UNIVERSIDADE FEDERAL DO RIO GRANDE DO SUL
FACULDADE DE CIÊNCIAS ECONÔMICAS
PROGRAMA DE PÓS-GRADUAÇÃO EM ECONOMIA

FILIPE STONA

FISCAL POLICY UNCERTAINTY: THEORY AND MEASUREMENT

Porto Alegre
2022

FILIPE STONA

FISCAL POLICY UNCERTAINTY: THEORY AND MEASUREMENT

Tese submetida ao Programa de Pós-Graduação em Economia da Faculdade de Ciências Econômicas da UFRGS, como requisito parcial para obtenção do título de Doutor em Economia.

Orientador: Prof. Dr. Marcelo S. Portugal

Porto Alegre
2022

CIP - Catalogação na Publicação

Stona, Filipe
Fiscal Policy Uncertainty: theory and measurement / Filipe Stona. -- 2022.
75 f.
Orientador: Marcelo S. Portugal.

Tese (Doutorado) -- Universidade Federal do Rio Grande do Sul, Faculdade de Ciências Econômicas, Programa de Pós-Graduação em Economia, Porto Alegre, BR-RS, 2022.

1. New Keynesian model. 2. Fiscal Policy Uncertainty . 3. Stochastic Volatility. 4. Two-agent model. I. S. Portugal, Marcelo, orient. II. Título.

Elaborada pelo Sistema de Geração Automática de Ficha Catalográfica da UFRGS com os dados fornecidos pelo(a) autor(a).

FILIPE STONA

FISCAL POLICY UNCERTAINTY: THEORY AND MEASUREMENT

Tese submetida ao Programa de Pós-Graduação em Economia da Faculdade de Ciências Econômicas da UFRGS, como requisito parcial para obtenção do título de Doutor em Economia.

Aprovada em: Porto Alegre, 01 de abril de 2021.

BANCA EXAMINADORA:

Prof. Dr. Marcelo S. Portugal – Orientador
Universidade Federal do Rio Grande do Sul (UFRGS)

Profa. Dra. Andreza A. Palma
Universidade Federal de São Carlos (UFSCAR)

Dr. Angelo M. Fasolo
Banco Central do Brasil (BCB)

Dr. Fernando Genta dos Santos
XP Asset

ACKNOWLEDGEMENTS

I would like to thank my advisor, Marcelo Portugal, for the continuous support and unyielding guidance during these years of studies. I am also indebted to João F. Caldeira, who guided me in my first years and shared his knowledge with me; Andreza Palma, who has become a great instructor since the day I applied to this PhD program; and Igor Morais, my advisor during my Master's studies, who encouraged me to persuade this goal.

Fellow PhD and MSc students at the PPGE have been an unfailing source of great support, both intellectually and psychologically. I am especially grateful for the companionship of my friends Felipe Wolk, Hugo Szmidt Neto and Vanessa Sulzbach, the best cohort I could hope for.

I am also extremely grateful to professor Jesús Fernández-Villaverde, for whom I hold the highest admiration and who has given me one of the most precious opportunities of my life by inviting me to attend the University of Pennsylvania as a visiting scholar. I also thank professors Oren Levintal, Dirk Krueger and José-Víctor Ríos-Rull, who gave me the opportunity to discuss my research projects. A special thanks to my friends André Victor Luduvice, Charu Gupta, Matheus Sesso, Thiago Curado, Ricardo Marto, Benjamin Larin, and Le Xu for making my days at Penn better.

I would like to thank our PhD professors and UFRGS for giving me excellent support and being a great source of intellectual development. In particular, I would like to thank Eduardo Horta, Flávio Ziegelmann, Marcelo Griebeler, and Hudson Torrent.

I would also like to thank Ederson Schumanski and Pedro Lutz Ramos for their support and advice in the last years of my journey as a PhD candidate and in my first professional years as an economist. I also thank numerous seminar audiences and academics for comments and discussions that allowed me to advance my research. In special, I would like to thank Angelo Fasolo, Fernando Genta, and Guilherme Moura for their comments.

Por fim, eu gostaria de agradecer minha famíliar por serem minhas maiores inspirações. Ao meu pai e minha mãe, Dirceo e Patrica, minhas irmãs, Deborah e Priscila, e a Luíza, pela compreensão e apoio que me permitiram concluir mais essa etapa da minha vida. Sem vocês, nada disso seria possível.

ABSTRACT

This dissertation demonstrates how to measure uncertainty as a stochastic volatility (SV) process, focusing on the effects of fiscal policy uncertainty on macroeconomic aggregates. Also, the study split the standard representative agent structure into a two-agent framework, allowing the understanding of how hand-to-mouth agents are affected by these shocks and highlighting an important transmission channel through the labor market. Furthermore, this was the first study to estimate and explore the effects of tax and spending uncertainty in Brazil. SV estimation underlines the importance of proper measurement of relevant aggregate tax rates, which are still limited for Brazil. In the first chapter, we explore the literature on SV and how it can be understood as an uncertainty measurement. We develop a survey about SV models and estimation procedures using particle filters, assessing their application in the macroeconomic literature, in the hope of cooperating with future studies on the same subject, which still has much room for improvement. To demonstrate the transmission channels of fiscal policy uncertainty, we build a two-agent new Keynesian (TANK) model with SV shocks. First, we show that a TANK model highlights the importance of the labor market on the transmission of uncertainty to households, reinforcing the real consequences that uncertainty shocks have on the economy. Next, we show that these shocks can be amplified by combining a fraction of hand-to-mouth agents on the economy and their risk-aversion characteristics. By extending our analysis to a developing country and comparing results for the US and Brazil, we are able to understand what attributes of a country lead to different results, which is mainly driven by the hand-to-mouth agent's wealth characteristics. The model also allows us to understand how government spending can be managed to ease the negative impact on households, demonstrating how a policy that generates similar outcomes on output can harm agents' constitution level in such a way that a government that expands spending to counterbalance a fall in activity ends up harming agents a lot more than if it chose to cut down on expenses.

JEL: E12, E23, E32, E52, E62.

Keywords: fiscal policy. Volatility. Uncertainty. New Keynesian model. Two-Agent model.

RESUMO

Esta tese se propõe a indicar como mensurar incerteza como um processo de volatilidade estocástica, focando nos efeitos de choques de incerteza fiscal nos agregados macroeconômicas. Além disso, o estudo se distancia do conceito padrão de um agente representativo para uma estrutura com dois agentes, o que permite entender como agentes *hand-to-mouth* são impactados por esse tipo de choque e destaca a importância do mercado de trabalho como canal de transmissão. Ainda, esse é o primeiro estudo a estimar e explorar os efeitos de incerteza quanto aos gastos do governo e taxação no Brasil. A estimação da volatilidade estocástica para esses agregados destaca a importância desses dados, que ainda são limitados no Brasil. No primeiro capítulo, a literatura sobre volatilidade estocástica é analisada, destacando como esse processo pode ser compreendido como uma métrica de incerteza. Desse modo, é desenvolvida uma survey sobre os modelos e procedimentos de estimação com filtro de partículas, assim como sua aplicação na literatura de macroeconomia, com a expectativa de cooperar com estudos futuros que busquem analisar esse tempo, o qual ainda tem muitas oportunidades de desenvolvimento. Para demonstrar os canais de transmissão dos choques de incerteza fiscal, construiu-se um modelo Novo Keynesiano de dois agentes (TANK) com choques de volatilidade estocástica. Primeiro, apontamos que um TANK permite que o mercado de trabalho se mostre como um meio relevante para que o choque atinja as famílias. Em seguida, mostramos que esses choques podem ser amplificados pala proporção de agentes com acesso a títulos nessa economia, assim como suas características de aversão a risco. Essa análise é possível a partir da extensão da análise para um país em desenvolvimento, comparando resultados para os Estados Unidos e Brasil. O modelo também permite entender o impacto da reação dos gastos do governo para diminuir o efeito do choque de incerteza nas famílias, de tal modo que um governo que busque expandir os gastos para contrabalancear a queda na atividade, acaba por prejudicar mais os agentes do que se optasse por conter gastos.
Keywords: Política fiscal. Volatilidade. Incerteza, Modelo Novo Keynesiano. DSGE. Volatilidade fiscal. Filtro de partículas.

CONTENTS

1 INTRODUCTION

The significant decrease of natural rates of interest in both advanced and emerging economies has put some pressure on monetary policy, which lost much room to stimulate the economy during a downturn. In a scenario of low growth, inflation, and interest rates, the government may be compelled to take action by using its fiscal capacity to stimulate the economy. The use of discretionary fiscal policy to reactivate economic performance when aggregate demand and interest rates are low has been described in the economics literature at least since Feldstein (2002), and every time this combination hits a country and the central bank seems to run out of ammunition to fight a downturn, this policy makes a comeback. However, fiscal policy decisions are not as easy to be taken as are monetary policy decisions. Fiscal adjustments are under the influence of politicians, who may have political interest in its usage and a different interpretation of what an active fiscal policy implies. Even with increased levels of public indebtedness, governments can be tempted to use fiscal instruments to fight back an economic slowdown, increasing the concern about debt sustainability. Furthermore, depending on the system of governance, some fiscal action has to be approved by the Congress, leading to a long waiting period of uncertainty.

Among the aggregate effects of uncertainty described by Bloom (), an increase in uncertainty may depress investment due to the "wait-and-see" approach. This behavior of firms would harm households, which would have to use precautionary savings to hedge against an unexpected decrease in their income. At the same time, Kaplan, Violante and Weidner (2014) highlight that a great fraction of individuals in the US are borrowing-constrained, with few liquid assets in their portfolios. Kuhn, Schularick and Steins (2017) also show the evolution of portfolio composition of households according to their income level, showing that those in the bottom half of the distribution have a high level of indebtedness and some illiquid assets, but little room for unexpected expenses. These is some evidence demonstrating why people without liquid capital are more exposed to uncertainty shocks.

This dissertation aims to demonstrate how fiscal policy uncertainty affects borrowing-constrained and unconstrained households. We consider a two-agent new Keynesian economy with Rotemberg pricing and Epstein-Zin preferences. The framework shows that, with simple modifications in an otherwise standard model with fiscal policy SV proposed by Fernández-Villaverde et al. (2015), we are able to observe responses neglected in the fully aggregated case. For instance, a TANK model highlights indirect channels of uncertainty transmission, such as the labor market. Furthermore, not only do we reinforce real effects that uncertainty shocks can have on the economy, demonstrating that they can be bigger than fiscal policy decisions, but we also demonstrate that the fraction of hand-to-mouth (HtM) agents in the economy risk-aversion characteristics can be a key factor to the outcome in the real economy. Also, we provide an alternative fiscal policy rule adapted to a developing economy, given the lack of data on fiscal revenue estimations. As highlighted by Gavin and Perotti (1997), fiscal policy outcomes present sharp differences between Latin American and industrial countries. We identify these discrepancies and redefine a rule that is appropriate for an emerging economy. Finally, the model also allows us to understand how government spending can be managed to ease the negative impact on households, demonstrating how a policy that generates similar outcomes on output can harm the consumption level of both agents.

Kaplan, Moll and Violante (2018) show that models with a representative agent mutate indirect channels of monetary policy transmission, which play a pivotal role in understanding their effects on consumption changes. Therefore, we also discuss the relevance of heterogeneous agent New Keynesian models (HANK) in the analysis of policy transmissions, illustrating that two-agent new Keynesian model (TANK) can account for aggregate results and indirect policy channels as highlighted by Kaplan, Moll and Violante (2018) and argued by Debortoli and Galí (2017). Fernández-Villaverde et al. (2015) demonstrate the relevance of monetary policy for fiscal volatility shocks, and considering that those central banks that react to uncertainty can deviate from their standard reaction rule, a two-agent framework plays a meaningful role in the analysis of their transmission channels of such shocks.

This study does not focus on wealth distribution and its reaction to uncertainty shocks, as in Bayer et al. (), but rather on borrowing-constrained households, even though we understand that the portfolio composition data presented in Kuhn, Schularick and Steins (2017) are a piece of relevant evidence of borrowing-constrained households in the US economy. According to Kuhn, Schularick and Steins (2017), housing and other non-financial assets make up more than 80 percent of the assets disclosed in the balance sheet of the bottom 50 percent, while the middle class and the top 10 percent have higher levels of bonds and financial assets in their portfolios. At the same time, Kaplan, Violante and Weidner (2014) show that HtM agents represent roughly 20 percent of the total US income, but three-quarters of these are wealthy households with sizable amounts of illiquid assets, which carry a transaction cost, such as housing. The authors also give evidence that the share of HtM agents can vary roughly from 20 to 50 percent, depending on the definition of HtM.

Also, our aim is to understand to what extent the results presented in this paper are related to country-specific characteristics, such as fiscal structure and political uncertainty. For this reason, the analysis is extended to a developing economy. Emerging markets are known for their political instability, which make them more prone to fiscal policy volatility shocks. Brazil has faced fiscal uncertainty about its fiscal policy path since 2011, when a fall in extraordinary revenues observed in previous years further increased the levels of public indebtedness. At the same time, Brazil has a completely different fiscal policy structure and higher income inequality, which can represent a higher fraction of HtM in the economy. For instance, while the US tax system focuses on capital income, the Brazilian system has lower capital taxes and higher income charges. Studies such as Kaplan, Violante and Weidner (2014) are scarce for Brazil, but according to the Brazilian National Survey on Household Budget[1], around 75 percent of households reported living paycheck to paycheck to make ends meet by the end of the month. Also, 80 percent of the portfolio composition accounts for consumption, 2.1 percent for the payment of liabilities, such as payment of loans, and 5.8 percent for increases in asset levels.

Related Literature. This paper is an intersection of three branches of the literature: two-agent models, uncertainty shocks, and fiscal policy. The growing literature on New Keynesian models with heterogeneous agents since Kaplan, Moll and Violante (2018) brought together the re-emergence of two-agent models. Galí, López-Salido and Vallés () introduce HtM consumers to study the effects of government spending under Taylor rule in a closed economy, which supports a positive comovement between consumption and investment, conditional on government spending shocks. As summarized by Coenen et al. (2012), this class of models are extensively used by central banks to analyze fiscal and monetary policies. Specifically with respect to fiscal policy, Fiscal... (2015) provides a great example of a model where some agents are borrowing-constrained and behave in an HtM fashion, spending their current labor income at all times, while the other agent has full access to financial markets and capital investments.

The... (2012) study the effect of fiscal policies on long-term growth when agents are uncertain about the distribution of future fiscal shocks. Agents have a tendency to put more probability on the worse scenario of fiscal distortions. The authors use a stochastic version of an exogenous growth model, assuming that the government finances its spending on debt and distortionary taxes on labor income. The greater the uncertainty, the worse the agents' view, making future tax expectations higher, discouraging work. In turn, Bi, Leeper and Leith () expand the nada (a) model with uncertainty about the timing and composition of fiscal consolidation, seeking to explain empirical studies that find an expansionary outcome, with an acceleration in production growth after a fiscal contraction, such as a cut in government spending.

While Bi, Leeper and Leith () consider uncertainty about systematic parts of the fiscal rule to study the timing and composition of a fiscal consolidation, Fernández-Villaverde et al. (2015) and Policy... (2014) consider fiscal policy uncertainty as a time-varying shock. Similar in many aspects, these two studies differ in the format of the shocks and in some specifications of the model. The results are qualitatively close, but Policy... (2014) note that the significance of uncertainty shocks is relatively small. On the other hand, the results of Fernández-Villaverde et al. (2015) demonstrate that the results of a volatility shock on fiscal policy are not only significant but also greater in a zero lower

[1]Pesquisa de Orçamentos Familiares (2008-2009) - POF, in Portuguese.

bound (ZLB) scenario. The fall in interest rates allows reducing the countercyclical effect of shocks of fiscal volatility at the level of economic activity. For the Brazilian case, where there is still a lot of room for lowering the interest rate level, this perspective suggests that the responses tend to be less corrosive at the production level, for example. It is also worth highlighting the work of Basu and Bundick (), which, instead of considering uncertainty about the fiscal policy rule, considers uncertainty on the demand side.

As highlighted by Moura (2015), few papers in Brazil analyze the issue of fiscal policy, and even fewer debate about volatility shocks on fiscal instruments. Among the few papers using a dynamic equilibrium model to discuss fiscal stimulus in Brazil, an uninvestigated topic, there are Carvalho and Valli (2011), Moura (2015) and Cavalcanti and Vereda (2015).

2 A SURVEY ON STOCHASTIC VOLATILITY MODELS

One of the current challenges of modern macroeconomics is the understanding of uncertainty shocks.[1] Ever since Bloom (), there has been a growing body of literature on the aggregate effects of such uncertainty shocks. However, there is still some contradiction in the literature concerning the scale and channels of such shocks. As uncertainty is not observable, the literature considers a broad range of proxies to define it, as in Surprise... (2016), Baker, Bloom and Davis (2016), Jurado, Ludvigson and Ng (2015), Fernández-Villaverde et al. (2011), and Bloom (). The main understanding in this literature is that periods of higher volatility are associated with greater uncertainty, as shown in Bloom (), who found that shocks on stock market volatility are associated with real and financial shocks.

This dissertation follows the great branch of the literature on the estimation of fiscal policy uncertainty as a stochastic volatility (SV) process. Thus, in this chapter, we present a survey on SV models and estimation procedures, as well as their application in the macroeconomic literature. In the next section, we discuss how the macroeconomic literature has estimated uncertainty, showing why we decided to follow the SV method. Then, we present the theoretical treatment of sequential Monte Carlo methods and the broad theory about how to deal with nonlinearities of stochastic filtering. We also present the algorithm for particle filters and show its variations. Finally, in the next chapter, we look in detail at the estimation procedure of the SV model.

2.1 MEASURING UNCERTAINTY

One of the most prominent introductions of uncertainty into the macroeconomic literature was made by Bernanke (1983), where he discusses the impact of "bad news" on investment decisions. The model developed by Bernanke (1983) states that the willingness of a firm to invest will not only depend on the current return of the most profitable project, but also on the expected severity of bad news that may come through in the future. Since he deals with irreversible investments, the implication of bad news outcome one period after the investment decision is that the firm would prefer to wait for a different project. Thus, when firms decide whether to invest in a new project, they also consider the probability of a bad news shock affecting their decision — once in every period the investor faces the option of waiting for a different project or investing. In this model, uncertainty is measured as the distribution of bad news shocks, since its realization can be observed by waiting. Thus, events that threaten the profitability of irreversible projects

[1] See Dou et al. (2017) for a critical review of macroeconomics models, which describes a number of model features and quantitative methodologies that are crucial to our understanding of the macroeconomic factors not incorporated by the standard New Keynesian models or still incipient in the literature.

tend to reduce the propensity to invest.

The extension of this understanding was made through the quantitative analysis of uncertainty. While Bloom () introduces uncertainty as a stochastic process, Justiniano and Primiceri (2008) and Fernández-Villaverde and Rubio-Ramírez (2007) introduce heteroskedastic shocks to the quantitative macroeconomic literature. However, these papers have a different focus; while Bloom () aims to discuss uncertainty effects on the economy using stock market volatility as a proxy, Justiniano and Primiceri (2008) are focused on the shifting volatility of US macroeconomic aggregates, and Fernández-Villaverde and Rubio-Ramírez (2007) inaugurate the usage of particle filtering for the estimation of nonlinear models. The combination of the understanding of stochastic volatility as a measure of uncertainty with the evidence that the Great Moderation is characterized by a lower volatility of the US macro variables and the methodological breakthrough regarding the usage of particle filtering for nonlinear estimation in economics enables the progress of studies on uncertainty.

As highlighted by Jurado, Ludvigson and Ng (2015), the literature on measures of uncertainty is still in its infancy. Fernández-Villaverde et al. (2011) show how interest rate volatility has an important effect on output, consumption, and investment. These authors also contribute to the literature by proving that second-order shocks, such as volatility shocks, need a third-order approximation so that innovation will enter as arguments in the policy functions with a coefficient different from zero. In turn, Jurado, Ludvigson and Ng (2015) provide a measure of time-varying uncertainty based on the decomposition of a stochastic volatility process where they define uncertainty as the conditional volatility of a disturbance that is unforeseeable from the perspective of economic agents. Also, in the closest related literature to this dissertation, Basu and Bundick (), Fernández-Villaverde et al. (2015), and Policy... (2014) also define uncertainty as a stochastic volatility process.

However, there are alternative views about uncertainty or how to measure it and its insertion into macro models. For instance, Baker, Bloom and Davis (2016) develop an uncertainty index based on newspaper coverage frequency and Surprise... (2016) creates a surprise and an uncertainty index, which are weighted averages of the surprises or squared surprises from a set of releases. Hansen and Sargent (2019) develop a model where agents construct a set of probability models, then uncertainty is interpreted in the sense that structured models are misspecified. Decision-makers respond to those assumptions by evaluating alternative unstructured models that are statistically near the structured model. Finally, Time-varying... (2019) consider a measure of uncertainty that uses forecast errors based on firm-specific production expectation.

So far, the benchmark for measuring uncertainty in macroeconomics is based on time-varying second-moment shocks. This kind of measure embeds nonlinear and/or non-normal dynamics, which usually take us to complex models and often lead to integrals that cannot be solved analytically. Sequential Monte Carlo (SMC) methods are alternative simulation-based algorithms for solving analytically intractable integrals, having become popular and essential knowledge to the study of uncertainty.

2.2 SEQUENTIAL MONTE CARLO METHODS

State-space models are universal dynamic systems that can be used to describe a wide range of processes. A generic state-space form of the time series x_t with N elements related to a vector s_t named state vector through a measurement equation can be represented as follows:

$$x_t = g(s_t, u_t) \tag{2.1}$$

$$s_t = f(s_{t-1}, v_t), \tag{2.2}$$

where v_t represent a collection of structural shocks and u_t represents measurement errors. Assuming that g and f are linear functions in variables and parameters and u_t and v_t are Gaussian distributed with a constant mean and variance, we could evaluate densities $p(x_t|s_t;\theta)$ and $p(x_t|x_{t-1};\theta)$, corresponding to measurement and transition densities, i.e., Eq. (2.1) and (2.2), depending on an unknown vector of parameters θ, using a Kalman filter. However, if any of these assumptions is not observed, the model will be nonlinear or non-Gaussian, and an alternative approach will be needed to evaluate its likelihood during an estimation process. Most of this section follows the large survey on the Sequential Monte Carlo method in economics by Creal (2012); however, I was also deeply inspired by Doucet, Freitas and Gordon (2001), Doucet and Johansen (2011), Fernández-Villaverde and Rubio-Ramírez (2007), Fernández-Villaverde et al. (2011), Herbst and Schorfheide (2015), and Künsch (2005), among others, whom I deeply recommend for the interested reader.

We are interested in the sequential approximation of distributions $\{p(s_{1:N}|x_{1:N})\}_{N\geq 1}$ and their marginal likelihoods, $\{x_{1:N}\}_{N\geq 1}$, known as an optimal filtering problem. Since $x_{1:N} = \{x_1, \ldots, x_N\}$ is a set of observables that depend on unobserved states $s_{0:N} = \{s_1, \ldots, s_N\}$. One can formulate the uncertainty about the state variable from the realization of the observations, which is the joint conditional probability distribution $p(s_{0:N}|x_{1:N};\theta)$ expressed as

$$p(s_{0:N}|x_{1:N};\theta) = \frac{p(s_{0:N}, x_{1:N};\theta)}{p(x_{1:N};\theta)}, \tag{2.3}$$

where the constant of integration in the denominator is the likelihood of the model. In order to estimate a state-space model with an unknown set of parameters θ, it is usually necessary to compute the likelihood of the model. Furthermore, a few other results that we obtain from the filter density are the one-step-ahead prediction density $p(s_t|x_{1:t-1};\theta)$, the updated filter density one time step later $p(s_t|x_{1:t;\theta})$, and the smoothing distribution $p(s_t|x_{1:N;\theta})$.

