UNIVERSIDADE FEDERAL DO RIO GRANDE DO SUL
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SÉRIE A: TRABALHO DE PESQUISA

# INSTANTANEOUS FREQUENCY DETECTION BY THE CONTRACTION MAPPING METHOD

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# Instantaneous Frequency Detection by the Contraction Mapping Method

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**Summary**

This paper provides the estimation of the instantaneous frequency of sinusoidal frequency modulated models. The estimation procedure is based on the Contraction Mapping Method (CM) applied to overlaping stretches of data. With some simple assumptions on the modulating process we show, by simulation studies, that the instantaneous frequency is very well captured by using two parametric families of linear filters.

## 1. Introduction

Here we want to use optimally the *Contraction Mapping Method* (see Kedem 1992, Kedem 1994, Kedem and Lopes 1992, Lopes and Kedem 1994) ideas for the frequency modulated (FM) process defined as

$$Z_t = Y_t + \xi_t = A\cos(\omega_c\, t + X(t) + \phi) + \xi_t, \quad \text{for } t \in \mathbf{Z}, \tag{1.1}$$

where

$$X(t) = B\sin(\omega_0\, t + \varphi)$$

is the sinusoidal modulating process, $A$ and $B$ are constants, $\omega_c$, $\omega_0 \in [0, \pi]$ and $\varphi$ and $\phi$ are uniformly distributed random variables independent of each other and of the noise process $\{\xi_t\}_{t\in\mathbf{Z}}$. For simplicity of the exposition we consider the noise process as being a Gaussian white noise.

Our goal is to estimate the *instantaneous frequency* given by

$$\omega(t) = \omega_c + B\,\omega_0 \cos(\omega_0\, t + \varphi), \quad \text{for } t \in \mathbf{Z}.$$

Consider $\{\mathcal{L}_\alpha(\cdot)\}_{\alpha\in\Theta}$ a parametric family of time invariant linear filters, where $\alpha$ is a finite dimensional parameter in the parameter space $\Theta$. Denote by $\{Z_t(\alpha)\}_{t\in\mathbf{Z}}$ the filtered process

$$Z_t(\alpha) \equiv \mathcal{L}_\alpha(Z)_t,$$

where $\{Z_t\}_{t\in\mathbf{Z}}$ is the zero-mean stationary process given by (1.1).

Then $\{\rho(\alpha)\}_{\alpha\in\Theta}$, defined by

$$\rho(\alpha) = \frac{\mathcal{R}\{E[Z_t(\alpha)\overline{Z_{t+1}(\alpha)}]\}}{E|Z_t(\alpha)|^2},$$

is a *"higher order correlation" (HOC)* family (see Kedem 1994) based on a parametrized first order autocorrelation. Here and elsewhere, $\mathcal{R}\{z\}$ denotes the real part of $z$ and $\overline{z}$ the complex conjugate of $z$.

Consider the updating procedure, based on $HOC$ and applied to the process (1.1), given by

$$\alpha_{k+1} = \rho(\alpha_k), \quad \text{for} \quad k \in \mathbf{N}. \tag{1.2}$$

We will analyze the effects of the above updating procedure caused by special families of filters applied to a time series. This procedure will converge to a value that give us important information about the process (1.1).

The outline of this paper is thus as follows. Some definitions related to the frequency modulated model are presented in Section 2 and the parametric families of filters are given in Section 3. In Section 4 we introduce the Contraction Mapping Method while in Section 5 is given the instantaneous frequency estimator. Some comments are given in Section 6.

## 2. The Frequency Modulated Model (FM)

Consider the stochastic process

$$Z_t = Y_t + \xi_t = A\cos(\omega_c\, t + X(t) + \phi) + \xi_t, \quad \text{for } t \in \mathbf{Z},$$

where

$$X(t) = B\sin(\omega_0\, t + \varphi) \tag{2.1}$$

is the sinusoidal modulating process, A and B are contants, $\omega_c, \omega_0 \in [0, \pi]$ are, respectively, the carrier and the modulating frequencies and $\varphi$ and $\phi$ are uniformly distributed random variables independent of each other and of the noise process $\{\xi_t\}_{t\in\mathbf{Z}}$. In the sequel the noise process is a Gaussian white noise for simplicity of the exposition but any stationary and ergodic process with a continuous spectral density function $f_\xi(\lambda)$ will do the same.

