The establishment of the time interval between inspections for a cold standby system with component repair

Angélica Alebrant Mendes[a]*, José Luis Duarte Ribeiro[b]

[a]Universidade Federal de Santa Maria, Santa Maria, RS, Brazil

[b]Universidade Federal do Rio Grande do Sul, Porto Alegre, RS, Brazil

*angelica.mendes@ufsm.br

Abstract

The time interval between inspections of cold standby systems is a crucial decision to ensure the appropriated system reliability and the lowest costs possible. This paper presents a model developed to establish the optimal time interval between inspections for a two-unit cold standby system with component repair and subject to periodic inspection, considering reliability and costs. A Markov chain is used to define possible states, their transition probabilities and the mean time to system failure, as a function of the time interval between inspections. Given the mean time to system failure, the steady state availability is determined. Finally, the costs related to the system maintenance are established and a cost function is developed and optimized for the time interval between inspections. Numerical examples are presented and results for different system parameters are compared. Besides optimizing the time interval between inspections, the analyses also reveal the effect of repair time on system availability and mean time to system failure.

Keywords

Cold standby. Periodic inspections. Availability. Markov process. Cost optimization.

1. Introduction

A cold standby system is a system with two or more components where *m* out of *n* components are operative and the other *n* - *m* components are in standby position and can start to work once one or more operative components fail. Cold standby systems have been widely applied in systems where operational safety is a key concern such as aircraft controls, nuclear power plants, and telecommunication networks (Billinton & Allan, 1984; Levitin et al., 1996). These configurations are used to increase system reliability. Once the operating component has failed, the component in the standby position assumes operation immediately, thus avoiding system failure. Usually, when a failure occurs in cold standby systems, the impacts on manufacturing costs, environmental safety, or even on human welfare, can be disastrous (Bloom, 2006).

However, to ensure the inherent high level of reliability, maintenance needs to be performed on cold standby systems. Periodic inspections are largely used as a preventive maintenance policy where the condition of the standby component(s) is/are checked periodically (Wang & Zhang, 2014). Every time that a failure is found during the inspection, a repair begins. Considering that a failure occurs, cold standby systems subject to periodic inspection are unprotected until the next inspection and during the time that the component is under repair (Alebrant Mendes & Ribeiro, 2016). Frequent inspections increase the availability of the system, but require higher costs of preventive maintenance. On the other hand, longer periods between inspections decrease inspection total costs, but can increase the costs of corrective maintenance (component and system repair, safety accidents) and downtime since there are longer periods where the system can be unavailable (Alebrant Mendes et al., 2014).

Consequently, the establishment of the optimal time interval between inspections and the understanding of the effect of the repair time in the system reliability are very important to ensure satisfactory system availability along with the lowest possible cost.

Recently, many researches have been studying the maintenance optimization for cold standby systems. There are many approaches related to maintenance procedures and the methods used to optimize systems. Considering only corrective maintenance at the component level, Ereau et al. (1997), Bloch-Mercier (2001), Gupta et al. (1994), Zhang & Wang (2007), and Jia & Wu (2009) applied the a Markov process aligned with Petri nets and Monte Carlo simulation, renewal process, and renewal process and cost function optimization, respectively. Hu & Xie (2008), Hsieh & Chiu (2002), Zhang & Wang (2011), and Smith & Dekker (1997) utilized component replacement for preventive maintenance and optimized the maintenance policy using Markov process and genetic algorithm, multistate deteriorating system, renewal reward theorem, and a derivation scheme for approximation.

Osaki (1972) developed a model to determine the appropriate maintenance policy of a cold-standby system with repair. In this system, preventive maintenance was applied to the operative unit to maintain the reliability level. Once the operative component has failed, the standby component becomes active and the failed component is sent to repair immediately. The author used Markov renewal process and Laplace-Stieltjes transforms to determine the amount of time until the first system-down as a function of the time between the preventive maintenance. Zhong & Jin (2014) developed a similar model for an equivalent system, with the exception that the component's failure probability fits a Weibull distribution instead of an exponential distribution. The authors used semi-Markov and regenerative point technique to determine the state transition probability while also using Laplace transforms to solve the Markov renewal equations. The optimal preventive maintenance cycle was defined by maximizing the mean time from the initial state until system failure.

Most of the studies analyze cold standby systems subject to continuous monitoring assuming that repair starts immediately after the failure occurrence. These papers aim to determine mainly the mean time to system failure and the system steady state availability. Just a few studies consider systems subject to periodic inspections where components and system states are verified only at inspections. However, this is a situation observed in many industrial scenarios, especially when the analysis of some components requires local inspection and access is difficult. In these cases, the components are only analyzed from time-to-time (Alebrant Mendes et al., 2017).

Only a few studies consider the application of periodic inspection in cold standby redundant systems. Mokaddis et al. (1993) used Markov renewal process to analyze a system with a deteriorating cold standby unit where failed components are found only after an inspection. Mokaddis et al. (1997) applied semi-Markov process to analyze a cold standby system where inspection is required to decide whether failed components need minor repair or major repair. These studies are limited to determine the mean time to system failure and the pointwise and steady state availability, do not considering the costs involved and do not optimizing the time interval between inspections.

