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On the influence of the shape of kappa distributions of ions and electrons on the ion-cyclotron instability

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The dispersion relation for ion-cyclotron waves propagating along the direction of the ambient magnetic field is investigated numerically by considering different forms of kappa functions as velocity distributions of ions and electrons. General forms of kappa distributions, isotropic and anisotropic, are defined and used to obtain the dispersion relations for ion-cyclotron waves. With suitable choice of parameters, the general forms reduce to anisotropic versions of the kappa distributions most frequently employed in the literature. The analysis is focused in cases with a small value of the kappa index, for which the non thermal character of the kappa distributions is enhanced. The results show the effects of the superthermal tails of the velocity distributions of both particle species (ions and electrons) on the growth rate of the ion-cyclotron instability. It is seen that different forms of anisotropic kappa distributions, which are used in the current literature, can have a significantly different effect on the growth rates of the instability. *Published by AIP Publishing*. [http://dx.doi.org/10.1063/1.5002136]

I. INTRODUCTION

The space environment of the solar system has begun to be explored by satellites in the 1960s, and already in the early years of exploration evidence started to pile up, showing that the plasma particles frequently have non thermal velocity distributions featuring power-law tails.[1–3] As a mathematical model for description of the observed non thermal features of velocity distributions in space, mathematical functions which are known as *kappa distributions* have been introduced and widely used.[4–10] However, it has been also observed that velocity distributions in space, in addition of being non thermal, frequently are also anisotropic.[11–15] For the mathematical description of the observed distributions, anisotropic kappa distributions have also been introduced. These anisotropic distributions may be classified either as *bi-kappa distributions* (BK), which are characterized by a single kappa index and anisotropic temperature parameters, or as *product-bi-kappa distributions* (PBKs), which are characterized by different kappa indexes along parallel and perpendicular directions, and also by anisotropic temperature parameters. Anisotropic kappa distributions can have important role on the analysis and understanding of space plasma phenomena.[16,17]

The anisotropies, and other non thermal features, usually displayed by the observed distribution functions in space plasmas often serve as a source of free energy that excites several distinct instabilities. One of the instabilities which may occur is the so-called ion-cyclotron (IC) instability, which is associated to dispersion of ion velocities along the perpendicular directions (relative to the ambient magnetic field) greater than the dispersion along the parallel direction. The IC instability has been subject to continued interest from the community of space plasma physicists along the recent decades, as seen in studies developed considering the case of bi-Maxwellian distributions,[18–20] and also as seen in several papers which have considered the occurrence of kappa distributions.[21–24]

The work of Shaaban *et al.* has dedicated special attention to the study of the influence of the electron population on the IC instability, and investigate the potential relevance of their findings for the explanation of the instability thresholds observed in the proton temperature anisotropy in the solar wind.[20] Ions and electrons are described by anisotropic Maxwellian distributions, and the influence of electrons on the growth rates of the instability is investigated considering situations where $T_{e\perp}/T_{e\parallel} > 1$, and situations with $T_{e\perp}/T_{e\parallel} < 1$.

In the sequence of the results obtained in Ref. 20, the influence of non thermal features in the electron distribution has been investigated in Ref. 24. Ions are described by anisotropic Maxwellian distributions, and electrons are described by BK distributions. The results obtained show that the non thermal features of the BK distribution contribute to enhance the effect of the electron population, in comparison with the case of bi-Maxwellian electrons.

Other instances of use of kappa distributions can be mentioned and briefly discussed. In Ref. 21, for instance, the IC instability was studied considering plasma particles (ions and electrons) with BK distributions, and the instability growth rate and instability thresholds were obtained for different values of parameters such as the plasma β, the ion temperature anisotropy, and the κ index. In Ref. 22, a similar study was made considering both BK and PBK distributions for both ions and electrons, with the electron temperature assumed to be isotropic.