The *instantaneous frequency* of the process is defined as the derivative with respect to the time of the *instantaneous phase* and it is given by

$$\omega(t) = \frac{d}{dt}(\omega_c t + B\sin(\omega_0 t + \varphi) + \phi) = \omega_c + B\,\omega_0 \cos(\omega_0 t + \varphi), \quad \text{for } t \in \mathbf{Z}. \qquad (2.2)$$

One considers the assumptions that the modulating signal varies slowly compared to the carrier, that is,

$$\omega_c >> \omega_0$$

and that the frequency support is in $(-\pi, \pi]$, that is,

$$-\pi < \omega_c - B\,\omega_0 < \omega_c + B\,\omega_0 < \pi.$$

In the literature the constant B is called the *modulation index*. The *instantaneous frequency* varies about the unmodulated carrier frequency $\omega_c$ at the rate $\omega_0$ of the modulating signal and with a maximum deviation of $B\omega_0$ radians. In Lopes (1994) and Lopes and Kedem (1991) another method based on the *Contraction Mapping (CM) Method* (see Section 4) is applied to the same process (2.1) to estimate $\omega_c$ and $\omega_0$.

Increasing the modulated signal amplitude corresponds to increasing the modulation index B. So the bandwidth of the FM wave will depend on B. If the modulation index is zero, the resulting process

$$Z_t = A\cos(\omega_c t + \phi) + \xi_t, \quad t \in \mathbf{Z}, \qquad (2.3)$$

is one sinusoid plus noise model already pursued in the works of Lopes and Kedem (1994), Kedem and Lopes (1992), and Lopes (1993, 1991).

In order to estimate the *instantaneous frequency* one needs to estimate the parameters $\omega_c$, $\omega_0$ and $B$. The novelty here is to employ families of linear filters in an updating procedure based on the *analysis of higher order correlations* (see Kedem 1994) where the sample autocorrelation function of first order is chosen to be observed and to produce the estimators. For an application of this procedure in the case of finite number of frequencies see Lopes (1991), Kedem and Lopes (1992) and Lopes and Kedem (1994).

The analysis for FM models (see also Lopes and Kedem 1996, Brillinger 1987) is much more complicated than the case of finite number of frequencies (see Kedem and Lopes 1994). In some sense we are facing an infinite and dense set of frequencies in $(-\pi, \pi]$.

## 3. Linear Filters

In this section we shall define two useful parametric families of filters. These two families will play an important role in the CM Method to analyze the model (2.1).

### 3.1. The Alpha Filter

**Definition 1:** The *alpha filter* is defined by the time invariant linear transformation

$$Y_t(\alpha) = Y_t + \alpha Y_{t-1}(\alpha), \quad \text{for } t \in \mathbf{Z}, \tag{3.1}$$

where $\alpha \in (-1,1)$.Its impulse response function (see Kedem 1994) is

$$h(n;\alpha) = \begin{cases} \alpha^n, & \text{for n } \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

and its corresponding squared gain function is given by

$$|H_\alpha(\lambda)|^2 = \frac{1}{1 - 2\alpha\cos(\lambda) + \alpha^2}, \quad -1 < \alpha < 1 \quad \text{and} \quad -\pi < \lambda \leq \pi.$$

The first order autocorrelation function of the filtered process $\{Y_t(\alpha)\}_{t\in\mathbf{Z}}$ (see Lopes 1991) is given by

$$\rho(\alpha) = \frac{E[Y_t(\alpha)Y_{t+1}(\alpha)]}{E[Y_t^2(\alpha)]} = \frac{\int_{-\pi}^{\pi}\cos(\lambda)|H_\alpha(\lambda)|^2\,dF_Y(\lambda)}{\int_{-\pi}^{\pi}|H_\alpha(\lambda)|^2\,dF_Y(\lambda)} =$$

$$= \frac{\frac{A^2}{2}\sum_{n=-\infty}^{\infty} J_n^2(B)|H_\alpha(\omega_c + n\omega_0)|^2\cos(\omega_c + n\omega_0)}{\frac{A^2}{2}\sum_{n=-\infty}^{\infty} J_n^2(B)|H_\alpha(\omega_c + n\omega_0)|^2}, \tag{3.2}$$

where $J_n(z)$ is the Bessel function of the first kind of order $n \in \mathbf{Z}$ with z any real number.