Assuming an initial distribution $p(s_0;\theta)$ of the state variable, in a further iteration t, the prediction step projects the last period filtering distribution $p(s_{t-1}|x_{1:t-1};\theta)$ forward. Thus, the one-step-ahead prediction density can be represented as

$$p(s_t|x_{1:t-1};\theta) = \int p(s_t|x_{t-1};\theta)p(s_{t-1}|x_{1:t-1};\theta)\mathrm{d}x_{t-1}. \tag{2.4}$$

Obtaining a new observation x_t, we can use the update step to compute the filtering distribution by applying the Bayes' rule:

$$p(s_t|x_{1:t};\theta) = \frac{p(s_t, x_t|x_{1:t-1};\theta)}{p(x_t|x_{1:t-1};\theta)}, \tag{2.5}$$

$$p(s_t|x_{1:t};\theta) = \frac{p(s_t, x_t;\theta)p(s_t|x_{1:t-1};\theta)}{\int p(x_t|s_t;\theta)p(x_t|x_{1:t-1};\theta)\mathrm{d}x_t}. \tag{2.6}$$

This is an iterative process, which we should keep up to the end of our sample. Also, one should note that the denominator in the update step is the conditional density of x_t given x_{t-1},

$$p(x_t|x_{1:t-1};\theta) = \int p_{t|t-1}(s_t|x_{1:t-1};\theta)\mathrm{d}x_t, \tag{2.7}$$

where $p_{t|t-1}$ is the conditional density of p at time t given the previous realization. If $p_{t|t-1}$ is available, we can obtain the likelihood from

$$p(x_{1:N}) = \prod_{t=1}^{N} p(x_t|x_{1:t-1}), \tag{2.8}$$

which can be factorized into a sequence of conditional distributions. However, due to the nonlinear and/or non-normal representation, these conditional densities will be no longer normal, requiring computationally intensive numerical methods to solve the integrals in this recursion, since they cannot be solved analytically. Thus, SMC is used to approximate the sequence of probability distributions.

Sequential Monte Carlo methods are a generic class of Monte Carlo methods sampled sequentially from a sequence of target probability densities to increase the dimension where each distribution is defined in the product space (DOUCET; JOHANSEN, 2011). The basic idea of SMC methods is that, instead of drawing from the unknown distribution $p(s_t,\theta)$, we can take random draws from a known proposal distribution from which it is easy to sample. After taking draws from that proposal, we will have a sequence of values for s_t, which we assume to be drawn from the wrong distribution, so we need to reweight them according to some importance weights. The filter approximates the filtering random variable $s_t|x_{1:t}$ by "particles" $s_t^1,\ldots,s_t^M$ with a discrete probability mass $\pi_t^1,\ldots,\pi_t^M$. The particle filter is based on the development of other methods, thus, for the rest of this section, we will introduce the importance sampling (IS), sequential importance sampling (SIS), and sequential importance sampling with resampling (SISR) methods, which are basically the core of the particle filter algorithm.

The main idea behind the IS method was described in the last paragraph. Precisely, we take random draws from an importance distribution $h_{0:t}(s_{0:t}|x_{1:t};\psi)$, where ψ denotes a vector of tuning parameters that approximates $h_{0:t}(\cdot|\cdot)$ to the target distribution $p(s_{0:t}|x_{1:t};\theta)$. After M independent samples $\{s^i_{1:t}\}_{i=1}^M$ from the proposal distribution, the random draws can be weighted according to the importance weights defined as

$$W_t^i = \frac{p(s^i_{0:t}|x_{1:t};\theta)}{\sum_{i=1}^M p(s^i_{0:t}|x_{1:t};\theta)}, \tag{2.9}$$

and the IS estimator is

$$\mathbb{E}[f(s_{0:t})] \approx \sum_{i=1}^M f(s^i_{0:t})W^i_t. \tag{2.10}$$

Thus, we can eliminate unnecessary randomness by choosing a sample where each $s^i_{0:t}$ appears MW^i times.[2] Also, it solves the sampling problem from a high-dimensional and complex probability distribution; however, the problem embedded in this approach is that the importance weights need to be recomputed at each iteration, increasing the computational burden.

SIS is a special case of IS that aims to address the computational complexity problem. The SIS algorithm assumes that $h_{0:t}$ can be factored, so we do not need to sample from a joint distribution, but from a sequence of conditional distributions,

$$\begin{aligned} h_{0:t}(s_{0:t}|x_{1:t};\psi) &= h_{t-1}(s_{0:t-1}|x_{1:t-1};\psi)h_t(s_{0:t-1}|s_{0:t-1}x_{1:t};\psi), && (2.11)\\ &= h_0(s_0|\psi)\prod_{t=1}^N h_t(s_t|y_{1:t-1};\psi). && (2.12)\end{aligned}$$

This way, to obtain particles $\{s^i_t\}_{i=1}^M$ at time t, we sample $\{s^i_0\}_{i=1}^M \sim h_0(s_0|\psi)$ at time 0 and $\{s^i_k\}_{i=1}^M \sim h_t(s_t|y_{1:t-1};\psi)$ at time $k = 2,\dots,N$, which means that we can append newly simulated values to the end of the old trajectories. This also allows us to split the weights as follows,

$$\begin{aligned} W_t &= \frac{p(s_{0:t-1}|x_{1:t-1};\theta)p(x_t|s_t;\theta)p(s_t|s_{t-1};\theta)}{h_{0:t-1}(s_{0:t-1}|x_{1:t-1};\psi)p(x_t|x_{1:t-1};\theta)h_t(s_t|s_{0:t-1},x_{1:t};\psi)} && (2.13)\\ &\propto W_{t-1}\frac{p(x_t|s_t;\theta)p(s_t|s_{t-1};\theta)}{h_t(s_t|s_{0:t-1},x_{1:t};\psi)} && (2.14)\\ &\propto W_{t-1}w_t, && (2.15)\end{aligned}$$

where w_t is known as the incremental importance weight, which does not require the past observations of the entire past trajectories $\{s^i_{0:t-2}\}_{i=1}^M$ to be calculated.

The computational difficulty of IS is solved by SIS at the expense of other drawbacks. While SIS avoids evaluating the importance weight completely in each period, the variance of the resulting estimates will increase exponentially with t. As pointed out by Creal

[2]See Künsch (2001).

(2012), as the number of iterations increases, all the probability mass will eventually be allocated to one particle. This problem is known as weight degeneracy. Doucet and Johansen (2011) also highlight that any SMC algorithm that relies upon the distribution of states will fail for a large enough N. The mitigation of this problem was introduced by Gordon, Salmond and Smith (1993), who added a resampling step within the SIS algorithm.

2.2.1 The Particle Filter

The combination of SIS with resampling techniques is the basis of the particle filter and can be seen as a generic SMC algorithm. As summarized by Creal (2012), resampling means that a new population of particles is replicated from the existing population in proportion to their normalized importance weights. Assuming that the weights have already been normalized, i.e., $\sum_{i=1}^{M} w_t^i = 1$, resampling consists in selecting new particle positions and weights $\{s_t^i, w_t^i\}_{i=1}^{M}$ such that the discrepancy between the resampled weights is reduced. Thus, we draw M random variables with replacement using some resampling algorithm with probabilities $\{w_t^i\}_{i=1}^{M}$.

Douc, Cappe and Moulines (2005) present the most popular algorithms for this task, comparing their performance in terms of reduction of the Monte Carlo variation. Multinomial resampling is the simplest approach and it is at the core of the Gordon, Salmond and Smith (1993) filter. It consists in drawing the new positions of the state variable independently from the common point mass distribution. However, according to Doucet and Johansen (2011), systemic resampling is the most widely used algorithm in the literature, since it is easy to implement and has a good empirical performance, even though Douc, Cappe and Moulines (2005) show that it does not dominate multinomial resampling in terms of variance. This method defines $U_i = U_1 + (i-1)/M$, where $U_1 \sim \mathcal{U}[0, \frac{1}{M}]$ and $i = 1, \ldots, M$, aiming to reduce the discrepancy between the empirical distribution of the generated draws and the uniform distribution.

Different specifications of the particle filter can be obtained from different choices of the incremental importance distribution $h(s_{0:t}|s_{1:t-1}, x_{1:t}; \psi)$ and resampling algorithms, which are chosen by the researcher. However, a generic particle filter algorithm can be defined as in algorithm 1. Note that after the definition of the incremental weights, the predictive density $p(x_t|x_{1:t-1}; \theta)$ can be approximated by $\frac{1}{M}\sum_{i=1}^{M} W_{t-1}^j w_t^j$. Furthermore, assuming that the measurement errors are normal, i.e., $u_t \sim \mathcal{N}(0, \sigma_u)$, the incremental weights take the form

$$w_t^i = \frac{1}{\sigma_u\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left[\frac{x_t - g(s_t^i, u_t)}{\sigma_u}\right]^2\right\}.$$

Given the output of the algorithm, we compute the approximation of the log-likelihood as

$$\ln p(x_{1:N};\theta) \approx \sum_{t=0}^{N} \ln\left(\frac{1}{M}\sum_{i=1}^{M} W_{t-1}^{i} w_{t}^{i}\right), \tag{2.16}$$

which will be essential during the estimation procedure for θ.

Algorithm 1 Generic Particle Filter

Step 0: Initialization. Set $t \to 0$. For $i = 1, \ldots, M$, draw the initial particles $s_0^i \sim h(s_0)$ and set $W_0^i = 1$.

Step 1: Prediction. For $i = 1, \ldots, M$, sample values for $\{s_t^i, W_t^i\}_{i=1}^{M}$. Draw

$$s_t^i \sim h_t(s_t|s_{t-1}^i, x_t; \psi)$$

from the conditional density and compute the incremental weights

$$w_t^i = \frac{p(x_t|s_t^i;\theta)p(s_t^i|s_{t-1}^i;\theta)}{h_t(s_t^i|s_{0:t-1}^i, x_{1:t};\psi)}.$$

Step 2: Filtering. For $i = 1, \ldots, M$, define the normalized weights

$$\widehat{W}_t^i = \frac{W_{t-1}^i w_t^i}{\frac{1}{M}\sum_{i=1}^{M} W_{t-1}^i w_t^i}$$

and assign weights to each drawn.

Step 3: Resample M particles $\{s_{t-1}^i\}_{i=1}^{M}$with probabilities $\{\widehat{W}_t^i\}_{i=1}^{M}$. If $t < N$, assign $W_{t-1} = \widehat{W}_t^i$, set $t \to t+1$ and go to step 1. Stop otherwise.

The popularity of particle filters has increased since Gordon, Salmond and Smith (1993) original paper, due to its simple implementation and good properties. For instance, Creal (2012) shows that many particle filters are consistent and asymptotically normal and that they will not accumulate past errors. This property ensures that a particle filter can be applied to a long stretch of time series and the precision of the estimator will not deteriorate as one obtains more observations. Also, its usage in stochastic volatility models increased its popularity for the analysis of financial time series. Since the variance of returns on assets tends to change over time, there are two well-known methods to measure this behavior: stochastic volatility (SV) and autoregressive conditional heteroscedastic (ARCH) models.

While SV considers that conditional variance follows an independent process, ARCH and GARCH models describe it as a function of past values of the dependent variable. As the likelihood of an SV model may not have analytical solution, fitting these models can be more burdensome and demanding than fitting ARCH/GARCH models. However, Künsch (2001) highlights that for a larger dataset, since multivariate GARCH models would be more difficult given the large number of parameters. Kim, Shephard and Chib (1998) also compare the performance of SV and GARCH models, showing that a simple SV model can fit the data as efficiently as its heavily parameterized GARCH counterpart. Fernández-Villaverde and Rubio-Ramírez (2010) also highlight that SV models allow unrelated shocks between measurement and standard deviation shocks, while GARCH models assume that shocks on the measurement equation will affect the volatility process. Another characteristic of the particle filter is its parallel nature of the computational implementation, in a way that a development on computational capacity will lead to computational gains. Thus, SV models are more flexible and their estimation can be less demanding, given the computational capacity available nowadays.

2.3 STOCHASTIC VOLATILITY ESTIMATION PROCEDURES

As introduced in this chapter, fiscal policy uncertainty will be modeled as a stochastic volatility process. Assuming that x_t is a fiscal instrument, such as government spending or capital income tax, the generic stochastic volatility model can be represented as

$$\begin{aligned} \mathsf{x}_t &= \Psi(\mathsf{x}_{t-1}, \mathsf{y}_{t-1}; \theta) + \exp\{\sigma_t\}\varepsilon_{\mathsf{x},t}, \quad \varepsilon_{\mathsf{x},t_t} \sim \mathcal{N}(0,1). \\ \sigma_t &= \Phi(\sigma_{t-1}, u_t; \theta), \quad u_t \sim \mathcal{N}(0,1), \end{aligned}$$

where θ is a vector of parameters and y_t represents the exogenous variables with explanatory power over fiscal policy, such as output gap and debt-to-output ratio. In this section, we describe briefly the procedures to estimate the parameters in θ and the particle filter procedure to approximate the likelihood function. In summary, we estimate θ using a Bayesian technique called Metropolis-Hastings algorithm and the particle filter to approximate the log-likelihood and to estimate those parameters. Herbst and Schorfheide (2015), Robert and Casella (2004), and Gamerman and Lopes (2006) are a useful source for a better understanding of this estimation method. However, since the particle filter is only a device to compute the likelihood, it is also possible to use it in a classical inference.

As described in Robert and Casella (2004), a Metropolis-Hastings algorithm starts with the objective density. A conditional density $q(\theta^{j-1}|\theta)$ defined with respect to the dominating measure for the model is then chosen. The Metropolis-Hastings algorithm can be implemented in practice when $q(\cdot|\theta)$ is easy to simulate and is either explicitly available or symmetric. The random walk chain is a common option and practical implementation of Metropolis-Hastings algorithm when you cannot find a good approximation density for the posterior. The random walk Metropolis-Hastings algorithm (RWMH) takes the following steps described in algorithm 2. Notice that we calculate the likelihood during Step 3, which means that one could rewrite algorithm 2 including algorithm 1 in Step 3.

The RWMH has a proposal distribution with a random walk form, so it generates candidate draws according to

$$\vartheta = \theta^{j-1} + \varphi,$$

Algorithm 2 Metropolis-Hastings estimation

Step 0: Chose starting value, θ^0;
Step 1: Take a candidate draw ϑ from the candidate generating density, $q(\vartheta|\theta^{j-1})$;
Step 2: Calculate the acceptance probability, $\alpha(\theta^{j-1}, \vartheta)$;
Step 3: Set $\theta^j = \vartheta$ with probability $\alpha(\theta^{j-1}, \vartheta)$ and $\theta^j = \theta^{j-1}$ with probability $1 - \alpha(\theta^{j-1}, \vartheta)$;
Step 4: Repeat steps 1 to 3 N_{MH} times.

where φ is the increment random variable with mean zero and variance cV. We use a normal proposal distribution with $V = 1$ and set c_n adaptively to achieve a desired acceptance rate. The proposal covariance scale factor is tuned according to the following rule[3]

$$c_n = c_{n-1} f(1 - R_{n-1}), \quad f(x) = 0.95 + 0.10 \frac{e^{20(x-0.40)}}{1 + e^{20(x-0.40)}},$$

where R_{n-1} is the rejection rate in the past draw and $f(x)$ considers a target acceptance rate of 30 percent, updating the value of c_n every 1,000 draws. Notice that once the acceptance rate is equal to the target, the scale factor will be immutable. We keep tuning it until the last burn-in draw. This procedure allows us to achieve an acceptance rate close to the target without a very exhaustive search within the burn-in period, saving significant computational time. However, these are settings that we choose based on literature recommendation, for the acceptance ratio, and after a search of the best specifications for our database, which can be deferentially selected by other researchers.

The acceptance probability $\alpha(\cdot, \cdot)$ ensures that the chain moves in the correct direction. Giving the symmetric nature of the proposal distribution, the acceptance probability takes the form

$$\alpha(\theta^{j-1}, \vartheta) = \min \left\{ \frac{p(\mathsf{x}^T|\vartheta)}{p(\mathsf{x}^T|\theta^{j-1})}, 1 \right\},$$

in a way that a draw ϑ is accepted with probability one if the posterior at ϑ has a higher value than the posterior at θ^{j-1}. Considering the ratio between accepted draws and total draws, we have the acceptance rate. As Gamerman and Lopes (2006) and Herbst and Schorfheide (2015) mention, there is no consensus in the literature for the optimal acceptance rate, suggesting, as a generic rule, an acceptance rate around 24 to 44 percent, which justifies the target set for the tuning function.

To obtain the estimate of the likelihood that will determine acceptance probability, we use a particle filter with a swarm of 40,000 particles. There is no rule to guide our choice for the number of particles, but the researcher faces a clear trade-off between accuracy and estimation time. In our case, moving from 20,000 to 40,000 particles, the filter takes 73 milliseconds more to evaluate the likelihood of a simulated database with 100

[3]This function was initially proposed by Tempered... (2019) in the context of the tempering particle filter.

observations (93 and 166 milliseconds, respectively), which means that for the estimation of 500,000 draws, the procedure can take roughly extra 7 hours. On the other hand, the higher the number of particles, the lower will be the log-likelihood variance, which is an important indication of accuracy, since this is a Monte Carlo simulation procedure.

Taking the stacking representation of all T observations of the observable variable as x^T and given that the state-space representation has a Markov structure, the likelihood function can be factorized as a function:

$$p(\mathsf{x}^T;\theta) = \prod_{t=1}^{T} p(\mathsf{x}_t|\mathsf{x}^{t-1};\theta),$$

which depends on the sequence $\{p(\sigma_t|\mathsf{x}^{t-1};\theta)\}_{t=1}^{T}$ that we cannot characterize analytically. The particle filter substitutes the density $p(\sigma_t|\mathsf{x}^{t-1};\theta)$ for an empirical draw from it, representing the distribution of the hidden state vector σ_t conditional on time t information x^t through a swarm of M particle simulations $\{\sigma^i_{t|t-1}, W^i_t\}_{i=1}^{M}$, where $\sigma^i_{t|t-1}$ is a draw at moment t condition in $t-1$ and W^i_t is the weight of each particle based on its success in predicting the time t observation, measured by $p(\mathsf{x}_t|\sigma^i_t,\theta)$. As mentioned in the previous section, a well-known problem in the literature is that $\{\sigma^i_{t|t-1}, W^i_t\}_{i=1}^{M}$ suffers from particle degeneracy, in the sense that, as the number of iterations increases, all the probability mass will be allocated to one particle. To mitigate this problem, we resampled the particles at the end of each step. Alternatively, one could define an effective sample size (in terms of number of particles) and only resample when it falls below a threshold, after time periods in which the importance weights are unstable. However, resampling at each step allows a new population of particles to be replicated from the existing population in proportion to their importance weights.[4]

2.4 SUMMARY AND CONCLUSION

In this chapter, we review Monte Carlo simulation methods usually implemented in economics to measure uncertainty. Specifically, we describe the estimation procedure for an SV model, which uses the particle filter, a class of SMC algorithm. We highlight a few different algorithms that preceded the particle filter in the literature, which helps to understand the challenges surrounding the algorithm and its mechanism. Also, we describe how the particle filter can be used to estimate a stochastic volatility model. The example used in this chapter will be estimated in the next two chapters. In the next chapter, a detailed fiscal policy rule will be described, and results will be shown in the following chapter.

It is also interesting to notice that stochastic volatility models are more pliable than their ARCH/GARCH counterparts. Those are much popular because of their

[4]See Douc, Cappe and Moulines (2005), Doucet and Johansen (2011), and Creal (2012) for a more detailed discussion about resampling.

simple implementation, with many ready-to-use packages in a wide range of statistical programming languages. However, in GARCH specifications, higher volatility is triggered only by large innovations, since there is only one shock driving the dynamics of level and volatility factors.

The SV approach gives a higher degree of freedom, since we can have two different sources of shock, in which the shock from the measurement equation is not related to the volatility. Thus, the researcher cannot separate a level shock from a volatility shock in GARCH class of models. Furthermore, as mentioned in Fernández-Villaverde and Rubio-Ramírez (2010), this constraint is very restrictive in the context of structural models, even though this may not seem a problem for reduced-form time series analysis. Thus, we believe that due to their advantages over GARCH models, SV specifications are getting more feasible thanks to computational advances, giving more freedom for researchers interested in structured models with volatility shocks.

3 MODELING FISCAL POLICY UNCERTAINTY

In this chapter we aim to understand fiscal policy rules, its estimation and how to insert this kind of feature in a quantitative model.

3.1 FISCAL POLICY RULES

Unlike monetary policy, which follows a Taylor rule that can be estimated for a wide range of countries, fiscal policy lacks a regular rule. Even for the United States, there are still few studies on rules-based fiscal policy. Taylor (2000) discusses the relevance of a rule in this context distinguishing discretionary changes in taxes and spending and change due to automatic stabilizers. The former is related to legislative and executive actions, posed by the government, while the latter is related to automatic changes in the fiscal instruments according to noncyclical factors, increasing spending on programs such as unemployment compensations and decrease in tax revenue as both employment and income fall in a recession.

Taylor (2000) focuses on discretionary policies, such as short-term impacts, i.e., deviations from potential GDP, which are relevant for a countercyclical fiscal policy. In his view, fiscal policy should work like monetary policy, keeping real GDP close to the potential GDP. However, Gavin and Perotti (1997) show that this is not the case for Latin American countries, where government spending is usually procyclical, with some evidence of asymmetric behavior, making public spending even more procyclical during bad times. Recession is thus associated with collapses in public spending. The authors raise a few hypotheses for this behavior, but this is a feature that persists over the years for many emerging economies, as demonstrated by Alesina, Campante and Tabellini (2008) and Vegh and Vuletin (2015).

Among the hypotheses raised in the literature, Alesina, Campante and Tabellini (2008) focus on political distortions to explain their effects, showing that voters face corrupt governments that can appropriate part of tax revenues for political rents. Another possibility mentioned in Gavin and Perotti (1997) is the *voracity effect*, which arises because interest groups compete for a share of tax revenue. In this case, any fiscal surplus will increase the pressure on the government to increase investments, but it ends up in wasteful spending. An underlying is that some agents will have more political power to appropriate public assets and keep them in the strong group, driving the procyclical fiscal policy.

While this fact can be incorporated into a model as a matter of sign and parameter estimation of the policy instrument rule, some other characteristics of a fiscal rule are more sensitive to the data. Dynamics... (2010) have shown that different feedback rules matter for policy analysis, Toward... (2014) demonstrate the importance of variables to which the fiscal instrument reacts. Even in the literature on macroeconomic models for the United States, there is a lack of consensus about fiscal rules. Comparing Dynamics... (2010), Policy... (2014), Fiscal... (2015), Fernández-Villaverde et al. (2015), and Leeper, Traum and Walker (2017), they all share common features, such as a reaction to deviations from potential GDP and/or debt level. However, it is not straightforward to define whether the government reacts to both at the same time or to the debt-to-GDP ratio or debt cycles, among some other characteristics observed by the authors. Thus, our model incorporates procyclicality and the composition discussion about the fiscal policy rule in a developing country.

3.2 FISCAL POLICY UNCERTAINTY: ESTIMATION

Our analysis focuses on three fiscal policy instruments: government spending as a share of output ($\tilde{g}_t$), tax rate on labor income (τ_t^w), and tax rate on capital income (τ_t^k). To understand the impact of uncertainty on the instruments, we adopt the idea introduced by Bloom () that uncertainty follows a stochastic volatility process. However, the fiscal policy rule can be highly sensitive to the data. Most of the literature about fiscal policy is centered on government spending, but Dynamics... (2010) formulate fiscal rules for capital, labor, and consumption that would be empirically estimated and also provide better data fitting. Since then, some other alternatives have emerged in the literature, such as those used by (FERNÁNDEZ-VILLAVERDE et al., 2015) and Policy... (2014).

The fiscal rule suggested by Dynamics... (2010) for capital and labor tax allows a response to the current cyclical position of the economy and to changes in the level of government debt. Furthermore, they also allow the shocks to be serially correlated, capturing the persistent nature of exogenous changes in instruments. Policy... (2014), in turn, assume that tax rates follow an AR(2) process and allow a response to the lagged debt-to-output ratio and lagged output position. As in Fernández-Villaverde et al. (2015) and Policy... (2014), Dynamics... (2010) assume that the response of these fiscal instruments to output cycles and debt will be positive, but the responses to government spending must be negative.

