

A NOTE ON LIBERAL EXTENSIONS WITH AUTOMORPHISMS

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ABSTRACT. We consider a ring R , a liberal extension S of R , a group G whose elements act as R -automorphisms on the ring S and an intermediate extension T of R such that $\sigma(T) = T$ for all $\sigma \in G$. It is shown that an ideal Q of T is a G -prime ideal (resp. G -primitive ideal) if and only if Q is the maximum G -subideal of a prime (resp. primitive) ideal of T .

0. INTRODUCTION.

Let K be a ring and let G be a group whose elements act as automorphisms on the ring K . If I is an ideal of K we denote by $\Gamma(I)$ the maximum G -subideal of I . If P is a prime ideal of K , then $\Gamma(P)$ is a G -prime ideal. Suppose that either K is a right Noetherian ring or G is finite and let Q be a G -prime ideal of K . Then, by the Zorn's lemma, there is a prime P of K with $\Gamma(P) = Q$. Hence, in this case the set of all the G -prime ideals of K is determined by the set of all the prime ideals.

Now, let R be a ring, S a liberal extension of R , T an intermediate extension and D a derivation of S such that $D/R = 0$ and $D(T) \subseteq T$. The determination of the D -prime ideals of the differential ring (T, D) was studied in ([3], section 4). An ideal Q of T is a D -prime ideal if and only if there exists a prime ideal P such that Q is the maximum D -subideal of P . Moreover, this correspondence is a one-to-one correspondence.

In this paper we consider a ring R , a liberal extension S of R and a group G whose elements act as R -automorphisms on the ring S . Let T be an intermediate extension such that $\sigma(T) = T$ for all $\sigma \in G$. The purpose of this paper is to study the determination of the G -

prime ideals of T as above.

§1 gives an introductory section to the material. In §2, we shall show that an ideal Q of T is G -prime if and only if Q is equal to $\Gamma(P)$, for some prime P . In §3, we shall show that a G -prime ideal $Q = \Gamma(P)$, P a prime, is G -primitive if and only if P is primitive. Finally, in §4, we apply our results to study radical questions in skew polynomial rings and skew group rings over T .

1. PREREQUISITES.

Throughout this paper every ring has an identity element and if $R \subset S$ is a ring extension, then R and S share the identity 1.

Let K be a ring. We say here that K is a G -ring if G is a group whose elements act as automorphisms on the ring K . An ideal I of a G -ring K is said to be a G -ideal if $\sigma(I) = I$ for every $\sigma \in G$. If I is a G -ideal of K , then K/I is a G -ring in the natural way. A G -ideal P of K is said to be a G -prime ideal if $AB \subseteq P$ for any G -ideals A and B implies that either $A \subseteq P$ or $B \subseteq P$. If I is an ideal of K we put $\Gamma(I) = \{a \in I : \sigma(a) \in I \text{ for all } \sigma \in G\}$. Then $\Gamma(I)$ is the maximum G -subideal of I . If P is a prime ideal of K we can easily see that $\Gamma(P)$ is a G -prime ideal.

Now, let K be a ring and σ an automorphism of K . In this case we say that K is a σ -ring rather than a (σ) -ring, where (σ) is the cyclic group generated by σ . Similar remarks can be made for the terminology σ -ideal and σ -prime ideal. For a σ -ring K our definitions of a σ -ideal and a σ -prime ideal coincide with those given in [4]. In [8], a σ -ideal is called a σ -invariant ideal. One should note that our definition of a σ -prime ideal differs from those given in [7] and [8]. A σ -prime ideal in the sense of [8] is called here a strongly σ -prime ideal, as in [7]. Thus a strongly σ -prime ideal is a σ -prime ideal. In the case studied here the converse is true (see Proposition 2.5).

An ideal P of a G -ring K is said to be (left) G -primitive in K if there is a maximal G -left ideal M of K such that $(M : K) = P$, where $(M : K) = \{x \in K : xK \subseteq M\}$ ([8], section 1). A G -primitive ideal of K is also a G -prime ideal.