### 3.2. The Complex Filter

**Definition 2:** The *complex filter* is defined by the transformation

$$Y_t(\alpha; M) = (1 + e^{i\theta(\alpha)}\mathcal{B})^M Y_t, \quad \text{for } t \in \mathbf{Z},$$

where M is a positive integer, $\alpha \in (-1,1)$, $\theta(\alpha) \in (-\pi,\pi)$ and $\mathcal{B}$ is the backward shift operator. One considers M as being sufficiently large so that we can entertain the approximation $\theta(\alpha) \approx \cos^{-1}(\alpha)$. We can rewrite $Y_t(\alpha; M)$ as

$$Y_t(\alpha; M) = \sum_{n=0}^{M} \binom{M}{n} e^{i\theta(\alpha)n} Y_{t-n}, \quad \text{for } t \in \mathbf{Z}, \ -\pi < \theta(\alpha) < \pi \ \text{ and } M \in \mathbf{N} - \{0\}. \tag{3.3}$$

The impulse response function of the above filter is

$$h(n;\alpha, M) = \begin{cases} \binom{M}{n} e^{i\theta(\alpha)n}, & \text{for } 0 \leq n \leq M \\ 0, & \text{otherwise} \end{cases}$$

and its transfer function is

$$H(\lambda;\alpha,M)=(1+e^{i(\theta(\alpha)-\lambda)})^M,\quad \text{for}\quad -\pi<\lambda\leq\pi.$$

The corresponding square gain function is given by

$$|H(\lambda;\theta(\alpha),M)|^2=4^M\cos^{2M}\left(\frac{\lambda-\theta(\alpha)}{2}\right),\quad \text{for}\quad -\pi<\lambda,\theta\leq\pi\ \text{ and }\ -1<\alpha<1.$$

The first order autocorrelation function of the filtered process $\{Y_t(\alpha;M)\}_{t\in\mathbf{Z}}$ (see Lopes 1991) is given by

$$\rho(\alpha,M)=\frac{\mathcal{R}\{E[Y_t(\alpha,M)\overline{Y_{t+1}(\alpha,M)}]\}}{E|Y_t(\alpha,M)|^2}=$$

$$=\frac{4^{M-1}A^2\sum_{n=-\infty}^{\infty}J_n^2(B)\{\cos^{2M}(\frac{\theta-\theta_n}{2})+\cos^{2M}(\frac{\theta+\theta_n}{2})\}\cos(\theta_n)}{4^{M-1}A^2\sum_{n=-\infty}^{\infty}J_n^2(B)\{\cos^{2M}(\frac{\theta-\theta_n}{2})+\cos^{2M}(\frac{\theta+\theta_n}{2})\}},\tag{3.4}$$

where $J_n(z)$ is the Bessel function of the first kind of order $n\in\mathbf{Z}$ with z any real number.

## 4. The CM Method when B=0

The CM Method can be more easily explained when the signal process $\{Y_t\}_{t\in\mathbf{Z}}$ is only one sinusoid with frequency, say, $\omega_c$ as in the expression (2.3), where $A>0$ and $\omega_c\in(0,\pi]$ are constantes, $\phi$ is a uniformly distributed random variable, that is, $\phi\sim\mathcal{U}((-\pi,\pi])$ and $\{\xi_t\}_{t\in\mathbf{Z}}$ is a zero mean stationary and ergodic stochastic process, independent of the phase $\phi$, with spectral distribution function $F_\xi(\omega)$ continuous at $\omega_c$. Considering the alpha parametric family of filters, as in the expression (3.1), the first order autocorrelation function of the filtered process $\{Y_t(\alpha)\}_{t\in\mathbf{Z}}$ is given by

$$\rho(\alpha)=\frac{E[Y_t(\alpha)Y_{t+1}(\alpha)]}{E[Y_t^2(\alpha)]}=\frac{\frac{A^2}{2}\frac{\cos(\omega_c)}{1-2\alpha\cos(\omega_c)+\alpha^2}+\sigma_\xi^2\frac{\alpha}{1-\alpha^2}}{\frac{A^2}{2}\frac{1}{1-2\alpha\cos(\omega_c)+\alpha^2}+\sigma_\xi^2\frac{1}{1-\alpha^2}}.\tag{4.1}$$