For $\mathsf{x} \in \{\tilde{g}, \tau^k, \tau^l\}$, the law of motion for each instrument used by Fernández-Villaverde et al. (2015) is given by

$$\begin{aligned} \mathsf{x}_t - \mathsf{x} =& \rho_{\mathsf{x}}(\mathsf{x}_{t-1} - \mathsf{x}) + \rho_{\mathsf{x},y}\tilde{y}_{t-1} \\ &+ \rho_{\mathsf{x},b}\left(\frac{b_{t-1}}{y_{t-1}} - \frac{b_s}{y_s}\right) + \exp\{\sigma_{\mathsf{x},t}\}\varepsilon_{\mathsf{x},t}, \quad \varepsilon_{\mathsf{x},t} \sim \mathcal{N}(0,1), \end{aligned} \tag{3.1}$$

where the log standard deviation of each policy instrument ($\sigma_{\mathsf{x},t}$) has a time-varying stochastic volatility process,[1]

$$\sigma_{\mathsf{x},t} = (1-\rho_\sigma)\sigma_s + \rho_\sigma \sigma_{t-1} + (1-\rho_\sigma^2)^{1/2}\eta_\sigma u_t, \quad u_t \sim \mathcal{N}(0,1). \tag{3.2}$$

Equation (3.1) assumes that demean fiscal policy reacts to its previous value x_{t-1}, lagged output gap $\tilde{y}_{t-1}$, and the lagged debt-to-output ratio (b_{t-1}/y_{t-1}) deviation from its mean value. Idiosyncratic shocks to uncertainty are independent of the first-movement fiscal policy shocks $\varepsilon_{\mathsf{x},t}$, which follow a normal distribution with zero mean and one standard deviation. The stochastic volatility process $\sigma_{\mathsf{x},t}$ in Equation (3.2) allows a shock to the second moment u_t, which will be interpreted as an increase in uncertainty about the future time path of fiscal policy, and η_σ will be the unconditional standard deviation of the fiscal volatility shock. The higher the term $(1-\rho_\sigma^2)^{1/2}\eta_\sigma$, the stronger the evidence of a time-varying volatility. Furthermore, σ_t follows an AR(1) process with steady-state value of σ_s.

We explore several specifications for our data. As initially explored by Gavin and Perotti (1997), it is well known that fiscal policy in emerging markets, and specifically in Latin American countries, does not necessarily have the same fiscal policy reaction in advanced countries. Specifically for Brazil, the literature is centered on the understanding of government spending reaction to output and debt, [2] mainly due to the lack of data on other fiscal instruments, as we highlight in Section 4.4. The same occurs for other developing countries, which cannot take advantage of the effective tax rate methodology developed by Effective... (1994) for industrial countries.

For $\mathsf{x} \in \{g, \tau^k, \tau^l\}$, the law of motion for each instrument used for Brazil is given by

$$\begin{aligned} \mathsf{x}_t - \mathsf{x} =& \rho_\mathsf{x}(\mathsf{x}_{t-1} - \mathsf{x}) + \rho_{\mathsf{x},y}(\tilde{y}_{t-1}) \\ &+ \rho_{\mathsf{x},b}\tilde{b}_{t-1} + \exp\{\sigma_{\mathsf{x},t}\}\varepsilon_{\mathsf{x},t}, \quad \varepsilon_{\mathsf{x},t} \sim \mathcal{N}(0,1), \end{aligned} \tag{3.3}$$

where $\tilde{b}_{t-1}$ is the lagged deviation of debt from its trend and $\sigma_{\mathsf{x},t}$ is the log standard deviation of each policy instrument that follows Eq. 3.2. This rule can be seen as a combination of those in Dynamics... (2010) and Fernández-Villaverde et al. (2015). However, the authors assume that capital and labor income tax will be a procyclical reaction to debt and output and a countercyclical reaction to government spending. Considering the results in Gavin and Perotti (1997), Vegh and Vuletin (2015), and Campos and Cysne (), we relaxed the hypothesis of a countercyclical reaction of government spending to the output. At the same time, we relaxed the procyclicality assumption about the reaction of labor and capital tax to lagged debt, since the literature from developing countries, as presented in Section 3.1, does not allow us to know whether the reaction goes in the same or opposite direction.

[1] We omitted the subscript x in the stochastic volatility process for the sake of clarity, but the parameter and σ were estimated according to the fiscal instrument.

[2] See Campos and Cysne ().

Equation (3.3) and (3.2) are estimated for each instrument independently and the posterior distribution of the parameters is characterized using the random walk Metropolis-Hastings algorithm with a sequential Monte Carlo method to obtain a numerical estimation of the likelihood. We draw 400,000 times from the posterior using a random walk Metropolis-Hastings algorithm. The draw was run after a search for appropriate initial conditions and an additional 100,000 burn-in draws.[3] We tune the covariance scale factor of the proposal density according to the rule used in Tempered... (2019) to induce the appropriate acceptance ratio of proposals. The tuning function allows us to calibrate the covariance scale factor during the burn-in period and, after achieving the target acceptance ratio, we keep making fine adjustments every 1,000 draws up to the last burn-in draw. Each evaluation of the likelihood is performed with 40,000 particles.

3.3 MODEL

The quantitative analysis will be based on a two-agent model with Epstein-Zin (EZ) preferences and fiscal policy volatility shocks. HtM agents do not have access to capital or to the bonds market, while bondholders (or savers) own capital in the economy, receive firms' profits, and have access to the financial market, buying governmental bonds to smooth consumption. Not only does this distinction change the way each agent behaves, mainly because of the inability of HtM agents to insure against unexpected shocks, but it also allows some fiscal instruments to have a direct or indirect impact according to the agent's type. For instance, the government levies capital duty, which is only part of the bondholder's budget, and labor, which will have a direct impact on both types of household members. The model also contains several frictions that introduce nominal and real rigidities in prices, wages, capital, and investments. A large part of the model is built upon the framework developed in Fernández-Villaverde et al. (2015, hereafter FGKR) and Schmitt-Grohé and Uribe (2005).

3.3.1 Households

Preferences. All households are assumed to have preferences given by an EZ aggregator between the period utility U_t and the continuation V_{t+1}:

$$V_t^{1-\psi} = (1-\beta)U_t(c_t, h_t)^{1-\psi} + \beta \mathbb{E}_t (V_{t+1}^{1-\gamma})^{\frac{1-\psi}{1-\gamma}}, \tag{3.4}$$

[3] All programs needed for the Metropolis-Hastings and the sequential Monte Carlo estimation were coded in C++-17 and compiled in GCC-9.1 with fully parallelized algorithm by using Intel TBB library to run on OSX-based machines. On a 2.6 GHz Intel Core i7 with six physical cores, each draw from the posterior using the sequential Monte Carlo estimation with 40,000 particles takes around 0.107 seconds. That implies a total of about 16 h for each simulation of 500,000 draws. Figure (B.1) presents the recursive mean of each parameter for the capital income tax estimation.

where $U_t = e^{\xi_t} c_t^{\eta}(1-h_t)^{1-\eta}$. The discount factor $\beta \in (0,1)$ and the elasticity of leisure η have the same value for both agents. [4] At the same time, the risk aversion γ and the inverse intertemporal elasticity of substitution ψ are calibrated according to the type of each agent. The consumption and hours worked will be agent-specific, as will become clear in the presentation of budget constraints. We omitted a superindex denoting c_t^h, h_t^h or c_t^b, h_t^b, for HtM or bondholders, respectively, for the sake of a clearer notation. The intertemporal preference shock ξ_t follows $\xi_t = \rho_\xi \xi_{t-1} + \sigma_\xi \varepsilon_{\xi,t}$, where $\varepsilon_{\xi,t} \sim \mathcal{N}(0,1)$.

Labor Supply. We assume that households supply differentiated labor services to the production sector through a representative labor aggregator.[5]. Each type of agent belongs to a different labor packer that is proportionally aggregate and supplied to the firms. That is, each type of household supplies labor $h_{j,t}$ to a continuum of unions $j \in [0,1]$, and firms regard each household labor service as an imperfect substitute for the labor of other households. Denoting the aggregate demand for composite labor services as $h_{d,t}$, the bundling technology is

$$h_{d,t} = \left(\int_0^1 h_{j,t}^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dj \right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}},$$

where $\varepsilon_w > 1$ measures the elasticity of substitution. The labor aggregator minimizes the cost of producing a given amount of the aggregate labor index,[6] taking each household's wage $w_{j,t}$ as given, where w_t is the aggregate nominal wage. Thus, the demand for labor services is

$$h_{j,t} = \left(\frac{w_{j,t}}{w_t} \right)^{-\varepsilon_w} h_{d,t}. \tag{3.5}$$

Considering this result, it is straightforward to define the aggregate wage index as $w_t = (\int_0^1 w_{j,t}^{1-\varepsilon_w} dj)^{\frac{1}{1-\varepsilon_w}}$, and the aggregate labor supply as the sum of labor by variety, $h_t = \int_0^1 h_{j,t} dj$.

Budget Constraint. A fraction μ of the agents in the economy is HtM, without access to any kind of asset in the economy. The budget constraint of the HtM agent is given by:

$$c_t^h + \Omega_t^h = (1 - \tau_t^w) \int_0^1 w_{j,t} h_{j,t}^h dj - AC_{j,t}^w, \tag{3.6}$$

[4]The... (2006) present some empirical micro evidence that the elasticity of leisure will be different for constrained and unconstrained agents (See Table 4 in their paper); however, the functional form used here already gives us flexibility to define risk aversion and the intertemporal elasticity of substitution differently. The addition of a third difference in the agents' preference could make their relationship poorly identifiable.

[5]Following the assumption of Optimal... (2000), the labor aggregator can be interpreted as a union, labor packer, or employment agency.

[6]It solves the following problem:

$$\max_{h_{j,t}} \quad w_t \left(\int_0^1 h_{j,t}^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dj \right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}} - \int_0^1 w_{j,t} h_{j,t} dj.$$

where Ω_t are lump-sum taxes. Wages are subject to a quadratic adjustment cost $AC^w_{j,t} = \frac{\phi_w}{2}\left(\frac{w_{j,t}}{w_{j,t-1}} - z_s\right)^2 y_t$, where z_s is the steady-state of the productivity shock z_t. A perfect competitive labor packer aggregates the different types of labor $h_{j,t}$ into homogeneous labor, so $h_{j,t} = h_{d,t}\int_0^1 \left(\frac{w_{j,t}}{w_t}\right)^{-\varepsilon_w} dj$.

The bondholder agent can trade one-period riskless bonds, invest in capital, and receive profits of the firms, facing the following budget constraint:

$$c^b_t + x_t + b_t + \Omega^b_t = (1-\tau^w_t)\int_0^1 w_{j,t}h^b_{j,t}dj + (1-\tau^k_t)r^k_t u_t k_{t-1} + \frac{R_{t-1}b_{t-1}}{\Pi_t} - AC^w_{j,t} + F_t, \quad (3.7)$$

where x_t is the investment level, u_t is the rate of utilization of capital, Ω_t are lump-sum taxes, and F_t are the profits of the firms in the economy. Besides the labor taxes τ^w_t levied on both agents, a bondholder also faces capital tax τ^k_t, since she is the owner of capital granted at a rate r^k_t to intermediate firms.

Investment decisions are subject to an adjustment cost. The law of motion of physical capital is described by

$$k_t = (1-\delta(u_t))k_{t-1} + \left(1 - \mathcal{S}\left[\frac{x_t}{x_{t-1}}\right]\right)x_t, \quad (3.8)$$

where the adjustment cost of investments assumes a quadratic form $\mathcal{S}\left[\frac{x_t}{x_{t-1}}\right] = \frac{\kappa}{2}\left(\frac{x_t}{x_{t-1}} - z_s\right)^2$, and $\delta(u_t) = \delta + \delta_1(u_t - 1) + \frac{1}{2}\delta_2(u_t-1)^2$ is the depreciation rate that depends on the capacity utilization rate.

Agents' Aggregation. Firms cannot distinguish the type of each agent on the labor market, nor their decision about consumption level, thus assuming that a fraction μ of the agents in the economy is HtM, and the aggregation of these variables is given by

$$h_t = \mu h^h_t + (1-\mu)h^b_t, \quad (3.9)$$

$$c_t = \mu c^h_t + (1-\mu)c^b_t. \quad (3.10)$$

The bondholder chooses processes for $c_t, b_t, u_t, k_t, x_t,$ and $w_{j,t}$ so as to maximize its utility function (3.4) subject to budget constraints, wage adjustment costs, the law of motion of capital (3.8), and demand for labor services (3.5). On the other hand, the HtM agent only chooses c_t and $w_{j,t}$, subject to related constraints. Appendix 6 presents the equilibrium conditions associated with households' problems.

3.3.2 Firms

The model's firms consist of perfectly competitive final-goods-producing firms and a continuum of monopolistically competitive intermediate goods producers. At every time t, a competitive firm produces the final consumption good y^d_t using the intermediate goods

$y_{i,t}$, $i \in [0,1]$ and the technology $y_t^d = (\int_0^1 y_{i,t}^{(\varepsilon_p-1)/\varepsilon_p} di)^{\varepsilon_p/(\varepsilon_p-1)}$, where ε_p is the elasticity of substitution. Expenditure minimization leads to the final input demand function given by

$$y_{i,t} = \left(\frac{p_{i,t}}{p_t}\right)^{-\varepsilon_p} y_{d,t},$$

where $p_{i,t}$ is the price of the intermediate goods and p_t is the price index for final goods, $p_t = (\int_0^1 p_{i,t}^{1-\varepsilon_p} di)^{1/(1-\varepsilon_p)}$.

The monopolistic firm produces intermediate goods using a production function $y_{i,t} = k_{i,t}^{\alpha}(z_t h_{i,t})^{1-\alpha}$. Each i intermediate good producers rent labor $h_{i,t}$ from both agents and capital $k_{i,t-1}$ from the saver to produce intermediate good $y_{i,t}$. Producers maximize profits by setting prices subject to a quadratic adjustment cost. Since firms are owned by bondholders, firms use their stochastic discount factor:

$$\max_{p_{i,t+s}} \quad \mathbb{E}_t \sum_{s=0}^{\infty} \left[\frac{\partial V_t/\partial c_{t+s}^b}{\partial V_t/\partial c_t^b}\right] \left[\frac{p_{i,t+s}}{p_{t+s}} y_{i,t+s} - mc_{i,t+s} y_{i,t+s} - AC_{i,t+s}^p\right] \tag{3.11}$$

subject to the final input demand function $y_{i,t}$ and a quadratic adjustment cost for prices, $AC_{i,t}^p = \frac{\phi_p}{2}\left(\frac{p_{i,t}}{p_{i,t-1}} - \Pi_s\right)^2 y_{i,t}$, where Π_s is the steady-state level of inflation.

3.3.3 Government and monetary policy

The government issues bonds b_t every period to satisfy its budget constraint, given by

$$\begin{aligned} b_t =& \frac{R_{t-1}b_{t-1}}{\Pi_t} + g_t - \tau_t^w w_t h_t - \tau_t^k r_t^k u_t k_{t-1} \\ &- z_t \left[\phi_{d,b}\left(\frac{b_{t-1}}{z_{t-1}y_s} - \frac{b_s}{y_s}\right) + \mu\Omega^h + (1-\mu)\Omega^b\right], \end{aligned} \tag{3.12}$$

where we consider that lump-sum tax stabilizes the debt-to-output ratio, imposing a passive fiscal/ active monetary regime. The government expenditure g_t, labor, and capital taxes (τ_t^w and τ_t^k) evolve endogenously according to the three rules defined in equations (3.3) and (3.2).

The monetary authority controls the short-term interest rates following a modified Taylor rule that responds to deviations of inflation from the steady state, the gap of observed output, and uncertainty shocks. We consider the results in Fernández-Villaverde et al. (2015), where the authors highlight that a standard Taylor rule does not generate the same effects from empirical estimations for the inflation and interest rates. They propose the rule given by

$$R_t = R_s \left(\frac{R_{t-1}}{R_s}\right)^{\phi_R} \left[\left(\frac{\Pi_t}{\Pi_s}\right)^{\phi_\Pi} \left(\frac{y_t}{y_s}\right)^{\phi_y} \left(\frac{e^{\sigma_{\mathrm{x},t}}}{e^{\sigma_{\mathrm{x},s}}}\right)^{\phi_\sigma}\right]^{1-\phi_R} \exp\{\sigma_m \Phi_t\}, \tag{3.13}$$

where the monetary policy shock Φ_t follows a $\mathcal{N}(0,1)$ process. The parameters ϕ_R are smoothing parameters used to capture gradual movements of interest rates, ϕ_Π and ϕ_y capture the responsiveness of the interest rate to deviations of inflation from its steady state and of the output gap from its trend, and ϕ_σ captures the reaction of the monetary authority to uncertainty shocks.

3.4 SUMMARY AND CONCLUSION

In this chapter, we showed how to empirically estimate fiscal policy uncertainty for both developed and developing countries and we incorporated these rules into a quantitative macro model. Our analysis is based on a two-agent New Keynesian model, with nominal and real rigidities in prices, wages, capital, and investments. A fraction of the population is classified as HtM, which means that these households consume all of its tax-free income in one time period, while bondholders can choose to save their income by buying government bonds or investing in capital. Not only does this mechanism incorporate a well-known fact highlighted by the empirical literature, but it also allows understanding the transmission mechanisms that can be hidden in a representative-agent model.

Another interesting feature of the model is the usage of Epstein-Zin preferences for the households. The main advantage of this setting is to disentangle the risk aversion and intertemporal elasticity of substitution parameters. This way, we will be able to characterize and distinguish both agents in the model more accurately. As described in the next chapter, while risk aversion is related to the state of the economy or to the economic environment to which the agent belongs, the intertemporal elasticity of substitution is a time decision, which may not be related to the former. However, the usual preferences setting does not allow analyzing this difference between both parameters. Since we are analyzing two different countries, this framework brings a wealth of information, allowing us to understand what features of the households can make uncertainty a more severe shock on agents and/or on the economy as a whole.

4 A TANK MODEL OF FISCAL POLICY UNCERTAINTY

This chapter aims to present the results for the empirical and quantitative model developed in the previous chapter. We present the results for the stochastic volatility estimation in Section 4.1. Next, in Section 4.2, we discuss the solution method and calibration strategy for the quantitative macro model developed in the previous chapter. In Section 4.3, we present the results for the US, and in Section 4.4, we expand our analysis to Brazil, comparing the effects of fiscal policy uncertainty in a developed and developing economy. Finally, we engage in a pseudo-optimal policy analysis in Section 4.6, aiming to understand the effects on both agents and on the economy given the behavior of the government. We are able to understand to what extent some country-specific reactions, such as fiscal structure and political uncertainty, are able to influence the responses to our shocks. Also, the model demonstrates how government spending can be managed to ease the negative impact of such shocks on households, demonstrating how a policy that generates similar outcomes on output can harm agents' constitution level.

4.1 FISCAL POLICY UNCERTAINTY: RESULTS

Table 4.1 presents the parameters estimated by Fernández-Villaverde et al. (2015) for the US economy and the estimations for Brazil. The reaction of fiscal policy instruments and the stochastic volatility process follow the description in Section 3.2 and the estimated parameters in Table 4.1. To estimate these parameters, Policy... (2014) and Fernández-Villaverde et al. (2015) construct their aggregate effective tax rates using national account information following the measurement of relevant aggregate tax rates in Effective... (1994) and Has... (2002). However, the aggregation of these data may be more challenging if we consider Brazilian data, as discussed by Azevedo and Fasolo (2015). Since the construction of aggregate tax rates is already worth a whole study, we took advantage of Azevedo and Fasolo (2015) using their database for estimation. The smoothed fiscal volatility shock to capital income tax rates and government spending are presented in Figure (4.1).

We adjusted the labor income tax by the unemployment rate, once the period of analysis exhibits a considerable decrease of unemployment levels in Brazil from 18.9 to 11.0 percent, which generates an upward trend for labor tax income, Since the main unemployment rate series for Brazil does not cover the whole tax sample, we selected the unemployment rate in the Metropolitan Area of the city of São Paulo (IPEADATA id code 37655). This series has a correlation of 0.97 with the official data considered by the government from the Brazilian National Household Survey. The main difference will be in levels, with mean unemployment rate varying from 13.9 to 9.6. Besides the tax data in Azevedo and Fasolo (2015), the government spending data represent the total expenditure reported in Table 1.4-A of the Central Government Fiscal Balance available on the Brazilian National Treasury website.[1] Among the three fiscal instruments, only one observation can be seen as an outlier. In the third quarter of 2010, the government spending rose suddenly, going back to its normal level in the subsequent quarter. This outlier precedes the 2010 Brazilian Presidential Election, held in October of that year. There are several models to incorporate jumps into stochastic volatility models[2]; however, in our specific case, modeling this outlier would generate a slight improvement in our analysis, which led us to interpolate the outlier instead of changing the whole model to accommodate a single observation. Finally, we used the Federal Government Net Debt data (SGS-BCB id code 4479) to calculate the government indebtedness level.

We can have a clear perspective on fiscal moments of high fiscal volatility by analyzing Figure (4.1). The first observation is that the size of these shocks is significantly different. While tax capital rate changes have medians greater than one throughout the sample, the estimation obtained for spending shows a variation lower than 0.5 percent. At the same time, in each chart, we have essentially two main moments of higher volatility. Starting with the panel on government spending volatility, in 2003:I, the government level of spending would have moved between 0.4 and 1.1 percentage points (considering a 95% confidence interval), while from 2010:I to 2010:II, we have two quarters of spending moving between 0.4 an 0.9 percentage points. In 2003, the Lula administration (2003 - 2010) inaugurated several social programs, which widely extended to the next administrations. On the other hand, capital tax revenue had its peaks in 1999:II and 2007:III. The first moment indicates that capital tax rate moved between 1.07 and 3.15 percentage points in 1999, which is related to a change in the tax rate for CPMF[3] to 0.38%, which took place in the third quarter of 1999 and represented approximately 83% of the total variation in capital income tax according to Azevedo and Fasolo (2015). The second moment of high volatility at the end of 2007 can be related to a decrease in capital tax revenue. Azevedo and Fasolo (2015) point out that capital tax income decreased by 0.42% due to the end of CPMF charges and a reduction in corporate tax rates[4].

The first thing we can highlight in Table 4.1 is the difference between fiscal rule parameters for Brazil and the US. Notice that the rules are not exactly the same, but these parameters still have the same meaning, i.e., $\rho_{\mathsf{x},b}$ is the reaction of the fiscal instrument to the lagged debt and $\rho_{\mathsf{x},y}$ is the reaction to the lagged output gap. Since the fiscal instruments considered in this paper are government spending, capital income tax, and labor income tax, $\mathsf{x} \in \{g, \tau^k, \tau^l\}$, each line in Table 4.1 demonstrates how each of these instruments behaves in the US and Brazil. While in the US, the reaction of labor income

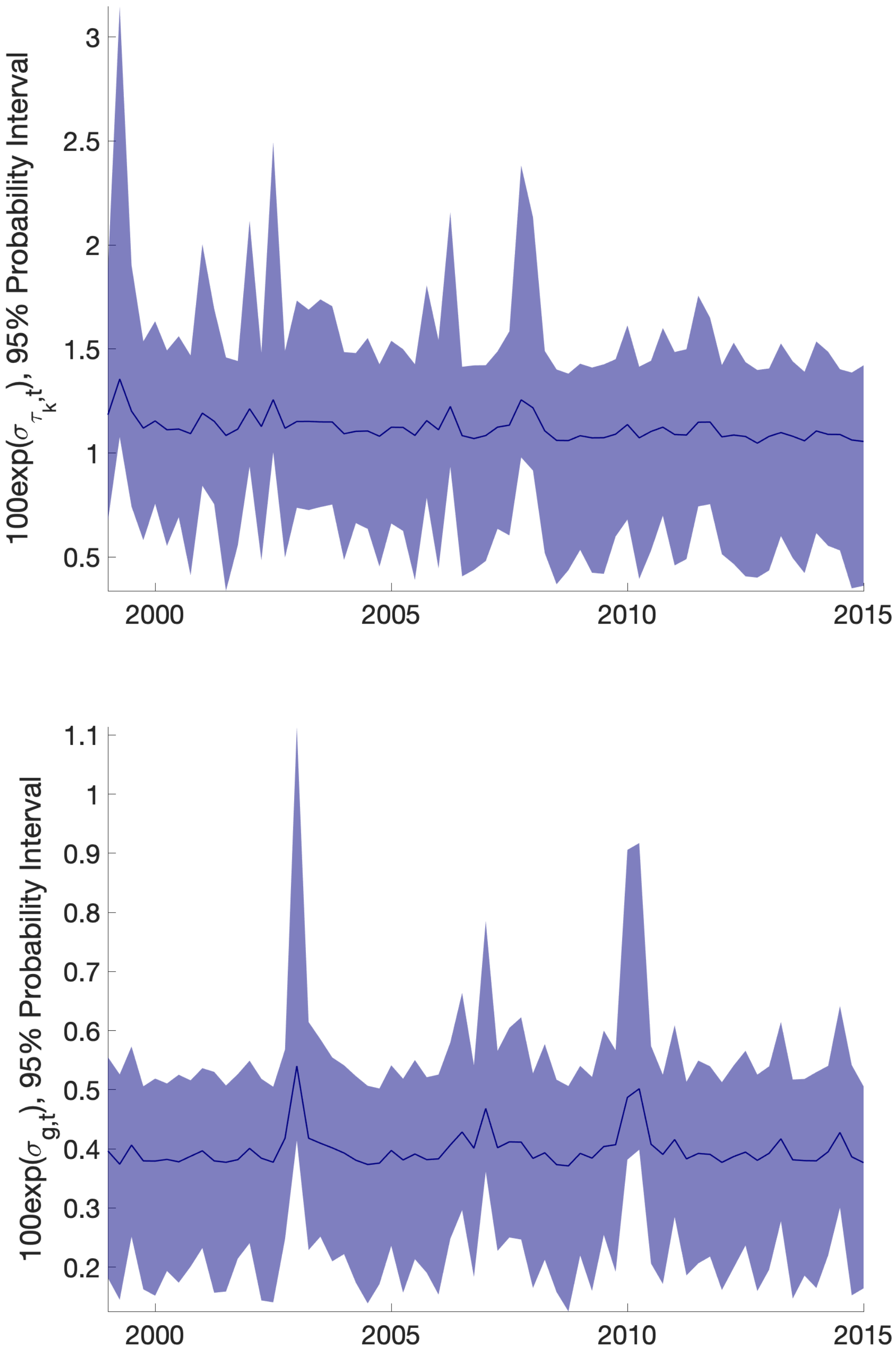
[1]http://www.tesouro.fazenda.gov.br/en/central-government-primary-balance.

