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The use of kriging methods and fractal Brownian functions to model the spatial distribution of grades in orebodies

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Abstract: An ore body model is constructed from a set of sample data obtained from various locations in the ore body, thus it is a manner of representing the true phenomena. Geostatistics provides a sequence of modelling procedures that are extremely powerful but are strongly dependent on prerequisites that are embedded in probability theory, eg. the gaussianity of the data, the existence of both variance and covariance and the intrinsic hypothesis. Fractal geometry is another way of modelling the spatial distribution of a geological variable in an ore body geometry. Fractals can be used to interpolate the Z(x) geological variable at unsampled sites. One modelling method should not be considered better or worse than another. The selection of the most appropriate method is based on the available data. This study checks the fractal properties of grades using both the variogram and the distribution methods as techniques to measure the fractalness. The aim is to check the suitability of the fractal geometry theory to model ore grades in a deposit. The study develops three dimensional geological models and global resources estimation using distinct kriging methods, specifically employing ordinary, robust, indicator, universal, median polish kriging and distance kriging. Adaptations on geostatistical libraries were carried out to make these libraries able to be processed by the CRAY Y-MP. A final confrontation of the results from geostatistical conditional simulation and fractional Brownian motion function simulation where carried out in a small part of the gold deposit where the real grades are known through exhaustive sampling.

1. INTRODUCTION

Presently, ore body modelling consists basically of fitting a mathematical model to represent spatially a certain geological variable, generally the grade. An ore body model is constructed from a set of sample data obtained from various locations in the ore body, which is one manner of representation of the true phenomenon. Checking the compatibility of the data with the modelling procedure is the most important factor when assessing confidence in the ore body model. The level of confidence in the model will depend on the adequacy of the data set and the validity of the basic assumptions used in the modelling.

Geostatistics provides a sequence of modelling procedures that are extremely powerful but dependent on prerequisites that are embedded in probability theory, e.g. the existence of both variance and covariance and the verification of the intrinsic hypothesis (Matheron, 1963). A variation of basic geostatistical method involves transformation of the original geological variable to adapt the population distribution to a certain model assumption (e.g. if a random variable is transformed to fit the hypothesis necessary for a specific kriging method, geostatistics can be used with confidence that the best model of the variable is obtained). On the other hand, if the variable does not fit the theoretical assumptions then the model will fail in its attempt to best represent the true spatial distribution of the variable in the ore body.

Fractal geometry is another way of modelling the spatial distribution of a geological variable in an ore body geometry. Fractals can be used to interpolate the geological variable, Z(x) at unsampled sites. The concepts of self similarity and self affinity that form the basic assumption of fractal geometry theory have to be investigated in the context of the geological variables it is proposed to be interpolated.

2. THEORETICAL ASPECTS

Ore reserve estimation consists fundamentally of defining an orebody with respect to its size, shape, grade distribution and other geological properties essential for reserve assessment, mine planning and optimum mill recovery. The main objective is to provide a mathematically valid three dimensional representation of the deposit with the maximum possible confidence.

Geostatistical ore modelling procedures are based primarily on methods of interpolation of spatial data and application of the best linear unbiased estimator, commonly known as kriging. In the last 20 years much has been done to improve the fundamentals of the theory proposed by Matheron (1963). Most of the newer research looks for better ways to measure and model the spatial continuity of a geological variable such as grade, density or ore thickness. Unfortunately, ore genesis and grade distribution are not well enough understood to permit a deterministic approach to estimate the spatial location and the grade distribution behaviour of a certain deposit. There is a lot of uncertainty about the values at unsampled sites, justifying the use of the geostatistical approach to the problem, based on a probabilistic model that incorporates and manages these uncertainties.

The definition of geostatistical procedures makes the subject seem easy to use at first glance. Initially, an experimental and omnidirectional variogram is modelled to obtain the spatial autocovariance of the data set. Using the modelled variogram to obtain the variables necessary to run the kriging interpolation system, it is possible to interpolate the variable at defined unsampled sites. The procedure sounds simple, but in fact is fraught with difficulty and uncertainty to the unwary or inexperienced geostatistician. Fitting a variogram model properly to the experimental values is the major problem. The theoretical concept of the variogram is relatively simple and is basically the attempt to quantify the variability of stationary phenomena using the variance of the differences between the values measured at varying distances apart. Each term in the last sentence can be a source of problems when one starts the variographic studies. David (1988) pointed out different errors associated with fitting a model to an experimental variogram, discussing each of them with logical solutions. This is the essential part of geostatistics - the generation of a variogram that is acceptable or of reasonable confidence, that properly represents the spatial behaviour of the variable. Until the spatial behaviour of the variable is well understood, no estimation procedure should be carried out.

Isaaks and Srivastava (1989) pp. 278-322 describe clearly all steps in the construction of the kriging system. Ordinary kriging attempts to minimise the σ_R^2 (estimation variance) without knowing the mean error, as one does not know the true average, or the exhaustive data set distribution. This is achieved by constructing a model of the available data using the average error and the error variance in relation to the model. The backbone of kriging is the use of a probability model where the bias and the estimation variance can be calculated, afterwards choosing appropriate weights for the nearby samples to ensure that the average error for the model is zero and the modelled error estimation variance is minimised. The main difference between kriging methods and other estimation procedures, is that kriging aims to minimise the variance of the error, defining it as an optimum estimator.

