49	IF d_{j45} THEN d_{k47} (0.5; 0.25; 0.25)
R5	IF d_{j05} THEN d_{k06} (1.0; 0.25; 1.0)	R27	IF d_{j38} THEN d_{k32} (1.0; 0.25; 1.0)	R50	IF d_{j47} THEN d_{k48} (1.0; 0.25; 0.75)
R6	IF d_{j06} *and* d_{j08} THEN d_{k19} (0.5, 0.5; 0.25; 1.0)	R28	IF d_{j30} THEN d_{k32}, d_{k39} (1.0; 0.25, 0.25; 1.0, 0.75)	R51	IF d_{j48} THEN d_{k49} (1.0; 0.25; 1.0)
R7	IF d_{j07} THEN d_{k08} (1.0; 0.25; 0.75)	R29	IF d_{j31} THEN d_{k32} (1.0; 0.25; 0.5)	R52	IF d_{j40} THEN d_{k52}, d_{k41} (1.0; 0.25, 0.25; 1.0, 0.75)
R8	IF d_{j09} THEN d_{k10} (1.0; 0.25; 0.75)	R30	IF d_{j32} THEN d_{k33} (1.0; 0.25; 0.75)	R53	IF d_{j52} THEN d_{k53} (1.0; 0.25; 1.0)
R9	IF d_{j10} THEN d_{k11} (1.0; 0.25; 1.0)	R31	IF d_{j34} THEN d_{k35} (0.5; 0.25; 0.5)	R54	IF d_{j53} THEN d_{k54} (1.0; 0.25; 0.75)
R10	IF d_{j12} THEN d_{k13} (1.0; 0.25; 1.0)	R32	IF d_{j35} THEN d_{k36} (1.0; 0.25; 1.0)	R55	IF d_{j54} THEN d_{k57} (1.0; 0.25; 0.5)
R11	IF d_{j13} THEN d_{k19} (1.0; 0.25; 0.75)	R33	IF d_{j36} THEN d_{k37} (1.0; 0.25; 0.5)	R56	IF d_{j06}*and* d_{j49} THEN d_{k55} (0.5, 0.5; 0.25; 1.0)
R12	IF d_{j14} THEN d_{k15}, d_{k17} (1.0; 0.25, 0.25; 0.25, 0.25)	R34	IF d_{j37} THEN d_{k37} (1.0; 0.25; 0.5)	R57	IF d_{j55} THEN d_{k58} (1.0; 0.25; 1.0)
R13	IF d_{j16} THEN d_{k17} (1.0; 0.25; 0.5)	R35	IF d_{j33} THEN d_{k57} (1.0; 0.25; 1.0)	R58	IF d_{j45} THEN d_{k56} (1.0; 0.25; 0.25)
R14	IF d_{j15} THEN d_{k19} (1.0; 0.25; 0.25)	R36	IF d_{j38} THEN d_{k40} (1.0; 0.25; 0.75)	R59	IF d_{j41} THEN d_{k58} (1.0; 0.25; 0.75)
R15	IF d_{j17} THEN d_{k19} (1.0; 0.25; 1.0)	R37	IF d_{j38} THEN d_{k40} (1.0; 0.25; 0.5)	R60	IF d_{j58} THEN d_{k60} (1.0; 0.25; 0.75)
R16	IF d_{j11} THEN d_{k19} (1.0; 0.25; 1.0)	R38	IF d_{j19} THEN d_{k39} (1.0; 0.25; 1.0)	R61	IF d_{j57} THEN d_{k60} (1.0; 0.25; 1.0)
R16	IF d_{j11} THEN d_{k19} (1.0; 0.25; 1.0)	R39	IF d_{j33} THEN d_{k39} (1.0; 0.25; 1.0)	R62	IF d_{j47} THEN d_{k56} (1.0; 0.25; 0.5)
R17	IF d_{j18} THEN d_{k19} (1.0; 0.25; 0.5)	R40	IF d_{j39} THEN d_{k40} (1.0; 0.25; 0.75)	R63	IF d_{j56} THEN d_{k59} (1.0; 0.25; 0.5)
R18	IF d_{j20} THEN d_{k22} (0.5; 0.25; 0.75)	R41	IF d_{j39} THEN d_{k41} (1.0; 0.25; 0.5)	R64	IF d_{j59} THEN d_{k60} (1.0; 0.25; 1.0)
R19	IF d_{j21} THEN d_{k22} (0.5; 0.25; 1.0)	R42	IF d_{j39} THEN d_{k42} (1.0; 0.25; 1.0)		
R20	IF d_{j24} THEN d_{k25}, d_{k44} (0.5; 0.25, 0.25; 0.25, 0.25)	R43	IF d_{j43} THEN d_{k44} (0.5; 0.25; 0.25)		
R21	IF d_{j25} THEN d_{k26} (1.0; 0.25; 0.75)	R44	IF d_{j44} THEN d_{k50}, d_{k51} (1.0; 0.25, 0.25; 0.5, 1.0)		
R22	IF d_{j22} THEN d_{k26}, d_{k28} (1.0; 0.25, 0.25; 0.5, 1.0)	R45	IF d_{j42} THEN d_{k41}, d_{k51} (1.0; 0.25, 0.25; 0.75, 1.0)		