In a recent study, Alebrant Mendes & Ribeiro (2016) established the system reliability, the expected time-to-failure, and the appropriate time interval between inspections for a cold standby system with periodic inspection using an analysis of expected exposure time for active and redundant components. Although the method developed provides a good approximation for practical applications, the authors did consider neither the actual maintenance costs nor the time-to-repair.

Courtois & Delsarte (2006) established an optimal time schedule of periodic inspections which maximized the mean time between system failures by applying suitable transformations and variable identification to obtain an analytic closed form expression. It is worth noting that these authors also did not take component repair into consideration. Kančev & Cepin (2011) evaluated risk and cost using an age-dependent unavailability modelling of test and maintenance for standby components. These authors have used Fault Tree Analysis to risk assessment and cost optimization to quantify the impact of sequential versus staggered testing and aging on system unavailability.

Even though many papers have been published about reliability analysis of cold standby systems, there is still a gap in the literature regarding to the optimization of the time interval between inspections of these systems. Despite the wide industrial application of cold standby systems subject to periodic inspections, the studies that consider this configuration are limited to determine the mean time to system failure and the pointwise and steady state availability, do not considering the costs involved and do not optimizing the time interval between inspections.

In view of the above discussion, the main objective of this paper is to optimize the time interval between inspections for a cold standby system with component repair the using discrete-time Markov process. The maintenance process of a cold standby system is a stochastic process, since it has n components and each one can be up or down (operating or failed state) at any time. Each combination of component states could represent a state in the state-space of a Markov Chain. The probabilities p_{ij} of moving from state i to state j are known and not depend on time n or the previous states (Ross, 2007). This scenario justifies the application of a Markov Chain.

This paper is organized as follows: in Section 2, the methodology including basic assumptions for system modeling is described and all possible system states are determined. In Section 3, Results and Discussion, the transition probabilities between states are calculated, the mean time to system failure and the system steady state availability are established, numerical examples are presented, and a sensitivity analysis of the effect of failure time and repair time on mean time to system failure is provided. Finally, Section 4 summarizes the key aspects of the article and includes concluding remarks.

2. Methodology

In this study, a discrete-time Markov process is used to define possible states, their transition probabilities and the mean time to system failure. Given the mean time to system failure, the steady state availability is determined. Finally, the costs related to the system maintenance are established and cost function is developed and minimized to optimize the time interval between inspections. Also, numerical examples and a sensitivity analysis comparing the results with different system parameters are presented and analyzed.

Notations used in this paper are presented in Chart 1.

Chart 1. Notations.

Notation	
U_i	component i, $i = 1, 2$
$F(t)$	cumulative distribution function (CDF) of component lifetime
$G(t)$	cumulative distribution function (CDF) of repair time for failed component
X	random variable denoting component lifetime
R	random variable denoting repair time
λ	parameter for the exponential distribution of $F(t)$, failure rate
α	parameter for the exponential distribution of $G(t)$, repair rate
τ	time interval between inspections
p_{ij}	transition probability from state S_i to S_j
MTSF	Mean time to system failure
MTTF	Mean time to component failure
MTTR	Mean time to component repair
A	Steady state availability
C_i	Cost of inspection per inspection
C_r	Cost of repair per repair
C_R	Cost of system repair
TC	Total cost per cycle
N_{insp}	Number of inspections
N_{rep}	Number of repairs
P_n	matrix of the state transitions probabilities
Q	transient part of matrix P_n
I	Identity matrix
N	matrix of the expected number of times that the process is in the transient state S_j if it is started in the transient state S_i
n_{ij}	ij-entry of the matrix N

2.1. System description and assumptions

The system studied in this paper is comprised of two identical components where one is operating and the other is in standby position. Since both components are identical, they have the same characteristics and the same probability distributions for failure time and repair time. In addition, one repairman is available and can repair only one component at a time. Also, inspections to check the status of the components are performed periodically and are independent from the availability of the repairman. The following assumptions were made in conducting this study:

Assumption 1. Initially, the two components are new. Component U_1 starts in an operating state while the component U_2 remains in an inoperative state. Once component U_1 fails, component U_2 assumes operation instantaneously and automatically until its failure. After the next inspection, the failed component U_1 is identified and sent to repair.

Assumption 2. The time to switch from component U_1 to component U_2 and vice-versa is negligible. The switch is perfect and does not affect the system reliability. These simplifications are acceptable in the practice for many systems that have switches much more reliable than their components. Also, it is assumed that components in cold standby position do not deteriorate or fail. Inspections are carried out just to check if the operating component has failed, and, consequently, the system is operating unprotected. Tests are not performed in the cold standby component.

Assumption 3. Periodic inspections are performed to check whether components are in the operating or failure state. During the inspections, no preventive maintenance is fulfilled and repair activities start only after a failed component is found. The inspections are performed in constant time intervals, even when a component has failed or is under repair. Since the state of the system and its components are verified only during the inspections, the system works with lower reliability during the time between the failure of the first component and its repair after the next inspection.

