X-PROBE CALIBRATION USING COLLIS AND WILLIAM'S EQUATION

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Abstract: A simple method for calibration of X-probe hot-wire anemometer, based on King's Law, has been proposed and analyzed. This method uses a simplified solution for the Collis and William's equation applied on the wires of the probe based at the geometric similarity. It utilizes a few calibration points for velocity and yaw angle, thereby it's very useful for manual calibrations. The results from evaluation of accuracy show a better solution when compared with other methods to solve the Collis and William's equations

Key Words: hot-wire anemometer, X-probe

1. Introduction

Hot-wire anemometry probes are velocity indirect measurement elements. In order to convert the voltage value measured on the wire to velocity it is necessary to use a function to correlate them. Normally, either look-up table or empirical relationship that describe the wire heat transfer as a flow function are used. When the measurement of two velocity components of the flow is necessary, a probe with two wires placed at 45º related to the flow direction (forming a X) is used. In this case, it is necessary to include in the transference functions the angle between the flow velocity and the axis probe. Alternatively, the axial sensibility coefficients can be calculated to combine the cooling effects of normal and tangential components on each probe wire (Hinze, 1975). When the look-up table is used (Lueptow et al., 1998 and Meyer, 1992), it is necessary a large number of calibration points (velocity and angle), becoming the non automatic calibration process very slow.

In this work is presented a simplification to the approach on the equations using the geometric similarity of wires. This allows the variables in the equation system that results when the function is applied on each wire, to be isolated. In this way, one avoid to use numeric methods to obtain the results. According to Bruun, 1988, in practice a successful calibration/evaluation procedure combines both computational simplicity with a good accuracy.

The equation used is a adaptation made by Collis and Williams (1959) of King´s Law and modified by Davies and Bruun (Bruun et al. 1990) to produce a directional response:

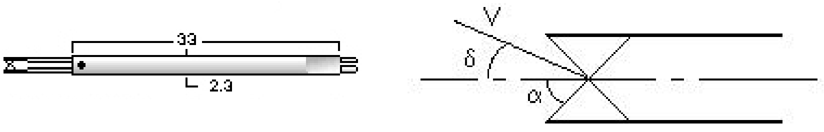
$$E^2 - Eo^2 = BV^n \cos^m(\alpha - \delta) \tag{1}$$

where E and Eo are respectively the mean voltage measure on the bridge at the velocity V and the voltage at the velocity zero; α is the angle between the wire and the probe axis, δ is the angle between the mean flow direction and the probe axis and B, n and m are the constants to be determined during the calibration processes using the method proposed by Hooper (Hooper, 1982).

2. Experimental technique

Hot-wire anemometer are used as the active part of a Wheatstone bridge. The flow on the wire causes a change in the heat transfer ratio, disbalancing the bridge by the change in the wire resistance. This causes a change in the bridge voltage to maintain constant the wire temperature. Thus, the flow velocity is directly related with the bridge voltage.

The hot-wire probe used was a Dantec X-probe 55P61, connect two constant temperature anemometry bridges (Dantec StreamLine CTA 90C10). The calibration was carried out using a 8.5 mm diameter nozzle, connect to a plenum chamber where the air flow was supplied by a centrifugal blower. The calibration facility was developed in the Fluid Mechanic Laboratory of Federal University of Rio Grande do Sul (Vicari, 1996). It allows the flow velocity control through a valve and the flow incidence angle related with the probe axis using a yaw scale. The actual flow velocity in the nozzle is calculated through the pressure measured in the plenum chamber. The probe calibration is carried out for the different flow velocity and incident angles. Before starting the blower of the calibration unit, Eo was measured by simply turning the bridge to operate mode. Figure 1 shows a scheme of the probe with the incident velocities.



a b

Figure 1. Two wire probe: a) probe, b) velocity scheme

3. Analytical relationships

Using Eq. (1) for two probe wires, as described in Fig. 1:

$$E_1^{\,2} - Eo_1^{\,2} = B_1 V^{n_1} \cos^{m_1}(\alpha - \delta) \tag{2}$$

$$E_2^{\,2} - Eo_2^{\,2} = B_2 V^{n_2} \cos^{m_2}(\alpha + \delta) \tag{3}$$

If the probe is already calibrated, the values to determinate are V e δ. Isolating V from (2) and (3):

$$V = \left[\frac{E_1^{\,2} - Eo_1^{\,2}}{B_1 \cos^{m_1}(\alpha - \delta)}\right]^{1/n_1} = \left[\frac{E_2^{\,2} - Eo_2^{\,2}}{B_2 \cos^{m_2}(\alpha + \delta)}\right]^{1/n_2} \tag{4}$$