[2]See Pitt and Shephard (1999) for a discussion about how to model outliers with SV models and how it can degenerate the particle filter estimation.

[3]Provisional Contribution on Financial Transactions, in Portuguese, *Contribuição Provisória sobre Movimentação Financeira (CPMF)*

[4]In Portuguese, *Imposto de Renda de Pessoa Jurídica* (IRPJ).

Figure 4.1 – Smooth volatility shock to capital tax and government spending



Note: 95% posterior probability intervals of the smoothed fiscal volatility shock to capital income tax rates and government spending for Brazil, $100exp(\sigma_{g,t})$ and $100exp(\sigma_{\tau_k,t})$, over the sample.

Table 4.1 – Fiscal Policy Instruments Parameters

	United States			**Brazil**		
	Labor	Capital	Government Spending	Labor	Capital	Government Spending
ρ_x	0.99	0.98	0.99	0.88	0.63	0.91
	[0.98, 0.99]	[0.92, 0.99]	[0.65, 0.97]	[0.80, 0.95]	[0.47, 0.77]	[0.87, 0.96]
$\rho_{x,y}$	0.040	0.043	-0.004	0.047	0.11	0.021
	[0.008, 0.045]	[0.003, 0.099]	[-0.018, 0.00]	[0.004, 0.159]	[0.02, 0.23]	[-0.025, 0.066]
$\rho_{x,b}$	0.003	0.003	-0.008	0.01	-0.002	-0.012
	[0.000, 0.008]	[0.003, 0.099]	[-0.018, 0.00]	[-0.01, 0.04]	[-0.03, 0.03]	[-0.021, -0.006]
σ_s	-6.01	-4.89	-6.20	-4.10	-4.56	-5.59
	[-6.20, -5.81]	[-7.35, -6.87]	[-6.53, -5.71]	[-4.74, -3.79]	[-5.15, -4.33]	[-5.99, -5.38]
ρ_σ	0.46	0.65	0.92	0.28	0.20	0.21
	[0.33, 0.58]	[0.39, 0.86]	[0.78, 0.99]	[0.03, 0.69]	[0.02, 0.69]	[0.02, 0.58]
η_σ	0.820	0.400	0.180	0.27	0.16	0.20
	[0.70, 0.97]	[0.22, 0.58]	[0.10, 0.29]	[0.02, 0.93]	[0.02, 0.88]	[0.02, 0.65]

Note: Parameters estimated following a Bayesian approach by combining the likelihood function with flat priors and sampling from the posterior with a Markov Chain Monte Carlo. For each parameter, we report the posterior mean and, in brackets, a 90 percent probability interval.

tax to output gap is almost the same size of capital tax outcome, in Brazil, the reaction of tax levied on capital is much stronger than on labor. At the same time, both reactions to debt are quite small. This shows that when compared with the US, Brazil's tax system is more centered on capital income tax than on labor.

At the same time, the reaction of people to whether the government is spending more or less than it is normal is small in both countries, showing that much of the government spending has been unproductive. We would expect greater negative parameters for a government that compromises on productive investment, because the cost for private agents to invest would then raise, discouraging them from doing it. Another interpretation is that the government can engage in public investments when the private sector avoids compromising on investments due to uncertainty about the economic outlook or tax reforms that can change the future behavior of the private sector. Either way, our empirical results demonstrate that this is not the case of Brazil or the US. However, another understanding of these results can be that the government spending behavior is tightly linked to politics and to the party leading the country. In this case, a better methodology would be a time-varying parameter estimation.

Since the main focus of this empirical exercise is to evaluate the stochastic volatility behavior of fiscal instruments, we assume that the reaction to the lagged debt is constant over governments and we focus on the parameters related to uncertainty. η_σ is the unconditional standard deviation of the fiscal volatility shock, while ρ_σ measures the persistence of this process and σ_s its steady-state value. First, we can notice that both ρ_σ and η_σ are greater for the US than for Brazil, regardless of the fiscal instrument, which tells us that fiscal policy uncertainty shocks in the US are stronger and tend to last longer. However, since we take the exponential of σ_t, the more negative σ_s is, the closer to zero the steady-state value of the standard deviation will be. Thus, the mean standard deviation of fiscal instruments is higher in Brazil than in the US, which helps to understand the dichotomy presented. Since the US has a lower steady-state standard deviation for the instruments, when an uncertainty shock hits any of those, its effect on the economy will be stronger and more persistent than in Brazil, which has, on average, a higher standard deviation of its fiscal policy mechanisms.

The comparison of instruments tells how strong an uncertainty that is specific to each of them can be. Parameters for labor second-order shocks are greater than capital or government spending, which demonstrates that uncertainty about this fiscal instrument tends to be greater and last longer. This can be associated with the fact that the legislation in Brazil is usually protective of workers' rights. However, considering the quantitative macro model presented in the previous section, we notice that even with greater parameter values for labor tax, the capital income tax second-order shocks will be more important, with stronger general equilibrium effects on the economy.

4.2 QUANTITATIVE ANALYSIS

Solution and Calibration. The characterization of the model's equilibrium conditions is relatively standard and it is dealt with in the Appendix 6. The model presented in this paper needs a higher-order approximation. As explained by Fernández-Villaverde et al. (2011), second-moment shocks that can generate precautionary behavior do not occur in a first-order approximation (which is equivalent to the traditional log-linearization), and second-order approximation would capture the volatility effect only indirectly, via cross product terms. Hence, to explore the direct role of the volatility shocks in the model, we need to consider cubic terms. Another issue is that a third-order approximation of this model gives explosive sample paths, so we use the pruning method developed by Andreasen, Fernández-Villaverde and Rubio-Ramírez (2018) for third-order approximation.

One of the key parameters in our model is the proportion of borrowing constraint agents in the economy. Following Kaplan, Violante and Weidner (2014), we assume that 40 percent of the households are HtM ($\mu = 0.4$)[5]. Some parameters are fixed before the estimation because data moments can be uninformative and the literature provides their proper estimation. We normalize the steady state of hours worked to 1/3 using η. The intertemporal elasticity of substitution (IES) is set to 2.5 for bondholders and 0.8 for HtM agents ($\psi = 0.40$ and 1.25, respectively)[6], considering the results in Vissing-Jørgensen (2002). The depreciation parameter δ_1 is used to set the steady-state capital utilization to

[5] Since there is no consensus about the fraction of HtM agents in the US economy, we also present results for a sensitivity test on this parameter (Figure 4.3).
[6] Due to the utility function definition, the IES is equal to $1/\psi$. See Swanson (2012) and Gourio (2012).

Table 4.2 – Summary Calibration and Fixed Parameters

	Description	Value	Source
Preferences			
β	Time-discount factor	0.9959	Calibration
γ_h	HtM Risk Aversion	370.34	Calibration
γ_b	Bondholder Risk Aversion	661.83	Calibration
ψ_h	HtM Inverse IES	0.40	Vissing-Jørgensen (2002)
ψ_b	Bondholder Inverse IES	1.25	Vissing-Jørgensen (2002)
μ	Fraction of HtM agents	0.40	Kaplan, Violante and Weidner (2014)
η	Consumption preferences	0.4034	Endogenous
Ω^h	HtM lump-sum tax steady state	-0.2263	Endogenous
Ω^b	Lump-sum tax steady state	0.3019	Endogenous
Technology			
α	Capital Share	0.36	FGKR
ε_p	Goods elasticity of substitution	21.0	FGKR
ε_w	Labor elasticity of substitution	21.0	FGKR
δ	Steady state depreciation	0.0451	Calibration
δ_1	Rate of capital depreciation	0.01	Jaimovich and Rebelo (2009)
δ_2	Rate of capital depreciation	0.0139	Calibration
κ	Investment cost	0.9419	Calibration
Taylor Rule			
ϕ_R	Smoothing of past interest rate	0.7584	Calibration
ϕ_Π	Response to inflation deviations	1.6966	Calibration
ϕ_y	Response to output gap deviations	0.0863	Calibration
ϕ_σ	Response to capital tax volatility	0.005	FGKR
Π_s	Steady state inflation	1.0090	Calibration
σ_m	Standard dev. of the monetary shock	2.0e-4	Calibration

Note: parameters fixed prior to the estimation are referred with their corresponding source. Parameters estimated with the SMM procedure are tagged with "Calibration." The consumption preference parameter η is tagged with "Endogenous" because we normalize the steady state of hours worked to 1/3 using η. We also set Ω^h and Ω^b endogenously to achieve a balanced HtM and government budget constraint, respectively.

1 from its first-order condition. The quadratic cost of changing nominal prices is set to $\phi_p = 200.0$, which implies all intermediate products prices are fixed for about six quarters in a linearized Calvo setting, and the measure of wage rigidity is set to $\phi_w = 2000.0$, both parameters following FGKR. The parameters calibrated with data are the discount factor β, risk aversion of each type of agent, i.e., γ_h and γ_b, two parameters related to the depreciation rule, δ and δ_2, the parameter κ that controls the curvature for the adjustment cost of the investment function, and five Taylor rule parameters, $\phi_R, \phi_\Pi, \phi_y, \sigma_m$, and Π_s. We present other fixed and calibrated parameters in Table 4.2.

Since our model has a large number of variables, the computation required to find theoretical moments is computationally inefficient. Thus, the only difference in our calibration procedure in comparison to Andreasen, Fernández-Villaverde and Rubio-Ramírez (2018) is that instead of calculating theoretical moments, we compute them via simulation. We applied the simulated method of moments (SMM) to calibrate the selected

Table 4.3 – Model Fit - selected moments

Moments	Data	Model	Moments	Data	Model
Mean					
R_t	0.8885	0.8912	Π_t	1.4028	1.2852
Standard Deviation			*Auto-Correlation*		
y_t	1.5278	1.8796	$corr(y_t, y_{t-1})$	0.8779	0.9178
c_t	1.2481	0.7818	$corr(c_t, c_{t-1})$	0.8909	0.8343
x_t	7.0489	6.9979	$corr(x_t, x_{t-1})$	0.8532	0.9511
w_t	0.9329	0.2090	$corr(w_t, w_{t-1})$	0.7074	0.9694
h_t	1.9474	1.6701	$corr(h_t, h_{t-1})$	0.9276	0.8315
u_t	3.2484	2.4141	$corr(u_t, u_{t-1})$	0.8993	0.8970
R_t	0.9365	0.7355	$corr(R_t, R_{t-1})$	0.9701	0.9842
Π_t	0.6079	0.7192	$corr(\Pi_t, \Pi_{t-1})$	0.9038	0.9349
Correlation					
$corr(y_t, c_t)$	0.8796	0.7932	$corr(y_t, u_t)$	0.8777	0.6230
$corr(y_t, x_t)$	0.9214	0.8232	$corr(y_t, R_t)$	0.2021	-0.4609
$corr(y_t, w_t)$	0.1196	0.5958	$corr(y_t, \Pi_t)$	0.1160	-0.2736
$corr(y_t, h_t)$	0.8702	0.8959	$corr(\Pi_t, R_t)$	0.6651	0.9113

Note: All data, except nominal interest rates and inflation, are in logs, HP-filtered, and multiplied by 100 for their expression in percentage deviation from the trend. Nominal interest rates and inflation are directly expressed in percentage points. We omitted the model fit for the mean and autocorrelation for lag 5 due to space constraints.

parameters of the model. As described by Estimating… (2012), SMM is an appropriate method to calibrate parameters of a nonlinear model, since it is computationally efficient and delivers accurate estimates even for short sample series. We present the model fit for selected moments in Table 4.3. We match the average of interest rates and inflation, standard deviation, lag 1 autocorrelation and correlation of all variables with output, and the correlation between interest rates and inflation, summing up 26 moments to calibrate 11 parameters. With a few exceptions, the model does a reasonable job on matching the mean, standard deviation, and autocorrelation, while the results for the correlation are mixed. As detailed in Section 3.2, the parameters for the fiscal rule are exogenously calibrated with an SV model. Data for the US economy are taken from St. Louis Fed's FRED database, and the time span 1970:Q1–2014:Q2 is the same used in Fernández-Villaverde et al. (2015) for the comparison of the results . The data series used to match the moments of variables y_t, c_t, x_t, w_t, h_t, u_t, R_t, and Π_t are the real gross domestic product (GDPC1), real personal consumption expenditures (PCECC96), real gross private domestic investment (GPDIC1), compensation per hour (HCOMPBS), hours of all persons (HOABS), capacity utilization of total industry (TCU), effective federal funds rate (FEDFUNDS), and implicit price deflator (GDPDEF).

4.3 THE EFFECT OF FISCAL VOLATILITY SHOCKS

This section presents estimation results, equilibrium dynamics and further quantitative results of the model.

4.3.1 Estimation Results

Parameters calibrated with SMM in Table 4.2 are the same ones chosen by FGKR, with exception of the risk aversion γ of each type of agent. A few differences can be noticed when comparing the results for the Taylor rule parameters, specifically regarding the responsiveness of the monetary authority to deviations of the steady-state inflation ϕ_{Π}, which takes the value of 1.35 in the results obtained by FGKR in comparison with 1.69 in our study. This difference can be explained by a slightly higher estimation of the steady-state inflation in our model (an annualized inflation rate 0.55 percent higher) and lower standard deviation of the innovation for the monetary policy shock σ_m.

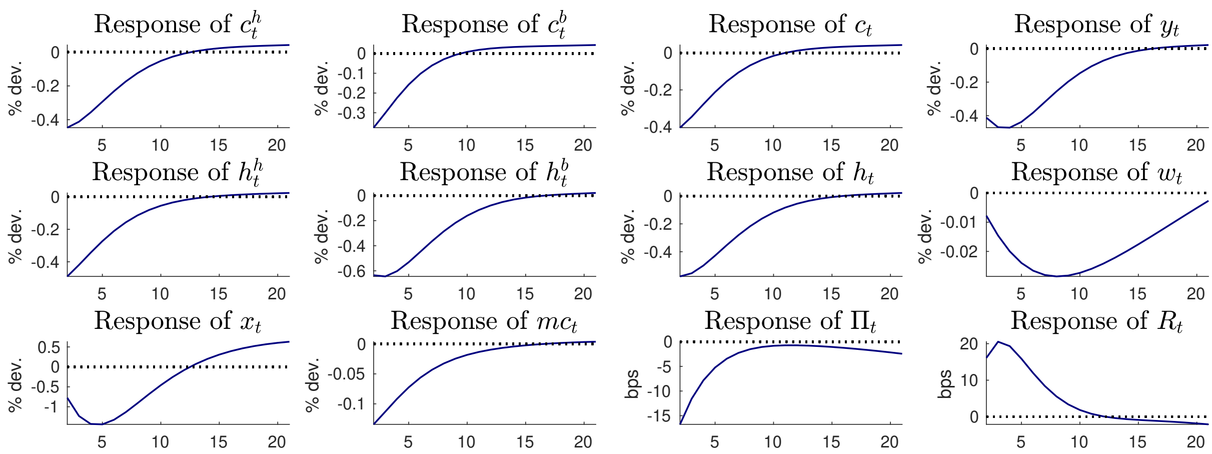
Our estimation for the risk-aversion parameters i in tune with other estimations in the macroeconomics representative agent literature. These parameters are at odds with the empirical micro literature, where, by confronting individuals with specific risk prospects, Barsky et al. (1997) obtain a measure of the relative risk-aversion parameter ranging from 3.8 to 15.7, while other macro studies with a representative agent with Epstein-Zin preference estimate values ranging from 80.0 to 600.0, depending on the model structure.[7] The bondholder presents a parameter value of 661.83, in line with Andreasen, Fernández-Villaverde and Rubio-Ramírez (2018). At the same time, the HtM agent presents a low parameter value of 370.34, which is reasonable given the fact that she does not have assets, such as capital or bonds. This result is in line with Guiso and Paiella (2008) self-selection argument, where the authors highlight that more risk-averse agents select themselves into occupations with low-income risk. However, the reasoning of these results is not completely straightforward. We will expand the discussion when we present the risk aversion results for Brazil, where bondholders are less risk-averse than HtM agents.

4.3.2 Equilibrium Dynamics

The estimated model allows us to investigate the outcome of fiscal policy volatility shock on business cycle fluctuations and observe heterogeneous effects between constrained and unconstrained agents. As pointed out by Debortoli and Galí (2017), a TANK model can approximate well the aggregate results of a New-Keynesian model with heterogeneous

[7] See Binsbergen et al. (2012), Rudebusch and Swanson (2012), and Andreasen, Fernández-Villaverde and Rubio-Ramírez (2018), for example.

Figure 4.2 – IRF to a Fiscal Policy Uncertainty Shock



Note: Generalized impulse response functions (GIRFs) for a positive two-standard-deviation innovation to a fiscal volatility shock to the capital income tax. Interest rate and inflation are presented in annualized basis points. In the first row, the plots present the reaction of consumption of HtM agent (c_t^h), bondholder (c_t^b), aggregate consumption, and output (c_t and y_t). In the second row, HtM, saver, and aggregate hours worked (h_t^h, h_t^b and h_t) and wages (w_t). Finally, investments (x_t), marginal cost (mc_t), inflation (Π_t), and interest rates (R_t).

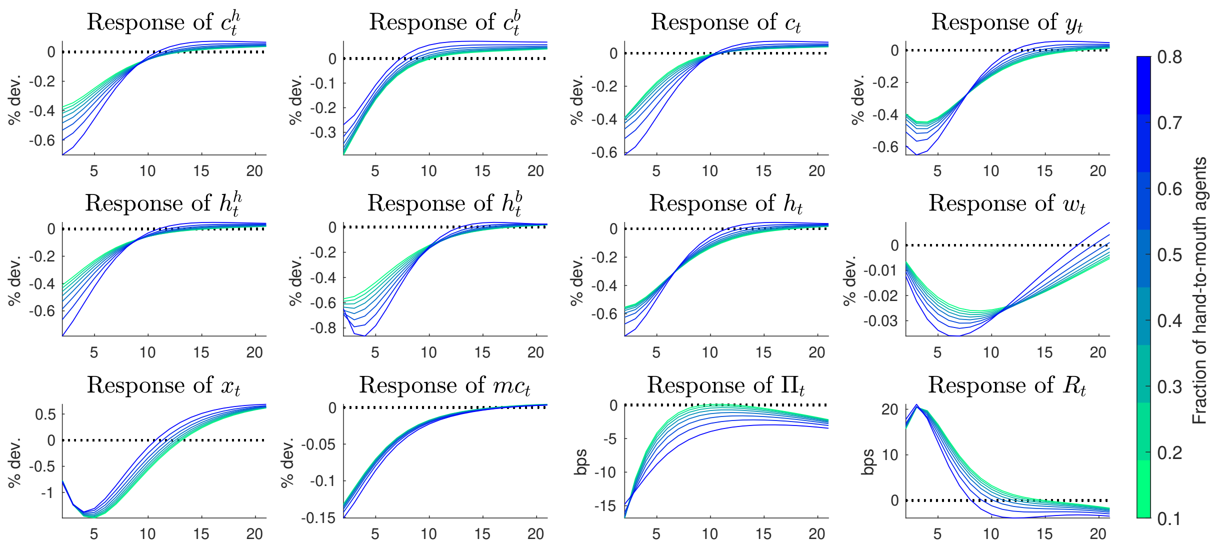
agents. This is relevant for the analysis of policy transmissions, highlighting indirect transmission channels that are absent in a standard representative agent model. Thus, we focus on the different effect of fiscal uncertainty on constrained and unconstrained agents as well as aggregate consequences of these shocks. Since volatility shocks on instruments other than capital tax rate present smaller responses, we overlook their results in this section.

Figure 4.2 presents the GIRFs for a positive two-standard-deviation innovation to a fiscal volatility shock to the capital income tax.[8] Bondholders invest less because of the increased probability of a high tax rate on capital income, and the lower marginal cost (higher markups) implies that firms will have a smaller output and require less capital. The high decline of marginal costs is related to the reaction of the monetary authority to the uncertainty shock, increasing the interest rate more than it would in a scenario where the Taylor rule does not account for the reaction to second-moment shocks. This prompts reaction of marginal costs and decreases labor demand from firms and hours worked by both agents. However, non-HtM agents can decrease their labor supply without losing as much consumption as HtM agents. At the same time, HtM agents would have to decrease consumption more drastically if they reduce labor supply by the same rate used by bondholders.[9] The proportionality between effects on consumption and hours worked is due to the utility kernel used, since non-separability leads individuals who work

[8]We used the closed-form solution in Andreasen, Fernández-Villaverde and Rubio-Ramírez (2018) to compute GIRFs for a third-order approximation.

[9]The... (2006) estimate that individuals with no assets will increase their labor supply if wages go down. We have not had this effect in our model so far, as we cannot distinguish between supply and demand labor responses.

Figure 4.3 – Sensitivity to μ



Note: Sensitivity test on the fraction of HtM agents in the economy μ given a positive two-standard-deviation innovation to a fiscal volatility shock to the capital income tax.

fewer hours to dedicate more time to home production, decreasing the aggregate level.

The contraction in output and investments presents a slightly higher drop than do the results in FGKR. Moreover, together with investment rates, the uncertainty shock generates a decrease followed by an overshooting reaction, in line with Bloom (). Firms diminish investment rates after facing uncertainty about future capital taxes, which fosters a wait-and-see attitude. The rapid activity slowdown bounced back after five quarters, a different reaction in comparison to the actual shock, which did not return to its steady-state value after 21 quarters (see Figure 4.6). It is also worth noticing that the decrease in consumption faced by HtM agents has the same size as that generated by an increase in labor taxes but with a different shape, since it generates a long-lasting reaction after an actual increase of labor taxes (Figure 4.7). The output reaction due to uncertainty about capital tax is also greater than the effects of an actual increase in labor taxes or government spending (Figure 4.8), which shows that volatility shocks should not be neglected by policymakers.

The effect of a change in the fraction of HtM agents in the economy can be seen in Figure 4.3. Kaplan, Violante and Weidner (2014) highlight that the estimation of this parameter in the literature is highly susceptible to the definition of an HtM individual. This analysis allows us to understand the impact of this parameter on the model and which variables are more sensitive to the fraction of a borrowing-constrained household in the economy. Furthermore, ranging μ from 0.1 to 0.8 by increments of 0.1, Figure 4.3 encompasses all possible values for μ referred in Kaplan, Violante and Weidner (2014) as well as some far-fetched cases.