Let K be a G -prime ring. Following [6], we consider the collection of all non-zero G -ideals of K and the totality of all right K -homomorphisms $f: I \rightarrow K$, where I is a G -ideal of K . On this set we define an equivalence relation in the obvious way and we denote by \mathcal{Q} the set of all equivalence classes. An element of \mathcal{Q} is denoted by $[f, I]$ where I is a G -ideal of K and $f: I_K \rightarrow K_K$. We can also de-

fine on Q a ring structure in the natural way. The map $a \rightarrow [a_\ell, K]$, where a_ℓ is the left multiplication $x \rightarrow ax$, is an embedding of K into Q and we consider $K \subseteq Q$. For every $q \in Q$ there exists a G -ideal I of K such that $qI \subseteq K$.

For every $\sigma \in G$ we define an automorphism σ^* of Q by $\sigma^*([f, I]) = [\sigma \circ f \circ (\sigma^{-1}/I), I]$. Then it can easily be verified that $\sigma^*/K = \sigma$ and Q is a G -prime ring. We denote by Z the center of Q and we put $C_G(K) = ZK$. We also denote by σ^* again the automorphism $\sigma^*/C_G(K)$ of $C_G(K)$. Then $C_G(K)$ is a G -prime ring which is an extension of K . The center of $C_G(K)$ is Z again and $[f, I] \in Z$ if and only if $f: I \rightarrow K$ is a K -bimodule map.

In this paper we consider a ring R , a liberal extension S of R and an intermediate extension T of R . We suppose that G is a group whose elements act as R -automorphisms on S and $\sigma(T) = T$ for every $\sigma \in G$. In this case we say that S is a G -liberal extension of R and T is a G -intermediate extension. The methods and results of [10] and [11] are used frequently here.

Let S be a G -liberal extension of R and suppose that S is a G -prime ring. Then R is prime and we can consider CR , the central closure of R , where C is the extended centroid of R . In the same way as in [11] it can easily be verified that we can consider $C \subseteq Z \subseteq C_G(S)$. Further, $\sigma^*/C = \text{id}_C$ for every $\sigma \in G$. Then CS is a G -prime ring and a G -liberal extension of CR . Finally, if T is a G -intermediate extension of R , then CT is a G -intermediate extension of CR .

The following propositions can be proved in a similar way to [10] and [11].

PROPOSITION 1.1. ([11], Proposition 6.5 and [10], Proposition 2.5). *Let R be a centrally closed prime ring, S a torsion-free G -liberal extension of R and T a G -intermediate extension. Then there are a torsion-free G -liberal extension S' of R and a non-zero ideal X of R such that $XS' \subseteq T \subseteq S' \subseteq S$. In this case, if we denote by α the bijection defined in [10] we have $\alpha(\Gamma(P)) = \Gamma(\alpha(P))$ for every prime ideal P of T with $P \not\subseteq XS'$, where $\alpha(\Gamma(P)) = \{x \in S' : XS'x \subseteq \Gamma(P)\}$.*

PROPOSITION 1.2. ([10], Theorem 3.2). *Let T be a G -intermediate extension of R and Q a G -prime ideal of T . Then $Q \cap R$ is a prime ideal of R and there exists a G -prime ideal H of S such that $H \cap R = Q \cap R$ and $H \cap T \subseteq Q$.*

Hereafter, we write $L(K)$, $J(K)$ and $B(K)$ for the prime (lower nil) radical, the Jacobson radical and the Brown-McCoy radical of a

ring K , respectively. If σ is an automorphism of K and V is any radical, then $V_\alpha(K)$ denotes the corresponding σ -radical of K (see [7] and [8]).

2. G-PRIME IDEALS.

We begin with the following

LEMMA 2.1. *Let K be a G -ring which is a semi-simple right Artinian ring. Then every G -prime ideal of K is a G -maximal ideal.*

Proof. It is enough to prove the result when F is a G -prime ring. Then it follows easily that F is G -simple by the structure theorem for semi-simple right Artinian rings.

Now we can prove our first main result.

