

# OPTIMIZATION OF PARAMETERS OF TUNED MASS DAMPERS FOR USE IN TALL BUILDINGS SUBJECTED TO THE WIND ACTION

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**Abstract.** This paper proposes a methodology to optimize the parameters of tuned mass dampers (TMD's) installed in a high-rise building subject to wind-induced vibration. The cost function is the minimization of the maximum displacement at the top-floor of the building, while the design variables are the spring stiffness and damping coefficients. The total mass of the TMD's are assumed to be between 2% to 10% of the total mass of the building. To carry out the proposed optimization, the Search Group Algorithm (SGA) is employed. Several different scenarios are taken into account, such as: (i) a single TMD (STMD) installed at the top, (ii) multiple TMD's (MTMD) positioned at the top-floor and also (iii) MTMD installed on different stories of the building. The effectiveness of the proposed method to optimize the parameters of TMD's in order to minimize the dynamic response of the building is shown in each case. At the end of this paper, a comparison among the results is carried out.

Keywords: Tuned mass damper (TMD), Stochastic optimization, Wind-action.

### **1** Introduction

The development of light and high resistance materials allied with the advancement of design and construction technologies resulted in the construction of increasingly light, tall and slender buildings. However, high-rise buildings are more flexible and may not have sufficient damping, making them susceptible to wind-induced vibrations.

It is known that the vibration that occurs in high buildings, due to the wind forces, causes discomfort to the residents and / or users, thus limiting the vibration through the inclusion of control devices improves the habitability and safety of this type of building.

The most well-known structural vibration control device is the tuned mass damper (TMD), which consists of an additional mass, with its own damping and stiffness, attached to the main structure and tuned to the fundamental mode of vibration of the building. According to Holmes [1], the system works by absorbing the energy of vibration through the movement of its mass, which is a secondary mass connected to the main frame system by viscous dampers.

A tuned mass damper structure can be convenently considered as a two degrees of freedom system, where the main structure is designed with only one degree of freedom and the TMD as an additional degree of freedom. The device uses its mass to oscillate at the same frequency as the main system, and the damper, for energy dissipation. Conveniently it can be assumed that the structure responds to the dynamic actions in its fundamental mode of vibration which does not change with the addition of the damping system.

This device has existed for many years, and has been used in many structures to suppress windinduced vibrations and seismic excitation. The efficiency of the TMD has already been studied by many researchers, who have reported that this type of device effectively promotes the comfort and convenience of users by reducing the amplitude of response in displacement and acceleration in a building.

Tanaka and Mak [2] analyzed the efficiency of the use of tuned mass dampers in the energy dissipation in a wind-excited tall building. The authors performed numerical and experimental analyzes, using wind tunnel test and the CAARC standard model, noting the effectiveness of the TMD and obtaining a reduction in the dynamic response of the building in the range of 30 to 60%. They also reported that the smaller the frequency range of the excitation, the more effective the control system, provided that the TMD is tuned to the natural frequency of the structure.

Mohebbi et al. [3] used genetic algorithms to obtain the optimum design of tuned mass dampers. They employed a shear building model to minimize the structural response to earthquakes. The study concluded that increasing the mass ratio between the structure and TMD improves the device performance. Notwithstanding, for multiple tuned mass dampers (MTMD), increase the number of devices did not significantly affect the response.

Elias and Matsagar [4] studied the application of a single TMD, MTMD located at the top of the building and MTMD distributed by floors (d-MTMD), according to the first modal forms of vibration of the structure. The authors concluded that there are significant improvements in the performance of the dynamic response control with the use of MTMD than to the use of only one TMD. Conversely, the use of d-MTMD resulted in more efficient solutions, besides its needs a reduced space for installation.

The structural parameters that influence the TMD efficiency are the mass ratio, the frequency tuning rate, obtained by the device stifness, and the damping coefficient.

The application of structural vibration control systems includes the determination of the structural parameters, quantity and location of these devices. Despite the basic concept, the TMD parameters must be achieved through optimization techniques for the best performance of the device.