Developing (4) in order to obtain δ:

$$\frac{\left(E_1^{\,2} - Eo_1^{\,2}\right)^{1/n_1}}{B_1^{1/n_1} \cos^{m_1/n_1}(\alpha - \delta)} = \frac{\left(E_2^{\,2} - Eo_2^{\,2}\right)^{1/n_2}}{B_2^{1/n_2} \cos^{m_2/n_2}(\alpha + \delta)} \quad \therefore \quad \frac{\left(E_1^{\,2} - Eo_1^{\,2}\right)^{1/n_1} B_2^{1/n_2}}{\left(E_2^{\,2} - Eo_2^{\,2}\right)^{1/n_2} B_1^{1/n_1}} = \frac{\cos^{m_1/n_1}(\alpha - \delta)}{\cos^{m_2/n_2}(\alpha + \delta)}$$

Developing the cosine summation:

$$\frac{\left(E_1^{\,2} - Eo_1^{\,2}\right)^{1/n_1} B_2^{1/n_2}}{\left(E_2^{\,2} - Eo_2^{\,2}\right)^{1/n_2} B_1^{1/n_1}} = \frac{\left(\cos\alpha \cos\delta + sin\alpha sin\delta\right)^{m_1/n_1}}{\left(\cos\alpha \cos\delta - sin\alpha sin\delta\right)^{m_2/n_2}}$$

For $\alpha = 45^{o}$, $sin\alpha = \cos\alpha = \frac{\sqrt{2}}{2}$:

$$\frac{\left(E_1^{\,2} - Eo_1^{\,2}\right)^{1/n_1} B_2^{1/n_2}}{\left(E_2^{\,2} - Eo_2^{\,2}\right)^{1/n_2} B_1^{1/n_1}} = \frac{\left[\frac{\sqrt{2}}{2}\left(\cos\delta + sin\delta\right)\right]^{m_1/n_1}}{\left[\frac{\sqrt{2}}{2}\left(\cos\delta - sin\delta\right)\right]^{m_2/n_2}} \quad \therefore \quad \frac{\left(E_1^{\,2} - Eo_1^{\,2}\right)^{1/n_1} B_2^{1/n_2}}{\left(E_2^{\,2} - Eo_2^{\,2}\right)^{1/n_2} B_1^{1/n_1}} = \frac{\sqrt{2}^{\frac{m_1}{n_1} - \frac{m_2}{n_2}}}{2} \frac{\left(\cos\delta + sin\delta\right)^{m_1/n_1}}{\left(\cos\delta - sin\delta\right)^{m_2/n_2}}$$

thus:

$$\frac{\left(\cos\delta + sin\delta\right)^{m_1/n_1}}{\left(\cos\delta - sin\delta\right)^{m_2/n_2}} = \frac{\sqrt{2}^{\frac{m_2}{n_2} - \frac{m_1}{n_1}}}{2} \frac{\left(E_1^{\,2} - Eo_1^{\,2}\right)^{1/n_1} B_2^{1/n_2}}{\left(E_2^{\,2} - Eo_2^{\,2}\right)^{1/n_2} B_1^{1/n_1}} \tag{5}$$

Introducing a new variable N defined as:

$$N = \frac{\sqrt{2}}{2}^{\frac{m_2}{n_2}-\frac{m_1}{n_1}} \frac{\left(E_1^{\,2} - Eo_1^{\,2}\right)^{1/n_1} B_2^{\,1/n_2}}{\left(E_2^{\,2} - Eo_2^{\,2}\right)^{1/n_2} B_1^{\,1/n_1}}$$

and

$$a = \frac{m_1}{n_1} \quad \text{e} \quad b = \frac{m_2}{n_2}$$

yields to:

$$(\cos\delta + sin\delta)^a = N(\cos\delta - sin\delta)^b \tag{6}$$

Solving the Eq.(6), based on the calibration data and the voltage measured to each wire its possible to obtain the value of the angle between the velocity vector and the probe axis. The velocity value is obtained by solving equation (4) for the angle δ calculated.