Some properties of the mapping $\rho(\cdot)$, defined by the expression (4.1) above, that will help to prove the existence of a contraction mapping are now presented.

**Proposition 4.1:** *The mapping $\rho(\cdot)$, given by the expression (4.1), is a mapping from $[-1,1]$ onto $[-1,1]$.*

**Proof:** Consider $\alpha\in(-1,1)$. Since $cos(\omega_c)\in(-1,1)$ and $(1-2\alpha\cos(\omega_c)+\alpha^2)>0$, by using the expression (4.1) we have

$$\frac{-A^2}{2}\frac{1}{1-2\alpha\cos(\omega_c)+\alpha^2}-\sigma_\xi^2\frac{1}{1-\alpha^2}<\frac{A^2}{2}\frac{\cos(\omega_c)}{1-2\alpha\cos(\omega_c)+\alpha^2}+\sigma_\xi^2\frac{\alpha}{1-\alpha^2}<$$

$$<\frac{A^2}{2}\frac{1}{1-2\alpha\cos(\omega_c)+\alpha^2}+\sigma_\xi^2\frac{1}{1-\alpha^2}$$

The goal is to estimate the instantaneous frequency by *the Contraction Mapping (CM) Method* (see Kedem 1992) based on sample autocorrelations and demodulate the baseband signal $\{Z_t\}_{t\in\mathbf{Z}}$. The novelty here is to apply the *CM Method* to overlaping stretches of data (see Kedem and Yakowitz 1990). The analysis is based on a single time series $\{Z_t\}_{t=1}^{N}$ with N observations. This time series is divided into several overlaping stretches of data, denoted by $N_1$, each stretch with the same number of observations, denoted by $N_2$, and a bandwidth, denoted by $b$, such that the total number of observations satisfies the following equation

$$N = (N_1 - 1)b + N_2.$$

The *CM Method* is applied to each stretch using the *alpha* and *complex* filters defined both in Section 3. In order to do this one needs the following assumptions.

*Assumptions* :

(1). *The model (2.1) is considered in the discrete time where* $t \in \mathbf{Z}$.
(2). *The random variables* $\phi$ *and* $\varphi$ *are uniformly distributed on* $(-\pi, \pi]$ *independent of each other and of the process* $\{\xi_t\}_{t\in\mathbf{Z}}$.
(3). *The modulating signal varies slowly compared to the carrier frequency* ($\omega_c >> \omega_0$).
(4). *The modulating signal must be centered on* $(-\pi, 0)$ *or* $(0, \pi)$.

The *assumption (3)* is very common in the engineering literature (see, for instance, Subba-Rao and Yar 1984).

After some simulations using both the alpha and complex filters, described in Section 3, we obtain the best stretch of frequency to detect the instantaneous frequency. Notice that each stretch is smaller than the method without overlaping the stretches (see Lopes 1994).

In Figure 1 we consider the process (2.1) with $A = \sqrt{2}$, $B = 760$, $\omega_c = 1.303988$, $\omega_0 = 0.000744$, $N_1 = 160$, $N_2 = 2,000$ and the process $\{\xi_t\}_{t\in\mathbf{Z}}$ is a Gaussian white noise process with $\sigma_\xi^2 = 1.0$ (a), $\sigma_\xi^2 < 1.0$ (b) and $\sigma_\xi^2 > 1.0$ (c). The filter used here is the alpha filter. Notice that the noise is almost annihilated by the action of the alpha filter.

In Figure 2 we consider the same process (2.1) with same parameters as in Figure 1 but now applying the complex filter. In Figure 2 we consider $N_1 = 500$ and $N_2 = 500$.