During one inspection it is expected that the professional interrupts the verification immediately in case of finding a hazardous situation. However, it may require more than electrical expertise and the proposed tool can help to indicate to the professional the needed to intervention through the simulation.

Table 4
Questionnaire to determinate $\mathbf{M_0}$.

		[X]	p_i = 1.0
1)	Has the installation more than 30 years old?	[ ]	p01
2)	Are there lack of periodic maintenance and inspections?	[ ]	p02
3)	Are there inconsistencies in the design of circuit breakers and/or RCCB?	[ ]	p03
4)	Are there reports of frequent breaker tripping?	[ ]	p04
5)	Have automatic switching devices problems in functional tests?	[ ]	p06
6)	Are there any problems between the neutral of the installation and the grounding of the system?	[ ]	p07
7)	Is the same neutral shared for more than one circuit?	[ ]	p09
8)	Are there circuits passing through materials used for thermal insulation?	[ ]	p12
9)	The levels of harmonic distortion are above the limit recommended by the standard?	[ ]	p14
10)	Is there a marked difference between the currents of each phase?	[ ]	p16
11)	Are there equipment or heat sources causing cable heating?	[ ]	p18
12)	Are thermal anomalies verified through thermographic inspections?	[ ]	p19
13)	Are there poorly executed cable splices, lack of signaling?	[ ]	p20
14)	Does the installation use aluminum conductors in low voltage circuits?	[ ]	p21
15)	Are loose connections found in outlets, switches, or do you have breaker terminals?	[ ]	p22
16)	Are there connections outside junction boxes or sealing problems in electrical cabinets?	[ ]	p23
17)	Are there conductors exposed to mechanical damage?	[ ]	p24
18)	Are there partially broken conductors?	[ ]	p25
19)	Is there excessive dust or moisture on electrical connections or terminals?	[ ]	p29
20)	Are there direct connections between copper and aluminum?	[ ]	p31
21)	Are there signs of oxidation on electrical connections?	[ ]	p32
22)	Are glowing connections found?	[ ]	p33
23)	Are there non-protected PVC cables exposed to direct sunlight?	[ ]	p34
24)	Are there wires exposed to sun showing color fading?	[ ]	p36
25)	Are there wires with cracks in their insulation?	[ ]	p37
26)	Are there any wires with signs of charred insulation?	[ ]	p40
27)	Are there wires with signs of melting?	[ ]	p42
28)	Are there damages resulting from rodent animals such as rats?	[ ]	p43
29)	Are there any cables with their cover damaged?	[ ]	p44
30)	Is there lack of protective conductor in outlets or high fault loop resistance?	[ ]	p45
31)	Is there no SPD in the main electrical panel?	[ ]	p46
32)	Are there live parts exposed, with risk of accidental contact?	[ ]	p50
33)	Is insulation resistance below recommended?	[ ]	p53

4.3. Automatic execution

Calculating complex systems represented by a Fuzzy Petri Net (FPN) manually can be a challenging task. Fortunately, several techniques have been proposed to facilitate computational implementation. In this technique, each place and transition in the FPN is assigned a unique index, and a matrix is created to represent the FPN's firing rules. The matrix contains rows for each transition and columns for each place. The values in the matrix represent the degree of membership of a given transition to a specific place. This membership degree can be represented as a real number between 0 and 1. To simulate the behavior of the FPN, a marking vector is created, which represents the current state of each place in the FPN. The marking vector is then multiplied by the matrix of firing rules to obtain a vector of firing weights for each transition. The transition with the highest firing weight is then fired, and the marking vector is updated accordingly. Matrix operators are used to manipulate the matrices and marking vectors. For example, to calculate the maximum firing weight for each transition, a maximum operator is applied to the firing weight vector. One such technique is the use of a matrix representation, which requires the use of some operators, such follow [12].