In order to solve Eq. (6), a numerical routine was developed, using Matlab v.5.3 that allows the evaluation of the necessary time to process the solution. The task was carried out using a personal computer with 1.7 GHz processor and 256 Mb memory RAM. The solution was obtained for an absolute error of 0.1% and integrated step 0.1 in the value of the angle. The time spent to obtain the solution is showing in the Fig. 2 as a function of the sample size. The sample size used in these tests is smaller than that used in practical experimental works, where the spectral analysis is made. Furthermore, the tests were made just for a measurement point. During a calibration or a measurement, it is necessary repeat the procedure at all the measurement points.

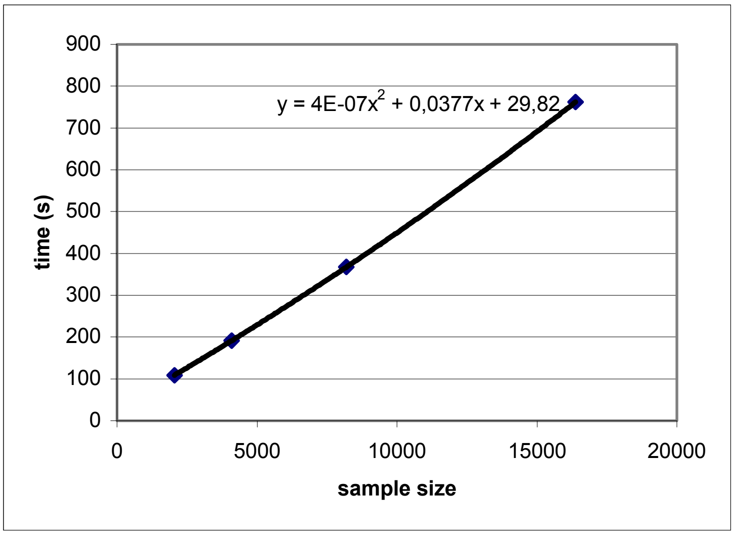


Figure 2. Processing time spent to solve Eq.(6).

In order to look for a simple direct solution for the Eq. (6), some mathematical developments are carried out hereafter.

Developing the sine and the cosine using series:

$$sin\delta = \delta - \frac{\delta^3}{3!} + \frac{\delta^5}{5!} - \frac{\delta^7}{7!} + ... \tag{7}$$

$$\cos\delta = 1 - \frac{\delta^2}{2!} + \frac{\delta^4}{4!} - \frac{\delta^6}{6!} + ... \tag{8}$$

For small angles, up to 30º, the series can be truncated in the second term with maximum error of 0.065% for the sine and 0.36% for the cosine.

Thus,

$$\left(1-\frac{\delta^2}{2!}+\delta-\frac{\delta^3}{3!}\right)^a = N\left(1-\frac{\delta^2}{2!}-\delta+\frac{\delta^3}{3!}\right)^b$$

$$\left(\frac{6-3\delta^2+6\delta-3\delta^3}{6}\right)^a = N\left(\frac{6-3\delta^2-6\delta+3\delta^3}{6}\right)^b$$

$$\left(6-3\delta^2+6\delta-3\delta^3\right)^a = N6^{a-b}\left(6-3\delta^2-6\delta+3\delta^3\right)^b$$

Introducing a new variable C, defined as:

$$C = N6^{a-b}$$

results:

$$\left(6-3\delta^2+6\delta-3\delta^3\right)^a = C\left(6-3\delta^2-6\delta+3\delta^3\right)^b \tag{9}$$

In order to simplify this solution, the exponent *a* can be eliminated:

$$\left(6-3\delta^2+6\delta-3\delta^3\right) = C^{1/a}\left(6-3\delta^2-6\delta+3\delta^3\right)^{b/a}$$

Introducing:

$$D = C^{1/a} \quad \text{and} \quad f = \frac{b}{a}$$

By the application of the logarithm in both side of equation the exponential function can be eliminated, thus:

$$\ln\left(6-3\delta^2+6\delta-3\delta^3\right) = \ln D + f\ln\left(6-3\delta^2-6\delta+3\delta^3\right)$$

Developing the logarithm with a two term series, valid for x>1:

$$\ln x = 2\left[\left(\frac{x-1}{x+1}\right)+\frac{1}{3}\left(\frac{x-1}{x+1}\right)^3+\ldots\right] \tag{10}$$