In Figure 3 we consider another simulation of the process (2.1) with parameters $A = \sqrt{2}$, $B = 890$, $\omega_c = 1.253988$, $\omega_0 = 0.000437$ with $\sigma_\xi^2 = 1.0$ (a), $\sigma_\xi^2 < 1.0$ (b) and $\sigma_\xi^2 > 1.0$ (c).

**Figure 1:** The instantaneous frequency $\omega(t)$ as given in expression (2.2) and its estimated value by the method based on overlaping stretches of data with the alpha filter. The parameters are given by $A = \sqrt{2}$, $B = 760$, $\omega_c = 1.303988$, $\omega_0 = 0.000744$, $N_1 = 160$, $N_2 = 2,000$ and
(a) $\sigma_\xi^2 = 1.0$ (SNR=0); (b) $\sigma_\xi^2 < 1.0$ (SNR>0); (c) $\sigma_\xi^2 > 1.0$ (SNR<0).

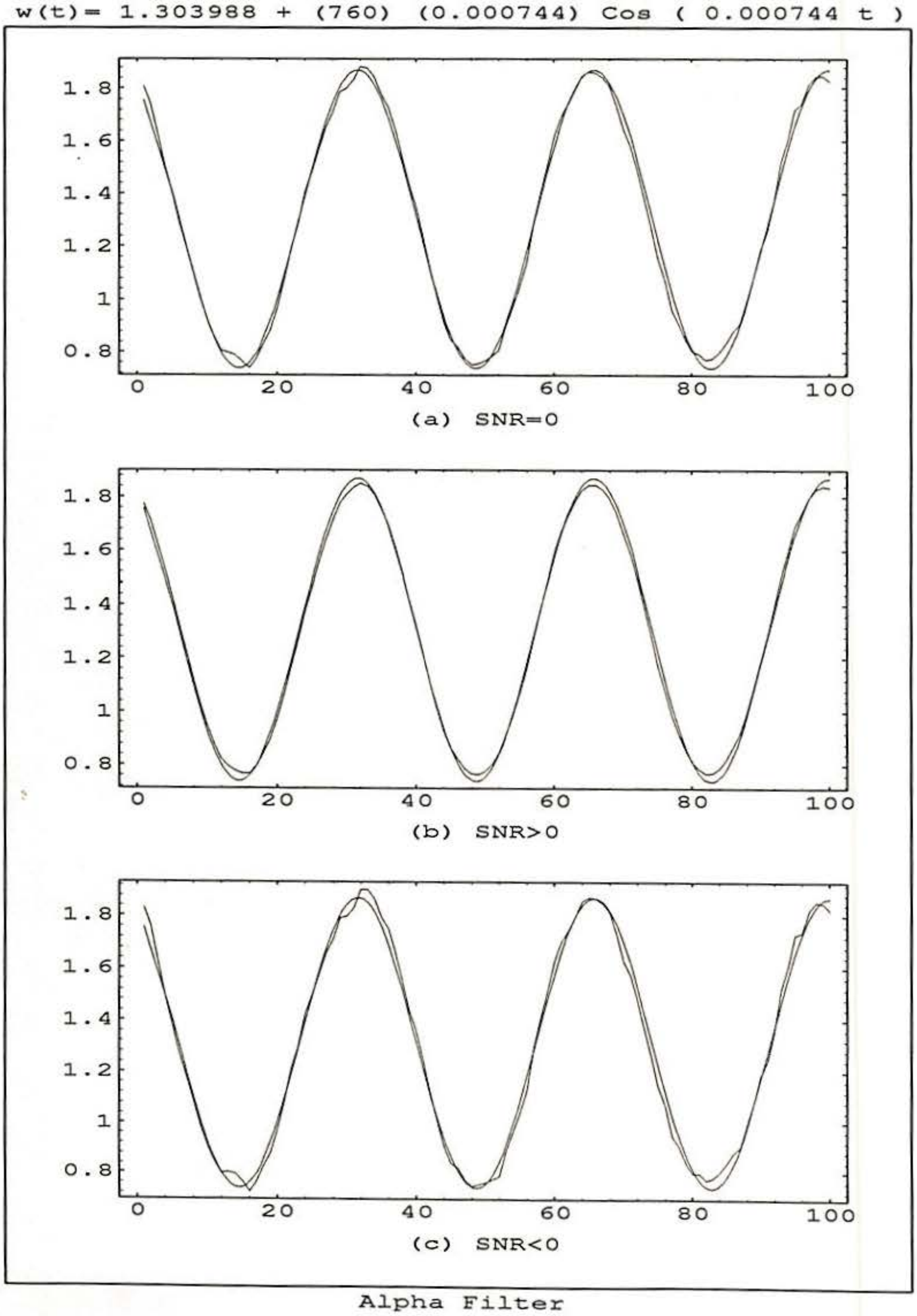


**Figure 2:** The instantaneous frequency $\omega(t)$ as given in expression (2.2) and its estimated value by the method based on overlaping stretches of data with the complex filter. The parameters are given by $A = \sqrt{2}$, $B = 760$, $\omega_c = 1.303988$, $\omega_0 = 0.000744$, $N_1 = 500$, $N_2 = 500$ and
(a) $\sigma_\xi^2 = 1.0$ (SNR=0); (b) $\sigma_\xi^2 < 1.0$ (SNR>0); (c) $\sigma_\xi^2 > 1.0$ (SNR<0).