THEOREM 2.2. *Let S be a G -liberal extension of R , T a G -intermediate extension and Q a G -prime ideal of T . Then there is a prime P of T such that $\Gamma(P) = Q$. Moreover, for every ideal I of T such that $I \cap R = Q \cap R$ and $I \supseteq Q$ we have $\Gamma(I) = Q$.*

Proof. It is enough to prove the second part. Choose an ideal P of T which is maximal with respect to $P \supseteq I \supseteq Q$ and $P \cap R = Q \cap R$. Then P is prime and we must show that $\Gamma(P) = Q$. On the other hand, by factoring out convenient ideals we may suppose that R is prime, S is a G -prime liberal extension and Q is a G -prime ideal of T with $Q \cap R = 0$ (Proposition 1.2).

Firstly, assume that R is a centrally closed prime ring, with center C and consider two cases.

Case I. If $T=S$ we may suppose that $Q=0$. Therefore, T is a torsion-free liberal extension of R because T is G -prime. By ([10], Proposition 2.4) $T \cong V \otimes_C R$ and $P = P' \otimes_C R$, where V is the centralizer of R in T and P' is a prime ideal of the Artinian algebra V . Further, since $L(T)$ is a nilpotent G -ideal of T we have $L(T) = 0$ and so $L(V) = 0$. Then V is a semi-simple Artinian algebra which is G -prime. By Lemma 2.1 V is G -simple. Therefore, $\Gamma(P') = 0$ and hence $\Gamma(P) = 0$.

Case II. If T is a G -intermediate extension we can apply Proposition 1.1. Then there are a torsion-free G -liberal extension S' of R and a non-zero ideal X of R such that $XS' \subseteq T \subseteq S' \subseteq S$. Hence $\alpha(P)$ is a prime ideal of S' and $\alpha(P) \cap T = P$. Further, as in ([10],

Proposition 2.5) we can easily verify that $\alpha(Q) = \{x \in S' : XS'xXS' \subseteq Q\}$ is a G-prime ideal of S' with $\alpha(Q) \subseteq \alpha(P)$ and $\alpha(Q) \cap T = Q$. By the case I, $\Gamma(\alpha(P)) = \alpha(Q)$ and so $\Gamma(P) = \Gamma(\alpha(P) \cap T) = \alpha(Q) \cap T = Q$.

In general, consider $CR \subseteq CT \subseteq CS$, where the elements of G act on CS as we said in section 1. The details in the following are similar to ([10], Theorem 3.3). There exists an ideal Q' of CT which is G-maximal with respect to $Q' \supseteq CQ$ and $Q' \cap T = Q$. Similarly, $(CP + Q') \cap T = P$ and there exists an ideal P' of CT which is maximal with respect to $P' \supseteq CP + Q'$ and $P' \cap T = P$. Hence $P' \supseteq Q'$, Q' is a G-prime ideal of CT , P' is a prime ideal of CT and $P' \cap CR = Q' \cap CR = 0$. Therefore, $\Gamma(P') = Q'$ and so $\Gamma(P) = \Gamma(P') \cap T = Q' \cap T = Q$.

COROLLARY 2.3. Let S be a G-liberal extension of R , T a G-intermediate extension and Q a G-prime ideal of T . If $I \supseteq Q$ is a G-ideal of T and $I \cap R = Q \cap R$, then $I = Q$. In particular, Q is a semi-prime ideal.

Proof. Let P be a prime ideal of T such that $P \supseteq I$ and $P \cap R = Q \cap R$. Then P is prime and $I = \Gamma(P) = Q$. Further, if H is the ideal of T with $H/Q = L(T/Q)$, then H is a G-ideal and $H \subseteq P$. Hence $H = Q$.

COROLLARY 2.4. Let S be a G-liberal extension of R , T a G-intermediate extension, Q a G-prime ideal of T and P a prime with $\Gamma(P) = Q$. Then the following conditions are equivalent:

- (i) Q is a G-maximal ideal of T .
- (ii) P is a maximal ideal of T .
- (iii) $Q \cap R$ is a maximal ideal of R .

Proof. (i) \rightarrow (iii). Suppose that Q is a G-maximal ideal. If $I \supseteq Q \cap R = P \cap R$ is a prime ideal of R there exists a prime ideal P' of T such that $P' \supseteq P$ and I is a minimal prime over $P' \cap R$. ([5], Corollary 4.2). Since $P' \cap R = \Gamma(P') \cap R = Q \cap R$, it follows that $I = Q \cap R$. Then $Q \cap R$ is a maximal ideal of R .