The optimal design parameters of tuned mass dampers subject to wind-induced vibration are determined through minimizing the displacement response at the top of the building. The optimization algorithm employed is the Search Group Algorithm (SGA), developed by Gonçalves, Lopez and Miguel [5]. Several different scenarios are taken into account: (i) a single TMD (STMD) installed at the top, (ii) multiple MTDs (MTMD) positioned at the top-floor and (iii) MTMD installed on different stories of the building.

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### 2 Mathematical Models

#### 2.1 Dynamic Analysis

The equilibrium equation governing the dynamic response of a finite element system is given by Eq. (1).

$$[M]{\ddot{x}(t)} + [C]{\dot{x}(t)} + [K]{x(t)} = {F(t)}$$
(1)

Where [M], [C] and [K] are the  $(n + N_tmd) x (n + N_tmd)$  mass, damping and stiffness matrices, respectively. The number of degrees of freedom is defined by n and the number of mass dampers tuned by  $N_ams$ .  $\{\ddot{x}(t)\}, \{\dot{x}(t)\}$  and  $\{x(t)\}$  are the acceleration, velocity and displacement vectors.  $\{F(t)\}$  represents the vector of the external excitation force. The structure will be modeled in shear building and it will be possible to add tuned mass dampers to each floor of the building as shown in Fig.1.



Figure 1. Shear Building with tuned mass dampers [6]

The mass and stiffness matrices, for this type of structure, with the inclusion of the tuned mass dampers are given by

	[]m₁	0		0	0	0	0		0	0	٦
	0	$m_2$		0	0	0	0		0	0	
	1 :	:	÷.,	:	:	-	-	:	-	-	ĺ
	0	0		m <sub>n-1</sub>	0	0	0		0	0	
м	0	0		0	$\mathbf{m}_{\mathbf{n}}$	0	0		0	0	
$\mathbf{M} =$	0	0		0	0	$m_{d_1}$	0		0	0	İ
	0	0	•••	0	0	0	$m_{d_2}$		0	0	
	1 :	÷		:	÷	÷	-	÷.,	:		
	0	0		0	0	0	0		m <sub>d<sub>N-1</sub>-1</sub>	0	
	0	0		0	0	0	0		0	m <sub>d<sub>Ntmd</sub></sub>	

	$\left[k_1 + k_2 + \sum_{i=1}^{N_{TMD}^{lafloor}} k_{d1i}\right]$	$-k_2$		0	$-k_{d11}$	-k <sub>d12</sub>		$-k_{dln}$	0	0		0		0	0		0
	-k <sub>2</sub>	$k_2+k_3+\sum_{i=1}^{N_{TMD}^{2nd \text{Floor}}}k_{d2i}$		0	0	0		0	-k <sub>d21</sub>	-k <sub>d22</sub>		$-k_{d2n}$		0	0		0
	1	1	15	1	1	1	••••	1.0	1.1	1		1	•••	0	0	1	0
	0	0		$k_n + \sum_{i=1}^{N_{TMD}^{nnd Floor}} k_{dni}$	0	0		0	0	0		0		$-k_{dn1}$	$-k_{dn2}$		-k <sub>dnn</sub>
	$-k_{d11}$	0		0	k <sub>d11</sub>	0	••••	0	0	0		0	•••	0	0		0
	$-k_{d12}$	0		0	0	<i>k</i> <sub>12</sub>	•••	0	0	0		0	•••	0	0		0
*		1	÷	÷	1	1	$\gamma_{\rm e}$	1	1.1	1	3	1	3	1.0	1	3	1
=	$-k_{d \ln}$	0		0	0	0		k <sub>d1n</sub>				0		0	0		0
	0	$-k_{d21}$		0	0	0		0	k <sub>d21</sub>			0		0	0		0
	0	$-k_{d22}$		0	0	0		0	0	<i>k</i> <sub>d22</sub>		0		0	0		0
			÷	1	1		÷		1		$\gamma_{\rm c}$	1	3	1	1	1	1
	0	$-k_{d2n}$		0	0	0		0	0	0		$k_{d2n}$		0	0		0
	1	1	÷	1	1		1		1	1	1	1	$\gamma_{\rm e}$	1	1	1	1
	0	0		$-k_{dn1}$	0	0		0	0	0				k <sub>dn1</sub>	0		0
	0	0		$-k_{dn2}$	0	0		0	0	0					k <sub>dn2</sub>		0
			÷	1	1	1	÷	1	1	1	÷	1	3	1	1	$\gamma_{\rm e}$	1
	L o	0		$-k_{dm}$	0	0		0	0	0		0		0	0		k dan