1) "OR" Operator $\bigoplus$: $\boldsymbol{A \oplus B = D}$, where A, B and C are all $(\boldsymbol{m \times n})$ matrices and $\boldsymbol{a_{ji}, b_{ji}}$ ***and*** $\boldsymbol{d_{ji}}$ are their elements, such that:

$$d_{ji} = max\left(a_{ji}, b_{ji}\right) \tag{4}$$

2) Composition Operator $\circ$: $\boldsymbol{A \circ B = D}$, where A, B and C are all $(\boldsymbol{m \times n})$ matrices and $\boldsymbol{a_{ji}, b_{ji}}$ ***and*** $\boldsymbol{d_{ji}}$ are their elements, such that:

$$d_{ji} = a_{ji} \times b_{ji} \tag{5}$$

3) "AND" Operator $\bigotimes$: $\boldsymbol{A \bigotimes B = D}$, where A, B and C are $(\boldsymbol{m \times p}), (\boldsymbol{p \times n})$ ***and*** $(\boldsymbol{p \times n})$ matrices, respectively, and $\boldsymbol{a_{ji}, b_{ji}}$ ***and*** $\boldsymbol{d_{ji}}$ are their elements, such that:

$$d_{ji} = {}^{max}_{1 \le k \le p}\left(a_{ji} \bullet b_{ji}\right) \tag{6}$$

Based on the statements presented, it is feasible to calculate the production rules, as shown in Table 5.

The use of matrix representation in FPNs has several advantages, such as simplifying the computation of the firing rules and enabling efficient simulation of the FPN. Additionally, it allows for the use of established matrix manipulation techniques, which have been extensively studied in the field of linear algebra. In this paper, the automatic execution algorithm proposed by Liu et al. [26] was chosen to be implemented in the MATLAB software for case studies in section 5. Implementing the algorithm in MATLAB allowed for the development of a user-friendly application that can be used to evaluate the safety and reliability of electrical installations.

4.3.1. Execution Algorithm

Input: I, O, W, U, Th_I and Th_O are $(m \times n)$ matrices; M_0 is a vector of length m representing the initial states of the model.

Output: M_k is a vector of length m representing the final state of all prepositions.

Step 1: Start - Let $k = 1$, Where k is an iterator.

Step 2: Compute the input enabled matrix $D^{(k)}$ that indicates the enabled input arcs of the transitions. Let ${x_{ij}}^{(k)}$ be the comparison result between the fuzzy truth and the input threshold of the transition t_i at the iteration k, then:

$$X^{(k)} = \left(x_{ij}^{(k)}\right)_{mxn} = M_k^{'} - Th_I, i = 1, 2, \ldots, m; j = 1, 2, \ldots, n \tag{7}$$

Where $M_k^{'} = [M_{k-1}, M_{k-1}, \ldots, M_{k-1}]_{m \times n} \circ I$. Here, $M_k^{'}$ is an assignment matrix, which is used to assign the fuzzy truth of input places to the input arcs of the transitions so that the fuzzy truth and the input threshold can be compared (the composition operator, $\circ$, is defined previously (5)).

Let ${d_{ij}}^{(k)}$ be the function of the comparison result ${x_{ij}}^{(k)}$, then:

Table 5
Fuzzy production rule calculation.

Rule Type	Transition	Linguistic Rule	Calculation
Simple	t_i	**If** $\alpha(p_i)$ **THEN** $\beta(p_i)$ $(1;\ \tau; \mu)$	$\beta(p_i) = (d_j)\mu$ If $(d_j)\mu \ge \tau$
AND	t_i	**If** $\alpha(p_{i1})$ **AND** $\alpha(p_{i2})$ **THEN** $\beta(p_i)$ $(w_1,\ w_2;\ \tau;\ \mu)$,	$\beta(p_i) = (d_{j1} \times w_1 + d_{j2} \times w_2)\mu$ If $(d_{j1} \times w_1 + d_{j2} \times w_2)\mu \ge \tau$
OR	t_{i1}	**If** $\alpha(p_{i1})$ **THEN** $\beta(p_i)$ $(1;\ \tau_{j1}; \mu_{j1})$	$\beta(p_i) = \max(d_{j1} \times \mu_1, d_{j2} \times \mu_2)$ *for* $d_{j1} \ge \tau_1, d_{j2} \ge \tau_2$
	t_{i2}	**If** $\alpha(p_{i2})$ **THEN** $\beta(p_i)$ $(1;\ \tau_{j2}; \mu_{j2})$	

$$D^{(k)} = \left(d_{ij}\right)_{mxn}^{(k)} d_{ij} = \begin{cases} 1, x_{ij} \geq 0 \\ 0, x_{ij} < 0 \end{cases} i = 1, 2, \ldots, m, j = 1, 2, \ldots, n. \tag{8}$$