Assumption 4. Times to failure and to repair are a random variable that follows an exponential probability distribution. Usually, repair times present high variability, combining many relatively short times to repair (associated with situations where the defect is readily identified and a part is replaced) with a few relatively long times to repair (associated with difficult to find and repair defects). This scenario can be adequately modeled by the exponential distribution due to its shape and coefficient of variation. Exponential probability distributions of times-to-failure were used to allow for the use of the Markov process. It is known that the exponential distribution is appropriate to model times-to-failure of many components such as electronic and microelectronic components (Bloom, 2006). However, if it is not the case, other approaches, such as Monte Carlo simulation, should be utilized (Ross, 2012).

Assumption 5. The repair of components is assumed to be perfect. In other words, after the repair, the component is "as good as new" and goes to the standby position. When a component is under repair, the system operates at a lower level of reliability because the failure of one component leads to system failure.

Considering these assumptions, four system states can be defined:

- S_0: a component works, while the other is in the standby position;

- S_1: a component works, while the other has already failed;

- S_2: a component works, while the other is under repair;

- S_3: both components have failed or one component has failed while the other is under repair. This state represents the system failure.

The system state transitions are illustrated in (Figure 1). The system starts to work and enters into S_0 at $t = 0$. Any or many periodic inspections can be performed while the system is in this state. Then, one component fails, and the standby component starts to work while the system goes into state S_1. If the operating component
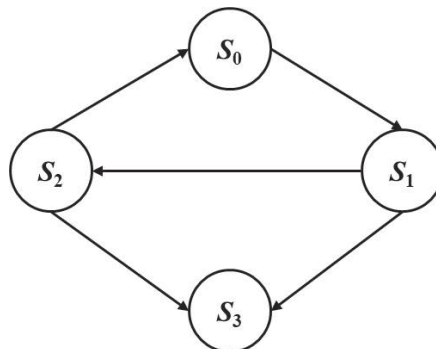

Figure 1. System state transitions.

fails before the inspection, the system moves into state S_3 which means that both components are in a state of failure. However, if the component survives until the next inspection, the component repair begins and the system goes into state S_2. While the system is in this state, any or many inspections can be carried out. If the working component survives until the repair is completed, the repaired component goes to the standby position and the system switches into state S_0, starting the whole process again. However, if the component fails before the repair is finished, the system fails and moves into state S_3. Once the system hits state S_3, the process stops. This state is called absorbing state (Ross, 2007). It is known that after the failure of the whole system, it can be repaired with a higher cost and restart the operation. However, for the initial purpose of this study, the state S_3 is considered the end of a cycle and, consequently, an absorbing state.

3. Results and discussion

In this section, the problem above is solved. To solve this problem the transition probabilities between states are calculated, the mean time to system failure and the system steady state availability are established, numerical examples are presented, and a sensitivity analysis of the effect of failure time and repair time on mean time to system failure is provided.

3.1. Problem formulation

Considering the probability of component failure equals to $F(t)=1-e^{(-\lambda t)}$ and the probability of repair equals to $G(t)=1-e^{-\alpha t}$, the transition probabilities of the system can be analyzed.

Initially, the system is in state S_0 with one component working and the other in the standby position. The system jumps to state S_1 when the working component fails. The standby component starts to work immediately and does not interfere in the transition probability. Thus, the transition probability from state S_0 to state S_1 is based on the probability of failure of the working component and can be expressed as:

$$p_{01} = \int_0^\infty \lambda e^{-\lambda t} = 1 \tag{1}$$

Given that the system is in state S_1, where one component has failed and the other is working, the system moves to state S_2 if the working component does not fail by the time of the next inspection. After the inspection, the repair starts and the system enters in state S_2. Therefore, the probability of the operating component lasting more than the time difference between the interval between inspections and the time-to-failure of the component that was previously working in that interval represents the transition probability from state S_1 to state S_2.

Since the time-to-failure of the first and, consequently, the second component are unknown, an approximation for the worse scenario is when the first component fails in the beginning of the time interval between inspections. In this case, it needs to work until the next inspection and the transition probability from state S_1 to state S_2 is represented by the probability of the operating component surviving the time interval between inspections (Equation 2).

$$p_{12} = P\{X > \tau\} = 1 - P\{X \le \tau\} = 1 - \int_0^\tau \lambda e^{-\lambda u} du = e^{-\lambda \tau} \tag{2}$$

If the system is in state S_1 and the working component fails before the next inspection, the repair does not start. Since both components have failed, the system fails. The transition probability of the system from state S_1 to state S_3 is represented by the probability of the operating component failing during the time difference between the inspection interval and the time-to-failure of the component that was previously working in that interval.