The fraction of HtM agents in the economy does not alter aggregate effects on investments and marginal costs, while other key aggregate variables, such as output and consumption, are sensitive to this ratio. The effect on output can vary from -0.4 to -0.6 percent, but its duration is almost independent of the ratio of HtM to non-HtM agents. A similar analysis can be made for the aggregate consumption, which is mainly led by HtM agents, who consume less as long as they work fewer hours. The smaller the portion of bondholders in the economy, the bigger the pressure of HtM agents on wages, which makes bondholders decrease labor supply. At the same time, since HtM agents do not have any asset to substitute labor income, they cannot decrease hours worked without losing consumption, which is not the case for bondholders, who face an almost unchangeable consumption reaction. This is possible because they will tune capital utilization fornot losing so much income after the uncertainty shock. Furthermore, a stochastic volatility shock to the capital tax rate resembles the results obtained for an income-risk shock in the heterogeneous agent model in Bayer et al. (). Even if we cannot account for the effects related to the distribution of wealth and income, our aggregate results are more in line with the data, which show a slower and slightly hump-shaped recovery. These results reinforce the statement of Debortoli and Galí (2017) that a TANK model can be viewed as a framework able to account for aggregate results of HANK models.

4.4 FISCAL VOLATILITY SHOCKS ON A DEVELOPING COUNTRY

In this section, we aim to explore the effects of fiscal volatility shocks on a developing country. This way, it would be possible to understand whether the sensitivity of the model to the share of HtM agents in the economy is suitable for changes according to country-specific characteristics. It would also be possible to identify which aggregate effect is more sensitive to a different macroeconomic scenario. The reaction of fiscal instruments to the debt-to-output ratio and to the output gap is one of the key differences that can emerge in the comparison between countries. Also, the time-varying process for the standard deviation is correlated with national political events, such as fiscal reform proposals and national public deficit administration and/or financial crises. Furthermore, parameters internally calibrated with the model, such as those concerning the Taylor rule, are considerably sensitive to the input data. As demonstrated in the previous section, the monetary policy is one of the main channels of fiscal policy volatility transmission and response to this shock can have alternative effects due to its calibration.

The current juncture of Brazil's national accounts, discussions on a proposal by the government for social security reform, and increased levels of public indebtedness have raised uncertainty about the evolution of the fiscal policy. The Brazilian economy is facing fiscal difficulty, which should be seen within the context of a hike in deficit since 2011. Before that, there were two moments that could identify the need for fiscal reform, as in 1999 and 2003. However, both episodes are related to exchange rate shocks, while since 2011 the debt increase has occurred due to increased government spending. The primary result of the public sector began to deteriorate more specifically in August 2011, with a fall in extraordinary revenues observed in previous years. In addition, it is possible to recognize an acceleration of this process, given that the gross debt of the general government has grown rapidly, from around 50 percent of GDP in 2014 to more than 70 percent in 2017.[10] Throughout 2017, for example, several proposals for reforms were sent to the Brazilian National Congress, generating doubts about fiscal adjustment directions. In addition to the limitation of public spending, there is a long debate about social security system reform, the need to raise taxes to finance the government deficit, and changes in the loan rates of the National Bank for Economic and Social Development (BNDES, in Portuguese). All these events are potential sources of fiscal volatility.

4.4.1 Estimation Results

We calibrate internally the same set of parameters of the model for the US economy using an SMM procedure. Results are reported in Table 4.5 and a summary of fit for selected moments in Table 4.4, which presents consistent results mainly for the standard deviation and autocorrelation, except for the moments for wages and for mixed correlation. The risk-aversion parameter for HtM agents is considerably higher than those presented by others in the macroeconomic literature with Epstain-Zin preferences, while bondholders' risk aversion is in line with the literature. Comparing with our prior results for the US, these parameters are switched, since HtM agents were less risk-averse than bondholders in the previously calibrated model. This result for Brazil runs counter to the argument in the literature, which claims that risk-averse agents self-select themselves into a low-income risk position.[11] There is a lack of studies in the literature that carry out this investigation for developing countries, but one hypothesis is that, in these countries, agents are not in an HtM situation because they want to, but because they can get out of it. This would mean that most HtM agents in emerging markets are poor, in contrast with the US, where Kaplan, Violante and Weidner (2014) highlight that many of these households are wealthy.

The curvature of the depreciation rule δ_2 is almost flat for Brazil, since this parameter tends to zero. Parameters related to the Taylor rule reinforce the Brazilian hyperinflation history, once the steady-state inflation is higher than that of the US. At the same time, the reaction to output gap deviations is smaller, with close parameters for the smoothness of interest rates. The reaction to deviations from the steady-state inflation is slightly smaller in Brazil, which can be due to the higher steady-state inflation

[10] Brazil is a developing country with a recent history of high levels of inflation and the historical level of indebtedness tolerated by Brazil can still be considered low, when compared with other countries. Thus, a debt level of 70 percent of GDP can be seen as excessive, as pointed out by Reinhart, Rogoff and Savastano (2003) and Fiscal... (2016) illustrate.

[11] See Guiso and Paiella (2008), among others.

Table 4.4 – Model Fit - selected moments

Moments	Data	Model	Moments	Data	Model
Mean					
R_t	3.6402	4.0196	Π_t	1.5861	1.6439
Standard Deviation			*Auto-Correlation*		
y_t	1.758	1.7623	$corr(y_t, y_{t-1})$	0.74388	0.69098
c_t	1.29	1.1464	$corr(c_t, c_{t-1})$	0.55825	0.48726
x_t	3.9761	4.5304	$corr(x_t, x_{t-1})$	0.60349	0.89865
w_t	2.0259	0.10741	$corr(w_t, w_{t-1})$	0.6087	0.94596
h_t	2.3639	2.0404	$corr(h_t, h_{t-1})$	0.71218	0.62762
u_t	1.6326	1.6537	$corr(u_t, u_{t-1})$	0.77985	0.69937
R_t	1.3517	1.3567	$corr(R_t, R_{t-1})$	0.67623	0.60661
Π_t	0.87585	0.24949	$corr(\Pi_t, \Pi_{t-1})$	0.49848	0.66197
Correlation					
$corr(y_t, c_t)$	0.77616	0.7664	$corr(y_t, u_t)$	0.90875	0.56632
$corr(y_t, x_t)$	0.67929	0.74621	$corr(y_t, R_t)$	0.12665	-0.76572
$corr(y_t, w_t)$	-0.38174	-0.050341	$corr(y_t, \Pi_t)$	0.23915	0.40929
$corr(y_t, h_t)$	0.49078	0.64953	$corr(\Pi_t, R_t)$	0.35504	-0.55523

Note: All data, except nominal interest rates and inflation, are in logs, HP-filtered, and multiplied by 100 for their expression in percentage deviation from the trend. Nominal interest rates and inflation are directly expressed in percentage points. We omitted model fit for the mean and autocorrelation for lag 5 due to space constraints.

and monetary shock standard deviation. These differences, along with those results for the fiscal instruments shape different reactions to a capital tax rate volatility shock.

4.4.2 Response to Volatility Shocks

The GIRFs for a positive two-standard-deviation innovation to a fiscal volatility shock to the capital income tax in Brazil are presented in Figure 4.4. The first remarkable difference from US results (Figure 4.2) is related to the size of responses. We keep two-standard-deviation innovations as in FGKR, where the authors verify that there is a probability of 10 percent of these shocks hitting the economy, but they argue that it cannot be considered an extreme event, since they show evidence that it is likely that at least three or four of those events hit the US economy from 1970 to 2014. Nevertheless, a positive two-standard-deviation innovation in Brazil generates effects that are slightly smaller in size compared with those presented previously for the US economy. The main reason for that is the size of unconditional standard deviation η_σ and the parameter ρ_σ, which controls the shock persistence. Since the database for Brazil is smaller than the one for the US, Table 4.1 shows that the confidence interval for these parameters is wider, and their values are smaller.

Broadly speaking, the same mechanism takes place, lowering marginal cost and

Table 4.5 – Summary Calibration for Brazil

	Description	Value	Source
Preferences			
β	Time-discount factor	0.9785	Calibration
γ_h	HtM Risk Aversion	1329.10	Calibration
γ_b	Bondholder Risk Aversion	373.46	Calibration
η	Consumption preferences	0.4034	Endogenous
Ω^h	HtM lump-sum tax steady state	-0.2263	Endogenous
Ω^b	Lump-sum tax steady state	0.3019	Endogenous
Technology			
δ	Steady state depreciation	0.0442	Calibration
δ_2	Rate of capital depreciation	0.003	Calibration
κ	Investment cost	0.9458	Calibration
Taylor Rule			
ϕ_R	Smoothing of past interest rate	0.7611	Calibration
ϕ_Π	Response to inflation deviations	1.5701	Calibration
ϕ_y	Response to output gap deviations	0.0097	Calibration
Π_s	Steady state inflation	1.0124	Calibration
σ_m	Standard dev. of the monetary shock	0.0034	Calibration

Note: Parameters estimated with an SMM procedure are tagged with "Calibration." The consumption preference parameter η is tagged with "Endogenous" because we normalize the steady state of hours worked to 1/3 using η. We also set Ω^h and Ω^b endogenously to achieve a balance between HtM and governmental budget constraint, respectively. The other parameters fixed prior to the estimation are reported in Section 4.2, and those estimated exogenously are shown in Table 4.1.

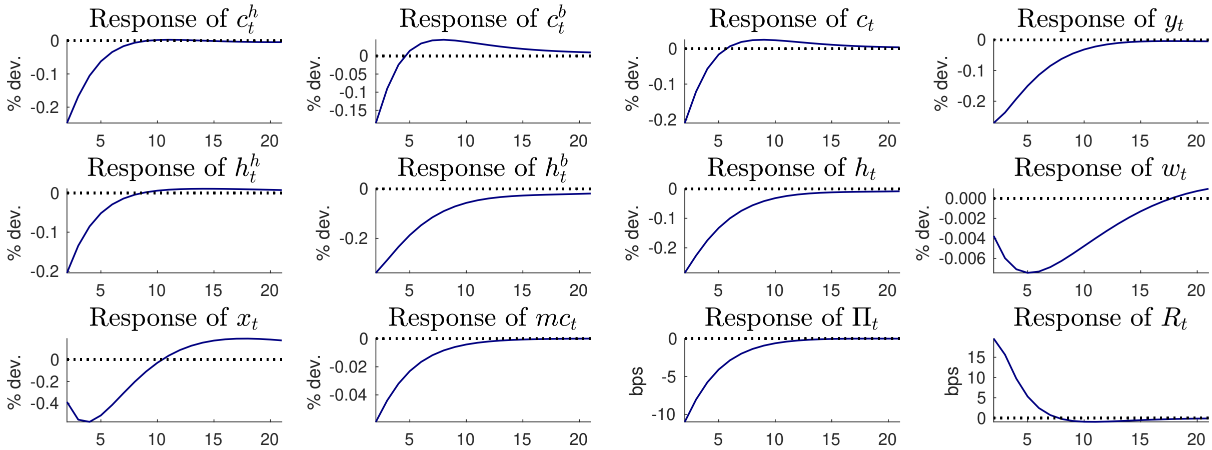
inflation. Yet, the monetary authority in Brazil reacts with less intensely than does the US Fed, pushing up interest rates even with inflation going down, due to the positive reaction to the uncertainty shock.

We present the role of this innovation in the Taylor rule in Figures B.2 and B.3 ,comparing the results of a standard Taylor rule, without any direct response of the monetary authority to the shock, and the Taylor rule in Eq. (3.13). While the effects on the US are relatively subtle, allowing the model to replicate empirical evidence of a drop in inflation, as documented by FGKR, in Brazil, the difference is much stronger. Without a direct response of the monetary authority to uncertainty about capital income tax, all observed effects would be negligible. Furthermore, this also shows that the model can be extremely sensitive to the parameter ϕ_σ that controls the responsiveness of interest rates to volatility shocks.[12]

Further effects that can be observed are the output decline, which takes more than 21 quarters to get back to its steady-state level, a more persistent result compared with the US, spawned by the high persistence of capital tax rate volatility. The decline of hours worked by HtM agents is still bigger than the bondholders' response, but it returns faster

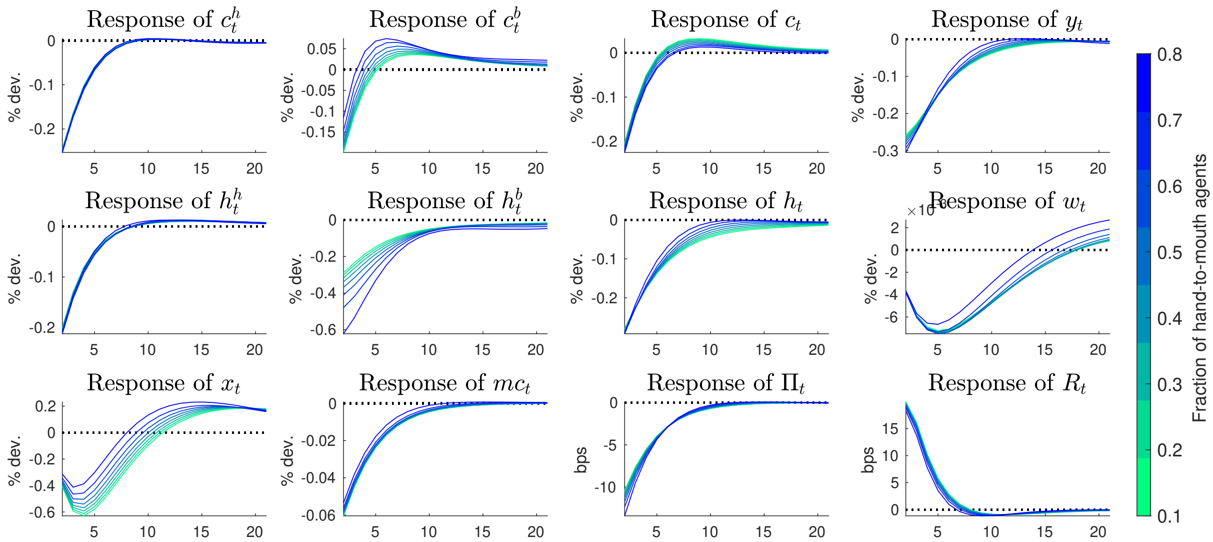
[12] An essential next step in this analysis would be the implementation of an empirical model, which we aim to include in an upcoming version of this paper.

Figure 4.4 – IRF to a Fiscal Policy Uncertainty Shock for Brazil



Note: GIRFs to a positive two-standard-deviations innovation to a fiscal volatility shock to the capital income tax in Brazil. Interest rate and inflation are in annualized basis points. In the first line the plots presents the reaction of consumption of HtM agent (c_t^h), bondholder (c_t^b), aggregate consumption and product (c_t and y_t). Second line, HtM, Saver and aggregate hours worked (h_t^h, h_t^b and h_t) and wages (w_t). Finally, investments (x_t), marginal cost (mc_t), inflation (Π_t) and interest rates (R_t).

Figure 4.5 – Sensitivity to μ in Brazil

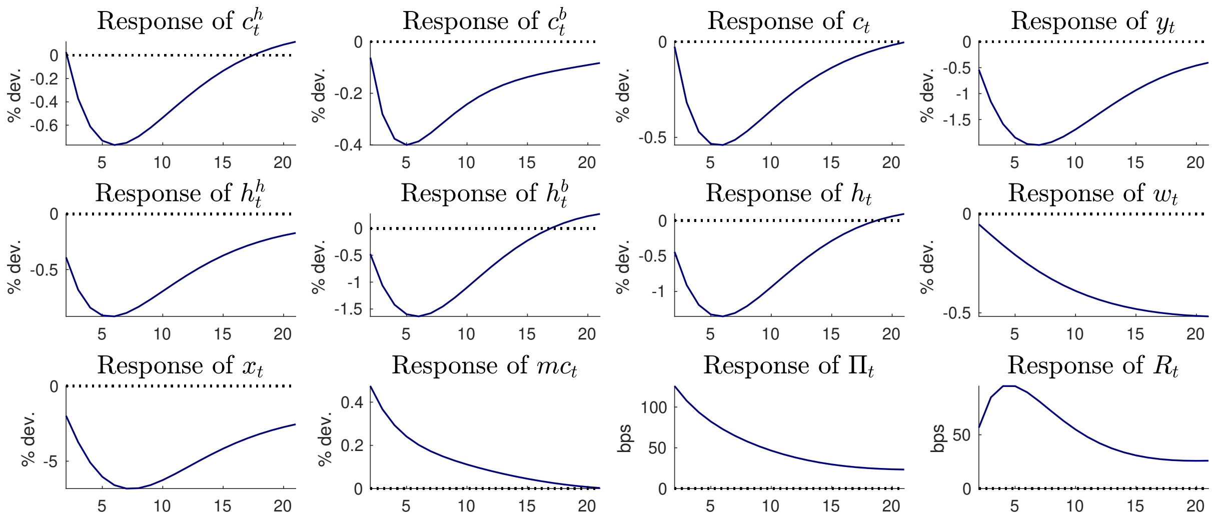


Note: Sensitivity test to the fraction of HtM agents on the economy μ given a positive two-standard-deviations innovation to a fiscal volatility shock to the capital income tax.

to the steady-state level. The opposite occurs in the reaction to consumption levels, where bondholders present a typical hump-shaped response to uncertainty shock, retrieving equilibrium in five quarters, while HtM agents face a slow recovery.

Finally, we are able to observe similar results for the sensitivity to the fraction of borrowing-constrained households in the economy (Figure 4.5). Since HtM agents do not have alternative sources of income, they push wages down, which makes bondholders work less. The higher the fraction of borrowing-constrained agents in this economy, the higher

Figure 4.6 – IRF to a Fiscal Policy Shock to Capital Tax



Note: GIRFs to a positive two-standard-deviations innovation to a fiscal shock to the capital income tax. Interest rate and inflation are in annualized basis points. In the first line the plots presents the reaction of consumption of HtM agent (c_t^h), bondholder (c_t^b), aggregate consumption and product (c_t and y_t). Second line, HtM, Saver and aggregate hours worked (h_t^h, h_t^b and h_t) and wages (w_t). Finally, investments (x_t), marginal cost (mc_t), inflation (Π_t) and interest rates (R_t).

this effect. Bondholders manage their budget to work less with lower capital utilization, as long as wages are too low. Also, for the set of parameters estimated for Brazil, investment level is more sensitive to μ than in the US. The same happens with the consumption response of bondholders, who are completely unresponsive to fiscal volatility shocks on the US.

4.5 FISCAL POLICY SHOCKS

In this section, we present the results for fiscal policy shocks on capital tax rate τ_t^k, labor tax rates τ_t^w, and government spending g_t, which will offer a benchmark comparison of the relevance of fiscal volatility shocks. Figure 4.6 presents the impulse response function to a fiscal policy shock to capital income tax. Comparing the effect of two-standard-deviation innovations of an actual fiscal shock to the uncertainty shocks will give us an understanding of the importance of the latter. First, it is important to highlight that two-standard deviations make sense for the uncertainty shock, but it would be rather unlikely for fiscal shocks. Even though this may not be the most likely size, we decided to keep it for a matter of comparison.

The effect on output for a capital income tax shock in Figure 4.6 is almost 5 times greater than the uncertainty shock on the same instrument. However, when we compare with labor tax and government spending shocks in Figure 4.7 and 4.8, we notice that the size of output response is much closer to the uncertainty shock. This means that uncertainty about fiscal policy can have a strong effect on the economy, comparable with actual fiscal shocks. Also, the distinction in timing is remarkable. While fiscal shocks last longer, uncertainty vanishes fast, which makes the cumulative effect greater for fiscal shocks. However, it is not hard to believe that uncertainty shocks may happen consecutively, which can be understood as an uncertainty trap.

Specific on government spending shocks, Galí, López-Salido and Vallés () introduced HtM households to obtain a positive response of consumption and investment to a government spending shock. They show that depending on the fraction of HtM agents on the economy (μ_t), price rigidities and labor market structure it is possible to generates a positive comovement of consumption and government spending, which is in line with empirical evidences for the US. However, Figure 4.8 goes on the opposite direction, showing a crowding-out effect of the government spending over consumption and investment. This suggests that the faction of HtM agents in Brazil could be higher then 40%, though a correlation analysis between the cyclical component of government spending to GDP and consumption shows a negative 0.36 result. At the same time, Ferreira (2015) finds a positive response of consumption to government spending shocks in a TVP-VAR with stochastic volatility estimated over 1996 to 2014. Since this analysis is beyond scope of this dissertation, we left this subject for further analysis of the interested reader.

Another important point is to understand how each agent reacts to the shock. In any case, the HtM agent is the most severely damaged, even for the government spending increase, where HtM agents may experience an increase in their consumption levels, but in the long run, the shock turns out to be negative. Comparing fiscal shocks, labor tax seems to be the most harmful to consumption levels due to its long-lasting effect, while capital tax drives consumption deeper in the short run, getting back to its steady state after 20 quarters.

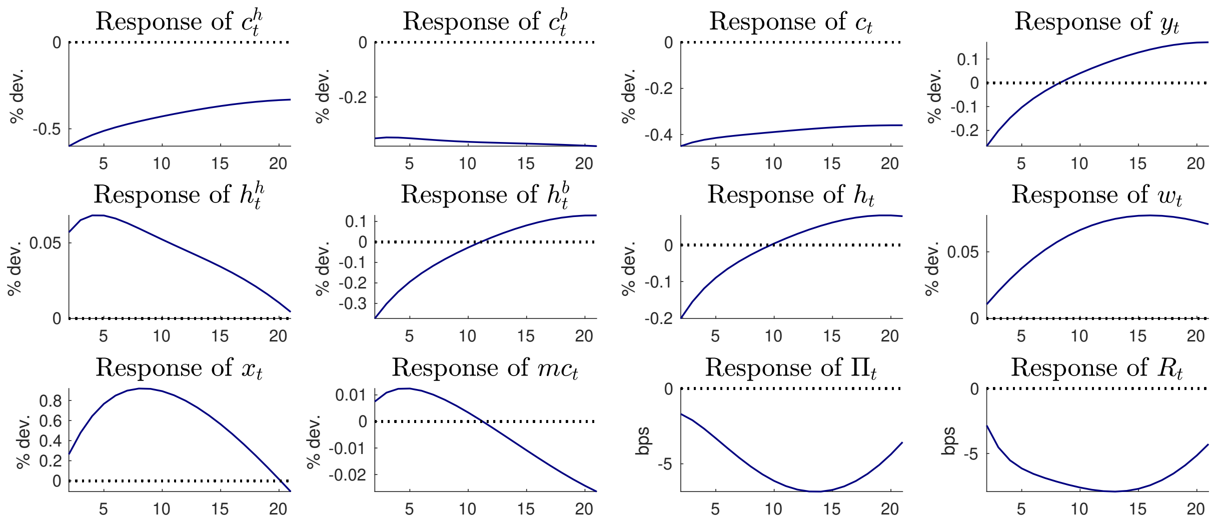
4.6 PSEUDO-OPTIMAL FISCAL POLICY

Considering that countries can diverge in their fiscal policy rule and in their reraction to its output growth or indebtedness, we perused a "Pseudo-Optimal" Fiscal Policy. Since we are not solving a welfare Ramsey problem, we cannot regard this exercise as an optimal policy prescription. In fact, we vary the parameter that captures the reaction of government spending to previous debt and output gap, considering a range in line with the literature.[13] We used an equally spaced grid of 6,400 points for $\rho_{g,y} \in \{-0.2, \dots, 0.2\}$ and $\rho_{g,b} \in \{0, \dots, 0.2\}$, where for each combination we solved the model and computed the cumulative GIRFs for three years. Thus, this procedure allowed us to identify optimal combinations for the fiscal policy rule on government spending in a specific parametric space reported by the empirical literature.

