

# SCALING FUNCTION FOR THE PRODUCTION OF VECTOR MESONS AND DVCS IN THE SATURATION SCHEME<sup>\* \*\*</sup>

F.G. BEN, M.V.T. MACHADO

High Energy Physics Phenomenology Group, GFPAE IF-UFRGS  
Caixa Postal 15051, CEP 91501-970, Porto Alegre, RS, Brazil

W.K. SAUTER

Instituto de Física e Matemática, Universidade Federal de Pelotas  
Caixa Postal 354, CEP 96010-900, Pelotas, RS, Brazil

*(Received March 6, 2019)*

In this work, we provide a scaling function that describes the cross section for all available data from DESY-HERA for the exclusive production of vector mesons  $\rho$ ,  $\phi$ ,  $J/\psi$  and Deeply Virtual Compton Scattering (DVCS). Our description is based on the geometric scaling phenomenon, in which cross sections are functions of a single variable  $\tau = Q^2/Q_{\text{sat}}^2$ , where  $Q^2$  is the photon virtuality and  $Q_{\text{sat}}^2$  represents the saturation scale and drives the energy dependence and nuclear effects. We briefly extend our results to nuclear targets to be tested in future EICs.

DOI:10.5506/APhysPolBSupp.12.921

## 1. Introduction

One of the striking consequences of high-energy deep inelastic electron–proton (or electron–nucleus) scattering (DIS) is the geometric scaling phenomenon. In this regime, the total  $\gamma^* p$  and  $\gamma^* A$  cross sections are not functions of the two independent variables  $x$  (Bjorken scale) and  $Q^2$  (photon virtuality), but are rather functions [2] of a single scaling variable,  $\tau_A = Q^2/Q_{\text{sat},A}^2$ . The saturation scale  $Q_{\text{sat},A}^2(x; A) \propto x G_A(x, Q_{\text{sat}}^2) / (\pi R_A^2)$  is connected to gluon saturation effects, but it extends to relative large virtualities [3]. For proton targets, it extends up to  $Q^2 \sim Q_{\text{sat}}^4(x) / \Lambda_{\text{QCD}}^2$ , provided one stays in the small- $x$  region. For nuclear targets, that kinematic window is further enlarged due to the nuclear enhancement of the saturation scale,

---

<sup>\*</sup> Presented at the Diffraction and Low- $x$  2018 Workshop, August 26–September 1, 2018, Reggio Calabria, Italy.

<sup>\*\*</sup> An extended version of this work has been published in [1].

$Q_{\text{sat},A}^2 \simeq A^{1/3} Q_{\text{sat},p}^2$ . It was proven for the first time in Ref. [4] that the DESY-HERA  $ep$  collider data on the proton structure function  $F_2$  present a scaling pattern at  $x \leq 0.01$  and  $Q^2 \leq 400 \text{ GeV}^2$ .

For lepton–nucleus interactions, in Ref. [5], the  $\gamma^* A$  cross section is obtained from the corresponding cross section for  $\gamma^* p$  process as a function of the scaling variable for a proton target. The following scaling curve for the photoabsorption cross section was considered [5]:

$$\sigma_{\text{tot}}^{\gamma^* p}(\tau_p) = \bar{\sigma}_0 [\gamma_E + \Gamma(0, \nu) + \ln(\nu)], \quad (1)$$

where  $\nu = a/\tau_p^b$ ,  $\gamma_E$  is the Euler constant and  $\Gamma(0, \nu)$  the incomplete Gamma function. The parameters for the proton case were obtained from a fit to the small- $x$   $ep$  DESY-HERA data, producing  $a = 1.868$ ,  $b = 0.746$ , and the overall normalization was fixed by  $\bar{\sigma}_0 = 40.56 \mu\text{b}$ . Their fit is presented in Fig. 1. The parameters for the nuclear saturation scale were determined by fitting the available lepton–hadron data using the relation in Ref. [5] and the same scaling function, Eq. (1). They obtained  $\delta = 1/\Delta = 0.79 \pm 0.02$  and  $\pi R_p^2 = [1.55 \pm 0.02] \text{ fm}^2$ .

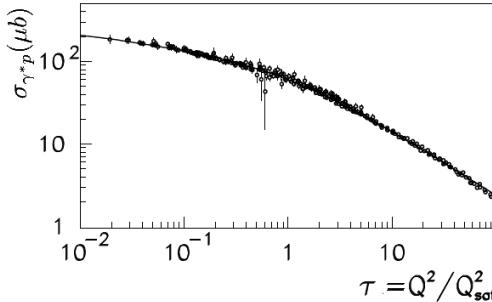


Fig. 1. Geometrical scaling for  $\gamma^* p$  together with the scaling expression Eq. (1). Figure from [5].