Since the time-to-failure of the first and, consequently, the second component are unknown, an approximation for the worse scenario is when the first component fails in the beginning of the time interval between inspections. In this case, the probability of jumping from state S_1 to state S_3 is represented by the probability of the operating component failing during the time interval between inspections (Equation 3).

$$p_{13} = P\{X \le \tau\} = \int_0^\tau \lambda e^{-\lambda u} du = 1 - e^{-\lambda \tau} \tag{3}$$

Given that the system is in state S_2, where one component is working and the other is under repair, the system switches to state S_0 when the repair is concluded and the operative component remains working. The repaired component takes place as the standby unit in S_0. Consequently, the probability of the system jumping from state S_2 to state S_0 is the probability of the component repair being concluded before the failure of the operating component.

Since the time-to-failure of the first and the second components are unknown, the remaining time-to-failure of the second component after the inspection is also unknown. The worse scenario is when the inspection is performed right before the failure of the operating component and the repair needs to be executed in a short period of time. On the other hand, the best scenario is when the inspection is performed right after the failure of the operating component and the repair can be executed in a time approximately equals to the time-to-failure of the component. Equation 4 represents an approximation of this probability, considering the best scenario.

$$p_{20} = P\{R \le t, R < X\} = \int_0^\infty e^{-\lambda t} \alpha e^{-\alpha t} du = \frac{\alpha}{\alpha + \lambda} \tag{4}$$

However, if the working component fails before the repair is concluded, the system also fails because there is no component available to continue the system's operation. This probability can be represented by the probability of the repair being longer than the time-to-failure of the working component. Following the same explanation given in the previous paragraph, the system transition probability from state S_2 to state S_3 can be approximated by Equation 5.

$$p_{23} = P\{X \le t, R > X\} = \int_0^\infty e^{-\alpha t} \lambda e^{-\lambda t} du = \frac{\lambda}{\alpha + \lambda} \tag{5}$$

It can be seen that the defined probabilities are in accordance with the Markov fundamental property that says $\sum_{j=0}^{N} p_{ij} = 1$. Matrix P shows the transition probabilities of the system.

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & e^{-\lambda \tau} & 1 - e^{-\lambda \tau} \\ \dfrac{\alpha}{\alpha + \lambda} & 0 & 0 & \dfrac{\lambda}{\alpha + \lambda} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3.2. Mean time to system failure and steady state availability

Equations 1-5 show that the time interval between inspections and the time to component repair affect the system reliability and, consequently, the system's time-to-failure. The mean time to system failure can be determined using a property of discrete-time Markov chains. This property says that the expected number of times that the process is in the transient state p_j, if it is started in the transient state p_i, is represented by the matrix N (Equation 6), where Q is the transient part of matrix P, and I is an identity matrix (Ross, 2007):

$$N = (I - Q)^{-1} \tag{6}$$

The sum of each line in N reveals the expected number of steps before absorption given that the chain began in the ith non-absorbing state. The mean time to system failure depends on how many times the system goes from state 0 to the other states and how long it takes to switch from one state to another. Since transition times are related to the components' failure time, we can approximate the mean time to system failure calculating Equation 7.

$$MTSF = \left(\sum_{j=0}^{2} n_{0j} \right) \frac{1}{\lambda} \tag{7}$$

According to the reliability theory, the steady state availability of a system under renewals can be determined by Equation 8 (Elsayed, 2012):

$$A = \frac{MTSF}{MTSF + MTTR} \tag{8}$$

The expected mean time to system failure is established in Equation 7 assuming state S_3 as an absorbing state or the end of the system cycle. Considering that after the system failure, it can restart the operation and start a new cycle after the repair of the first failed component, Equation 8 can be used to determine the system availability.

3.2.1. Numerical example

A numerical example is presented to validate the model developed in this study. A sensitivity analysis is conducted to verify the effect of parameters λ and α over the mean time to system failure.

This system is based on real systems installed in a petrochemical company that operates in southern Brazil. This is a large company comprising several operational systems and some of them are in a difficult-to-inspect position and are protected by redundancy. In order to protect confidential information, numbers concerning costs, mean time between failures (MTBF), and mean time-to-failure (MTTF) were modified, but the average proportion [between MTBF/mean time-to-repair (MTTR) and among costs] observed in the petrochemical company was preserved.

Letting parameter $\alpha = 1$ and using the Equations 1 to 7, it is possible to determine the mean time to system failure to any value of τ. Figure 2 presents a sensitivity analysis of parameter λ over mean time to system failure as a function of the time interval between inspection. It shows that λ, as expected, has an important effect in the mean time to system failure, especially for values smaller than 10% of α value. Also, the mean time to system failure decreases as the time interval between inspections increase and in the long time tends to the system's mean time-to-failure without inspections and repair (MTTF of U_1 + MTTF of U_2).
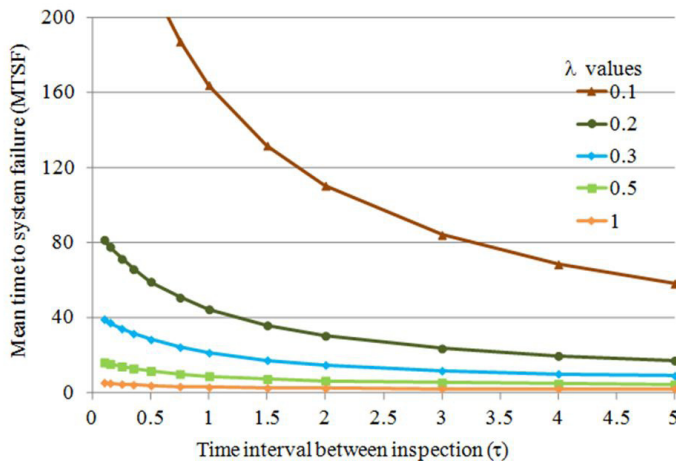