The damping matrix is given by the Rayleigh matrix (by Eq. 4)) [7], which combines the mass matrix and the stiffness matrix with two damping factors. These factors can be obtained by the matrix form presented in Eq. (5), and its calculation depends on the damping ratio of the system connected to the two main vibration frequencies of the structure.

$$[C] = a_0[M] + a_1[K]$$
(4)

$$\begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = 2 \frac{\omega_m \omega_n}{\omega_n^2 - \omega_m^2} \begin{bmatrix} \omega_n & \omega_m \\ -\frac{1}{\omega_n} & \frac{1}{\omega_m} \end{bmatrix} \begin{pmatrix} \zeta_m \\ \zeta_n \end{pmatrix}$$
(5)

Where  $\zeta_i$  is the damping ratio of the system;  $\omega_i$  main frequencies of the structure and  $a_0$  and  $a_1$  are the Rayleigh damping factors. The resolution of the dynamic equation will be done by the Newmark direct integration method.

### 2.2 Wind force simulation

Wind action is considered as a stochastic process, that is, defined through an unlimited sequence of random variables. This approach covers a portion of average speed and a portion of floating charge, resulting from the effect of the various bursts of nonconforming size and intensity.

As the numerical example of this research will be located in Brazilian territory, the premises of ABNT NBR 6123/88 [8] will be followed. The component of the overall force in the direction of the wind, called drag force is given by

$$F_a = C_a q(z) A_e \tag{6}$$

Where  $C_a$  is the force coefficient (drag), q(z) the dynamic pressure of the wind, and  $A_e$  the effective frontal area, given by the area of the orthogonal projection of the building. The drag coefficient is determined by figures 4 and 5 of ABNT NBR 6123/88 [8]. The dynamic wind pressure is calculated from the characteristic wind speed under normal conditions of pressure and temperature, by Eq (7).

$$q(z) = 0.613V_k(z)^2 \tag{7}$$

Where  $V_k(z)$  is characteristic wind velocity. The main direction in which the wind blows, the longitudinal direction is the only one that has expressive average velocities, and, therefore, will be the only orientation considered for the calculation of the force of the wind. The characteristic wind velocity is given by the sum of the static and dynamic plots, can be written as

$$V_{\chi}(t) = V + v_{\chi}(t) \tag{8}$$

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Where  $v_x(t)$  is dynamic plots. The part corresponding to the wind static force  $(\overline{V})$  will be calculated following the procedures described by ABNT NBR 6123/88 [8]. Initially, the design velocity is defined, which corresponds to the average speed over 10 minutes of time interval at 10 meters of soil height, given by

$$\bar{V}_{P} = 0.69 V_{0} S_{1} S_{3} \tag{9}$$

Where  $V_0$  is the basic wind speed, which corresponds to the speed of a gust of 3s, exceeded on average once in 50 years, 10 m above the ground, in an open field and flat.  $V_0$  must be adequate to the location of the construction, and can be obtained by the isopleths of the basic wind speed of the Brazilian wind standard.