(iii) \rightarrow (ii). It is clear from ([10], Theorem 3.3).

(ii) \rightarrow (i). As in the first part we can see that $P \cap R$ is a maximal ideal of R . Then it follows easily by Corollary 2.3.

PROPOSITION 2.5. Let S be a liberal extension of R , T an intermediate extension and σ a R -automorphism of S such that $\sigma(T) = T$. Then the ideal Q of T is a σ -prime ideal if and only if Q is a

strongly σ -prime ideal.

Proof. A strongly σ -prime ideal is always a σ -prime ideal. Conversely, let Q be a σ -prime ideal of T . We may suppose that $Q \cap R = 0$ and so R is a prime ring. If A and B are ideals of T such that $A \supseteq Q$, $B \supseteq Q$, $\sigma(B) \subseteq B$ and $AB \subseteq Q$, then either $A \cap R = 0$ or $B \cap R = 0$.

Suppose that $B \cap R = 0$. We have $\sigma^i(B) \cap R = 0$ and $\sigma^i(B) \subseteq \sigma^{i-1}(B)$, for all integer i . Thus $B' = \bigcup_{i \in \mathbb{Z}} \sigma^i(B)$ is a σ -ideal of T , $B' \cap R = 0$ and $B' \supseteq B \supseteq Q$. Then $B = Q$ by Corollary 2.3.

Now, assume that $B \cap R \neq 0$ and put $X = \{x \in T: xB \subseteq Q\}$. Then X is an ideal of T such that $X \supseteq A$ and $XB \subseteq Q$. Hence $X \cap R = 0$ and $\sigma^{-1}(X) \subseteq X$. As above, $X' = \bigcup_{i \in \mathbb{Z}} \sigma^i(X)$ is a σ -ideal of T with $X' \supseteq X \supseteq A \supseteq Q$ and $X' \cap R = 0$. Therefore, $X' = A = Q$.

REMARK 2.6. It is easy to see that the ideal P such that $\Gamma(P) = Q$ in Theorem 2.2 need not be unique. In fact, if K is a field,

$S = \bigoplus_{i=1}^n Ke_i$, where e_1, e_2, \dots, e_n is a family of orthogonal idempotents whose sum is one, and σ is the K -automorphism of S defined by $\sigma(e_i) = e_{i+1 \pmod n}$, then S is a σ -prime ring and $\Gamma(\bigoplus_{i \neq j} Ke_i) = 0$ for all j .

3. G-PRIMITIVE IDEALS.

We begin this section with the following

PROPOSITION 3.1. *Let K be a G -ring and Q a G -primitive ideal of K . Then there exists a primitive ideal P of K with $\Gamma(P) = Q$.*

Proof. Let M be a maximal G -left ideal of K with $(M : K) = Q$ and take a maximal left ideal M' of K such that $M' \supseteq M$. Then $\Gamma(M') = \{x \in M': \sigma(x) \in M' \text{ for all } \sigma \in G\} = M$ and so the ideal $P = (M' : K)$ is a primitive one with $\Gamma(P) = Q$.

Next we shall need the following

LEMMA 3.2. *Let S be a G -liberal extension of R and T a G -intermediate extension. Suppose that S is a G -prime ring, Q is a G -prime ideal of T with $Q \cap R = 0$ and L is a left ideal of R with $(L : R) = 0$. Then $TL + Q \neq T$.*

Proof. By the assumption, R is a prime ring. Suppose that R is a centrally closed prime ring with center C and L is any left ideal of R . Let S' be a torsion-free G -liberal extension of R and X a non-zero ideal of R such that $XS' \subseteq T \subseteq S'$. Since $\alpha(Q) = \alpha(\Gamma(P)) = \Gamma(\alpha(P))$, for some prime P , $\alpha(Q)$ is a G -prime ideal of S' . By factoring out from S' and T the ideals $\alpha(Q)$ and Q respectively, we may suppose that $Q = 0$. If $TL = T$, then $S'L = S'$, which is a contradiction ([11], Proposition 5.5, (i)).