Another look on the effect due to non thermal distributions on electromagnetic instabilities in the ion cyclotron

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range appears in a recent paper by Shaaban *et al.*[25] which contains an investigation about the growth rates of the electromagnetic ion-cyclotron instability (EMIC), obtained considering a combination of a bi-Maxwellian distribution for the core ions, and a BK distribution for the halo population of ions.[25] The analysis has assumed that the electron distribution is isotropic and Maxwellian, so that the influence of non thermal features in the electron distribution was not a part of the investigation. Regarding the influence of the non thermal features of the ion distribution, the results obtained have shown that the decrease of the kappa index of the halo distribution enhances the growth rate of the instability.[25]

Distributions with power-law velocity tails have also been employed in the studies of instabilities outside of the ion-cyclotron range. For instance, bi-kappa distributions were also considered in the analysis of the electromagnetic electron-cyclotron instability (EMEC), which is due to anisotropy in the electron temperature.[26] For the investigation of the EMEC instability, growth rates obtained considering bi-Maxwellian distribution functions were compared with growth rates obtained considering bi-kappa distribution functions, considering two different situations.[26] In one of the situations, denoted as case A in Ref. 26, the growth rates obtained in the Maxwellian case were compared with growth rates obtained considering BK distributions with the same temperature as the Maxwellian distribution. The results obtained in this case have shown that, for most of the parameters considered, the EMEC growth rates obtained with the BK distributions are smaller than those of the Maxwellian case, except for very small wave numbers.[26] Another situation considered, denoted as case B in Ref. 26, considered BK distributions having the same thermal velocity as the Maxwellian distribution, but larger temperature. That is, in case B the temperatures of the BK distribution were dependent on the κ index. In case B, it was obtained that the BK distribution leads to larger growth rates than those obtained in the Maxwellian case.[26]

The two different models of BK distributions which were considered in Ref. 26 were also discussed in a paper by Lazar, Fichtner, and Yoon,[27] from a different point of view. The emphasis of Ref. 27 has been on the physical mechanisms which could lead to these different velocity distributions, either the model A, which features an increased population of electrons at small velocities, or model B, with a comparatively higher population at the large velocity tail.

Other recent example of use of kappa distributions in the analysis of electromagnetic instabilities can be found in Ref. 28, which considered plasmas with electrons and ions described by either bi-Maxwellian or bi-kappa distributions, both with a drifting velocity along the magnetic field. These types of distributions have been utilized in Ref. 28 for the study of the spectral power of whistler and firehose fluctuations, and their dependence on the electron parallel temperature and the electron thermal anisotropy. The general conclusion has been that the spectrum of fluctuations is enhanced in the presence of suprathermal particles, and that the enhancement is more noticeable with the decrease of the kappa index.[28]

Another relatively recent analysis which is of interest for the present study appeared in a paper by dos Santos *et al.*.[23] That work contains an investigation of the IC instability considering kappa distributions for the ions, and emphasizing the role played by non thermal electron distributions. Most of the analysis is made considering PBK ion distributions, and different forms of electron distribution, with isotropic electron temperatures. A limited number of cases also discuss the comparison between results obtained considering PBK or BK ion distributions.[23]

The results obtained in Ref. 23 indicate that when both ions and electrons are described by PBK distributions, the decrease in the kappa index of the electron distribution leads to decrease in the growth rate of the IC instability and on the range of unstable wave-numbers. However, for BK electron distributions, the characteristics of the instability appear to be quite insensitive to the electron kappa index.[23] The results which compare PBK and BK distributions for the ions indicate that the growth rates of the IC instability are decreased if the ion distribution is changed from PBK to BK, regardless of the shape of the electron distribution.[23]

In the present paper, we develop further the analysis of the IC instability in the case of plasma particles with kappa distributions. We consider extended definitions of BK and PBK distributions, therefore enhancing the range of shapes of kappa distributions, in comparison with the analysis made in dos Santos *et al.*,[23] and also with the analyses made in Refs. 20 and 24.

By suitable choices of two convenient parameters, the extended definitions of BK and PBK distributions used in this paper may correspond to either one or another of two familiar forms of kappa distributions. One of these particular cases considers anisotropic versions of kappa distributions with kinetic temperature independent of the kappa index.[4–6,26] The other limiting case corresponds to anisotropic kappa distributions with the kinetic temperatures dependent on the kappa index, a formulation of kappa distribution which is also familiar in the literature.[9,10,26] In addition to considering different forms of anisotropic kappa distributions for the ions, which are the driving force for the IC instability, the present paper also investigates the effect of temperature anisotropy on the electron distribution, which has not been discussed in Ref. 23.