The velocity as a function of the height of the building is given by Eq.(10), where the parameters b and p are defined by table 21 of ABNT NBR 6123/88 [8], for a time period of 600s.

$$\overline{V}(z) = b\overline{V}_{P} \left(\frac{z}{10}\right)^{p}$$
(10)

The floating velocity will be calculated by the spectral representation method. In this method it is possible through the superposition of harmonic waves conceived by Shinozuka and Jan [9] by Eq.(12), to reach the floating component of the wind.

$$\Delta \vec{V}(t) = \sqrt{2S_{w}(f_{j})\Delta f_{j}\cos(2\pi f_{t} + \phi_{j})}$$
(11)

Among the variables of the equation we have  $\phi$ , which is the phase angle. It is a random variable, with a uniform probability distribution function, ranging from 0 to  $2\pi$ . Already *f* is the frequency corresponding to a whirl; and  $\Delta f_j$  is obtained by  $f_{j+1} - f_j$ , it is the division interval of the frequency band of interest.

For the calculation of the power spectral density  $S_w$  of the longitudinal component of the floating part of the wind speed, the model proposed by Kaimal [10] will be used, given by

$$\frac{\mathrm{fS}_{\mathrm{w}}}{\mathrm{u}_{*}^{2}} = \frac{200\mathrm{n}}{\left(1+50\mathrm{n}\right)^{5/3}} \tag{12}$$

Where:

$$n = \frac{fz}{\bar{V}(z)}$$
(13)

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$$u_* = 0.4 \frac{\overrightarrow{v}^{(z_{ref})}}{\ln\left(\frac{z_{ref}}{z_0}\right)}$$
(14)

Where  $z_{ref}$  is the reference height to the ground, considered equal to 10 meters; *n* the dimensionless frequency, related to the Taylor hypothesis;  $z_0$  the roughness length and  $u_*$  the friction velocity;

After calculating the velocities, the next step is to calculate the correlation length. The vertical correlation length between two points will be calculated according to the expression given by Miguel et al [11]. The studied building will be inserted into the correlation plane and the velocity for each node of the structure will be obtained through linear interpolation.

### **3** Numerical Examples

The numerical example is a 40-story building. This structural model was proposed by Liu et al. [12], and is 40 meters in both width and length. The total height of the building is 160 meters, with 4 meters for each floor. The mass for each storey is 9.80E5 kg. The stiffness decreases linearly as the height increases, on the first floor is equal to 2.13E9 N / m and on the top floor is equal to 9.98E8 N /

m. The damping is given by the Rayleigh damping matrix, with parameters a0 and a1 equal to 0 and 0.02 respectively. The natural frequency of the building is 1.64 rad / s.

The longitudinal wind force simulation was made considering that the building will be located in the city of Porto Alegre - RS. The drag coefficient is 1.35, considering low turbulence winds.

In Fig.2 and Fig.3 are shown respectively the Kaimal spectrum for the generated wind, and the force over time for the height of 160m. The maximum response to wind-induced vibration, without the TMD, is 0.47 meters of displacement and 1.034 m /  $s^2$  of acceleration, both for the fortieth floor.



Figure 2. Kaimal Spectrum for Longitudinal Wind



Figure 3. Wind force X time for height equal 160m

The different perspectives on analysis are: (i) Scenario 1: Only one TMD at the top of the building; (ii) Scenario 2: Multiple TMDs spread horizontally at the top of the building with the same

parameters; (iii) Scenario 3: Multiple TMDs distributed horizontally on top of the building with different parameters; (Iv) Scenario 4: Multiple TMDs distributed vertically, one on each floor of the building with different parameters.

#### 3.1 Scenario 1: Only one TMD at the top of the building

In this scenario, the optimization is performed considering the inclusion of only one tuned mass damper to the highest floor of the building, that is, the fortieth floor. The definition of the top floor is due to the fact that the fundamental vibration mode of the building generates larger movements on the higher floors.

To optimize the damper parameters, its mass was fixed in relation to the total mass of the building, defined by the symbol  $\mu$  and called mass ratio. Different mass ratios for TMD were evaluated, ranging from 2 to 10%. In Fig.4 it can be observed the reason for the decreased response in displacement to the top floor of the building in relation to mass ratio of the TMD.