Now, suppose that R is any prime ring and L is a left ideal of R with $(L : R) = 0$. Consider the central closure of R , $CR \subseteq CT \subseteq CS$ and take a G -prime ideal Q' of CT such that $Q' \supseteq CQ$ and $Q' \cap T = Q$. If $CL = CR$, then $1 = \sum_i c_i x_i$, $c_i \in C$, $x_i \in L$. Take a non-zero ideal U of R such that $Uc_i \subseteq R$ for all i . Hence, $U \subseteq \sum_i Uc_i x_i \subseteq L$ and so $U \subseteq (L : R) = 0$, a contradiction. Thus $CL \neq CR$ and from the first part $CTL + Q' \neq CT$. Therefore, $TL + Q \neq T$.

THEOREM 3.3. *Let S be a G -liberal extension of R , T a G -intermediate extension and P a primitive ideal of T . Then $\Gamma(P)$ is a G -primitive ideal of T .*

Proof. By factoring out convenient ideals we may suppose that R is a prime ring, S is a G -prime liberal extension of R and P is a primitive ideal of T such that $P \cap R = 0$. Then R is a primitive ring by ([10], Theorem 4.6). Let M be a maximal left ideal of R with $(M : R) = 0$. From the above lemma, $TM + \Gamma(P) \neq T$ and so there exists a maximal G -left ideal M' of T such that $M' \supseteq TM + \Gamma(P)$ and $M' \cap R = M$. Then $(M' : T)$ is a G -primitive ideal of T with $\Gamma(P) \subseteq (M' : T)$ and $(M' : T) \cap R = (M : R) = 0$. It follows that $\Gamma(P) = (M' : T)$ by Corollary 2.3.

Combining the above results and ([10], Theorem 4.6) we have the following corollary. The last part follows easily as in Corollary 2.3.

COROLLARY 3.4. *Let S be a G -liberal extension of R , T a G -intermediate extension, Q a G -prime ideal of T and P a prime with $Q = \Gamma(P)$. The following conditions are equivalent:*

- (i) Q is a G -primitive ideal of T .
- (ii) P is a primitive ideal of T .
- (iii) $Q \cap R$ is a primitive ideal of R .

In this case, Q is a semi-primitive ideal.

4. APPLICATIONS.

Let K be a G -ring. The intersection of all the G -prime (G -primitive, G -maximal) ideals of K is called the G -prime (G -Jacobson, G -Brown-McCoy) radical of K and denoted by $L_G(K)$ ($J_G(K)$, $B_G(K)$).

This definition was given in [8] for a special case. But for $L_\sigma(K)$ is not actually the same because, as we already pointed out, the concept of a σ -prime ideal differs from ours.

Suppose that an ideal Q of K is a G -prime ideal if and only if $Q = \Gamma(P)$ for a prime P . Then we have $L(K) = \Gamma(L(K)) \subseteq \Gamma(P) = Q \subseteq P$ and so $L(K) = L_G(K)$. Similar remarks can be made for $J_G(K)$ and $B_G(K)$. Thus, the following is clear.

COROLLARY 4.1. Let S be a G -liberal extension of R and T a G -intermediate extension. Then $L_G(T/A) = L(T/A)$, $J_G(T/A) = J(T/A)$ and $B_G(T/A) = B(T/A)$, for every G -ideal A of T .

Let K be a G -ring again. We denote by $[KG]$ the skew group ring over K whose elements are of the type $\sum_{\sigma \in G} u_\sigma a_\sigma$, where $a_\sigma \in K$ and u_σ is a basis element, and the multiplication is defined by $au_\sigma = u_\sigma(a)$, for all $a \in K$ and $\sigma \in G$. In [2] it is proved that if G is a totally ordered group then $L([KG]) = [L_G(K)G]$. Then we have

COROLLARY 4.2. Let S be a G -liberal extension of R and T a G -intermediate extension. If G is a totally ordered group then $L([TG]) = [L(T)G]$.

The following example is also given in [2].

EXAMPLE 4.3. Let F be a field and $X = (X_i)_{i \in \mathbb{Z}}$ a set of indeterminates. Put $A = F[X]$ the polynomial ring over X and σ the F -automorphism defined by $\sigma(X_i) = X_{i+1}$, for all $i \in \mathbb{Z}$. On the ring $K = A/P$, where P is the ideal generated by $\{X_i^n; i \in \mathbb{Z}\}$, is defined the automorphism induced by σ , which is again denoted by σ . Then K is a local ring with the maximal ideal M generated by $\{X_i + P, i \in \mathbb{Z}\}$. Therefore, M is the unique prime ideal of K . Furthermore, 0 is a σ -prime ideal of K and $0 \neq \Gamma(M) = M$.