Using two term for the series, for a range of –30º to 30º, the maximum error is 10.38%. Using three term, the maximum error decrease for 5%, yet the polynomial resulting is 15º grade. Therefore,

$$2\left(\frac{6+6\delta-3\delta^2-\delta^3-1}{6+6\delta-3\delta^2-\delta^3+1}\right)+\frac{2}{3}\left(\frac{6+6\delta-3\delta^2-\delta^3-1}{6+6\delta-3\delta^2-\delta^3+1}\right)^3 = E+$$
$$2f\left(\frac{6-6\delta-3\delta^2+\delta^3-1}{6-6\delta-3\delta^2+\delta^3+1}\right)+\frac{2}{3}f\left(\frac{6-6\delta-3\delta^2+\delta^3-1}{6-6\delta-3\delta^2+\delta^3+1}\right)^3$$

and simplifying,

$$2\left(\frac{5+6\delta-3\delta^2-\delta^3}{7+6\delta-3\delta^2-\delta^3}\right)+\frac{2}{3}\left(\frac{5+6\delta-3\delta^2-\delta^3}{7+6\delta-3\delta^2-\delta^3}\right)^3 = E+$$
$$2f\left(\frac{5-6\delta-3\delta^2+\delta^3}{7-6\delta-3\delta^2+\delta^3}\right)+\frac{2}{3}f\left(\frac{5-6\delta-3\delta^2+\delta^3}{7-6\delta-3\delta^2+\delta^3}\right)^3 \tag{11}$$

Equation (11) does not have a simple analytical solution. Since the subject of this work is to find an explicit solution for the independent variable in order to allow utilize in simple computer programs to convert experimental data, it is not possible to employ Eq. (11) directly.

4. Probe calibration

The probe used in the experiment, was calibrated using a technique describe by Hooper (Hooper, 1980), to a moderate velocity range of 3-30 m/s and to a range of –30º to 30º with 5º steps of the angle value at a constant velocity of 15 m/s. The results are showed in the Tabs. 1, 2 and 3.

Table 1. Velocity calibration

Flow velocity	*E1*	*E2*
3.120	1.711	1.698
5.094	1.803	1.783
6.976	1.860	1.842
9.837	1.939	1.923
14.965	2.047	2.038
19.887	2.136	2.127
26.101	2.218	2.197
29.868	2.265	2.251

Table 2. Angle calibration

Flow velocity	*angle*	*E1*	*E2*
15.05	30	2.123	1.847
15.05	25	2.118	1.889
15.05	20	2.106	1.926
15.05	15	2.099	1.965
15.05	10	2.085	1.990
15.05	5	2.073	2.017
15.05	0	2.051	2.036
15.05	-5	2.025	2.061
15.05	-10	1.992	2.074
15.05	-15	1.972	2.085
15.05	-20	1.938	2.098
15.05	-25	1.892	2.106
15.05	-30	1.864	2.110

Table 3. Calibration constants

	wire 1	*wire 2*
n	0.479656042	0.489013131
B	0.745331205	0.712213849
m	0.385937040	0.387621756

As described before, the 55P61 probe consists of two 45º inclined wires with the same geometry and structural composition. So, it is possible to consider that the calibration parameters B, n and m for both wires of the probe do not differ strongly. This can be confirmed in the Tab. 3. Using a relation between m and n parameters of each wire, an important simplification can be made.

$$\frac{m_1}{n_1} = 0.804612 \quad \text{e} \quad \frac{m_2}{n_2} = 0.792661$$

By using the approximation:

$$\frac{m_1}{n_1} \cong \frac{m_2}{n_2} = 0.8 = \frac{m}{n} \tag{12}$$

This simplification introduces an error that should be verified always the probe is calibrated. In this paper this verification is shown in Tables 4 and 5, and in Figures 3, 4, 5 and 6.