The analysis is made considering the dispersion relation for low frequency electromagnetic waves with parallel propagation, and is concentrated on the dependence of the growth rate of the IC instability on the shape of ion and electron distributions. Several combinations of velocity distributions for ions and electrons are considered, for a range of parameters which are of interest for space plasmas. The motivation for the work is the observation that the study of instabilities in plasmas described by velocity distributions with power-law tails, and particularly the study of the influence of non thermal features of the electron distribution on the growth rates of instabilities driven by anisotropy in the ion distribution, is presently of great interest for the plasma physics community. The literature contains several examples of recent studies on this subject, and the present paper aims to fill in unexplored gaps of previous analysis.

The paper is organized as follows: In Sec. II we briefly describe the theoretical formulation and the dispersion relation for electromagnetic waves propagating parallel to the ambient magnetic field, and introduce the generalized forms of BK and PBK distributions. For completicity, we also write in Sec. II the expressions obtained after evaluation of the velocity integrals which appear in the dispersion relation, one for each of the different forms of velocity distribution considered in the analysis. In Sec. III we present and discuss results obtained by the numerical solution of the dispersion relation, for several combinations of distribution functions. Final remarks and a discussion on future perspectives appear in Sec. IV.

II. THEORETICAL FORMULATION

For parallel propagation, and in the case of velocity distributions which are even along the parallel direction, the dispersion relation is obtained from the following determinant:

$$\det\begin{pmatrix} \varepsilon_{xx}-N_\parallel^2 & \varepsilon_{xy} & 0 \\ -\varepsilon_{12} & \varepsilon_{xx}-N_\parallel^2 & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix} = 0, \tag{1}$$

where we have taken into account that for $k_\perp \to 0$, $\varepsilon_{yy} = \varepsilon_{xx}$ and $\varepsilon_{xz} = \varepsilon_{yz} = 0$, and where $N_\parallel$ is the parallel component of the refraction vector, $\mathbf{N} = c\mathbf{k}/\omega$. The symbols $\mathbf{k}$ and ω represent the wave vector and the angular wave frequency, respectively.

The determinant given by Eq. (1) can be separated into two minor determinants, and the dispersion relation for parallel propagating electromagnetic waves is given by the following:

$$N_\parallel^2 = \varepsilon_{xx} \pm i\varepsilon_{xy}, \tag{2}$$

where the ε_{ij} are the components of the dielectric tensor. Using textbook expressions for these components, Eq. (2) can be written as follows:

$$N_\parallel^2 = 1 + \frac{1}{2}\sum_\beta \frac{\omega_{p\beta}^2}{\omega^2}\,\omega \int d^3v \frac{v_\perp L(f_\beta)}{\omega - s\Omega_\beta - k_\parallel v_\parallel}, \tag{3}$$

where $s = \pm 1$, Ω_β and $\omega_{p\beta}$ are, respectively, the angular cyclotron frequency and the angular plasma frequency of particles of species β, and where

$$L = \left(1 - \frac{k_\parallel v_\parallel}{\omega}\right)\frac{\partial}{\partial v_\perp} + \frac{k_\parallel v_\perp}{\omega}\frac{\partial}{\partial v_\parallel}.$$

The dispersion relation (2) depends on the equilibrium velocity distributions of the plasma particles, which we denote as f_β. For convenience, we introduce a symbol for the integral quantity appearing in the dispersion relation

$$J(s,f_\beta) = \omega \int d^3v \frac{v_\perp L(f_\beta)}{\omega - s\Omega_\beta - k_\parallel v_\parallel}. \tag{4}$$

It is seen that the contribution of each type of particle for the dispersion relation requires only the evaluation of the quantity $J(s,f_\beta)$, which depends on the distribution function f_β. Let us then introduce the different forms of velocity distributions, of interest for the present work.