The literature on optimal fiscal policy with recursive preferences is summarized by

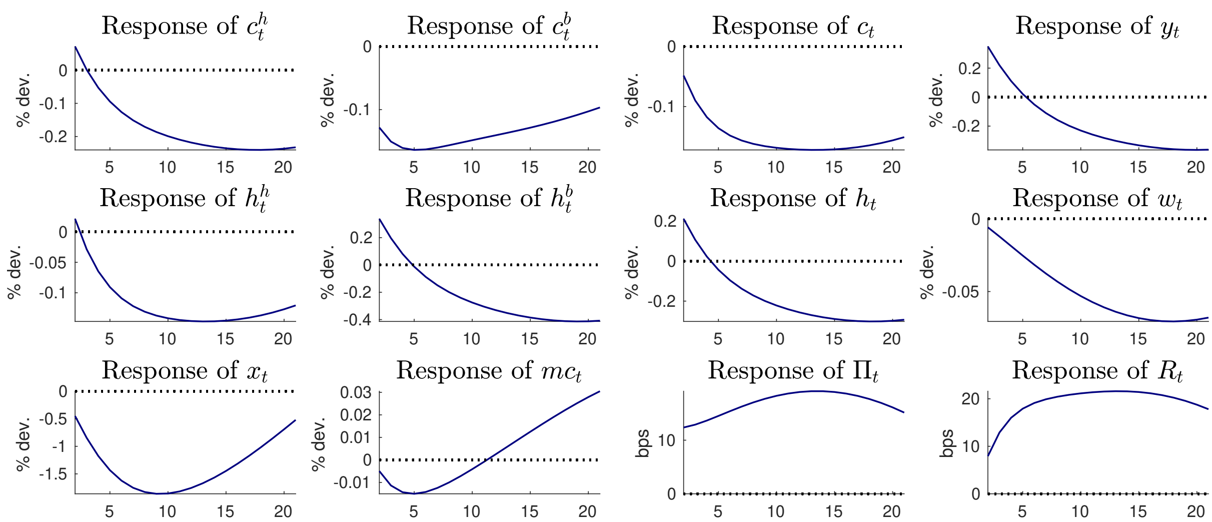
[13]See Vegh and Vuletin (2015).

Figure 4.7 – IRF to a Fiscal Policy Shock to Labor Tax



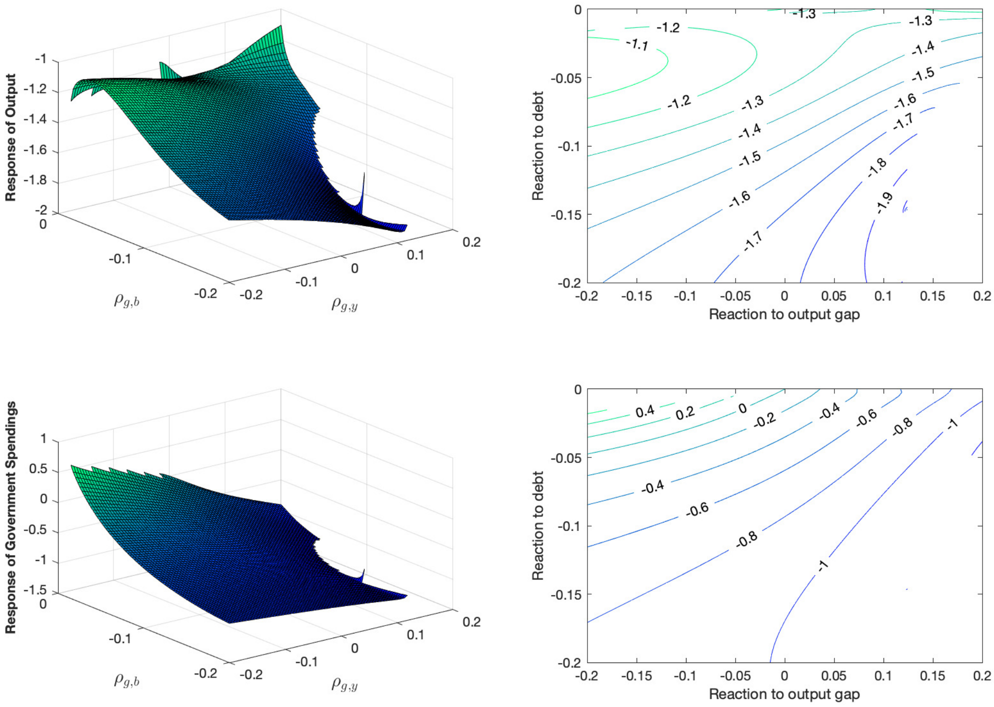
Note: GIRFs to a positive two-standard-deviations innovation to a fiscal shock to the labor tax. Interest rate and inflation are in annualized basis points. In the first line the plots presents the reaction of consumption of HtM agent (c_t^h), bondholder (c_t^b), aggregate consumption and product (c_t and y_t). Second line, HtM, Saver and aggregate hours worked (h_t^h, h_t^b and h_t) and wages (w_t). Finally, investments (x_t), marginal cost (mc_t), inflation (Π_t) and interest rates (R_t).

Figure 4.8 – IRF to a Fiscal Policy Shock to Government Spending



Note: GIRFs to a positive two-standard-deviations innovation to a fiscal shock to the government spending. Interest rate and inflation are in annualized basis points. In the first line the plots presents the reaction of consumption of HtM agent (c_t^h), bondholder (c_t^b), aggregate consumption and product (c_t and y_t). Second line, HtM, Saver and aggregate hours worked (h_t^h, h_t^b and h_t) and wages (w_t). Finally, investments (x_t), marginal cost (mc_t), inflation (Π_t) and interest rates (R_t).

Figure 4.9 – Pseudo-Optimal Fiscal Policy - Government Spending and Output



Note: Response of government spending and output gap to a capital income tax volatility shock. The left column presents the three-dimensional representation of all combinations of $\rho_{g,y}$ and $\rho_{g,b}$, i.e., the reaction of government spending to previous debt and output gap, and on the left, its contour representation. The level of the plot is the cumulative GIRFs three years after the realization of the uncertainty shock.

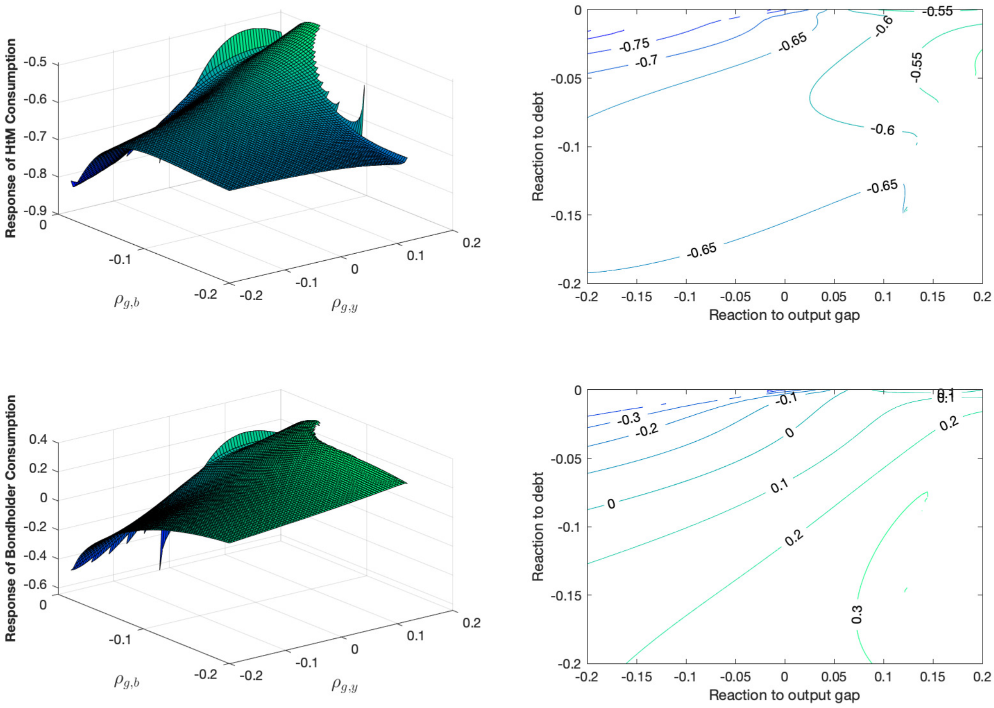
Karantounias (2018), who demonstrates that optimal fiscal policy prescriptions change dramatically depending on the Household preferences. According to Karantounias (2018), the government should manipulates the returns of its portfolio to minimize the welfare costs of taxes. In this way, the government would absorb the fiscal risk in order to attenuates utility volatility by taxing lass in bad times. However, he also stresses that the connection between risk-aversion and intertemporal elasticity of substitution is a key factor for this result. What we aim to understand in this section is how should the government behave in order to minimize utility loses generated by fiscal uncertainty. extending the understanding in Karantounias (2018) to the relationship between state-contingent debt and the existence of the different types of agents on the economy.

The first thing we need to understand is what effects capital income tax volatility shocks have on output considering several different reactions by the government, i.e., procyclical or not, as well as a positive or negative reaction of the spending level to current indebtedness. A remarkable result in Figure (4.9) is that, regardless of the government's policy decision, uncertainty will have a negative impact on output. Figure (4.9) presents the response of government spending and output gap to a capital income tax volatility shock, showing, in the left column, the three dimensional representation of all combinations of $\rho_{g,y}$ and $\rho_{g,b}$, i.e., the reaction of government spending to previous debt and output gap, and on the right, its contour representation. The level of the plot shows cumulative GIRFs three years after the realization of the uncertainty shock. It is possible to identify two quasi-optimal results for the output. First, when the parameter that indicates government behavior towards past debt is close to -0.05 and the parameter for response to the output gap is negative, i.e., countercyclical behavior. In this case, the output would have decreased around -1.1 percent given the steady-state level. On the other hand, a procyclical government could achieve a similar result if it had a weaker concern with the previous debt level. The main distinction would be that, in the first case, the government would end up with a net spending close to zero, while in the second case, the government decreases its spending level.

At the same time, when we perform the same analysis for household consumption level in Figure (4.10), there is a striking distinction in the outcome concerning government's behavior. The first thing to notice is that an HtM household will always face a negative response to its consumption level, while the bondholder is more responsive to fiscal policy. Concerning both spots where the government decision is less harmful to output, while the first one results in a contraction of consumption, the second one can even lead to a positive 0.1 percent deviation from its steady-state level for the bondholder, and a slightly less negative reaction to the HtM consumption level. This happens because when the government decides to shrink its spending, income taxes are also reduced, therebydecreasing the effects of uncertainty on households.

This result is in line with Karantounias (2018), since the optimal policy in this exercise would be the government's decision to absorb the risk, spending less and decreasing its indebtedness level. At the same time, we show how different kinds of agents are affected by the government's fiscal policy reaction parameters. With a lower debt level, the tax level on capital would slightly decrease, while capital income tax would be lower proportionally to the alternative outcome, when the government decides to spend more in reaction to the lower output gap. With lower tax on both capital and labor, the bondholder ends up with a higher consumption level after three years, while the HtM agent will only be affected by the lower level of labor tax. At the same time, a more accurate interpretation is that higher government spending leads to a higher cost of production and lower markups. A higher cost of production would push up inflation, leading the monetary authority to increase interest rates. Bondholders would use precautionary savings and buy debt to avoid greater losses. Since firms would face a greater decrease in their markup, capital utilization and labor demand would diminish, making both agents work less. However, when the government spends less and less pressure is put on markups, firms will go back to investing in capital earlier and demand labor. Thus, the consumption level of agents will return faster to its steady state, but ay a much slower pace for HtM agents.

Figure 4.10 – Pseudo-Optimal Fiscal Policy - Bondholder and Hand-to-Mouth Consumption



Note: Response of consumption for both agents, the bondholder and theHtM agent, to a capital income tax volatility shock. The left column presents the three-dimensional representation of all combinations of $\rho_{g,y}$ and $\rho_{g,b}$, i.e., the reaction of government spending to the previous debt and output gap, and on the left, its contour representation. The level of the plot is the cumulative GIRFs three years after the realization of the uncertainty shock.

5 GOVERNMENT SPENDING UNCERTAINTY

So far, we have focused on the effects of tax uncertainty, since it presents the most prominent effects on our framework, which is also a common feature of other counterparts in the literature. However, it is fairly plausible to assume that uncertainty about government spending can have significant effects on aggregate macroeconomic variables. As presented in Figure (4.8), an actual increase in government spending has a negative effect on aggregate consumption and investment level of the economy, and also a persistent increase in inflation, prompting the monetary authority to react with higher interest rates. Considering these real effects, it is reasonable to believe that in the presence of uncertainty about future government spending, some agents may prefer to wait for the government's decision.

In this chapter, we aim to explore the real effects of an exogenous stochastic government spending volatility shock. Considering the time-varying stochastic volatility process estimated in Section 4.1, we incorporate the filtered $\sigma_{g,t}$ from Eq. (3.2) into a structural vector autoregression (SVAR) with aggregate macroeconomic variables to evaluate the real effects of volatility shocks. The empirical framework is able to show relevant effects on the response of consumption, output, and monetary policy to a shock on government spending volatility. Given these results, we try to incorporate the empirical responses into the model by matching the impulse response of both frameworks.

It is important to highlight that our theoretical model assumes smooth decision rules and policy functions, without discontinuities. Thus, any uncertainty raised about government spending cannot transit to a default, since it would generate a kink on the debt and output level of the economy. In this case, we would need to use a different kind of solution method other than perturbation, which does not accommodate this situation. Albeit a relevant situation for many emerging economies, we kept the same framework designed in Section 3.3 for ease of comparison.

Related Literature. There are several uncertainty measures in the literature, but the economic policy uncertainty index (PUI) by Baker, Bloom and Davis (2016) are one of the most widely used measures in empirical papers. Broadly speaking, the PUI is a measure of uncertainty based on the number of articles about economic policy uncertainty in major newspapers. Kaviani et al. (2020) use the PUI to investigate the effect of policy uncertainty on corporate credit spreads. They highlight that policy uncertainty can affect firms directly, through its specific impact on a given firm, or indirectly, through the economic environment. Firms more sensitive to government policies may depend largely on government spending, either on an industry with higher regulation by the government, or on the payment of higher effective tax rates. Those three kinds of firms would be affected differently to an increasing level of spending, but before the actual policy, Kaviani et al. (2020) argue that all three would face a change in their borrowing costs due to a higher policy uncertainty.

In the same way as Kaviani et al. (2020), Gulen and Ion (2015) find evidence of a negative relationship between policy uncertainty measured by the PUI and investments. According to the authors, a doubling in the level of policy uncertainty can lead to a decrease by approximately 8.7% in quarterly investment rates relative to the average investment rate. A counterfactual analysis for the 2007-2009 period suggests that uncertainty may be accountable for roughly one third of the 32% fall in capital investments during the crisis.

An alternative commonly used proxy to measure uncertainty effects is elections. Julio and Yook (2012) highlight that election outcomes are relevant to corporate decisions, as they have implications for industry regulations, monetary and trade policy, taxation, etc. According to the authors, in the period leading up to the election, investment expenditures decline by an average of 4.8%, controlling for growth opportunities and economic conditions. Also, Julio and Yook (2012) document that the election-year drop in investment is followed by a small temporary increase in investment in the year following the election, but in a smaller scale than that of the earlier decline.

The aggregate measurement of uncertainty, based on stochastic volatility models, as described in Chapter 2, is used by Bloom (), Basu and Bundick (), and Fernández-Villaverde et al. (2015), and it will also be employed in the current chapter. Caldara et al. (2020) work on the news-based and the stochastic volatility measure of uncertainty to understand what changes in trade policy uncertainty affect economic activity. Thus, they find that both expected future tariffs and uncertainty about future tariffs reduce investment and output, leading also to a reduction in exporters' investments to a greater extent than in those of non-exporters, which is consistent with firm-level evidence.

The most closely related studies to the current chapter can be found in Ricco, Callegari and Cimadomo (2016) and Kim (2019), who investigate the empirical effects of uncertainty about government spending policy on economic activity. Ricco, Callegari and Cimadomo (2016) argue that uncertainty about fiscal policy is related to private agents' expectations in a system with dispersed information where the government has potentially superior information on its procedures and future plans. Bearing this in mind, they develop an index of precision of fiscal policy communication derived from forecasters' disagreement on the future path of federal fiscal spending, based on the Philadelphia Fed's Survey of Professional Forecasters, named 'fiscal news.'. In order to estimate the effects of fiscal news shocks, they incorporate the index into an expectations threshold VAR model together with a number of macroeconomic variables. Kim (2019) follows a similar strategy, using both the disagreement of the Philadelphia Fed forecasting measure and the PUI, provided by Baker, Bloom and Davis (2016), in a proxy SVAR model. A similar empirical strategy, with different uncertainty measure, is also pursued by Basu and Bundick () and Fernández-Villaverde et al. (2015), estimating a VAR with stochastic volatility and aggregate macro variables.

Concerning the results of these papers, Ricco, Callegari and Cimadomo (2016) provide evidence of the power of fiscal policy in periods of high (or low) disagreement, while Kim (2019) shows that an increase in government spending policy uncertainty has a negative, sizable, and prolonged effect on economic activity. Ricco, Callegari and Cimadomo (2016)'s study emphasizes the benefits of clear communication and lower uncertainty about future government's decisions. They argue that fiscal policy communication can be used as a forward guidance tool to coordinate economic agent's expectations and thus consumption, investment, and savings decisions. On the other hand, Kim (2019) focuses on the negative impacts of uncertainty on the real economy, also demonstrating that a recursive VAR tends to underestimate the adverse effects of such shocks. In order to address this issue, we will provide a detailed description of our simple empirical strategy in the next section and compare with other frequently used approaches described in the literature.

5.1 EMPIRICAL STRATEGY

In order to estimate the empirical effects of government spending uncertainty shocks on macroeconomic variables, we used a structural vector autoregression (SVAR) assuming a long-run restriction as identification strategy. The literature on identification strategies of VAR models with uncertainty still does not have a consensus. Several authors follow a standard recursive VAR, such as Fernández-Villaverde et al. (2015), Basu and Bundick (), and Caldara et al. (2020). Contrary to this approach, Ludvigson, Ma and Ng (forthcoming) argue that a recursive structure is unsatisfactory as an identification strategy for a study on uncertainty and business cycles, since it rules out the idea that uncertainty could covary contemporaneously with real activity both because it is an exogenous impulse driving business cycles and because it responds endogenously to first-moment shocks.

At the same time, Ludvigson, Ma and Ng (forthcoming) propose a novel SVAR identification strategy that allows for simultaneous feedback between uncertainty and real activity using shock-based restrictions. Alternatively, Piffer and Podstawski (2017), Kim (2019), and Mumtaz and Theodoridis (2020) use sign-restrictions or instrumental variables (IV) to identify exogenous shocks. Considering our research interest, the sign restriction VAR does not seem appropriate, once it requires several assumptions because a direct imposition of signs on the variables of interest can generate artificial impulse responses. At the same time, a proxy VAR with a weak IV problem can generate unreliable results. Thus, we impose long-run restrictions, implying that an uncertainty shock does not have a long-run effect on macro variables. By doing so, we avoid some of the criticisms by Ludvigson, Ma and Ng (forthcoming) about standard recursive identification, even though the long-run effects of uncertainty shocks have not been theorized yet.

We follow Basu and Bundick (), with uncertainty about government spending ordered first, which assumes that uncertainty shocks can have an immediate impact on output and its components, but non-uncertainty shocks do not affect the implied stock market volatility on impact. Also, this ordering is in line with the theoretical model presented in Chapter 3. We used the same variables selected for the calibration of the model in the previous chapter, detailed in Section B.1. We use the filtered estimates of $\sigma_{g,t}$ in Eq. 3.2, where we assume that the law of motion of g_t is given by

$$\begin{aligned} g_t - g =& \rho_g(g_{t-1} - g) + \rho_{g,y}\tilde{y}_{t-1} \\ &+ \rho_{g,b}\left(\frac{b_{t-1}}{y_{t-1}} - \frac{b_s}{y_s}\right) + \exp\{\sigma_{g,t}\}\varepsilon_{g,t}, \quad \varepsilon_{g,t} \sim \mathcal{N}(0,1), \end{aligned} \tag{5.1}$$

where the log standard deviation of each policy instrument ($\sigma_{\text{x},t}$) has a time-varying stochastic volatility process,

$$\sigma_{g,t} = (1-\rho_\sigma)\sigma_{g,s} + \rho_\sigma\sigma_{t-1} + (1-\rho_\sigma^2)^{1/2}\eta_\sigma u_{g,t}, \quad u_{g,t} \sim \mathcal{N}(0,1). \tag{5.2}$$

Equation (3.3) assumes that demean fiscal policy reacts to its previous value g_{t-1}, to the lagged output gap $\tilde{y}_{t-1}$, and to the lagged debt-to-output ratio (b_{t-1}/y_{t-1}) deviation from its mean value. Idiosyncratic shocks to uncertainty are independent of first-movement fiscal policy shocks $\varepsilon_{g,t}$, which follow a normal distribution with zero mean and one standard deviation. The stochastic volatility process $\sigma_{g,t}$ in Equation (3.2) allows a shock to the second moment u_t, which will be interpreted as an increase in uncertainty about the future time path of fiscal policy, and η_σ will be the unconditional standard deviation of the fiscal volatility shock.

The SVAR response is directly targeted in our calibration, following Basu and Bundick ()'s strategy of combining SMM and the impulse response matching estimator. The SMM procedure is described in Section (4.3) and is in line with Andreasen, Fernández-Villaverde and Rubio-Ramírez (2018), where we incorporate the VAR-based impulse response estimator of the structural parameters of our model summarized in Guerron-Quintana, Inoue and Kilian (2017). Formally, our estimator is the solution to the following problem:

$$J(\hat{\theta}) = \min_{\theta}[\hat{\Psi} - \Psi(\theta)]'\mathbf{V}[\hat{\Psi} - \Psi(\theta)] + [\zeta - \zeta(\theta)]'\mathbf{W}[\zeta - \zeta(\theta)]'. \tag{5.3}$$

Here, $\mathbf{W}$ is a positive definite weighting matrix and $\zeta(\theta)$ contains the model's simulated moments. We follow Andreasen, Fernández-Villaverde and Rubio-Ramírez (2018)'s implementation by letting $\mathbf{W}_T = diag(\hat{\mathbb{S}}_{mean}^{-1})$ in a preliminary first step to obtain $\hat{\theta}_{step1}$, where $\hat{\mathbb{S}}_m^{-1}ean$ denotes the long-run variance of the data ζ when re-centered around its sample mean. In the second step, $\mathbf{W}_T = \hat{\mathbb{S}}_{step1}^{-1}$ is set to the optimal weighting matrix, where $\hat{S}_{step1}^{-1}$ is the long-run variance of simulated moments parameterized with $\hat{\theta}_{step1}$. Both long-run variances are estimated by the Newey-West estimator using 10 lags. However, in the first step, we only run the SMM estimation. Also, $\hat{\Psi}$ in Eq. (5.1) denotes the

Table 5.1 – Summary Calibration and Fixed Parameters

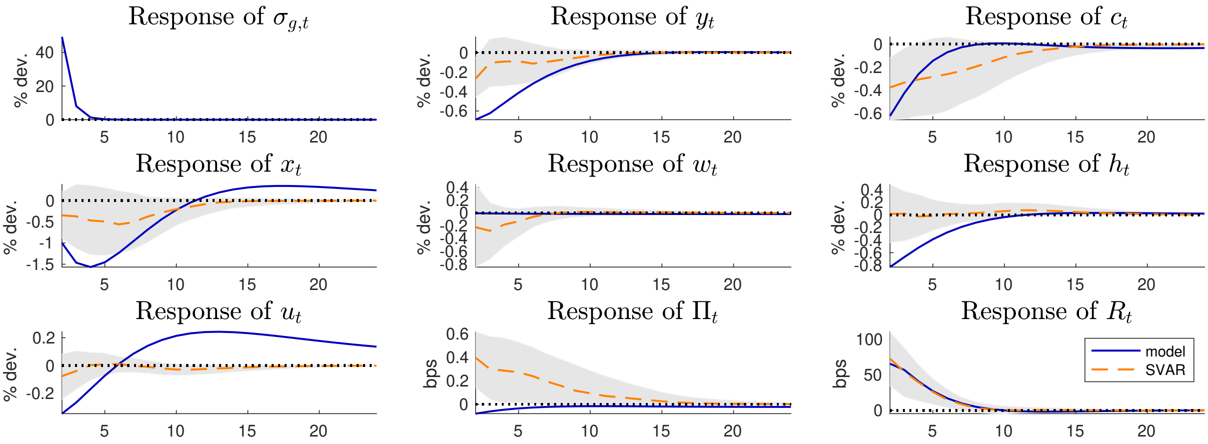
	Description	Value	Source
Preferences			
β	Time-discount factor	0.9865	Calibration
γ_h	HtM Risk Aversion	197.92	Calibration
γ_b	Bondholder Risk Aversion	509.54	Calibration
μ	Fraction of HtM agents	0.40	Kaplan, Violante and Weidner (2014)
η	Consumption preferences	0.4220	Endogenous
Technology			
α	Capital share	0.41	Calibration
ε_p	Goods elasticity of substitution	2.22	Calibration
ε_w	Labor elasticity of substitution	16.85	Calibration
δ	Steady state depreciation	0.0462	Calibration
δ_1	Rate of capital depreciation	0.01	Jaimovich and Rebelo (2009)
δ_2	Rate of capital depreciation	0.0076	Calibration
κ	Investment cost	0.9998	Calibration
Taylor Rule			
ϕ_R	Smoothing of past interest rate	0.8583	Calibration
ϕ_Π	Response to inflation deviations	1.66897	Calibration
ϕ_y	Response to output gap deviations	0.3494	Calibration
ϕ_σ	Response to capital tax volatility	0.0285	Calibration
Π_s	Steady state inflation	1.05	Calibration
Adjustment costs			
ϕ_p	Prices quadratic adjustment cost	4,335.73	Calibration
ϕ_w	Wages quadratic adjustment cost	4,660.99	Calibration

Note: parameters fixed prior to the estimation are referred with their corresponding source. Parameters estimated with SMM&IR matching procedure are tagged as "Calibration," The consumption preference parameter η is tagged as "Endogenous" because we normalize the steady state of hours worked to 1/3 using η.

vector of empirical impulse responses and $\Psi(\theta)$ is the model-implied impulse responses to uncertainty shock. Given the empirical results and the literature, we select the response of output, consumption, investment, inflation, and interest rate. Also, instead of matching the response on a long horizon, we restrict it to six quarters, with most responses still far from zero. Finally, $\mathbf{V} = \mathbf{\Sigma}_T^{-1}$ is the bootstrap covariance matrix estimator of the structural impulse response, as described by Guerron-Quintana, Inoue and Kilian (2017). We only incorporate the IR matching in the second step after estimating the optimal weighting matrix of the SMM estimator.