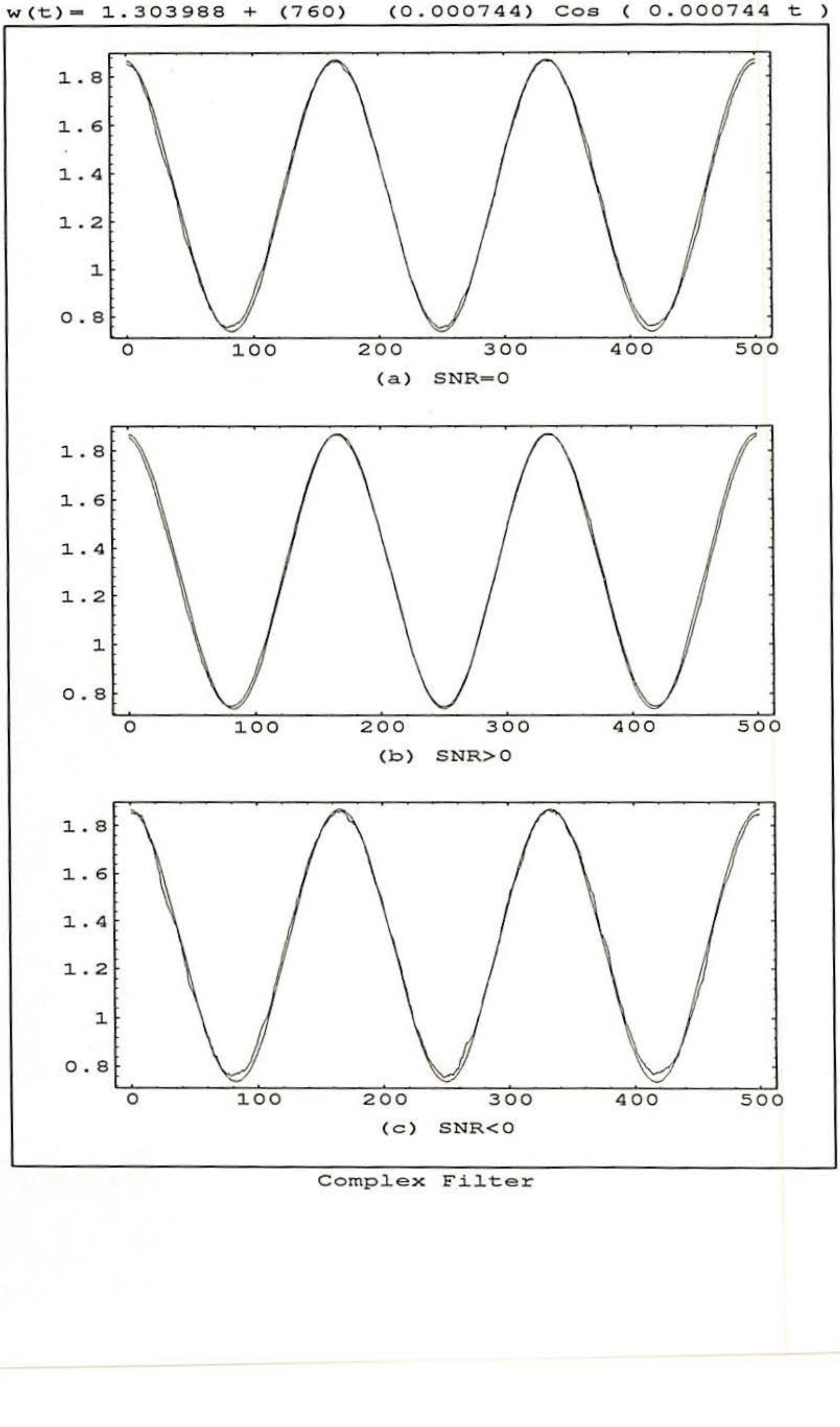
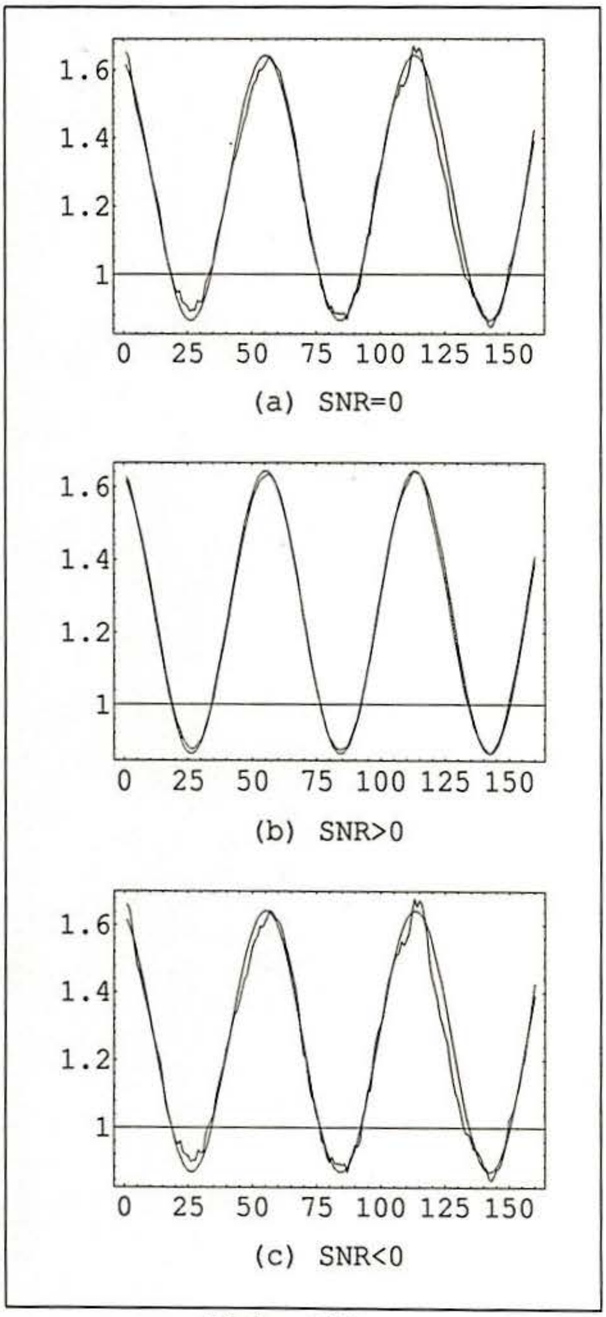