Step 3: If $D^{(k)}$ is a non-zero matrix, the input weight matrix $\Theta^{(k)}$ is computed by (9), otherwise, go to step 8:

$$\Theta^{(k)} = D^{(k)} \circ W. \tag{9}$$

Step 4: Compute the vector of equivalent fuzzy truth values of transitions:

$$\Gamma^{(k)} = \left(\Theta^{(k)}\right)^T M_{(k-1)} \tag{10}$$

Step 5: Compute the output enable matrix $E^{(k)}$ indicating the enabled output arcs of the transitions. Let $y_{ij}^{(k)}$ be the comparison result between the equivalent fuzzy truth value and the output threshold of transition t_i at iteration k, then:

$$Y^{(k)} = \left(y_{ij}^{(k)}\right)_{mxn} = N^{(k)} - Th_O \; i = 1, 2, \ldots, m, j = 1, 2, \ldots, n. \tag{11}$$

Where:

$$N^{(k)} = \left[\left(\Gamma^{(k)}\right)^T, \left(\Gamma^{(k)}\right)^T, \ldots, \left(\Gamma^{(k)}\right)^T\right]_{m \times n}^T \circ O. \tag{12}$$

Let $e_{ij}^{(k)}$ be the function of comparsion result $y_{ij}^{(k)}$, then:

$$E^{(k)} = \left(E_{ij}\right)_{mxn}^{(k)} e_{ij} = \begin{cases} 1, y_{ij} \geq 0 \\ 0, y_{ij} < 0 \end{cases} \quad i = 1, 2, \ldots, m, j = 1, 2, \ldots, n. \tag{13}$$

Step 6: If $E^{(k)}$ is a non-zero matrix, the output certainty factor, matrix $\Psi^{(k)}$, is computed by (14); otherwise, go to step 9.

$$\Psi^{(k)} = E^{(k)} \circ U \tag{14}$$

Step 7: Compute new marking M_k.

$$M_k = M_{k-1} \bigoplus \left(\Psi^{(k)} \bigotimes \Gamma^{(k)}\right) \tag{15}$$

Step 8: If $M_k = M_{k-1}$, go to step 9, otherwise go to step 1 (let $k = k + 1$).

Step 9: End.

5. Case studies

The proposed method was implemented in the MATLAB software to show its functionality and potential. The first step of the method is to conduct an electrical system verification to identify the system's characteristics, design, risk factors, pathological manifestations, malfunctions, and defects. This verification is essential to gather relevant information that will serve as input for the Fuzzy Petri net (FPN) model developed. The FPN model developed can simulate the behavior of the electrical system and estimate potential hazards and risks associated with the system. By using the FPN model, the simulations can predict the system's response and estimate the possibility of ignition caused by a critical fault. It is important to note that the term possibility is used instead of probability due to the uncertainties related to the fuzzy set theory used in the method. The application of the proposed method is summarized in Fig. 5.

Simulations were performed initially to find the relevance of each verification and identify which situations represent a higher hazard. The second case study involved evaluating the fire possibility of two dwellings and comparing the results with a classical risk indexing approach. In the third case study, the implementation of a new protection device, an arc fault circuit interrupter (AFDD), was simulated to estimate the safety improvement in a hypothetical facility.

5.1. *Case study 1 - ranking the hazards*

Evaluating the effect of each factor on the fire hazard makes it possible to build a hierarchy of prepositions that refer to the most dangerous situations. To evaluate the model response and the significance of each input factor, simulations were performed using the thirty-three items of the questionnaire individually. In the first thirty-three simulations, a threshold value of $\tau = 0.25$ was applied to it. However, the result of many factors was insignificant, which limited the evaluation of the effects. An added thirty-three simulations were performed using $\tau = 0.0$ to increase the sensitivity of the model. Results are shown in Table 6.