In Ref. [6], it was demonstrated that the data on diffractive DIS,  $\gamma^* p \rightarrow Xp$ , and other diffractive observables present geometric scaling on the variable  $\tau_D = Q^2/Q_{\text{sat}}^2(x_P)$ , in region  $x_P < 0.01$ , where  $x_P = (Q^2 + M_X^2)/(Q^2 + W^2)$ . Moreover, the total cross sections for  $\rho$ ,  $\phi$  and  $J/\psi$  are shown to present scaling on the variable  $\tau_V = (Q^2 + M_V^2)/Q_{\text{sat}}^2(x_P)$ . Nevertheless, Ref. [6] provides no theoretical or phenomenological expression for the scaling function. In this work, we extend the approach presented in Ref. [5] to exclusive (diffractive) processes to describe also the observed scaling features demonstrated in Ref. [6]. We are able to provide a reasonable description for the available data for  $V = \rho, \phi, J/\psi$  and real photons. The results are improved by allowing a global fit using the universal scaling expression which depends on very few parameters. We then extend our results to nuclear targets.

## 2. Cross sections for exclusive vector meson production and DVCS

In general, one may obtain the total cross section for a hadronic interaction from the elastic scattering amplitude  $a(s, b)$  through the optical theorem. In terms of the impact parameter, one can write  $\sigma_{\text{tot}} = 2 \int d^2b \text{Im } a(s, b)$  and  $\sigma_{\text{el}} = \int d^2b |a(s, b)|^2$ . In the eikonal approach,  $a(s, b) = i(1 - e^{-\Omega(s, b)})$ , where the eikonal  $\Omega$  is a real function. Thus,  $P(s, b) = e^{-2\Omega(s, b)}$  gives the probability that no inelastic interaction takes place at impact parameter  $b$ . Assuming for simplicity a Gaussian form for the eikonal,  $\Omega(s, b) = \nu(s) \exp(-b^2/R^2)$ , analytical expressions for total and elastic cross sections are generated

$$\sigma_{\text{tot}} = 2\pi R^2 [\ln(\nu) + \gamma_E + \Gamma(0, \nu)] , \quad (2)$$

$$\sigma_{\text{el}} = \pi R^2 \left[ \ln\left(\frac{\nu}{2}\right) + \gamma_E - \Gamma(0, 2\nu) + 2\Gamma(0, \nu) \right] . \quad (3)$$

From this, one can easily identify that Eq. (1) relies on the total cross section from the eikonal model, Eq. (2), with the following identification:  $\bar{\sigma}_0 = 2\pi R^2$  and  $\nu = a/\tau_p^b$ . The  $a$  and  $b$  parameters absorb the lost information when using a oversimplified photon wave-function overlap  $\Phi^{\gamma^*\gamma^*} \propto \delta(r - 1/Q)$  within the color dipole framework. We then construct the scaling function to describe exclusive diffractive processes starting from Eq. (3). The main point is to associate the exclusive vector meson production and DVCS process as a quasi-elastic scattering.

For vector meson production, we have to include information related to the meson wave function, and in the DVCS case information on the real photon appearing in the final state. Adding this new information will modify the overall normalization in Eq. (3) and possibly also the parameters  $a$  and  $b$  considered in Ref. [5]. We then write

$$\sigma(\gamma^* p \rightarrow Ep) = \frac{\bar{\sigma}_E}{2} \left[ \ln\left(\frac{\nu}{2}\right) + \gamma_E - \Gamma(0, 2\nu) + 2\Gamma(0, \nu) \right] , \quad (4)$$

where  $\bar{\sigma}_E = \bar{\sigma}_V$  in the case of vector mesons and  $\bar{\sigma}_E = \bar{\sigma}_{\text{DVCS}}$  for DVCS process. In both cases,  $\nu = a/\tau^b$ , with  $\tau = (Q^2 + M_V^2)/Q_{\text{sat}}^2$  for exclusive production of mesons and  $\tau = Q^2/Q_{\text{sat}}^2$  for DVCS. The overall normalization may be estimated from the inspection of the overlap functions for the total and vector meson production cross sections [1, 7], which leads to

$$\bar{\sigma}_{\text{DVCS}} = \left( \alpha_e \sum_f e_f^2 \right) \bar{\sigma}_0 , \quad (5)$$

$$\bar{\sigma}_V = \frac{4\pi \hat{e}_f^2 f_V^2}{M_V^2 \left( \sum_f e_f^2 \right)} \bar{\sigma}_0 . \quad (6)$$

### 3. Results

Let us now compare the scaling curve, Eq. (4), to the available experimental data in small- $x$  lepton–proton collisions. The data sets we have considered are presented in Refs. [8–11]. The values of parameters  $M_V$ ,  $f_V$  and  $\hat{e}_V$  were taken from Ref. [7]. We perform a fit to the experimental data using MINPACK routines [12] for choices of sets of parameters, described in the following. Our results are presented in Figs. 2 and 3. Explicitly, the scaling variable is  $\tau = \tau_V = (Q^2 + M_V^2)/Q_{\text{sat}}^2(x)$  for exclusive production of mesons and  $\tau = Q^2/Q_{\text{sat}}^2(x)$  for DVCS, with  $Q_{\text{sat}}^2(x) = [(x_0/\bar{x})^\lambda]$  GeV<sup>2</sup> as discussed in the introduction section.