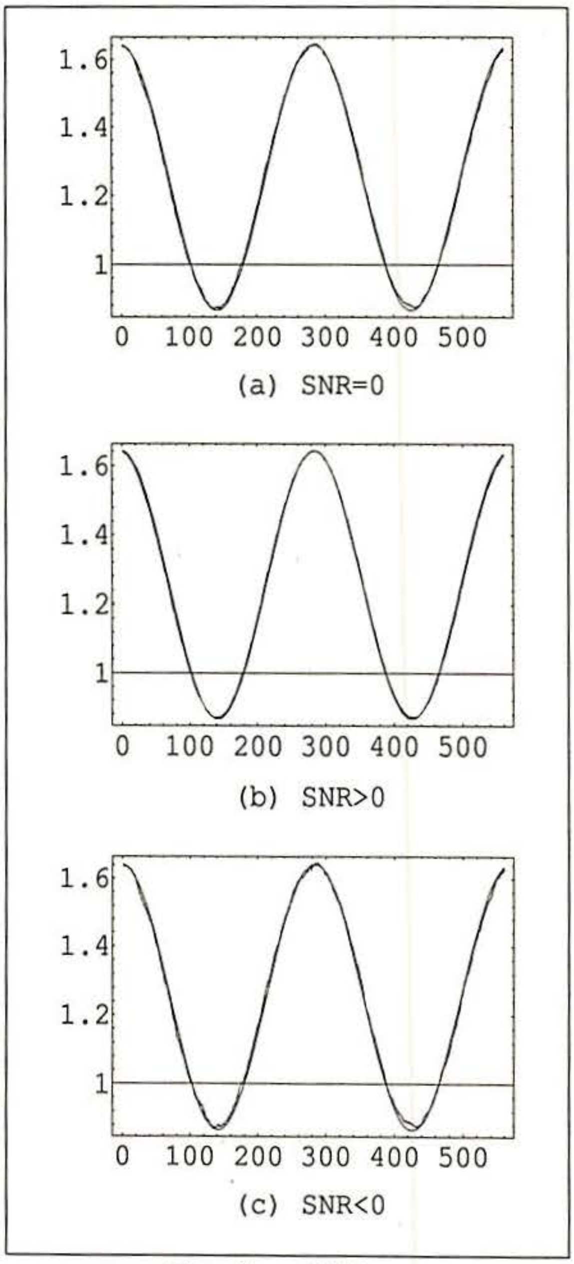