Figure 4. Ratio for decreased displacement on the 40<sup>th</sup> floor X TMD mass ratio

It can be noticed that the best result was obtained for the ratio of 6%. From 6% on, the increase in mass ratio decreased the effectiveness of the damper and the tuning frequency of the TMD (table 1). The tuning frequency of the damper has become irrelevant for mass ratios greater than 7%. In particular, the stiffness was no longer necessary and only the damping and mass of the devices became sufficient to decrease the displacement of the structure.

μ	K (N)	C(N-s/m)	ω (rad/s)	Max. Displacement (m)
0.02	2012481.97	823929.856	1.60	0.3507
0.03	2150720.17	1663382.37	1.35	0.3478
0.04	1691503.632	2319926.627	1.04	0.3456
0.05	1037846.106	2670134.344	0.73	0.3442
0.06	439645.909	2806815.534	0.43	0.3436

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0.07	0	2809464.307	0	0.3438
0.08	0	2627009.546	0	0.3462
0.10	0	348891.7148	0	0.3489

## **3.2** Scenario 2: Multiple TMDs spread horizontally at the top of the building with the same parameters

In this scenario, MTMDs were considered horizontally distributed at the top of the building and with identical parameters for each one. A mass ratio of 6% was used and three different cases were considered, with 2, 5 or 10 tuned mass dampers.

Although different parameters were obtained for each damper when comparing the cases of 2, 5 or 10 TMDs, as the values of the parameters of each case are summed, it is noted that all the sums have very close values, both for the stiffness and for damping (see table 2). The sum value is also close to the value obtained for the scenario 1 analysis using this same mass ratio.

The maximum displacement for the building with vibration control is identical to that found in scenario 1. Therefore, despite the division into MTMDs, the results showed a tendency to remain the same, only with the division between the number of parameters mass dampers considered for the floor.

N° of TMD	K(N) for each TMD	K(N) total	C(N-s/m) for each TMD	C(N-s/m) total	Max. Displacement
2	219984.36	439968.72	1403569.98	2807139.96	0.34361
5	88428.97	442144.88	561566.00	2807830.04	0.34362
10	44006.13	440061.35	280679.642	2806796.42	0.34361

Table 2. Results for scenario 2

# **3.3** Scenario 3: Multiple TMDs distributed horizontally on top of the building with different parameters

For the analysis of this third scenario it was also considered only dampers on the highest floor of the building. The dynamic routine for this scenario was developed enabling the optimization algorithm choose from 0 to 10 dampers, so that the search for the optimal number of dampers generate the lowest result for the displacement at the top of the building. In this scenario the stiffness and damping parameters may be different for each TMD.

The best result was the inclusion of four parallel mass dampeners, and the parameters between them differ greatly, a fact that can be observed by approximating the natural frequency of each TMD, where the values ranged from 0.37 to 2.62 (see table 3). In addition to this, the results show one of the MTMDs advantages: It is possible to embrace different tuning frequencies increasing the tune range with the building.

TMD	K (N)	C (N-s/m)	ω (rad/s)				
1	81226.53	1108750.11	0.37				
2	4042256.55	185323.10	2.62				
3	91922.96	3358258.60	0.39				
4	1000310.33	127370.57	1.30				

Table 3. Results for scenario 3

The maximum displacement was 0.3343 m, which is more beneficial for the building than the response obtained in scenario 2.

# **3.4** Scenario 4: Multiple TMDs distributed vertically, one on each floor of the building with different parameters

Scenario 4 analyzed multiple tuned mass dampers. It was possible to include a single damper per floor of the building. Besides that, the dampers could have different parameters from each other. The analysis, made for the 6% mass ratio, resulted in 20 TMDs spread over different floors of the building. The parameters found for each TMD and the location floor are shown in the table 4. The maximum displacement obtained for was 0.3595, the worst result among all scenarios for mass ratio equal to 6%.