In the first step of our estimation, we use the standard deviation and autocorrelation of eight variables to estimate nine parameters of our model, i.e. , β, δ, δ_2, α, κ, ϕ_R, ϕ_Π, ϕ_y, and Π_s. After computing these parameters and the optimal weighting matrix $\mathbf{W}$, we incorporate γ_b, γ_h, ε_p, ε_w, ϕ_σ, ϕ_p, and ϕ_w to match the empirical impulse response of the SVAR.

Figure 5.1 – Empirical and model-implied impulse response to government uncertainty shock



Note: GIRFs for a positive two-standard-deviation innovation to a fiscal volatility shock to government spending in Brazil. Interest rate and inflation are presented in annualized basis points. In the first row, the plots present the reaction of the stochastic volatility process ($\sigma_{g,t}$), aggregate consumption, and output (c_t and y_t). The second row shows the response of investments, wages, and hours worked (w_t, x_t, and h_t). Finally, in the last row, we present the reaction of capital utilization (u_t), inflation (Π_t), and interest rates (R_t). The shaded area represents the confidence interval of half standard deviation.

5.2 MAIN RESULTS

The main results of the SVAR estimation are in line with the literature, i.e., a higher stochastic volatility on government spending generates a decrease in consumption, output, and output level. At the same time, the empirical estimation also suggests that the monetary authority reacts with higher interest rates in the presence of an uncertainty shock. As we are estimating stochastic volatility from the aggregate data on government spending, higher volatility is followed by an actual change in spending behavior. Thus, the monetary authority reaction can be interpreted as an anticipation to the actual shock.

We take the empirical responses to the model, matching the impulse response of output, consumption, investments, inflation, and interest rates with a government spending volatility shock, as presented in Figure (5.1). While our strategy does a fairly reasonable job of matching the response of interest rates and consumption, it overestimates the reaction of the output and investments. The model is also unable to match the slightly higher inflation response, once the monetary authority also reacts with higher interest rates. This behavior also forces the cost of changing nominal prices and wages to their upper bound, implying high price and wage rigidities (see Table (5.1)).

The set of parameters presented in Table (5.1) should be analyzed with caution. As already mentioned, the high price and wage rigidities only make sense in the context of an uncertainty shock and of a higher investment cost κ. A similar result is also presented by Basu and Bundick () and may bring information about some missing channels in our model. These results are in line with the literature, showing higher credit spread and borrowing costs, as presented by Kaviani et al. (2020). This also suggests that it may be relevant to include a more consistent credit channel in the model, since the monetary policy reaction is already well calibrated. An alternative model would include a term structure of interest rates, where the borrowing cost of firms would be higher than the monetary policy interest rates. However, in frameworks like those of Binsbergen et al. (2012) and Kung (2015), the long-term interest rate depends on the expectation of future inflation and consumption. However, the model underestimates the future inflation response to an actual increase of government spending, such that there is a negative impact on the slope of the interest rate (see Figure B.4).

5.3 SUMMARY AND CONCLUSION

In this chapter, we explore the empirical relationship between government spending volatility shocks and macroeconomic variables. Even though it is a very preliminary and still superficial analysis, it demonstrates that uncertainty about government spending can have meaningful and downward effects on consumption and output level. The monetary authority's reaction as a transmission channel is described in our model. However, we highlight that other credit channels could also play an important role in this analysis. Furthermore, uncertainty may have an endogenous effect on price rigidities and investment costs, overcome by our calibration strategy by setting high and implausible adjustment costs.

Further analysis should consider a larger dataset instead of focusing solely on government spending data, which are more accessible than tax data for Brazil. This would provide a better empirical analysis and tighter confidence intervals. Also, there is a growing empirical literature on the estimation of uncertainty shocks, with new identification strategies as in Ludvigson, Ma and Ng (forthcoming), or alternatives with proxy VAR identification, which would impose the challenge of finding the appropriate instrumental variable for Brazil.

From a theoretical point of view, we believe that a model with more elaborate credit market and firm heterogeneity could provide a better explanation about the transmission of uncertainty. Also, it seems important to consider that the higher the debt level of the government, the higher the risk of default, which could explain the term premium absent in the extension of the model developed in this chapter, generating higher borrowing costs for the private sector.

6 FINAL REMARKS

This dissertation indicates how to measure uncertainty as a stochastic volatility process, focusing on the effects of fiscal policy uncertainty on macroeconomic aggregates. Also, the study split the standard representative agent structure into a two-agent framework, which allows understanding how HtM agents are affected by these shocks and highlights an important transmission channel through the labor market. Furthermore, this was the first study to estimate and explore the effects of tax and spending uncertainty in Brazil. The stochastic volatility estimation underscores the importance of proper measurement of relevant aggregate tax rates, which are still limited for Brazil.

In the first chapter, we explore the literature on stochastic volatility and how it can be understood as an uncertainty measurement. We develop a survey about SV models and estimation procedures, as well as their application in the macroeconomic literature, hoping to cooperate with future studies on the same subject, which still has much room for improvement. The analysis of the stochastic volatility of capital tax demonstrates that there is a high probability of relevant tax changes that have real effects in a fifteen years' time. A pertinent observation is that higher tax volatility generates real negative effects on output, regardless of the sign of the actual shock (increase or decrease in the tax charges). Our empirical analysis also explores a known feature of emerging markets, which does not necessarily follow the same fiscal policy reaction of advanced countries. Precisely, there are many cases of developing countries, such as Brazil, following procyclical fiscal policy rules, increasing the estimation challenge.

The quantitative macroeconomic model presented in the second chapter follows the New Keynesian literature, containing several frictions that introduce nominal and real rigidities in prices, wages, capital, and investment. Also, we use a two-agent model with Epstein-Zin preferences that allow the differentiation between household types according to their risk aversion and intertemporal elasticity of substitution. The results in chapter 3 demonstrate that this is an important feature of the model, once our calibration distinguishes significantly the risk aversion of agents when comparing results for the US and Brazil. This distinction between IES and risk aversion shows that HtM agents in the US tend to self-select themselves into a situation where they are unable to engage in precautionary savings, which does not mean that they are poor, as demonstrated by Kaplan, Violante and Weidner (2014). However, the risk-aversion calibration for Brazil indicates that HtM agents are more risk-averse than bondholders, a feature that may indicate that a greater portion of HtM agents in Brazil are poor. Finally, the two-agent setting also demonstrates to be relevant when we assume a different portion of HtM agents in the economy, which changes the size of responses to an uncertainty fiscal shock.

As some developing countries have unconventional fiscal policy reactions to the output gap and debt level, we perused a pseudo-optimal fiscal policy analysis, trying to understand what the best reaction of a government toa tax rate volatility shock would be. The analysis shows that a government that decides to react in a countercyclical way to an uncertainty shock, which drives aggregate output down, spending less because of higher tax uncertainty, generates smaller contraction on consumption level for both agents. At the same time, a decision rule that indicates the government will increase spending due to higher tax volatility drives the aggregate output to a slightly better outcome but drags consumption to a much lower level.

In the last chapter of this dissertation, we focus on government spending volatility, which does not have a significant role in the standard calibration of our model. In order to quantify empirical reactions to government spending volatility, we incorporate filtered stochastic volatility of government spending into a structural vector autoregression with aggregate macroeconomic variables to evaluate the real effects of volatility shocks. Although it is a very preliminary and still superficial analysis, it demonstrates that uncertainty about government spending can have meaningful and downward effects on consumption and output level. The SVAR response is directly targeted at our calibration, combining SMM and the impulse response matching estimator. This procedure shows that volatility shocks increase investment costs and may have endogenous channels that could be explored in future research.

REFERENCES

HOWARD Kung. ISSN 00028282.

ALESINA, A.; CAMPANTE, F. R.; TABELLINI, G. Why is Fiscal Policy Often Procyclical? *Journal of the European Economic Association*, v. 6, n. 5, p. 1006–1036, 09 2008. ISSN 1542-4766.

ANDREASEN, M. M.; FERNáNDEZ-VILLAVERDE, J.; RUBIO-RAMíREZ, J. F. The pruned state-space system for non-linear dsge models: Theory and empirical applications. *The Review of Economic Studies*, v. 85, n. 1, p. 1–49, 2018.

AZEVEDO, C. F.; FASOLO, A. M. *Effective Tax Rates on Consumption and Factor Incomes: a quarterly frequency estimation for Brazil.* [S.l.], 2015.

BAKER, S. R.; BLOOM, N.; DAVIS, S. J. Measuring Economic Policy Uncertainty*. *The Quarterly Journal of Economics*, v. 131, n. 4, p. 1593–1636, 07 2016. ISSN 0033-5533.

BARSKY, R. B. et al. Preference Parameters and Behavioral Heterogeneity: An Experimental Approach in the Health and Retirement Study*. *The Quarterly Journal of Economics*, v. 112, n. 2, p. 537–579, 05 1997. ISSN 0033-5533.

BASU, S.; BUNDICK, B. Uncertainty shocks in a model of effective demand. *Econometrica*, Blackwell Publishing Ltd, v. 85, n. 3. ISSN 1468-0262.

BAYER, C. et al. Precautionary savings, illiquid assets, and the aggregate consequences of shocks to household income risk. *Econometrica*, v. 87, n. 1, p. 255–290.

BERNANKE, B. S. Irreversibility, Uncertainty, and Cyclical Investment*. *The Quarterly Journal of Economics*, v. 98, n. 1, p. 85–106, 02 1983. ISSN 0033-5533.

BI, H.; LEEPER, E. M.; LEITH, C. Uncertain fiscal consolidations. *The Economic Journal*, v. 123, n. 566. ISSN 1468-0297.

BINSBERGEN, J. H. van et al. The term structure of interest rates in a DSGE model with recursive preferences. *Journal of Monetary Economics*, v. 59, n. 7, p. 634–648, 2012.

BLOOM, N. The impact of uncertainty shocks. *Econometrica*, Blackwell Publishing Ltd, v. 77, n. 3. ISSN 1468-0262.

CALDARA, D. et al. The economic effects of trade policy uncertainty. *Journal of Monetary Economics*, v. 109, p. 38–59, 2020. ISSN 0304-3932. SI:APR2019 CRN CONFERENCE. Disponível em: <https://www.sciencedirect.com/science/article/pii/S0304393219302004>.

CAMPOS, E. L.; CYSNE, R. P. A Time-Varying Fiscal Reaction Function for Brazil. *Estudos EconÃ ´micos (SÃPaulo)*. ISSN 0101-4161.

CARVALHO, F. A. de; VALLI, M. *Fiscal Policy in Brazil through the Lens of an Estimated DSGE Model.* [S.l.], 2011.

CAVALCANTI, M.; VEREDA, L. Fiscal policy multipliers in a dsge model for brazil. *Brazilian Review of Econometrics*, v. 35, n. 2, 2015.

COENEN, G. et al. Effects of fiscal stimulus in structural models. *American Economic Journal: Macroeconomics*, v. 4, n. 1, p. 22–68, January 2012.

CREAL, D. A survey of sequential monte carlo methods for economics and finance. *Econometric Reviews*, Taylor Francis, v. 31, n. 3, p. 245–296, 2012. Disponível em: <https://doi.org/10.1080/07474938.2011.607333>.

DEBORTOLI, D.; GALÍ, J. Monetary policy with heterogeneous agents: Insights from tank models. *Manuscript*, 2017.

DOU, W. W. et al. Macroeconomic models for monetary policy: A critical review from a finance perspective. 2017.

Douc, R.; Cappe, O.; MOULINES, E. Comparison of resampling schemes for particle filtering. In: *ISPA 2005. Proceedings of the 4th International Symposium on Image and Signal Processing and Analysis, 2005.* [S.l.: s.n.], 2005. p. 64–69. ISSN 1845-5921.

DOUCET, A.; FREITAS, N. de; GORDON, N. *Sequential Monte Carlo Methods in Practice.* [S.l.: s.n.], 2001.

DOUCET, A.; JOHANSEN, A. M. A tutorial on particle filtering and smoothing: Fifteen years later. In: CRISAN, D.; ROZOVSKII, B. (ed.). *The Oxford Handbook of Nonlinear Filtering.* Oxford: Oxford University Press, 2011. cap. 24, p. 656–704.

DYNAMICS of fiscal financing in the United States. *Journal of Econometrics*, v. 156, n. 2, p. 304 – 321, 2010. ISSN 0304-4076.

EFFECTIVE tax rates in macroeconomics: Cross-country estimates of tax rates on factor incomes and consumption. *Journal of Monetary Economics*, v. 34, n. 3, p. 297 – 323, 1994. ISSN 0304-3932.

ESTIMATING nonlinear DSGE models by the simulated method of moments: With an application to business cycles. *Journal of Economic Dynamics and Control*, v. 36, n. 6, p. 914 – 938, 2012. ISSN 0165-1889.

FELDSTEIN, M. *The Role for Discretionary Fiscal Policy in a Low Interest Rate Environment.* [S.l.], 2002. (Working Paper Series, 9203).

FERNÁNDEZ-VILLAVERDE, J. et al. Fiscal volatility shocks and economic activity. *American Economic Review*, v. 105, n. 11, p. 3352–84, November 2015.

FERNÁNDEZ-VILLAVERDE, J.; RUBIO-RAMíREZ, J. F. Estimating macroeconomic models: A likelihood approach. *The Review of Economic Studies*, v. 74, n. 4, p. 1059–1087, 2007.

FERNáNDEZ-VILLAVERDE, J. et al. Risk matters: The real effects of volatility shocks. *American Economic Review*, v. 101, n. 6, p. 2530–61, October 2011.

FERNáNDEZ-VILLAVERDE, J.; RUBIO-RAMíREZ, J. *Macroeconomics and Volatility: Data, Models, and Estimation.* [S.l.], 2010. (Working Paper Series, 16618). Disponível em: <http://www.nber.org/papers/w16618>.

FERREIRA, D. The time-(in)variant interplay of government spending and private consumption in brazil. *Economia Aplicada*, v. 19, n. 3, p. 429–454, 2015. ISSN 1980-5330.

FISCAL limits in developing countries: A DSGE Approach. *Journal of Macroeconomics*, v. 49, p. 119 – 130, 2016. ISSN 0164-0704.

FISCAL stimulus and distortionary taxation. *Review of Economic Dynamics*, v. 18, n. 4, p. 894 – 920, 2015. ISSN 1094-2025.

GALí, J.; LóPEZ-SALIDO, J. D.; VALLéS, J. Understanding the effects of government spending on consumption. *Journal of the European Economic Association*, Blackwell Publishing Ltd, v. 5, n. 1. ISSN 1542-4774.

GAMERMAN, D.; LOPES, H. F. *Markov Chain Monte Carlo: stochastic simulation for Bayesian inference*. 2. ed. Boca Raton, FL: Chapman & Hall/CRC, 2006.

GAVIN, M.; PEROTTI, R. Fiscal Policy in Latin America. In: *NBER Macroeconomics Annual 1997, Volume 12*. [S.l.]: National Bureau of Economic Research, Inc, 1997, (NBER Chapters). p. 11–72.

GORDON, N.; SALMOND, D.; SMITH, A. Novel approach to nonlinear/non-gaussian bayesian state estimation. *IEE Proceedings F (Radar and Signal Processing)*, v. 140, p. 107–113(6), April 1993. ISSN 0956-375X. Disponível em: <https://digital-library.theiet.org/content/journals/10.1049/ip-f-2.1993.0015>.

GOURIO, F. Disaster risk and business cycles. *American Economic Review*, v. 102, n. 6, p. 2734–66, May 2012.

GUERRON-QUINTANA, P.; INOUE, A.; KILIAN, L. Impulse response matching estimators for dsge models. *Journal of Econometrics*, v. 196, n. 1, p. 144–155, 2017. ISSN 0304-4076. Disponível em: <https://www.sciencedirect.com/science/article/pii/S0304407616301762>.

GUISO, L.; PAIELLA, M. Risk Aversion, Wealth, and Background Risk. *Journal of the European Economic Association*, v. 6, n. 6, p. 1109–1150, 12 2008. ISSN 1542-4766.

GULEN, H.; ION, M. Policy Uncertainty and Corporate Investment. *The Review of Financial Studies*, v. 29, n. 3, p. 523–564, 09 2015. ISSN 0893-9454. Disponível em: <https://doi.org/10.1093/rfs/hhv050>.

HANSEN, L. P.; SARGENT, T. J. *Structured uncertainty and model misspecification*. [S.l.], 2019. 2018-77 p. (Becker Friedman Institute for Economics Working Paper).

HAS fiscal policy helped stabilize the postwar U.S. economy? *Journal of Monetary Economics*, v. 49, n. 4, p. 709 – 746, 2002. ISSN 0304-3932.

HERBST, E. P.; SCHORFHEIDE, F. *Bayesian Estimation of DSGE Models*. [S.l.]: Princeton University Press, 2015.

JAIMOVICH, N.; REBELO, S. Can news about the future drive the business cycle? *American Economic Review*, v. 99, n. 4, p. 1097–1118, September 2009.

JULIO, B.; YOOK, Y. Political uncertainty and corporate investment cycles. *The Journal of Finance*, v. 67, n. 1, p. 45–83, 2012. Disponível em: <https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1540-6261.2011.01707.x>.

JURADO, K.; LUDVIGSON, S. C.; NG, S. Measuring uncertainty. *American Economic Review*, v. 105, n. 3, p. 1177–1216, March 2015.

JUSTINIANO, A.; PRIMICERI, G. E. The time-varying volatility of macroeconomic fluctuations. *American Economic Review*, v. 98, n. 3, p. 604–41, June 2008.

KAPLAN, G.; MOLL, B.; VIOLANTE, G. L. Monetary policy according to hank. *American Economic Review*, v. 108, n. 3, p. 697–743, March 2018.

KAPLAN, G.; VIOLANTE, G. L.; WEIDNER, J. The wealthy hand-to-mouth. *Brookings Papers on Economic Activity*, v. 45, n. 1 (Spring), p. 77–153, 2014.

KARANTOUNIAS, A. G. Optimal Fiscal Policy with Recursive Preferences. *The Review of Economic Studies*, v. 85, n. 4, p. 2283–2317, 01 2018. ISSN 0034-6527.

KAVIANI, M. S. et al. Policy uncertainty and corporate credit spreads. *Journal of Financial Economics*, v. 138, n. 3, p. 838–865, 2020. ISSN 0304-405X. Disponível em: <https://www.sciencedirect.com/science/article/pii/S0304405X20301872>.

KIM, S.; SHEPHARD, N.; CHIB, S. Stochastic volatility: Likelihood inference and comparison with arch models. *The Review of Economic Studies*, [Oxford University Press, Review of Economic Studies, Ltd.], v. 65, n. 3, p. 361–393, 1998. ISSN 00346527, 1467937X. Disponível em: <http://www.jstor.org/stable/2566931>.

KIM, W. Government spending policy uncertainty and economic activity: Us time series evidence. *Journal of Macroeconomics*, v. 61, p. 103124, 2019. ISSN 0164-0704. Disponível em: <https://www.sciencedirect.com/science/article/pii/S016407041830140X>.

KUHN, M.; SCHULARICK, M.; STEINS, U. I. *Income and Wealth Inequality in America, 1949-2016*. [S.l.], 2017.

KUNG, H. Macroeconomic linkages between monetary policy and the term structure of interest rates. *Journal of Financial Economics*, v. 115, n. 1, p. 42–57, 2015. ISSN 0304-405X. Disponível em: <https://www.sciencedirect.com/science/article/pii/S0304405X14001901>.

KüNSCH, H. R. A tutorial on particle filtering and smoothing: Fifteen years later. In: BARNDORFF-NIELSEN, O. E.; COX, D. R.; KLüPPELBERG, C. (ed.). *Complex Stochastic Systems*. [S.l.]: Chapman and Hall/CRC, 2001. cap. 3, p. 109–173.

KüNSCH, H. R. Recursive monte carlo filters: Algorithms and theoretical analysis. *The Annals of Statistics*, Institute of Mathematical Statistics, v. 33, n. 5, p. 1983–2021, 2005. ISSN 00905364. Disponível em: <http://www.jstor.org/stable/3448632>.

LEEPER, E. M.; TRAUM, N.; WALKER, T. B. Clearing up the fiscal multiplier morass. *American Economic Review*, v. 107, n. 8, p. 2409–54, August 2017.

LUDVIGSON, S.; MA, S.; NG, S. Uncertainty and business cycles: Exogenous impulse or endogenous response? *American Economic Journal: Macroeconomics*, forthcoming.

MOURA, G. Multiplicadores fiscais e investimento em infraestrutura. *Revista Brasileira de Economia*, v. 69, n. 1, 2015. ISSN 0034-7140.

MUMTAZ, H.; THEODORIDIS, K. Dynamic effects of monetary policy shocks on macroeconomic volatility. *Journal of Monetary Economics*, v. 114, p. 262–282, 2020. ISSN 0304-3932. Disponível em: <https://www.sciencedirect.com/science/article/pii/S0304393219300650>.

OPTIMAL monetary policy with staggered wage and price contracts. *Journal of Monetary Economics*, v. 46, n. 2, p. 281 – 313, 2000. ISSN 0304-3932.

PIFFER, M.; PODSTAWSKI, M. Identifying Uncertainty Shocks Using the Price of Gold. *The Economic Journal*, v. 128, n. 616, p. 3266–3284, 12 2017. ISSN 0013-0133. Disponível em: <https://doi.org/10.1111/ecoj.12545>.

PITT, M. K.; SHEPHARD, N. Filtering via simulation: Auxiliary particle filters. *Journal of the American Statistical Association*, Taylor Francis, v. 94, n. 446, p. 590–599, 1999. Disponível em: <https://www.tandfonline.com/doi/abs/10.1080/01621459.1999.10474153>.

POLICY risk and the business cycle. *Journal of Monetary Economics*, v. 68, p. 68 – 85, 2014. ISSN 0304-3932.

REINHART, C.; ROGOFF, K.; SAVASTANO, M. A. Debt intolerance. *Brookings Papers on Economic Activity*, v. 34, n. 1, p. 1–74, 2003.