Figure 2. Mean time to system failure (MTSF) as a function of the time interval between inspections (τ) and failure rate (λ).

In order to verify the effect of the time-to-repair on mean time to system failure, a sensitivity analysis was conducted. First, the value of the parameter λ (failure rate) was set to 1, as a reference value. Then, the mean time to system failure was calculated using Equations 1 to 7 and different values for the parameter α, which represents the repair rate. Finally, curves for α values equal to 1, 2, 10, and 100 were plotted in (Figure 3).

Figure 3 confirms that as the repair rate (α) increases, meaning faster repair, so does the mean time to system failure. This stems from the fact that higher repair rates imply in shorter times to repair and, consequently, the system operates in an unprotected condition (works with only one component) for a shorter period of time.
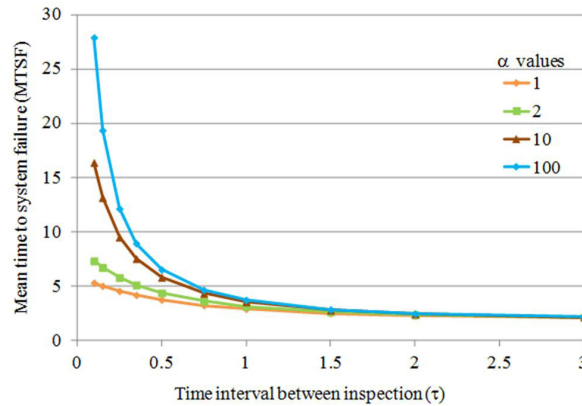
Figure 3. Mean time to system failure (MTSF) as a function of the time interval between inspections (τ) and repair rate (α).

In that way, times interval between inspections considerably shorter than the component's MTTF are recommended (e.g., τ = 0.5 or less) to ensure a lower probability of system failure. Similar conclusions can be seen in Figure 4 that shows the system steady state availability (A) as a function of the time interval between inspections (τ) and the repair rate (α).
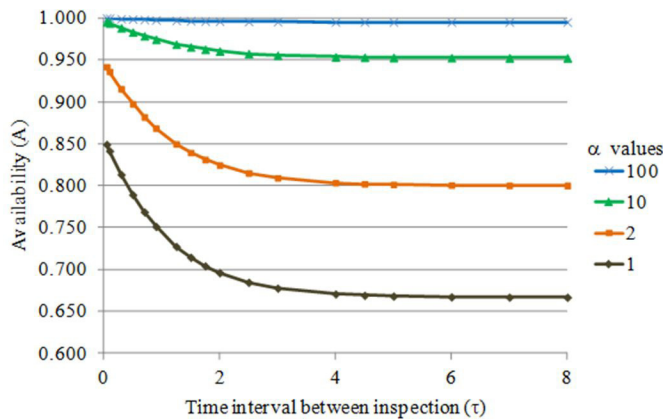

Figure 4. Steady state availability (A) as a function of the time interval between inspections (τ) and repair rate (α).

The model developed in this paper enables the analysis of the effects of time-to-repair, time-to-failure, and the time interval between inspections on the system availability. By using this model, it is possible to determine the appropriate time interval between inspections given the system mean time-to-failure or availability target. For example, when λ = 1 and α = 100 (which represents mean time-to-repair equals to 0.01 of components' MTTF). To achieve a system mean time-to-failure target of 10 or an availability of 0.9990, the time interval between inspections has to be approximately 0.3 (which represents the time interval between inspections equals to 0.3 of components' MTTF).

3.3. Cost function optimization

Optimizing the measure of performance per unit of time is equivalent to optimize the measure of performance over a long period. Thus, the cost model in this paper is based in the establishment of the maintenance total cost per cycle. In this application, a cycle is the period of time elapsed from the beginning of the system operation until its failure (Equation 9).

$$TC = \frac{E[cost\,in\,cycle]}{E[length\,of\,the\,cycle]} \qquad (9)$$

Given that when the entire system fails, the system repair begins immediately, three elements were considered to determine the costs associated with a cold standby system subject to periodic inspections: *i*) Cost of periodic inspection (C_i); *ii*) Cost of component repair (C_r); and *iii*) Cost of system repair (C_R).

