III Semana Acadêmica da Pós-Graduação em Matemática

Universidade Federal do Rio Grande do Sul - UFRGS

Porto Alegre, 05 a 08 de Novembro de 2018.

CADERNO DE RESUMOS

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Problema de Plateau assintótico em $\mathbb{H}^2 \times \mathbb{R}$

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Resumo
Nesta comunicação gostaria de apresentar a definição de bordo assintótico na variedade $\mathbb{H}^2 \times \mathbb{R}$, do mesmo modo explicar o que seria o problema de Plateau no bordo assintótico de $\mathbb{H}^2 \times \mathbb{R}$ e possíveis generalizações para superfícies CMC-H com $0 \leqslant H \leqslant 1/2$.

Nordhaus-Gaddum inequalities for l_2.

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Resumo
In this note, we consider a simple graph $G = (V(G), E(G))$ on n vertices and $e(G)$ edges, where $V(G) = \{v_1, v_2, ..., v_n\}$ is the vertex set and $E(G)$ is the edge set.

The *Laplacian matrix* of G is the matrix $L(G) = D(G) - A(G)$, where $A(G)$ is the *adjacency matrix* and $D(G)$ is the *diagonal matrix* with the degree sequence on its main diagonal. It is well-known that $L(G)$ is symmetric and positive semidefinite. The eigenvalues of $L(G)$ are called the *Laplacian eigenvalues* of G, and are denoted by $l_1(G) \geq l_2(G) \geq \cdots \geq l_n(G) = 0$.

The *complement of a graph* G is a graph G^c on the same vertex set such that two distinct vertices of G^c are adjacent if and only if they are not adjacent in G. Let $\rho = \rho(G)$ be an invariant in a graph G, we denote by $\rho^c = \rho(G^c)$ the same invariant in G^c.

This work is loosely inspired by the seminal paper of V. Nikiforov [1], where bounds for the (partial sums of) eigenvalues of graphs are obtained and numerous problems on the topic are proposed. Instead of dealing with the adjacency matrix of a graph G, here we are concerned with the Laplacian matrix of G. More precisely, we narrow down to investigate Nordahaus-Gaddum inequalities for its second largest eigenvalue.

In 1956, E. Nordhaus and J. Gaddum [2] gave lower and upper bounds on the sum and the product of the chromatic number of a graph and its complement, in terms of the order of the graph. Since then, any bound on the sum and/or the product of a graph invariant of G and the same invariant of G^c is called a *Nordhaus-Gaddum type inequality.* In general these inequalities are quite elegant as they reveal extremal values for a graph parameter and its complement. On the other hand, it may be quite difficult to be obtained.

A spectral graph invariant is a graph parameter defined using eigenvalues of the matrices associated with the graph, including the eigenvalues themselves. Many Nordhaus-

Gaddum type inequalities involve eigenvalues of the adjacency, Laplacian and signless Laplacian matrices of graphs.

In 2013, Nordhaus-Gaddum type inequalities for graph parameters were surveyed by M. Aouchiche and P. Hansen [3], where it may be seen that relations of a similar type have been proposed for many other graph invariants, in several hundred papers.

We present here Nordhaus-Gaddum type inequalities for the second largest eigenvalue of the Laplacian matrix. More precisely, we have studied the following conjecture.

Conjecture 1. Let G be a graph of order $n \geq 2$. Then

$$n \leq l_2 + l_2^c \leq 2n - 2.$$

Conjecture 2. Let G be a graph of order $n \geq 4$, with $2 \leq e(G) \leq \binom{n}{2} - 2$. Then

$$n - 2 \leq l_2 \, l_2^c \leq n(n-2).$$

We show that the conjectures are valid for trees and unicyclic graphs. Moreover, we show that these bounds are the best possible presenting graphs satisfying the equality.

Referências

[1] Nikiforov, Vladimir. Extrema of graph eigenvalues. *Linear Algebra and its Applications*, 482 (2015): 158-190.

[2] Nordhaus, E. A., and Jerry W. Gaddum. On complementary graphs. *The American Mathematical Monthly*, 63.3 (1956): 175-177.

[3] Aouchiche, Mustapha, and Pierre Hansen. A survey of Nordhaus-Gaddum type relations. *Discrete Applied Mathematics*, 161.4 (2013): 466-546.

Operadores hipercíclicos

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Resumo

O objetivo da apresentação é após definir operador hípercíclico, apresentar um exemplo exposto no artigo: On orbits of elements de S. Rolewicz em 1969. Aí está o exemplo:

Teorema: Seja X ou ℓ^p $(1 \leq p < \infty)$ ou c_0. Para todo número real $a > 1$ arbitrário, existe um operador T e um elemento x_0 tal que a órbita $orb(x_0, T)$ é densa em X.

Demonstração. Sejam B o shift à esquerda

$$B(x_1, x_2, x_3, ...) = (x_2, x_3, ...)$$

e F o shift à direita

$$F(x_1, x_2, ...) = (0, x_1, x_2, ...).$$