We start with the definition of an isotropic kappa distribution, characterized by the index κ_β and by parameters α_β and $w_{\beta,\kappa}$[29]

$$f_{\beta,K}(\mathbf{v}) = \frac{1}{\pi^{3/2}\kappa_\beta^{3/2} w_{\beta,\kappa}^3}\frac{\Gamma(\kappa_\beta+\alpha_\beta)}{\Gamma(\kappa_\beta+\alpha_\beta-3/2)} \times \left(1 + \frac{v^2}{\kappa_\beta w_{\beta,\kappa}^2}\right)^{-(\kappa_\beta+\alpha_\beta)}. \tag{5}$$

The distribution function (5) is normalized such that $\int d^3v f_{\beta,K} = 1$. The parameter $w_{\beta,\kappa}$ has the same physical dimension as the particle thermal velocity $v_\beta = \sqrt{2T_\beta/m_\beta}$, where m_β and T_β are, respectively, the mass and the temperature of particles of species β, written in units of energy. The parameter $w_{\beta,\kappa}$ must reduce to the thermal velocity in the limit $\kappa_\beta \to \infty$.

Important limiting cases are as follows. If the parameter α_β is taken as $\alpha_\beta = 1$ and $w_{\beta,\kappa}^2 = [(\kappa_\beta - 3/2)/\kappa_\beta]v_\beta^2$, the distribution function (5) becomes the kappa distribution as defined in the paper by Summers and Thorne.[4–6] We will call this distribution as the kappa distribution of type I, or KI distribution. For KI distributions, the value of the second moment is $\langle v^2\rangle = 3v_\beta^2/2$, and is therefore independent of the parameter κ_β.

If one takes $\alpha_\beta = 0$ and $w_{\beta,\kappa}^2 = v_\beta^2$, the distribution function (5) becomes the kappa distribution as defined in the paper by Leubner,[9,10] which we will identify as the kappa distribution of type II, or KII distribution. For KII distributions, the value of the second moment is dependent on κ_β, and is given by

$$\langle v^2\rangle = \frac{3}{2}\frac{\kappa_\beta}{\kappa_\beta - 5/2}v_\beta^2.$$

An anisotropic kappa distribution with isotropic kappa parameters, also depending on parameters α_β and $w_{\beta,\kappa}$, may be defined as follows:

$$f_{\beta,BK}(\mathbf{v}) = \frac{1}{\pi^{3/2}\kappa_\beta^{3/2} w_{\beta,\kappa,\perp}^2 w_{\beta,\kappa,\parallel}}\frac{\Gamma(\kappa_\beta+\alpha_\beta)}{\Gamma(\kappa_\beta+\alpha_\beta-3/2)} \times \left(1 + \frac{v_\parallel^2}{\kappa_\beta w_{\beta,\kappa,\parallel}^2} + \frac{v_\perp^2}{\kappa_\beta w_{\beta,\kappa,\perp}^2}\right)^{-(\kappa_\beta+\alpha_\beta)} \tag{6}$$

with the same normalization of distribution (5).

This distribution can be considered as a generalized *bi-kappa distribution* (BK). Similar to what was done in the case of isotropic kappa distributions, if the parameter α_β is taken as $\alpha_\beta = 1$ and $w_{\beta,\kappa,\perp}^2 = [(\kappa_\beta - 3/2)/\kappa_\beta]v_{\beta\perp}^2$, $w_{\beta,\kappa,\parallel}^2 = [(\kappa_\beta - 3/2)/\kappa_\beta]v_{\beta\parallel}^2$, with $v_{\beta\perp} = \sqrt{2T_{\beta\perp}/m_\beta}$ and $v_{\beta\parallel} = \sqrt{2T_{\beta\parallel}/m_\beta}$, one obtains the bi-kappa distribution of type I, or BKI distribution. For the BKI distribution, the average values of $v_\parallel^2$ and $v_\perp^2$ are as follows:

$$\langle v_\parallel^2\rangle = \frac{1}{2}v_{\beta\parallel}^2, \quad \langle v_\perp^2\rangle = v_{\beta\perp}^2.$$

These results show that the parallel and perpendicular kinetic temperatures of a BKI distribution are independent of the κ index of the distribution. This type of distribution corresponds to that characterizing the so-called "case A" of Ref. 27.