The mathematical summary of steps described above can be expressed as a system of linear equations, denoted in the geostatistics jargon as the ordinary kriging system in expression 1:

$$\begin{aligned} &\sum_{j=1}^{n} w_j \widetilde{C}_{ij} + \mu = \widetilde{C}_{i0} \quad \forall\, i = 1,\ldots,n \\ &\sum_{j=1}^{n} w_j = 1 \end{aligned} \tag{1}$$

Transforming all the mathematics into words, we should simply say that the minimisation of the error variance requires $(n+1)^2$ covariances. These covariances are obtained from the spatial continuity model (variogram) we have chosen in our random function model.

3. OBJECTIVES

This study aims to determine the fractal properties of gold grades in a large low grade gold deposit using both the variogram and the distribution methods as techniques to measure the fractalness. The aim is to check the suitability of the fractal geometry theory to model and simulate ore grades in a deposit. The fractal model will be compared with the geostatistical model.

The deposit to be studied in this research is the Morro do Ouro Mine, Rio Paracatu, Brazil. The project aims to determine the global and selective local gold resources of the deposit using geostatistical and fractal geometry concepts. The main objectives are:

- Analysing the gold grade dataset by applying intensive exploratory data analysis techniques.
- The application of different spatial continuity measurement techniques including the traditional variogram, the relative and robust variogram, fractal plots and correlograms, in order to determine spatial continuity of the gold grades.
- The use of ordinary, indicator, robust and median polish kriging for global, local and selective resources estimates. A comparison of the results from different estimation methods against a true dataset sampled in a dense grid (exhaustive data set) will be carried out.
- The use of conditional simulation based on geostatistics and fractal geometry to simulate different grade spatial distribution scenarios.
- Apply the concepts of fractal geometry to measure spatial continuity and use related theory to interpolate, simulate and construct spatial distribution of grades.
- Compare and analyse the results defining the potential of using these methods for ore resources assessment and simulation of mining block grades.

4. STEPS INVOLVED ON THIS PROJECT

Geostatistical modelling of spatial data should be divided in three main sub areas:

1. exploratory data analysis of the statistical characteristics of the data set including descriptive statistics and trend analysis;
2. spatial continuity measurements using the variogram and its variations that have evolved in the last decade;
3. kriging, using a method chosen on the basis of the data set characteristics to estimate global and recoverable reserves and simulate mining.

Since the most important task in this subject is obtaining the most accurate result in modelling a certain variable, many authors (including Cressie (1991)) suggest that the intense use of exploratory data analysis tools be used to identify different populations and outliers. Normally the original raw data set, obtained from DDH (diamond drill holes) logging, includes mixed populations that are not separated before the ore body is modelled.

This project was planned in steps defined as follow:

- review of the theoretical aspects in modelling procedures using geostatistics and fractal geometry,
- establishment of a case study,
- comprehensive data analysis using exploratory data analysis techniques,
- measurements of spatial continuity using variograms, correlograms and fractal plots,
- creation of three dimensional geological models and global resources estimation using distinct kriging methods, specifically employing ordinary, robust, indicator, universal, median polish kriging and distance kriging ;
- adapt geostatistical libraries to let the libraries run at CRAY Y-MP for creating three dimensional grade distribution models using fractal geometry and kriging,

- comparison of the results of geostatistics conditional simulation and fractional Brownian motion function simulation in a small part of the gold deposit where the real grades are known through exhaustive sampling.

5. RESULTS UNTIL THIS STAGE AND FUTURE STEPS

RTZ international mining group from England, operating a gold mine located in central Brazil supplied the data set necessary for this study. Important libraries of geostatistics and fractal methods obtained from Stanford University in Fortran 77 were adapted to run at CESUP CRAY Y-MP.

Spatial continuity measurements in 2D and 3D are working appropriately including: variograms, covariograms, correlograms and relative variograms explained in details in Deutsch and Journel (1992), pp. 39-43. Post script files generated by these routines are visualised using Xgam (Chu, 1993) and Ghostview softwares both available at graphical visualisation laboratory at CESUP. A variogram ($\gamma(h)$) calculated at vertical direction is presented (Figure 1) This figure shows the spatial continuity of gold grades measured vertically. A nugget and a spherical variogram models were fitted to the experimental variogram (Journel and Huijbregts 1978, pp. 161-170).

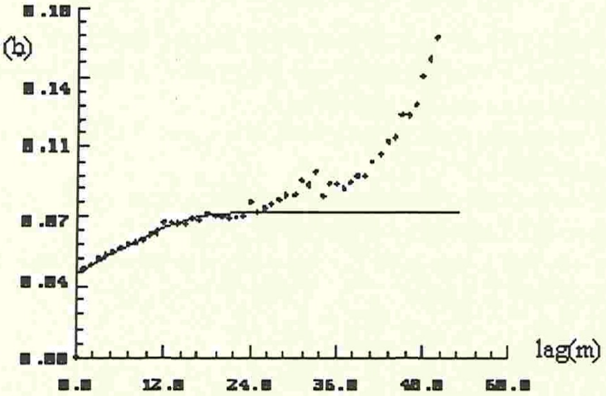

Figure 1 Experimental variogram (dots) and the respective fitted model (continuous line)

Fractal dimension and fractal simulation will be tried as to model the gold grade distribution spatially. Conditional simulation using geostatistical methods will also be tried and compared against fractal simulation results.

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