Hereafter, we suppose that S is a liberal extension of R , σ is a R -automorphism of S and T is a σ -intermediate extension, i.e., $\sigma(T) = T$. We denote by $T[X; \sigma]$ the skew polynomial ring over T which is defined by $aX = X\sigma(a)$ for all $a \in T$.

COROLLARY 4.4. For every σ -ideal A of T we have

$$(i) \quad L((T/A)[X;\sigma]) = L(T/A)[X;\sigma]$$

$$(ii) \quad J((T/A)[X;\sigma]) = I[X;\sigma] \text{ where } I = J((T/A)[X;\sigma]) \cap (T/A).$$

Proof. (i) is clear by ([7], Theorem 1.3) and Corollary 4.1. To show (ii) we may suppose that $A = 0$. If Q is a σ -primitive ideal of T , then $J_\sigma(T/Q) = 0$ and so $J(T[X;\sigma]/Q[X;\sigma]) = 0$ by ([8], Proposition 3.9 and Corollary 3.8). Put $I = \{a \in T: Xa \in J(T[X;\sigma])\}$. If $a \in I$, $Xa \in Q[X;\sigma]$ and so $a \in Q$. Hence $I \subseteq J_\sigma(T) = J(T)$. The assertion follows from ([1], question 2, p.333).

COROLLARY 4.5. The following conditions are equivalent:

(i) R is a Jacobson ring.

(ii) T is a Jacobson ring.

(iii) T is a σ -Jacobson ring.

(iv) $T[X;\sigma]$ is a Jacobson ring.

Proof. Let R be a Jacobson ring, Q a σ -prime ideal of T and P a prime with $\Gamma(P) = Q$. By Proposition 1.2 we may suppose that $Q \cap R = 0$ and R is prime. Then there exists a set of primitive ideals of R , $\{H_i\}$ say, such that $\bigcap_i H_i = 0$. For every i there is a prime P_i of T with $P_i \supseteq P$ and $P_i \cap R = H_i$ by ([5], Corollary 4.2). Furthermore, $\Gamma(P_i) \cap R = H_i$ and so $\Gamma(P_i)$ is σ -primitive. Since $\bigcap_i \Gamma(P_i) \supseteq \Gamma(P) = Q$ and $(\bigcap_i \Gamma(P_i)) \cap R = \bigcap_i H_i = 0$, we have $\bigcap_i \Gamma(P_i) = Q$. Then $J_\sigma(T/Q) = 0$. Therefore, T is a σ -Jacobson ring. Consequently, (i) implies (iii) and repeating this way for $\sigma = \text{id}$, (i) implies (ii).

Now, let T be a σ -Jacobson ring and H a prime ideal of R . Then there exists a σ -prime ideal Q of T with $Q \cap R = H$. Since Q is an intersection of σ -primitive ideals of T , H is an intersection of primitive ideals of R . Then R is a Jacobson ring. Similarly, (ii) implies (i). Therefore, (i), (ii) and (iii) are equivalent conditions.

The rest is clear. If $T[X;\sigma]$ is a Jacobson ring, then it is known that T is a Jacobson ring and the converse follows from ([8], Proposition 4.15).

Finally, we have the following

COROLLARY 4.6. The following conditions are equivalent:

(i) R is a Brown-McCoy ring.

(ii) T is a Brown-McCoy ring.

(iii) T is a σ -Brown-McCoy ring.

Further, $T[X;\sigma]$ is a Brown-McCoy ring if and only if R is a Brown-McCoy ring and for every maximal ideal M of T , σ is PI modulo $\Gamma(M)$.

Proof. The first part can be proved in a similar way to Corollary 4.5. If $T[X;\sigma]$ is a Brown-McCoy ring, clearly T is a Brown-McCoy ring and so it is a σ -Brown-McCoy ring. Then the result follows easily from the first part and ([9], Theorem 3.2 and Theorem 3.5).

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