Applying (12) in (5) yields to:

$$\left(\frac{\cos\delta + sin\delta}{\cos\delta - sin\delta}\right)^{m/n} = \frac{\sqrt{2}^{\,0}}{2}\,\frac{\left(E_1{}^2 - Eo_1{}^2\right)^{1/n_1} B_2{}^{1/n_2}}{\left(E_2{}^2 - Eo_2{}^2\right)^{1/n_2} B_1{}^{1/n_1}}$$

thus

$$\frac{\cos\delta + sin\delta}{\cos\delta - sin\delta} = \left[\frac{\left(E_1{}^2 - Eo_1{}^2\right)^{1/n_1} B_2{}^{1/n_2}}{\left(E_2{}^2 - Eo_2{}^2\right)^{1/n_2} B_1{}^{1/n_1}}\right]^{n/m}$$

Introducing a new variable M, defined as

$$M = \left[\frac{\left(E_1{}^2 - Eo_1{}^2\right)^{1/n_1} B_2{}^{1/n_2}}{\left(E_2{}^2 - Eo_2{}^2\right)^{1/n_2} B_1{}^{1/n_1}}\right]^{n/m}$$

yields to:

$$\cos\delta + sin\delta = M\left(\cos\delta - sin\delta\right)$$

Developing this expression, yields to an equation which allow a direct solution to the angle δ

$$\delta = \tan^{-1}\left(\frac{M-1}{M+1}\right) \tag{13}$$

Again, the velocities are calculated using Eq. (4).

5. Results

Equations (6), (9), (11) e (13) were tested with experimental data from the calibration facility for the velocities 15 and 10 m/s. Equations (6), (9) e (11) don't have direct solution than they were solved through the numerical method. The velocities were calculated by Eq. (4). Tables 4 and 5 show the results.

Table 4. Comparison of calibration conditions and results obtained by several equations – Flow velocity 15 m/s

Experimental		***Equation (6)***		***Equation (9)***		***Equation (11)***		***Equation (13)***	
δ	***V***	δ	***V***	δ	***V***	δ	***V***	δ	***V***
-30	15.07	-29.89	14.89	-29.82	14.83	-32.63	17.44	-30.00	14.98
-20	15.09	-21.48	14.86	-21.46	14.85	-25.24	16.98	-21.61	14.92
-10	15.12	-11.02	14.89	-11.02	14.89	-14.16	15.96	-11.16	14.93
-5	15.12	-6.54	14.90	-6.54	14.90	-8.62	15.48	-6.68	14.94
0	15.12	-0.08	14.85	-0.80	15.00	-1.01	15.05	-0.95	15.04
5	15.12	5.85	14.84	5.85	14.84	7.87	14.51	5.69	14.86
10	15.14	9.87	14.82	9.87	14.82	12.92	14.40	9.71	14.85
20	15.15	19.53	14.85	19.52	14.85	23.48	14.50	19.35	14.87
30	15.15	28.97	14.80	28.91	14.80	31.96	14.64	28.79	14.81

Table 5. Comparison of calibration conditions and results obtained by several equations – Flow velocity 10 m/s

Experimental		***Equation (6)***		***Equation (9)***		***Equation (11)***		***Equation (13)***	
δ	***V***	δ	***V***	δ	***V***	δ	***V***	δ	***V***
-30	9.85	-27.61	9.70	-27.57	9.68	-30.73	11.33	-27.74	9.75
-20	9.84	-20.64	9.57	-20.63	9.57	-24.45	10.90	-20.77	9.61
-10	9.84	-11.39	9.42	-11.39	9.42	-14.60	10.13	-11.52	9.45
-5	9.82	-6.15	9.31	-6.15	9.31	-8.12	9.65	-6.29	9.34
0	9.82	-0.58	9.30	-0.58	9.30	-0.71	9.31	-0.72	9.32
5	9.82	5.32	9.29	5.32	9.29	7.18	9.09	5.16	9.30
10	9.81	9.64	9.27	9.64	9.27	12.64	9.01	9.47	9.29
20	9.81	20.80	9.36	20.79	9.36	24.71	9.16	20.61	9.38
30	9.82	28.52	9.46	28.46	9.46	31.58	9.35	28.33	9.47

The next figures show the error assessment for each equation, based on de real value measured in the calibration facility. Figures 3 and 4 show the error assessment results of angle and velocity, respectively for velocity 15 m/s, whilst Figs. 5 and 6 show the results for velocity 10 m/s. The result for Eq. (6) is the best value possible using the Collis and William's equation, therefore the others values should be compared with these.