The bi-kappa distribution of type II, or BKII distribution, is obtained by assuming $\alpha_\beta = 0$, $w_{\beta,\kappa,\perp}^2 = v_{\beta\perp}^2$, and $w_{\beta,\kappa,\parallel}^2 = v_{\beta\parallel}^2$. In the case of the BKII distribution, the average values of $v_\parallel^2$ and $v_\perp^2$ are given by

$$\langle v_\parallel^2\rangle = \frac{1}{2}\frac{\kappa_\beta}{\kappa_\beta - 5/2}v_{\beta\parallel}^2, \quad \langle v_\perp^2\rangle = \frac{\kappa_\beta}{\kappa_\beta - 5/2}v_{\beta\perp}^2$$

which means that the kinetic temperatures of BKII distributions are dependent on the kappa index. The BKII distribution, obtained from Eq. (6) with use of $\alpha_\beta = 0$, $w_{\beta,\kappa,\perp}^2 = v_{\beta\perp}^2$, and $w_{\beta,\kappa,\parallel}^2 = v_{\beta\parallel}^2$, is similar to the distribution considered in "case B" of Ref. 27. However, there is a difference. The high-velocity limit of the BKII distribution used here contains terms which are proportional to $v_\parallel^{-\kappa_\beta}$ and $v_\perp^{-\kappa_\beta}$, while the high-velocity limit of the distribution of case B in Ref. 27 contains terms which are proportional to $v_\parallel^{-(\kappa_\beta+1)}$ and $v_\perp^{-(\kappa_\beta+1)}$.

Anisotropic kappa distributions with anisotropic kappa indexes, which are known as *product-bi-kappa distributions* (PBK), can be defined in similar way[30]

$$f_{\beta,PBK}(\mathbf{v}) = \frac{1}{\pi^{3/2}\kappa_{\beta\perp}\kappa_{\beta\parallel}^{1/2}w_{\beta,\kappa,\perp}^2 w_{\beta,\kappa,\parallel}} \times \frac{\Gamma(\kappa_{\beta\perp}+\alpha_\beta)\Gamma(\kappa_{\beta\parallel}+\alpha_\beta)}{\Gamma(\kappa_{\beta\perp}+\alpha_\beta-1)\Gamma(\kappa_{\beta\parallel}+\alpha_\beta-1/2)} \times \left(1+\frac{v_\parallel^2}{\kappa_{\beta\parallel}w_{\beta,\kappa,\parallel}^2}\right)^{-(\kappa_{\beta\parallel}+\alpha_\beta)} \times \left(1+\frac{v_\perp^2}{\kappa_{\beta\perp}w_{\beta,\kappa,\perp}^2}\right)^{-(\kappa_{\beta\perp}+\alpha_\beta)} \quad (7)$$

with the same normalization of distribution (5).

If the parameter α_β is taken as $\alpha_\beta = 1$ and $w_{\beta,\kappa,\parallel}^2 = [(\kappa_{\beta\parallel}-3/2)/\kappa_{\beta\parallel}]v_{\beta\parallel}^2$, $w_{\beta,\kappa,\perp}^2 = [(\kappa_{\beta\perp}-3/2)/\kappa_{\beta\perp}]v_{\beta\perp}^2$, one obtains the product-bi-kappa distribution of type I, or PBKI distribution.[17] For the PBKI distribution, the average values of $v_\parallel^2$ and $v_\perp^2$ are the same as those obtained in the case of BKI distributions, namely

$$\langle v_\parallel^2\rangle = \frac{1}{2}v_{\beta\parallel}^2, \quad \langle v_\perp^2\rangle = v_{\beta\perp}^2.$$

The product-bi-kappa distribution of type II, or PBKII distribution, is obtained by assuming $\alpha_\beta = 0$ and $w_{\beta,\kappa,\parallel}^2 = v_{\beta\parallel}^2$, $w_{\beta,\kappa,\perp}^2 = v_{\beta\perp}^2$. The average values of $v_\parallel^2$ and $v_\perp^2$, in the case of the PBKII distribution, are as follows:

$$\langle v_\parallel^2\rangle_\beta = \frac{1}{2}\frac{\kappa_{\beta\parallel}}{\kappa_{\beta\parallel}-3/2}v_{\beta\parallel}^2, \quad \langle v_\perp^2\rangle_\beta = \frac{\kappa_{\beta\perp}}{\kappa_{\beta\perp}-2}v_{\beta\perp}^2.$$