The results of the simulations conducted using the proposed risk assessment method based on Fuzzy Petri nets (FPNs) demonstrating that the system becomes more suitable for detailed assessment when using the most sensitive model with a threshold value of $\tau = 0.0$. By using this model, it was possible to identify sixteen factors that can significantly increase the likelihood of a fire risk ($p60 > 0.25$).

The simulations further revealed that Glowing Connections (p33) represent an imminent fire risk with a p60 value of 1.0. Similarly, Oxidation Spot (p32) is likely to turn into a fire outbreak, with a p60 value of 0.75. These results highlight the effectiveness of the proposed method in identifying the critical factors that contribute to the fire hazard and the potential dangers associated with specific factors.

5.2. *Case study 2 – comparison with a typical indexing method*

The conventional approach to electrical installation verifications is based on a binary pass or fail outcome checklist that relies on compliance with applicable standards. However, such a checklist is inadequate for risk management, and fire risk indexing techniques have been proposed as an alternative approach. Fire risk indexing involves assigning values to selected variables based on professional judgment and past experience, which are then operated on by some combination of arithmetic functions to arrive at a single value representing the likelihood of a fire outbreak. One limitation of fire risk indexing is that it does not account for the interdependent effects of failure mechanisms when calculating the overall risk as the sum of individual factor contributions [9].

To better illustrate the comparison between the index method and the fuzzy method, we can consider two dwellings, A and B, which do not comply with standards and would not be approved based on the binary

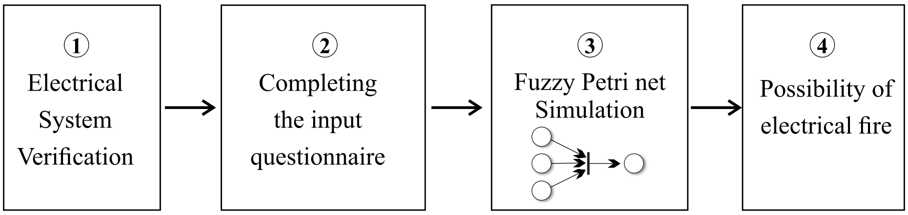


Fig. 5. The application of the proposed method.

Table 6
System response for each hazard form checklist.

Single factor impact	Fire Outbreak Risk (p60)		Single factor impact	Fire Outbreak Risk (p60)	
	τ = 0.25	τ = 0.0		τ = 0.25	τ = 0.0
p01 – Aged Dwelling	0.00	0.11	p25 - Broken Conductors	0.56	0.56
p02 - No Inspections	0.00	0.11	p29 - Moisturized Connections	0.38	0.38
p03 - Improper OCPDs Design	0.28	0.35	p31 - Improper Materials	0.38	0.38
p04 - Gross Overload	0.21	0.21	p32 - Oxidation Spots	0.75	0.75
p06 - Protection failure	0.28	0.35	p33 - Glowing Connections	1.00	1.00
p07 - Floated Neutral	0.00	0.16	p34 - UV cable exposed	0.00	0.09
p09 - Shared Neutral	0.32	0.32	p36 - Sheath Color Fading	0.00	0.19
p12 - Thermal Insulation	0.32	0.32	p37 - Insulation Cracks	0.38	0.38
p14 – Harmonic Distortions	0.00	0.11	p40 - Carbonization Signs	0.42	0.42
p16 - Unbalanced Load	0.21	0.21	p42 - Melting Signs	0.42	0.42
p18 - Heating Source	0.21	0.21	p43 - Rodents Sings	0.00	0.09
p19 - Wiring Overheating	0.42	0.42	p44 - Damaged Insulation	0.38	0.38
p20 - Poor Quality	0.00	0.14	p45 - Poor Ground or Bonding	0.00	0.13
p21 - Aluminum Wiring	0.00	0.19	p46 - No SPD	0.00	0.09
p22 - Loose Connections	0.38	0.38	p50 - Exposed Live Parts	0.00	0.09
p23 - Unprotected Connections	0.00	0.19	p53 - Stray Currents	0.38	0.38
p24 - Unprotected Conductors	0.00	0.09			

pass or fail outcome. In dwelling A, several factors were identified, including an aged dwelling (p01), no periodic inspections (p02), improper OCPD design (p03), floated neutral (p07), and harmonic distortion (p14). In contrast, dwelling B had unprotected connections (p23), unprotected conductors (p24), and oxidation spots over connections (p32). The fire risk indexing approach, which considers the individual contribution as shown in Table 6, suggested that installation A had a higher likelihood of a fire outbreak than installation B. However, this approach does not account for the interdependent effects of failure mechanisms.