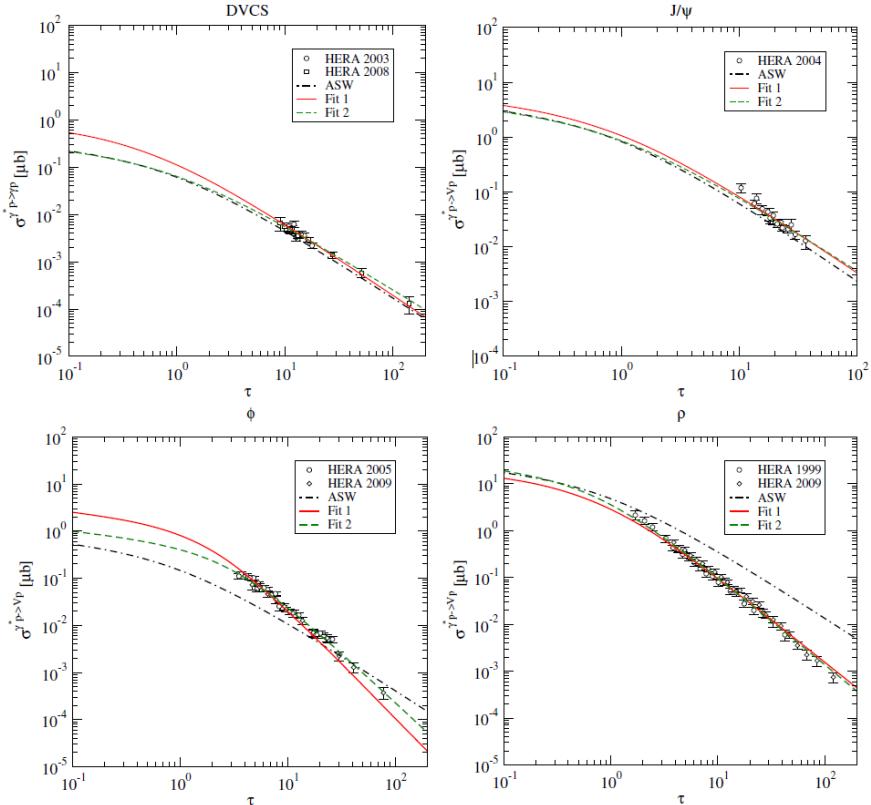


Fig. 2. The cross sections for DVCS,  $\phi$ ,  $J/\psi$  and  $\rho$  as a function of  $\tau$ . Figure from [1].

We use two different choices to perform the fits. The first one, labeled Fit 1 in the figures, adjusts all the three parameters ( $a$ ,  $b$  and  $\bar{\sigma}_0$ ). The other one, labeled Fit 2 in the figures, fits  $a$ ,  $b$  parameters, maintaining fixed  $\bar{\sigma}_0 = 40.56$   $\mu\text{b}$ . In general, both fits describe in good agreement the

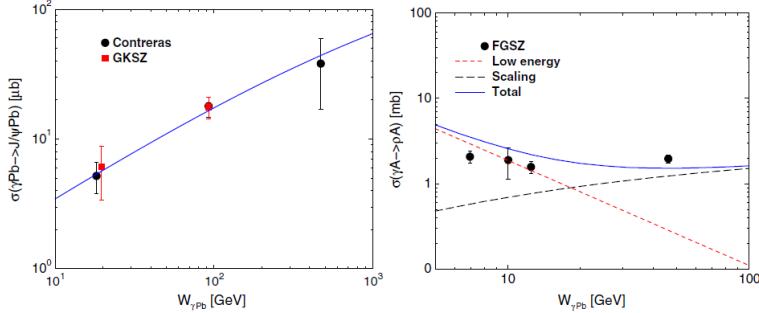


Fig. 3. The cross sections for nuclear production of  $J/\psi$  and  $\rho$  as a function of the corresponding photon–nucleus energy. Figure from [1].

available data for all observables (with the exception of  $\phi$  meson) for photon–proton interactions. It is very clear that the quality of fit for Fit 1 and Fit 2 are somewhat equivalent. Fit 2 is a straightforward extension of the celebrated scaling curve presented in Ref. [5] for the inclusive case. The overall normalization  $\bar{\sigma}_0$  is common to inclusive and exclusive photon–target processes.