The cost of periodic inspections comprises costs of manpower, tools and materials required to perform the inspection, even if there are no components in the failure state. The cost of component repair includes costs of manpower, tools, replacement of parts, and materials utilized to repair a failed component. The cost of system repair is the costs incurred to reactivate the system to its completely operational condition after a system failure. The cost of downtime referring to the production losses during the time that the system is down, as well as a loss of sales opportunity and monetary fees for delivery delay, are also included in the cost of system repair.

The expected cost in the cycle as a function of the time interval between inspections (τ) can be determined by the sum of the cost of each inspection multiplied by the number of inspections performed in the cycle, plus the cost of each component repair multiplied by the number of repairs carried out in the cycle, plus the cost of the system repair. The expected length of the cycle corresponds to the sum of the mean time to system failure and the mean time to system repair. Since the system can restart the operation after the repair of the first component, the mean repair time of only one component was considered.

$$TC = \frac{C_i N_{insp} + C_r N_{rep} + C_R}{MTSF + MTTR} \tag{10}$$

The number of inspections performed in the cycle (N_{insp}) is obtained dividing the mean time to system failure (MTSF) calculated using Equation 7 by the time interval between inspections (τ) as Equation 11. Usually, the result is not an integer and for these cases the value was approximated to the closest lower integer, which indicates the number of inspections that was concluded until the system failure.

$$N_{insp} = \frac{MTSF}{\tau} \tag{11}$$

In order to determine the number of component repairs in the cycle, Markov chain properties are used again considering that the state of system failure is an absorbing state. Using Equation 6 and considering state *j* as the state of component repair, the expected number of component repairs is given by n_{ij}.

The total cost in Equation 10 can be minimized and the optimal time interval between inspections can be found by setting values for the cost of inspection, component repair, and system repair and making the time interval between inspections the only variable in the equation. The minimum of the total cost curve is obtained by numerical search technique.

This problem has a complex model and a complex cost objective function, but the optimization itself is only for one decision variable, i.e., the inspection interval. Therefore, a simple one-dimensional numerical search is sufficient.

Giving that the optimization problem involves the minimization of Equation 10 (objective function) and that all the other parameters, excepted the inspection interval, are predetermined by the specific scenario, the minimum of Equation 10 (total cost) can be obtained by numerical search. The numerical search is an algorithmic procedure that allows for iterating to points that maximize or minimize an objective function

3.3.1. Numerical example

A numerical example of the optimization of the time interval between inspections is presented next. The cost parameters utilized are related accordingly real scenarios where the cost of inspection is smaller than the cost of component repair, which is much smaller than the cost of system repair. Table 1 shows the effect of the time interval between inspections (τ) over the total cost per cycle (TC), the steady state availability (A), the mean time to system failure (MTSF), the expected number of inspection per cycle (N_{insp}), and the expected number of component repairs per cycle (N_r). Figure 5 shows the total cost per cycle and the optimal time interval between inspections in months along with the system availability as a function of τ. The parameters utilized are:

Table 1. The time interval between inspections (τ), the total cost per cycle (TC), the steady state availability (A), the mean time to system failure (MTSF), the expected number of inspection per cycle (Ninsp), and the expected number of component repairs per cycle (Nr) for λ=1, α=10, Ci=$10.00, Cr=$50.00, and CR=$500.00.

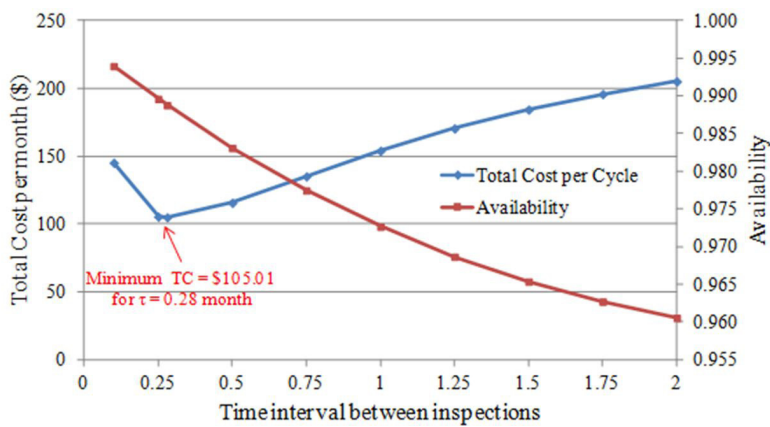| τ	TC	A	MTTF	Ni	Nr
0.1	145.23	0.994	16.373	163	5.10
0.25	105.45	0.990	9.516	38	2.67
0.28	105.01	0.989	8.807	31	2.41
0.5	115.70	0.983	5.810	11	1.35
0.75	135.16	0.977	4.333	5	0.83
1.0	153.98	0.973	3.558	3	0.55
1.25	170.47	0.969	3.092	2	0.39
1.5	184.36	0.965	2.789	1	0.28
1.75	195.80	0.963	2.582	1	0.21
2.0	205.09	0.961	2.435	1	0.15

Figure 5. The total cost per month (TC) and steady state availability (A)as a function of the time interval between inspections (τ).

a) Equal parameters of exponential distribution for times-to-failure for each component: failure rate $\lambda = 1$ (this parameter represents MTTF of 1 month).

b) Equal parameters of exponential distribution for times-to-repair: repair rate $\alpha = 10$ (this parameter represents MTTR of 0.1month).

c) $C_i = \$10.00$ and $C_r = \$50.00$: the cost of component repair is higher than the cost of inspection.

d) $C_R = \$500.00$: the cost of system repair is higher than the other costs. It means that unavailability and system failure incur higher costs than those associated to inspection and repair.