In contrast, the assessment based on the response model developed using the fuzzy method considers these interdependencies and provides a more realistic result. As shown in Table 7, the FPN-based method indicates that dwelling B has a higher overall risk of a fire outbreak than dwelling A. This outcome demonstrates the value of using the FPN-based method to account for the complex interactions between failure mechanisms, leading to a more accurate assessment of the overall risk.

Table 7
Results comparison between FPN and a common risk indexing technique.

Issues Found	Individual contribution in risk	Installation	
		A	B
Aged Dwelling	p01 (0.11)	**[X]**	
No Inspections	p02 (0.11)	**[X]**	
Improper Design OCPDs	p03 (0.35)	**[X]**	
Floated Neutral	p07 (0.16)	**[X]**	
Harmonic Distortions	p14 (0.11)	**[X]**	
External Heating Source	p18 (0.21)	**[X]**	
Unprotected Connections	p23 (0.18)		**[X]**
Unprotected Conductors	p24 (0.09)		**[X]**
Oxidation Spots	p32 (0.75)		**[X]**
Common Risk Indexing Technique (sum of individual contributions) – Overall risk:		**1.05** >	1.02
FPN (Fuzzy production rules) - Overall risk:		0.35<	**0.75**

Issues Found Individual contribution in risk Installation.

The conducted simulation emphasizes the distinction between system responses when employing a static indexing system (with fixed weights) and a dynamic system, like the proposed one. Ignition mechanisms involved in electrical fires exhibit strong correlations with each other. The influence of a specific factor on the overall risk can vary based on the combination of other factors, considering processes such as failure feedback loops. The Fuzzy Petri Net-based method accounts for the interdependencies between failure mechanisms, offering a more realistic and accurate response for complex systems such as these.

5.3. *Case study 3 - safety impact of AFDD*

The implementation of protection devices, such as Arc Fault Detection Device (AFDDs), can contribute to reducing the chances of certain electrical faults and ultimately improving safety. However, estimating the impact of such devices on overall safety is not a straightforward task, as the interactions involved are complex. It is important to note that reducing the occurrence of a particular fault by 50% does not necessarily translate to a 50% improvement in safety. The proposed model can represent such scenarios by altering the certainty factor, which indicates the degree of belief in a particular transition.

To illustrate the capability of the model, a simulation was conducted to evaluate the impact of introducing AFDDs. Specifically, the device can reduce the risk of series arcing, which is represented in the model by the certainty factor matrix U. The model accounts for both contact and non-contact arc faults, represented by factors p27 and p38, respectively. Three transitions and rules are associated with these factors, namely 25, 34, and 37, with corresponding certainty factors μ (27–25), μ (38–34), and μ (38–37). Reducing the likelihood of arc fault occurrence involves reducing these certainty factors, as shown in Table 8. It is important to note that AFDDs cannot eliminate every risk of arc faults, which is reflected in the nonzero certainty factors for each transition.

To evaluate the risk reduction provided by the AFDD, the model was run for each preposition individually using the modified certainty factors, as shown in Table 6. The results are presented in Table 9, which shows that the AFDD can significantly reduce the fire risk in some cases. For instance, the risk of broken conductors can be reduced by up to 75% with the introduction of an AFDD.

Based on the outcomes of the simulations, the introduction of AFDD only affects some of the failure occurrences, resulting in an average risk reduction of 14%. This is not surprising since many of these defects do not have a direct relationship with arc faults. However, this example demonstrates the potential of the proposed model to analyze the impact of protection devices on electrical fire risk. To assess the actual impact of AFDD on safety, it is essential to identify the most common dwelling setups, considering the combination of risk factors and the most frequent defects.

6. Conclusions

In conclusion, modeling electrical fires is a complex task due to the numerous factors involved and their interactions. Traditional modeling methods often struggle to capture this complexity. However, Fuzzy Petri

Table 8
AFDD Certainty Factors adjusts.

t_i	μ_{ij}	μ_{ij} No AFDD	μ_{ij} AFDD
t_{25}	μ_{27-25}	1.0	0.25
t_{34}	μ_{38-34}	0.5	0.0
t_{37}	μ_{38-37}	0.5	0.0

Table 9
AFDD risk reduction evaluation by FPN.