TMD floor	K (N)	C (N-s/m)	ω (rad/s)
2	164362.49	4847433.19	1.18
6	5841340.14	2335625.63	7.05
8	2722166.90	5202838.84	4.81
10	6882842.72	427211.19	7.65
11	6852385.60	378648.55	7.63
12	9902147.67	6209663.71	9.18
13	8647240.52	1363032.61	8.58
14	9994462.48	7313555.55	9.22
15	368727.44	100.00	1.77
17	26854.18	3653075.34	0.48
19	4614788.15	5325631.43	6.26
20	100.00	2502956.86	0.03
21	48230.59	59799.16	0.64
26	5473105.90	7927132.33	6.82
28	8915558.73	17269.14	8.71
35	4333993.47	2132540.27	6.07
36	258455.35	15250.14	1.48
37	7145341.15	3494860.78	7.79
38	87391.70	6375325.21	0.86
40	6559911.89	5662521.37	7.47

Table 4.	Results	for	scenario	4
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### **3.5** Comparison between scenarios

A possible analysis is the comparison between the results obtained for each scenario. As mentioned all the maximum displacement values for each scenario were very similar (see Fig.5). Obviously the best result was for the third scenario, which considered multiple TMDs, only on the top floor of the structure.



Figure 5. Maximum displacements for each building floor when  $\mu = 6\%$ 

The same analysis of Fig.4 was performed for the 2% mass ratio (fig.6). Nonetheless, with the change in the mass ratio, the results maintained the trend regardless of the scenario, without any prominence.



Figure 6. Maximum displacements for each building floor when  $\mu = 2\%$ 

Another important factor to consider is the acceleration generated on each floor of the building. Using a mass ratio of 6%, approximately 30% decrease in the acceleration of the structure without vibration control was conceivable. It should be noted, as for the case of the displacement for this same mass ratio, the results were close, but with positive highlight for the third scenario, which produced the smallest acceleration along all floors (Fig.7).



Figure 7. Maximum acceleration for each building floor when  $\mu = 6\%$ 

Changing to a lower mass ratio ( $\mu = 2\%$ ), the response due to the acceleration was very similar to that obtained for  $\mu = 6\%$ , with the best result for the third scenario, which decreased by 28.5% the maximum acceleration at the top (Fig.8). The negative highlight for this case was the fourth scenario, which despite providing a decrease in acceleration, presented the highest responses for each floor.



Figure 8. Maximum acceleration for each building floor when  $\mu = 2\%$ 

### 4 Conclusions

In this research was presented tuned mass dampers as structural vibration control device subject to dynamic wind action. The determination of the device parameters was made through the use of a structural optimization algorithm, the Search Group Algorithm, always aiming at the smallest displacement of the structure.

Four different analysis scenarios were proposed for a 40-story, 160-meter high building. It was decided to lease the building to the city of Porto Alegre - RS, due to the basic high wind speed in the

region and the tendency of higher buildings in the capital of Rio Grande do Sul. The suggested scenarios investigated the effect of the inclusion of a single TMD, multiple TMDs located on the top floor of the building and multiple TMDs distributed over different floors of the building.

Although small, there was a difference in results for different mass ratios. The best results were for the mass ratio of 6%. The use of only one TMD at the top, for equal mass ratios or higher than 7%, dispensed the stiffness parameter. Differing from the other results, in this case only the mass and the damping coefficient was required to obtain the smallest possible displacement.

Comparing the scenarios showed that it was more beneficial for the building to include multiple TMDs positioned on the 40th floor with different stiffness and damping parameters. The results in all scenarios were assertive in decreasing the displacement caused by wind-induced vibrations, although the horizontal distribution of the TMDs across the different floors of the building was the worst result obtained.

It was observed that different vibration control solutions through the tuned mass damper are possible, and may even be better than the classic single TMD solution at the top. Whereas all results were positive, the choice of the best scenario will depend on external factors such as design, space required and installation of the tuned mass dampers.

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