**Figure 3:** The instantaneous frequency $\omega(t)$ as given in expression (2.2) and its estimated value by the CM method using the alpha and the complex filter, respectively. The parameters are given by $A = \sqrt{2}$, $B = 890$, $\omega_c = 1.253988$, $\omega_0 = 0.000437$ and (a) $\sigma_\xi^2 = 1.0$ (SNR=0); (b) $\sigma_\xi^2 < 1.0$ (SNR>0); (c) $\sigma_\xi^2 > 1.0$ (SNR<0).

```
w(t)=  1.253988 + (890) (0.000437) Cos ( 0.000437 t )
```




Alpha Filter




Complex Filter

From Figures 1, 2 and 3 we observe that when $\sigma_\xi^2 > 1.0$ we have more noise than signal and the estimation is noisier. When $\sigma_\xi^2 < 1.0$ we have less noise in the model and the estimations are better.

One observes that for the same simulated model as in Figure 3 when the alpha filter is applied the results are worse than using the complex filter. We just show the graph of some simulated results but the studies have shown that the complex filter has a better perfor-mance. And if we introduce less noise in the model the estimation becomes much better for both family of filters but the best results are still obtained by using the complex filter.

## 6. Conclusion

We consider the Contraction Mapping Method based on sample autocorrelations function to estimate the instantaneous frequency of sinusoidal frequency modulated models. The procedure was based on overlaping stretches of data subject to the restriction that the modulating signal varies slowly compared to the carrier frequency (that is, $\omega_c >> \omega_0$) and that the modulating signal is centered on the intervals $(-\pi, 0)$ or $(0, \pi)$. In the simulation studies we consider both the alpha and the complex filters. The Contraction Mapping Method is originally created for sinusoidal plus noise models but works also well for FM models. The reason is that with assumption (3), in each stretch of data, the frequency modulated wave is approximatedly one sinusoidal wave. From Figures 1, 2 and 3 one can see that the instantaneous frequency $\omega(t)$ is better detected using the complex filter than the alpha filter.

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