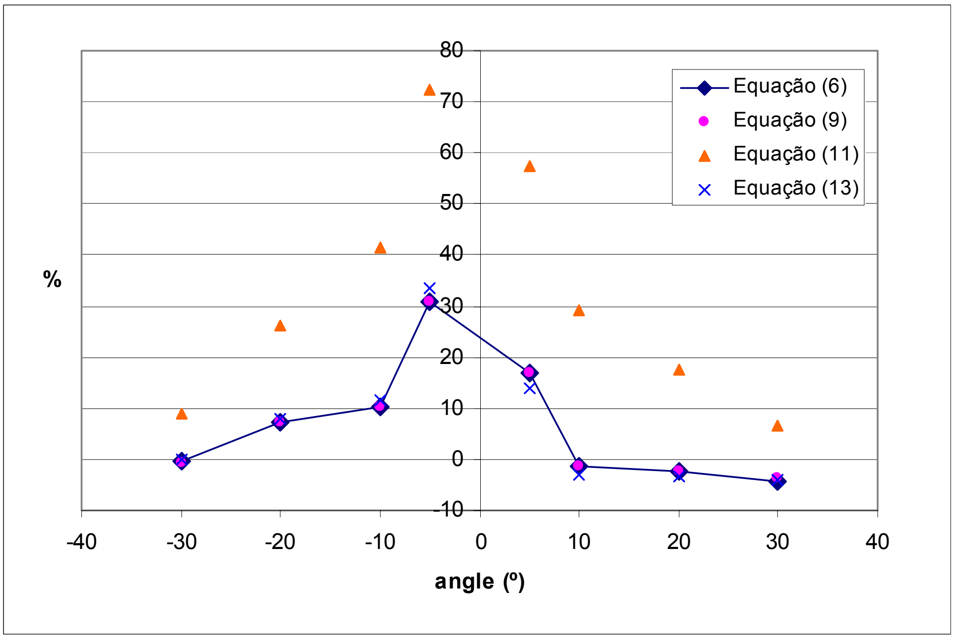


Figure 3. Angle error for 15 m/s

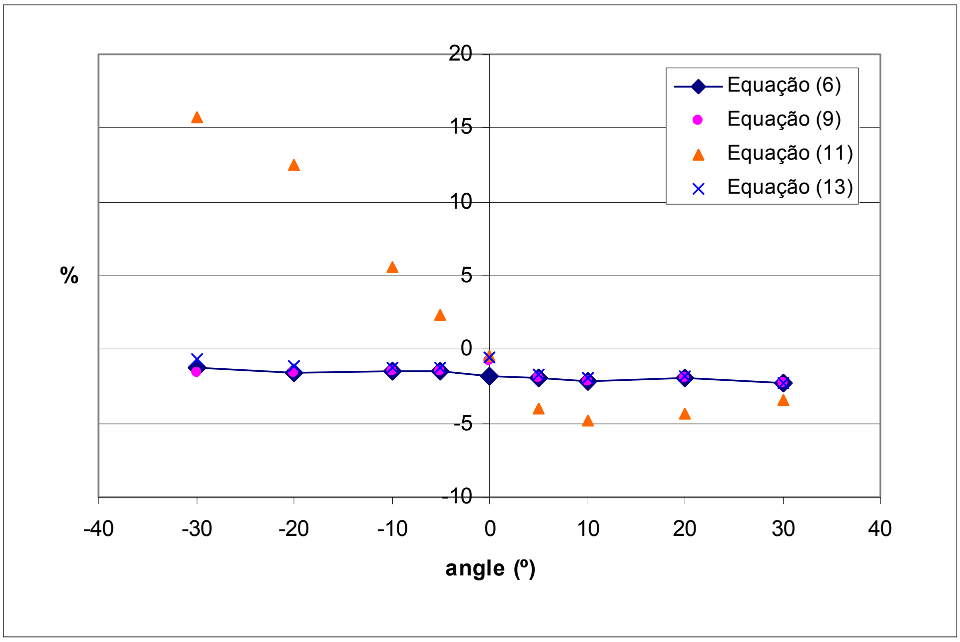


Figure 4. Velocity error for 15 m/s

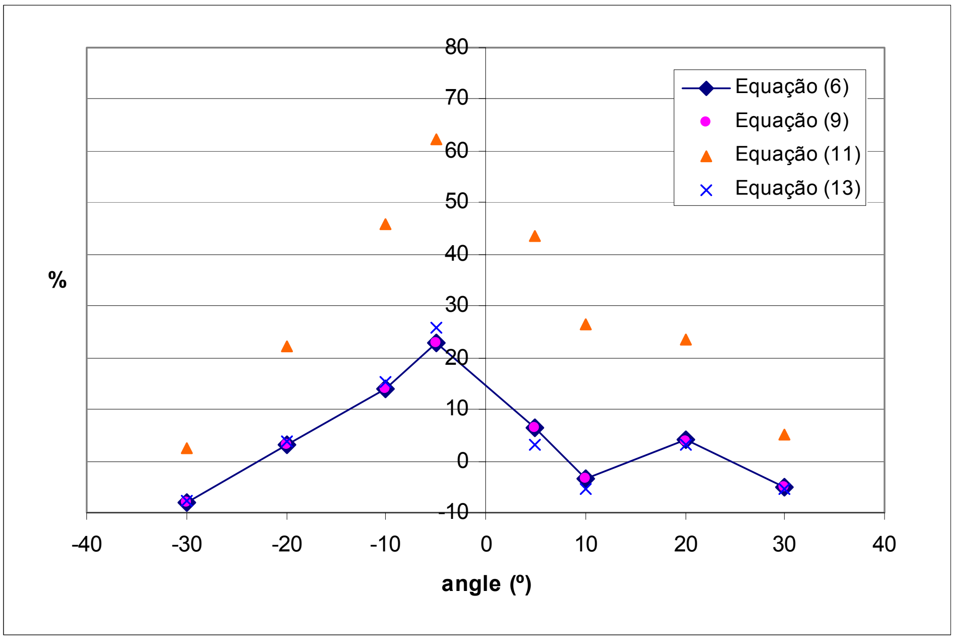


Figure 4. Angle error for 10 m/s

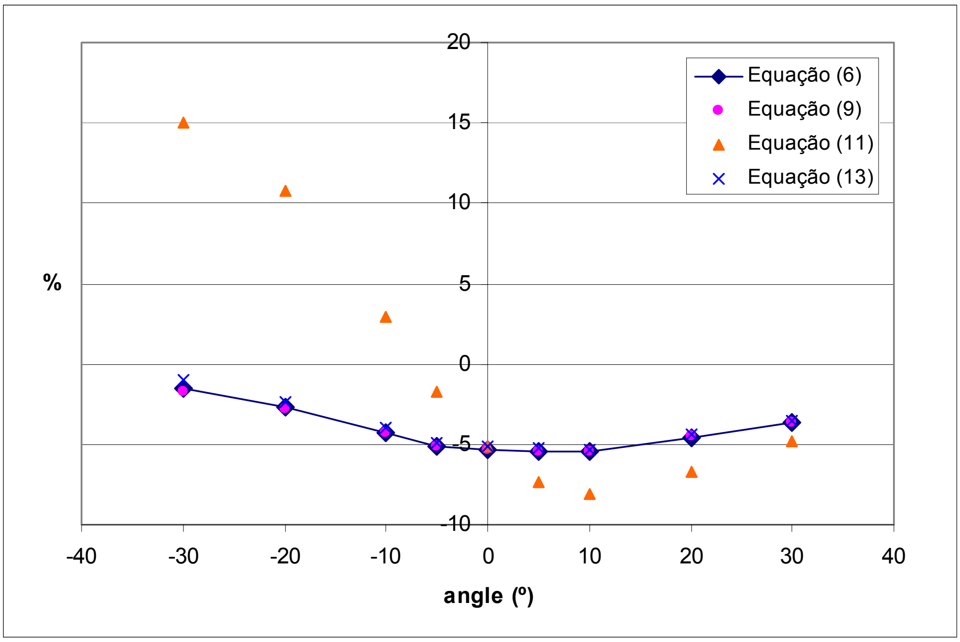


Figure 5. Velocity error for 10 m/s

5. Conclusions

This work propose a simplified approach for Collis and William's equation adapted by Davies e Bruun and applied for inclined two wire probes in order to allow its direct solution and reduce the process time in data reduction from experimental works. The simplification is based on the geometric similarity from two wires.

The X-probe calibration using tables for manual process is very slow because it is necessary a large number of angle measurement for different velocities.

A good numerical solution for the X-probe using Eq. (6) is a slow process, mainly when the work involve large data files.

The solution for the Eq. (9), developing in sine and cosine series, presents a small error related with the complete solution Eq. (6), yet it is a transcendent equation and must be solved numerically. Equation (11), developing in logarithm series, can be simplified for a polynomial, yet presents the biggest error, mainly for the angle determination.

The simplified solution for Eq. (13) is extremally fast, owing the fact the variables are explicit and the error is almost the same as obtained with the complete solution (Eq. 6).

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