Figure 5 shows that the minimum total cost per cycle is $105.01 for a time interval between inspections of 0.28 month. Its respective availability is 0.986. Higher levels of availability can be obtained decreasing the time interval between inspections and, consequently, increasing the total cost or improving the parameters of failure rate and/or repair rate of the system's components.

3.2.2. Sensitivity analysis

A sensitivity analysis was conducted to analyze the effect of different parameters C_i and C_r in the total cost and in the availability of the system studied. Figure 6a and Figure 6b present the effect of C_i and C_r, respectively, on the total cost and the optimal time interval between inspections along with the system availability.

Accordingly to Figure 6a, higher costs of inspection (C_i) cause higher minimum TCs and longer optimal time interval between inspections. Figure 6b demonstrates that costs of repair (C_r) have a small effect on TC and that the optimal time interval between inspections remains the same. Since the system availability is independent of C_i and C_r, it has the same behavior over τ in both Figures 6a and 6b.
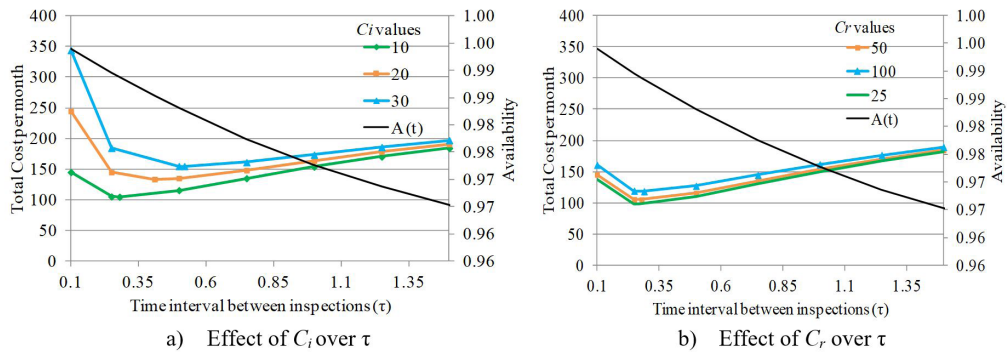
a)   Effect of C_i over τ

b)   Effect of C_r over τ

Figure 6. Sensitivity analysis of the effects of C_i and C_r over the total cost per month (*TC*), as a function of the time interval between inspections (τ). (a) Effect of C_i on τ; (b) Effect of C_r on τ.

4. Conclusion

This paper presented a model to determine the optimal time interval between inspections for a two-unit cold standby system with component repair and subject to periodic inspections using the discrete-time Markov process.

Using the method and the equations developed and tested in this paper, it is possible to optimize the time interval between periodical inspections considering its probability of failure and the costs incurred to perform the inspection and the repairs, if the system fails. A shorter time interval between inspections could avoid failures, but represent a higher cost of preventive maintenance. On the other hand, longer time interval between inspections increases the probability of failures and, consequently, the costs of corrective maintenance. Also, a balance between both costs is needed to reduce operational costs without compromise safety.

Besides optimizing the time interval between inspections, the analyses also reveal the effect of repair time on system availability and mean time to system failure, helping to show when improvements are needed in maintenance tasks.

For future research directions, cold standby systems with more than two components should be analyzed. Also, a more encompassing option is modelling a system comprised of components having probabilities of failure that follow a Weibull distribution, since Weibull distributions can provide better characterization for components with deterioration.

Acknowledgements

References

Alebrant Mendes, A., & Ribeiro, J. L. D. (2016). An accessible way to establish reliability and expected time-to-failure for cold standby redundant systems subject to periodic inspections. *Quality and Reliability Engineering International*, *32*(5), 1663-1675. http://dx.doi.org/10.1002/qre.1898.

Alebrant Mendes, A., Coit, D. W., & Ribeiro, J. L. D. (2014). Establishment of the optimal time interval between periodic inspections for redundant systems. *Reliability Engineering & System Safety*, *131*, 148-165. http://dx.doi.org/10.1016/j.ress.2014.06.021.

Alebrant Mendes, A., Ribeiro, J. L. D., & Coit, D. (2017). Optimal time interval between periodic inspections for a two-component cold standby multistate system. *ieee Transactions on Reliability*, *66*(2), 559-574. http://dx.doi.org/10.1109/TR.2017.2689501.

Billinton, R., & Allan, R. (1984). *Reliability of power systems*. London: Pitman. http://dx.doi.org/10.1007/978-1-4615-7731-7.