A PBK distribution has two sources of anisotropy along parallel and perpendicular directions, the temperature variables $T_{\beta\perp}$ and $T_{\beta\parallel}$, and the kappa indexes $\kappa_{\beta\perp}$ and $\kappa_{\beta\parallel}$.

Using the distribution given by Eq. (6), the integral J appearing in the dispersion relation, given by Eq. (3), becomes as follows:[31]

$$J(s,f_{\beta,BK}) = 2\frac{\Gamma(\kappa_\beta-1/2)}{\Gamma(\kappa_\beta+\alpha-3/2)} \times\left\{-\frac{\Gamma(\kappa_\beta+\alpha-3/2)}{\Gamma(\kappa_\beta-1/2)}\left(1-\frac{w_{\beta,\kappa,\perp}^2}{w_{\beta,\kappa,\parallel}^2}\right) + \frac{\Gamma(\kappa_\beta+\alpha-1)}{\Gamma(\kappa_\beta)}\left[\zeta_\beta^0-\left(1-\frac{w_{\beta,\kappa,\perp}^2}{w_{\beta,\kappa,\parallel}^2}\right)\zeta_\beta^s\right]Z_{\kappa_\beta}^{(\alpha-1)}(\zeta_\beta^s)\right\}, \quad (8)$$

where

$$\zeta_\beta^0 = \frac{\omega}{k_\parallel w_{\beta,\kappa,\parallel}}, \quad \zeta_\beta^s = \frac{\omega - s\Omega_\beta}{k_\parallel w_{\beta,\kappa,\parallel}} \quad (9)$$

and where we have used the definition of the plasma dispersion function for κ distributions, of order m

$$Z_\kappa^{(m)}(\zeta) = \frac{1}{\pi^{1/2}}\frac{\Gamma(\kappa)}{\kappa^{1/2}\Gamma(\kappa-1/2)} \times \int_{-\infty}^{\infty}\frac{ds}{(s-\zeta)(1+s^2/\kappa)^{\kappa+m}} \quad (10)$$

which reduces to the distribution defined by Summers and Thorne[5,6] in the case $m = 1$. The plasma dispersion function can be written in terms of the Gauss hypergeometric function ${}_2F_1(a,b,c,z)$, as follows:

$$Z_\kappa^{(m)}(\zeta) = \frac{i\Gamma(\kappa)\Gamma(\kappa+m+1/2)}{\kappa^{1/2}\Gamma(\kappa-1/2)\Gamma(\kappa+m+1)} \times {}_2F_1\left[1,2\kappa+2m;\kappa+m+1;\frac{1}{2}\left(1+\frac{i\zeta}{\kappa^{1/2}}\right)\right] \quad (11)$$

for $\kappa > -m - 1/2$. In the limit $\kappa \to \infty$, the function $Z_\kappa^{(m)}$ becomes the well known Z function[32]

$$Z(\zeta) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty}dt\frac{e^{-t^2}}{t-\zeta}. \quad (12)$$

Using the distribution given by Eq. (7), the integral J appearing in the dispersion relation, given by Eq. (3), becomes as follows:[31]