RICCO, G.; CALLEGARI, G.; CIMADOMO, J. Signals from the government: Policy disagreement and the transmission of fiscal shocks. *Journal of Monetary Economics*, v. 82, p. 107–118, 2016. ISSN 0304-3932. Disponível em: <https://www.sciencedirect.com/science/article/pii/S0304393216300411>.

ROBERT, C.; CASELLA, G. *Monte Carlo Statistical Methods*. 2. ed. New York: Springer, 2004. 649 p. (Springer Texts in Statistics).

RUDEBUSCH, G. D.; SWANSON, E. T. The bond premium in a dsge model with long-run real and nominal risks. *American Economic Journal: Macroeconomics*, v. 4, n. 1, p. 105–43, January 2012.

SCHMITT-GROHé, S.; URIBE, M. *Optimal Fiscal and Monetary Policy in a Medium-Scale Macroeconomic Model: Expanded Version*. [S.l.], 2005. (Working Paper Series, 11417).

SURPRISE and uncertainty indexes: Real-time aggregation of real-activity macro-surprises. *Journal of Monetary Economics*, v. 82, p. 1 – 19, 2016. ISSN 0304-3932.

SWANSON, E. T. Risk aversion and the labor margin in dynamic equilibrium models. *American Economic Review*, v. 102, n. 4, p. 1663–91, June 2012.

TAYLOR, J. B. Reassessing discretionary fiscal policy. *Journal of Economic Perspectives*, v. 14, n. 3, p. 21–36, September 2000.

TEMPERED particle filtering. *Journal of Econometrics*, v. 210, n. 1, p. 26 – 44, 2019. ISSN 0304-4076. Annals Issue in Honor of John Geweke "Complexity and Big Data in Economics and Finance: Recent Developments from a Bayesian Perspective".

THE labor-supply elasticity and borrowing constraints: Why estimates are biased. *Review of Economic Dynamics*, v. 9, n. 2, p. 242 – 262, 2006. ISSN 1094-2025.

THE market price of fiscal uncertainty. *Journal of Monetary Economics*, v. 59, n. 5, p. 401 – 416, 2012. ISSN 0304-3932. Carnegie-NYU-Rochester Conference Series on Public Policy - Robust Macroeconomic Policy at Carnegie Mellon University on November 11-12, 2011.

TIME-VARYING business volatility and the price setting of firms. *Journal of Monetary Economics*, v. 101, p. 82 – 99, 2019. ISSN 0304-3932.

TOWARD a Taylor rule for fiscal policy. *Review of Economic Dynamics*, v. 17, n. 2, p. 294 – 302, 2014. ISSN 1094-2025.

VEGH, C. A.; VULETIN, G. How is tax policy conducted over the business cycle? *American Economic Journal: Economic Policy*, v. 7, n. 3, p. 327–70, August 2015.

VISSING-JøRGENSEN, A. Limited asset market participation and the elasticity of intertemporal substitution. *Journal of Political Economy*, v. 110, n. 4, p. 825–853, 2002.

APPENDIX A - EQUILIBRIUM CONDITIONS

This appendix aims to represent the equilibrium conditions for the TANK model developed in Section 4.6. First, we need to understand that the stochastic discount factor for each agent is:

$$m_{t+1} = \frac{\partial V_t/\partial c_{t+1}}{\partial V_t/\partial c_t} = \frac{U_{c,t+1}}{U_{c,t}} \frac{U_t^{\psi}}{U_{t+1}^{\psi}} \beta \frac{V_{t+1}^{\psi-\gamma}}{\left(\mathbb{E}_t V_{t+1}^{1-\gamma}\right)^{\frac{\psi-\gamma}{1-\gamma}}},$$

which can be rewritten considering the FOC w.r.t. c_t,

$$m_{t+1} = \frac{\lambda_{t+1}}{\lambda_t} \beta \frac{V_{t+1}^{\psi-\gamma}}{\left(\mathbb{E}_t V_{t+1}^{1-\gamma}\right)^{\frac{\psi-\gamma}{1-\gamma}}}. \tag{1}$$

where λ_t is the Lagrangian multiplier associated with the budget constraint.

Focusing on a symmetric equilibrium, the first-order conditions of the saver problem of maximizing expected utility with respect to c_t, b_t, x_t, k_t, u_t and $w_{j,t}$ are

$$\lambda_t = (1-\beta)(1-\psi) U_t^{-\psi} U_{c,t}, \tag{2}$$

$$\frac{1}{R_t} = \mathbb{E}_t \left\{ m_{t+1} \left(\frac{1-\tau_{t+1}^k}{\Pi_{t+1}} \right) \right\} \tag{3}$$

$$1 = \mathbb{E}_t \left[m_{t+1} q_{t+1} \kappa \left(\frac{x_{t+1}}{x_t} - z_s \right) \left(\frac{x_{t+1}}{x_t} \right)^2 \right] + q_t \left[1 - \frac{\kappa}{2} \left(\frac{x_t}{x_{t-1}} - z_s \right)^2 - \kappa \left(\frac{x_t}{x_{t-1}} - z_s \right) \frac{x_t}{x_{t-1}} \right] \tag{4}$$

$$q_t = \mathbb{E}_t \left\{ m_{t+1} \left[q_{t+1}(1-\delta(u_{t+1})) + (1-\tau_{t+1}^k) u_{t+1} r_{t+1}^k \right] \right\} \tag{5}$$

$$q_t = \frac{(1-\tau_t^k) r_t^k}{\delta'(u_t)} \tag{6}$$

$$\phi_w y_t \left(\frac{w_t}{w_{t-1}} - z_s \right) \frac{w_t}{w_{t-1}} = \phi_w \mathbb{E}_t \left\{ m_{t+1} y_{t+1} \left(\frac{w_{t+1}}{w_t} - z_s \right) \frac{w_{t+1}}{w_t} \right\} + \frac{1-\eta}{\eta} \frac{c_t}{1-h_t} \varepsilon_w h_t + (1-\varepsilon_w)(1-\tau_t^w) w_t h_t, \tag{7}$$

where q_t is the multiplier associated with the investment adjustment constraint. The FOC of the HtM agent will be equivalent to Equations 2 and 7, since she does not have access to the bond market and cannot invest on capital.

Equilibrium conditions associated to firm's problem will be marginal cost and FOC for factor inputs,

$$mc_t = \left(\frac{w_t}{z_t(1-\alpha)} \right)^{1-\alpha} \left(\frac{r_t^k}{\alpha} \right)^{\alpha} \tag{8}$$

$$\frac{k_{t-1}}{h_t} = \frac{\alpha w_t}{r_t^k (1-\alpha)}, \tag{9}$$

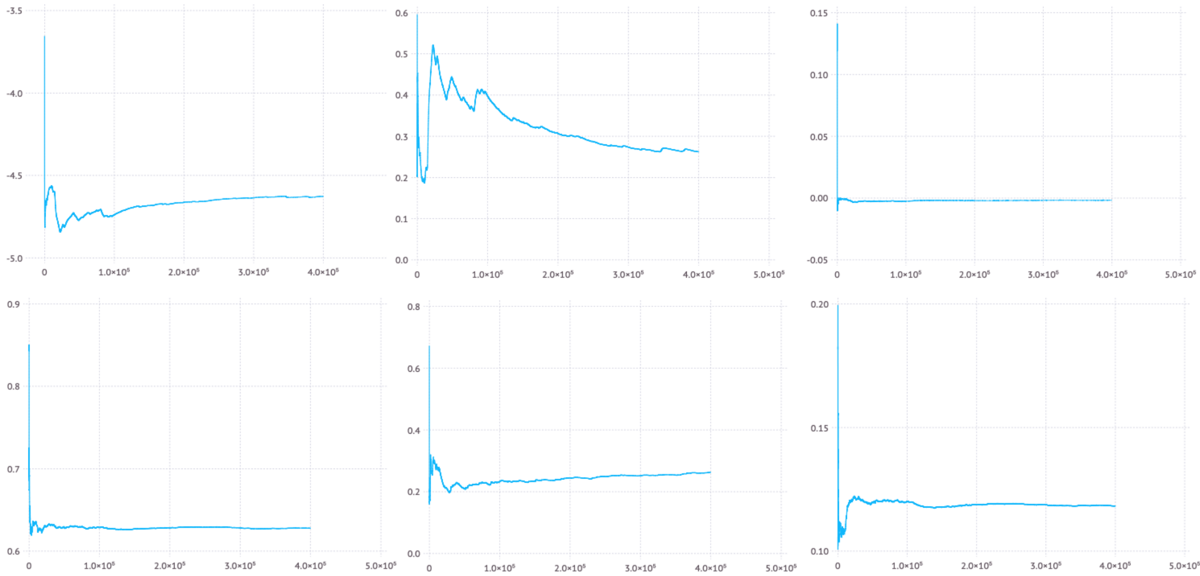
which are associated with the first stage of the problem, and the expended Phillips Curve is given by

$$\begin{aligned} \phi_p \Pi_t (\Pi_t - \Pi_s) =& (1-\varepsilon_p) + \varepsilon_p mc_t + \frac{\phi_p \varepsilon_p}{2} (\Pi_t - \Pi_s)^2 \\ &+ \phi_p \mathbb{E}_t \left\{ m_{t+1} \frac{y_{t+1}}{y_t} \Pi_{t+1} (\Pi_{t+1} - \Pi_s) \right\}. \end{aligned} \tag{10}$$

Thus, given these equilibrium conditions, we are able to solve the model through a third-order approximation.

APPENDIX B - FURTHER RESULTS

Figure B.1 – Recursive average of tax income capital parameters



Note: The recursive average indicates the convergence of parameters. In the first row we present results for σ_s, η_σ and ρ_b, while the second row has the following parameters ρ_x, ρ_σ and ρ_y, according to Eqs. (3.1) and (3.2).

B.1 DATA AND ESTIMATION FOR BRAZIL

In this Appendix, we aim to analyze in detail the data used for the estimation of the SV model and the calibration of the structural model for Brazil. As mentioned in Section 4.4, the labor tax income we obtained from Azevedo and Fasolo (2015) was adjusted by the unemployment rate. Since the main unemployment rate series for Brazil does not cover the whole tax sample, we selected the unemployment rate in the Metropolitan Area of the city of São Paulo (IPEADATA id code 37655). This series has a correlation of 0.97 with the official data considered by the government based on the Brazilian National Household Survey. The main difference will be in levels, with mean unemployment rate varying from 13.9 to 9.6. Besides the tax data in Azevedo and Fasolo (2015), the government spending data represent the total expenditure reported in Table 1.4-A of the Central Government Fiscal Balance available on the National Treasury website[1]. Among the three fiscal instruments, only one observation can be seen as an outlier. In the third quarter of 2010, the government spending rose suddenly, going back to its normal level in the next quarter. This outlier preceded the 2010 Brazilian Presidential Election, held in

[1] http://www.tesouro.fazenda.gov.br/en/central-government-primary-balance.

October of that year. There are several models for incorporation of jumps on stochastic volatility models; however, in our specific case, modeling this outlier would generate minimal improvement in our analysis, which led us to interpolate the outlier instead of changing the whole model to accommodate a single observation. Finally, we used the Federal Government Net Debt data (SGS-BCB id code 4479) to calculate the government indebtedness level.

Table 4.4 presents the model fit for the selected moments. We match the standard deviation and the autocorrelation among variables y_t, c_t, x_t, w_t, h_t, u_t, R_t, and Π_t, the correlation of these variables with y_t, the mean of interest rates and inflation, and the correlation between R_t and Π_t. The data for Brazil are taken from several sources, as specified next, and the period used goes from 1999:Q1 to 2014:Q4. We limited our data series to cover the same length of the data used for the fiscal instruments, where we are limited to the data provided in Azevedo and Fasolo (2015). We gave the same data treatment for both US and Brazil series, but some data for Brazil are more limited than those available for the US economy. An example of this limitation is the compensation-per-hour data series, and, therefore, we selected the average nominal salary for the industry in the State of São Paulo as the closest data available with sufficient size and frequency. In order to make the comprehension of each series counterpart clear, we use the same code of the corresponding US data at the St. Louis FRED database after each variable name with its specification and source in a footnote. The data used are the real gross domestic product (GDPC1), the real gross private domestic investment (GPDIC1), the real personal consumption expenditures (PCECC96)[2], the effective SELIC rate (FEDFUNDS)[3], the broad consumer price index, IPCA, (GDPDEF)[4], the average nominal wage in the industry of the State of São Paulo (HCOMPBS)[5], Industry: hours of all persons (HOABS)[6], and capacity utilization: total industry (TCU)[7].

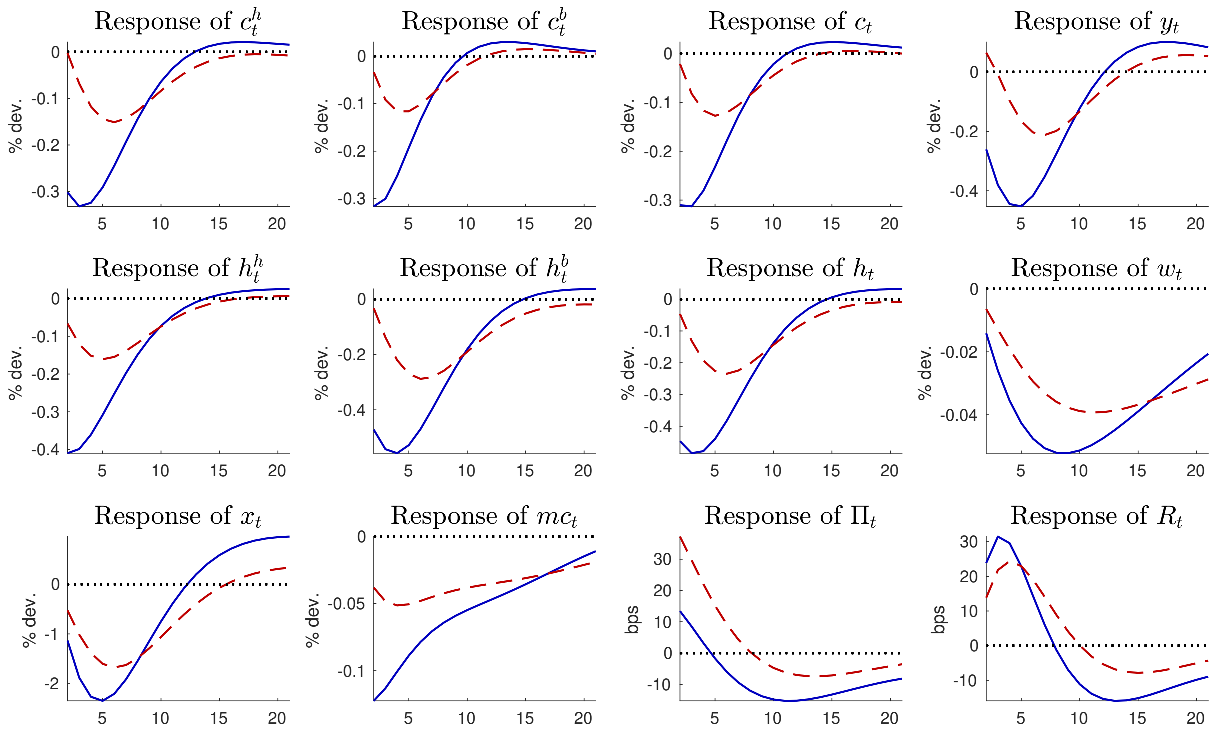
B.2 MONETARY POLICY RULE

Figures B.2 and B.3 present the role of this innovation on the Taylor Rule comparing the results of a standard Taylor Rule, without any direct response of the monetary authority to the shock, and the Taylor Rule on Eq. (3.13). While the effects on the US are relatively subtle, allowing the model to replicate empirical evidences of a drop on inflation, as

[2]Chained indexes (Source: System of Quarterly National Accounts, IBGE)
[3]Percent, Not Seasonally Adjusted (Source: BCB-SGS id code 4390)
[4]Index 1993 = 100, (Source: (IBGE/SNIPC, IPEADATA id code 36482)
[5]Index 2006 = 100, (Source: *Federação e Centro das Indústrias do Estado de São Paulo, Levantamento de Conjuntura*, Fiesp, IPEADATA id code 33689)
[6]Index 2006 = 100, Seasonally Adjusted, (Source: *Confederação Nacional da Indústria*, IPEADATA id code 33209)
[7]Percent of Capacity, Seasonally Adjusted Monthly, (Source: *Confederação Nacional da Indústria*, IPEADATA id code 33211)

documented by FGKR, in Brazil the difference is much stronger. Without a direct response of the monetary authority to uncertainty about capital income tax all observed effects would be negligible. Furthermore, this also shows that the model can be extremely sensible to the parameter ϕ_σ that controls the responsiveness of interest rates to volatility shocks.

Figure B.2 – The role of the Alternative Taylor Rule



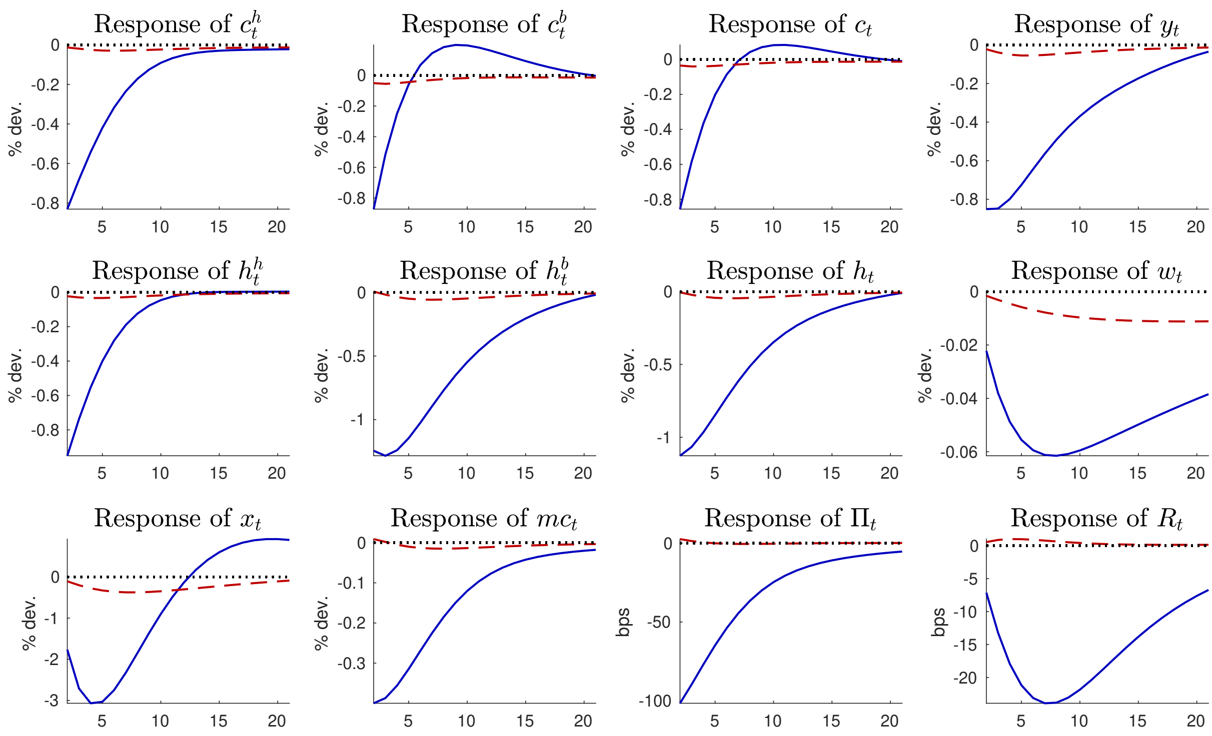
Note: GIRFs to a positive two-standard-deviations innovation to a fiscal volatility shock to the capital income tax for the US. Solid blue (dashed red) for the IRFs to a fiscal volatility shock with $\phi_\sigma = 0.005$ ($\phi_\sigma = 0$) on the Taylor Rule, which means that the monetary authority is directly reacting (or not) to the volatility shock.

B.3 INCLUDING A TERM STRUCTURE OF INTEREST RATES

As mentioned in Section 5.2, an alternative framework to understand the effect of uncertainty on borrowing costs is through the term structure of interest rates. Following Binsbergen et al. (2012) and Kung (2015), a standard setting is to assume that the central bank's short term interest rates and the forward rates are defined by the linkage to the previous maturity rate and expectation about future inflation. In our model, it is straightforward to consider the household intertemporal condition related to the first-order condition with respect to bonds,

$$\frac{1}{R_t} = \mathbb{E}_t \left\{ m_{t+1} \left(\frac{1}{\Pi_{t+1}} \right) \right\} \tag{11}$$

Figure B.3 – The role of the Alternative Taylor Rule in Brazil



Note: GIRFs to a positive two-standard-deviations innovation to a fiscal volatility shock to the capital income tax for Brazil. Solid blue (dashed red) for the IRFs to a fiscal volatility shock with $\phi_\sigma = 0.005$ ($\phi_\sigma = 0$) on the Taylor Rule, which means that the monetary authority is directly reacting (or not) to the volatility shock.

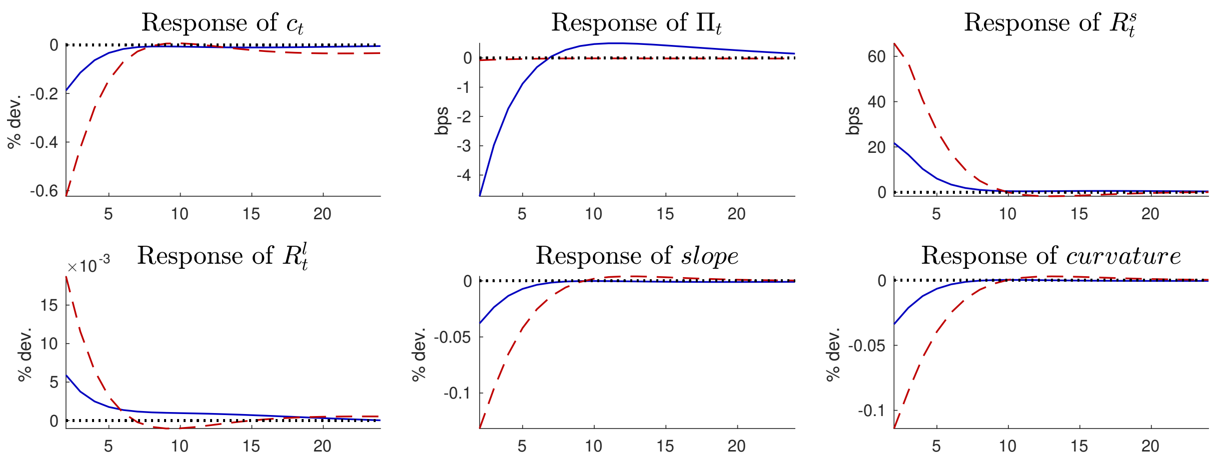
where the bond price can be computed recursively using

$$\frac{1}{R_{t,t+m}} = \mathbb{E}_t \left\{ m_{t+1} \left(\frac{1}{\Pi_{t+1} R_{t+1,t+m}} \right) \right\} \tag{12}$$

with $R_{t,t+m}^{-1}$ being the time-t price of an m-period nominal bond, $R_{t,t+1} = R_t$ and $R_{t+1,t+1} = 1$.

Considering this structure, Kung (2015) demonstrates that movements in nominal yields are driven by the conditional mean and variance of the nominal stochastic discount factor (m_{t+1}), which, in turn, depends on inflation and consumption growth. However, simulating our model based on this new feature does not generate an increase in the slope of the yield curve (see Figure B.4), suggesting that the uncertainty about the impact of government spending on inflation expectation is still underestimated.

Figure B.4 – Government spending uncertainty effect on the yield curve



Note: GIRFs for a positive two-standard-deviation innovation to a fiscal volatility shock to the government spending for Brazil. Solid blue (dashed red) line for the IRFs to a fiscal volatility shock with standard calibration presented in Chapter 3, with $\phi_\sigma = 0.005$ (versus the calibration presented in Section 5.2) for the Taylor rule. In the first row, we present the reaction of consumption (c_t), inflation (Π_t), and short-term interest rates defined by the monetary authority (R_t^s), and in the second row, we present the long-term interest rate for 5 years, 20 quarters, maturity bond (R_t^l), the slope of the yield curve ($slope = R_t^{20} - R_t^1$) and curvature ($curvature = (R_t^1 + R_t^{14} + R_t^{20})/3$).