Bloch-Mercier, S. (2001). Optimal restarting distribution after repair for a Markov deteriorating system. *Reliability Engineering & System Safety*, *74*(2), 181-191. http://dx.doi.org/10.1016/S0951-8320(01)00076-X.

Bloom, N. B. (2006). *Reliability Centered Maintenance: implementation made simple*. New York: McGraw-Hill. 291 p.

Courtois, P.-J., & Delsarte, P. (2006). On the optimal scheduling of periodic tests and maintenance for reliable redundant components. *Reliability Engineering & System Safety*, *91*(1), 66-72. http://dx.doi.org/10.1016/j.ress.2004.11.013.

Elsayed, A. E. (2012). *Reliability Engineering*. 2nd ed. Hoboken: John Wiley & Sons Inc. 850 p.

Ereau, J. F., Saleman, M., Valette, R., & Demmou, H. (1997). Petri nets for the evaluation of redundant systems. *Reliability Engineering & System Safety*, *55*(2), 95-104. http://dx.doi.org/10.1016/S0951-8320(96)00039-7.

Gupta, R., Goel, R., & Chaudhary, A. (1994). Analysis of a two-unit standby system with fixed allowed down time and truncated exponential lifetime distributions. *Reliability Engineering & System Safety, 44*(2), 119-124. http://dx.doi.org/10.1016/0951-8320(94)90002-7.

Hsieh, C. C., & Chiu, K. C. (2002). Optimal maintenance policy in a multistate deteriorating standby system. *European Journal of Operational Research, 141*(3), 689-698. http://dx.doi.org/10.1016/S0377-2217(01)00263-6.

Hu, R., & Xie, J. (2008). Optimal Maintenance Policies for a Cold Standby Redundant System with Two Units. In *Proceedings of the 2nd IEEE International Conference on Service Operations and Logistics, and Informatics (IEEE/SOLI)* (pp. 1774-1778).

Jia, J., & Wu, S. (2009). Optimizing replacement policy for a cold-standby system with waiting repair times. *Applied Mathematics and Computation, 214*(1), 133-141. http://dx.doi.org/10.1016/j.amc.2009.03.064.

Kančev, D., & Cepin, M. (2011). Evaluation of risk and cost using an age-dependent unavailability modelling of test and maintenance for standby components. *Journal of Loss Prevention in the Process Industries, 24*(2), 146-155. http://dx.doi.org/10.1016/j.jlp.2010.12.003.

Levitin, G., Lisnianski, A., Ben-Haim, H., & Elmakis, D. (1996). Power system structure optimization subject to reliability constraints. *Electric Power Systems Research, 39*(2), 145-152. http://dx.doi.org/10.1016/S0378-7796(96)01106-6.

Mokaddis, G. S., Labib, S. W., & El-Said, K. H. M. (1993). Availability analysis of a two-dissimilar-unit deteriorating standby system with inspection. *Microelectronics and Reliability, 33*(14), 2119-2141. http://dx.doi.org/10.1016/0026-2714(93)90005-J.

Mokaddis, G. S., Tawfek, M. L., & Elhssia, S. A. M. (1997). Cost analysis of a two dissimilar-unit cold standby redundant system subject to inspection and two types of repair. *Microelectronics and Reliability, 37*(2), 335-340. http://dx.doi.org/10.1016/0026-2714(95)00216-2.

Osaki, S. (1972). Reliability Analysis of a Two-Unit Standby-Redundant System with Preventive Maintenance. *IEEE Transactions on Reliability, R-21*(1), 24-29. http://dx.doi.org/10.1109/TR.1972.5216167.

Ross, S. M.(2007). *Introduction to probability models* (9th ed.). San Diego: Academic Press. 782 p.

Ross, S. M.(2012). *Simulation* (5th ed.). San Diego: Academic Press. 328 p.

Smith, M. A. J., & Dekker, R. (1997). Preventive maintenance in a 1 out of n system: The uptime, downtime and costs. *European Journal of Operational Research, 99*(3), 565-583. http://dx.doi.org/10.1016/S0377-2217(96)00321-9.

Wang, G. J., & Zhang, Y. L. (2014). Geometric process model for a system with inspections and preventive repair. *Computers & Industrial Engineering, 75*, 13-19. http://dx.doi.org/10.1016/j.cie.2014.06.007.

Zhang, Y. L., & Wang, G. J. (2007). A deteriorating cold standby repairable system with priority in use. *European Journal of Operational Research, 183*(1), 278-295. http://dx.doi.org/10.1016/j.ejor.2006.09.075.

Zhang, Y. L., & Wang, G. J. (2011). An optimal repair-replacement policy for a cold standby system with use priority. *Applied Mathematical Modelling, 35*(3), 1222-1230. http://dx.doi.org/10.1016/j.apm.2010.08.009.

Zhong, C., & Jin, H. (2014). A novel optimal preventive maintenance policy for a cold standby system based on semi-Markov theory. *European Journal of Operational Research, 232*(2), 405-411. http://dx.doi.org/10.1016/j.ejor.2013.07.020.