The geometrical scaling present in the lepton–proton cross sections for exclusive processes, as quantified by Eq. (4), is translated to the scattering on nuclear targets at high energies. Following the same arguments given in Ref. [5], the atomic number dependence is absorbed in the nuclear saturation scale and on the overall normalization related to the nuclear radius. Therefore, the cross section for lepton–nuclei scattering takes the following form:

$$\sigma^{\gamma^* A \rightarrow EA}(\tau_A) = \frac{\pi R_A^2}{\pi R_p^2} \sigma^{\gamma^* p \rightarrow Ep}(\tau = \tau_A), \quad (7)$$

where the scaling variable in nuclear case is  $\tau_A = \tau_p [\pi R_A^2 / (A \pi R_p^2)]^\Delta$ . In particular, we expect that for large  $\tau_A$ , the relation is  $\sigma(\gamma^* A \rightarrow EA) \propto R_A^2 \tau_A^{-b} = R_A^2 \tau_p^{-b} (A^{1/3})^{\frac{b}{\delta}}$ . Figure 3 shows the cross sections for nuclear production of  $J/\psi$  and  $\rho$  as a function of the corresponding photon–nucleus energy, together with a plot of the prediction from Eq. (7), using the parameters adjusted for  $\gamma^* p$  collisions. As the current data on nuclear targets are quite scarce at small- $x$  region, the above scaling formula can be tested in future measurements in EICs or in ultraperipheral heavy-ions collisions. The robustness of the geometric scaling treatment for the interaction is quite impressive and similar scaling properties have been proved theoretically and experimentally, for instance in charged hadron production [13] and in prompt photon production [14] on  $pA$  and  $AA$  collisions in colliders energy regime.

#### 4. Summary and conclusions

We demonstrate that simple considerations on the scope of the geometric scaling phenomenon and the eikonal model allow us to describe the available data on DVCS and vector meson production on proton target with a universal scaling function that depends on very few parameters. Our results can be extrapolated to nuclear targets to be tested in future EICs or in ultra-peripheral collisions.

This work was financed by the Brazilian funding agency CNPq and in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior — Brasil (CAPES) — Finance Code 001.

#### REFERENCES

- [1] F.G. Ben, M.V.T. Machado, W.K. Sauter, *Phys. Rev. D* **96**, 054015 (2017).
- [2] S. Munier, R. Peschanski, *Phys. Rev. Lett.* **91**, 232001 (2003).
- [3] E. Iancu, K. Itakura, L. McLerran, *Nucl. Phys. A* **708**, 327 (2002);  
A.H. Mueller, D.N. Triantafyllopoulos, *Nucl. Phys. B* **640**, 331 (2002).
- [4] A.M. Staśto, K. Golec-Biernat, J. Kwieciński, *Phys. Rev. Lett.* **86**, 599 (2001).
- [5] N. Armesto, C.A. Salgado, U.A. Wiedemann, *Phys. Rev. Lett.* **94**, 022002 (2005).
- [6] C. Marquet, L. Schoeffel, *Phys. Lett. B* **639**, 471 (2006).
- [7] H. Kowalski, L. Motyka, G. Watt, *Phys. Rev. D* **74**, 074016 (2006).
- [8] A. Aktas *et al.* [H1 Collaboration], *Eur. Phys. J. C* **44**, 1 (2005);  
S. Chekanov *et al.* [ZEUS Collaboration], *Phys. Lett. B* **573**, 46 (2003);  
S. Chekanov *et al.* [ZEUS Collaboration], *Phys. Lett. B* **659**, 796 (2008).
- [9] C. Adloff *et al.* [H1 Collaboration], *Eur. Phys. J. C* **13**, 371 (2000);  
J. Breitweg *et al.* [ZEUS Collaboration], *Eur. Phys. J. C* **6**, 603 (1999);  
F.D. Aaron *et al.* [H1 Collaboration], *J. High Energy Phys.* **1005**, 032 (2010).
- [10] S. Chekanov *et al.* [ZEUS Collaboration], *Nucl. Phys. B* **718**, 3 (2005).
- [11] A. Aktas *et al.* [H1 Collaboration], *Eur. Phys. J. C* **46**, 585 (2006)  
[arXiv:hep-ex/0510016]; S. Chekanov *et al.* [ZEUS Collaboration], *Nucl. Phys. B* **695**, 3 (2004).
- [12] J.J. Moré, B.S. Garbow, K.E. Hillstrom, Argonne National Laboratory Report ANL-80-74, 1980.
- [13] M. Praszalowicz, A. Francuz, *Phys. Rev. D* **92**, 074036 (2016); L. McLerran, M. Praszalowicz, *Phys. Lett. B* **741**, 246 (2015).
- [14] C. Klein-Bösing, L. McLerran, *Phys. Lett. B* **734**, 282 (2014).