Single factor impact	Fire Outbreak Risk (p60)			Single factor impact	Fire Outbreak Risk (p60)		
	No AFDD	AFDD	Risk Reduction		No AFDD	AFDD	Risk Reduction
p01 – Aged Dwelling	0.11	0.11	0%	p25 - Broken Conductors	0.56	0.14	75%
p02 - No Inspections	0.11	0.11	0%	p29 - Moisturized Connections	0.38	0.09	75%
p03 - Improper OCPDs Design	0.35	0.35	0%	p31 - Improper Materials	0.38	0.38	0%
p04 - Gross Overload	0.21	0.21	0%	p32 - Oxidation Spots	0.75	0.75	0%
p06 - Protection failure	0.35	0.35	0%	p33 - Glowing Connections	1.00	1.00	0%
p07 - Floated Neutral	0.16	0.16	0%	p34 - UV cable exposed	0.09	0.09	0%
p09 - Shared Neutral	0.32	0.32	0%	p36 - Sheath Color Fading	0.19	0.09	50%
p12 - Thermal Insulation	0.32	0.32	0%	p37 - Insulation Cracks	0.38	0.09	75%
p14 – Harmonic Distortions	0.11	0.11	0%	p40 - Carbonization Signs	0.42	0.42	0%
p16 - Unbalanced Load	0.21	0.21	0%	p42 - Melting Signs	0.42	0.42	0%
p18 - Heating Source	0.21	0.21	0%	p43 - Rodents Sings	0.09	0.09	0%
p19 - Wiring Overheating	0.42	0.42	0%	p44 - Damaged Insulation	0.38	0.38	0%
p20 - Poor Quality	0.14	0.09	33%	p45 - Poor Ground or Bonding	0.13	0.13	0%
p21 - Aluminum Wiring	0.19	0.09	50%	p46 - No SPD	0.09	0.09	0%
p22 - Loose Connections	0.38	0.19	50%	p50 - Exposed Live Parts	0.09	0.09	0%
p23 - Unprotected Connections	0.19	0.09	50%	p53 - Stray Currents	0.38	0.38	0%
p24 - Unprotected Conductors	0.09	0.09	0%				

nets (FPNs) provide a more effective approach to model system responses, utilizing their reasoning and knowledge representation capabilities, as well as their ability to manage uncertainty.

We propose a risk assessment method based on FPNs to address the challenges of modeling electrical fires. This methodology enables the development of a model that can predict the likelihood of electrical fires and identify critical factors contributing to their occurrence. FPNs allow for the examination of interactions between numerous factors, such as environmental conditions, system configurations, and human behavior. This leads to a more comprehensive understanding of the complex phenomenon of electrical fires.

The model developed using the proposed method can be used to suggest targeted interventions to reduce the risk of electrical fires. For example, the model can identify potential faults in electrical systems and recommend maintenance programs to address them. Regular maintenance can prevent faults from escalating into more serious problems that could result in fires. Moreover, FPNs can be utilized to evaluate the safety impacts of system modifications, which can help identify any additional measures needed to reduce the risk of electrical fires.

It is important to note that incorporating fuzzy logic concepts into classical risk assessment approaches does not always improve accuracy. Instead, it enables the analysis of safety and reliability in situations with uncertain data that were previously unfeasible. By incorporating fuzzy logic, these approaches provide a more flexible and comprehensive way to evaluate risk, supporting decision-making even in situations where data are scarce or uncertain.

However, it is crucial to recognize that the initial state and confidence equivalent in this paper are based on abstract data from expert opinions, resulting in a certain level of subjectivity. To achieve more accurate and reliable outcomes, it is essential to incorporate new information to refine and update the risk assessment process. Updating the risk assessment model based on the latest information and evidence can improve prediction accuracy and reduce subjectivity. Thus, it is important to consider the iterative nature of the risk assessment process when using FPNs to model electrical fires.

In summary, the proposed risk assessment method based on FPNs provides a valuable tool to address the challenges posed by the complexity of electrical fire modeling. By identifying critical factors and assessing their importance, this method can help prioritize interventions and ensure that resources are used effectively. Consequently, the use of FPNs to model the failure modes and mechanisms involved in electrical fires is a valuable approach that can enhance the safety of residential buildings and their occupants. This comprehensive method allows for better decision-making and resource allocation, ultimately contributing to the protection of lives and property.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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