$$J(s,f_{\beta,PBK}) = 2\frac{\kappa_{\beta\perp}}{\kappa_{\beta\perp}+\alpha_\beta-2} \times\left\{-\frac{\kappa_{\beta\perp}+\alpha_\beta-2}{\kappa_{\beta\perp}} + \frac{\kappa_{\beta\parallel}+\alpha_\beta-1/2}{\kappa_{\beta\parallel}}\frac{w_{\beta,\kappa,\perp}^2}{w_{\beta,\kappa,\parallel}^2} + \frac{\Gamma(\kappa_{\beta\parallel}-1/2)}{\Gamma(\kappa_{\beta\parallel}+\alpha_\beta-1/2)}\frac{\Gamma(\kappa_{\beta\parallel}+\alpha_\beta)}{\Gamma(\kappa_{\beta\parallel})} \times\left[\frac{\kappa_{\beta\perp}+\alpha_\beta-2}{\kappa_{\beta\perp}}(\zeta_\beta^0-\zeta_\beta^s)Z_{\kappa_\parallel}^{(\alpha)}(\zeta_\beta^s) + \frac{\kappa_{\beta\parallel}+\alpha_\beta}{\kappa_{\beta\parallel}}\frac{w_{\beta,\kappa,\perp}^2}{w_{\beta,\kappa,\parallel}^2}(\zeta_\beta^s)Z_{\kappa_\parallel}^{(\alpha+1)}(\zeta_\beta^s)\right]\right\}, \quad (13)$$

where

$$\zeta_\beta^0 = \frac{\omega}{k_\parallel w_{\beta,\kappa,\parallel}}, \quad \zeta_\beta^s = \frac{\omega - s\Omega_\beta}{k_\parallel w_{\beta,\kappa,\parallel}}. \tag{14}$$

Let us also introduce the well known bi-Maxwellian velocity distribution

$$f_{\beta,M} = \frac{1}{\pi^{3/2} v_{\beta\perp}^2 v_{\beta\parallel}} e^{-v_\perp^2/v_{\beta\perp}^2} e^{-v_\parallel^2/v_{\beta\parallel}^2} \tag{15}$$

normalized such that $\int d^3v f_{\beta,M} = 1$.

With use of the bi-Maxwellian distribution function, the J integral becomes

$$J(s, f_{\beta,M}) = 2\left\{\zeta_\beta^0 Z(\zeta_\beta^s) - \left(1 - \frac{u_{\beta\perp}^2}{u_{\beta\parallel}^2}\right)\left[1 + \zeta_\beta^n Z(\zeta_\beta^s)\right]\right\}. \tag{16}$$

Equation (16) is the limiting form of Eq. (8) for $\kappa_\beta \to \infty$, and also the limiting form of Eq. (13) in the limit $\kappa_{\beta\parallel} \to \infty$ and $\kappa_{\beta\perp} \to \infty$. Another comment which is pertinent here is that Eq. (8), in the isotropic limit given by $w_{\beta,\kappa,\perp} = w_{\beta,\kappa,\parallel}$, provides the form of the integral J to be used in the case of isotropic kappa distribution given by Eq. (5).

III. NUMERICAL ANALYSIS

For the numerical analysis of the effect of the shape of the particle velocity distributions on the dispersion relation of ion-cyclotron waves, we start by considering the case of $\beta_{i\parallel} = 2.0$ and $v_A/c = 1.0 \times 10^{-4}$, with $v_A = B_0/\sqrt{4\pi n_{i0} m_i}$ and $\beta_{i\parallel} = v_{i\parallel}^2/v_A^2$. We also consider $T_{e\parallel} = T_{i\parallel}$, and the ion mass equal to the mass of a proton, $m_i = m_p$.

Using these parameters, we solve the dispersion relation considering some combinations of the different forms of velocity distribution functions for ions and electrons, those which have been introduced in Sec. II. The use of these different forms of velocity distributions is widespread in the literature, and has been the motivation for a number of recent theoretical analyses.[27,33–36] In the ensuing figures we plot the quantity $z_i = \omega_i/\Omega_i$, the imaginary part of the frequency (ω_i) obtained from the dispersion relation for ion-cyclotron waves, divided by the ion-cyclotron angular frequency (Ω_i), *versus* the normalized wave number, $q = kv_A/\Omega_i$, where v_A is the Alfvén velocity. Figure 1 shows z_i *versus* the normalized wave number, for different values of the temperature ratio, by assuming the cases of $T_{\beta\perp}/T_{\beta\parallel} = 1.0$, 2.0, 3.0, 4.0, 5.0, 6.0, and 7.0, with $\beta = e, i$. That is, for Fig. 1 we assume that ions and electrons have anisotropic distribution functions, with the same temperature anisotropy.

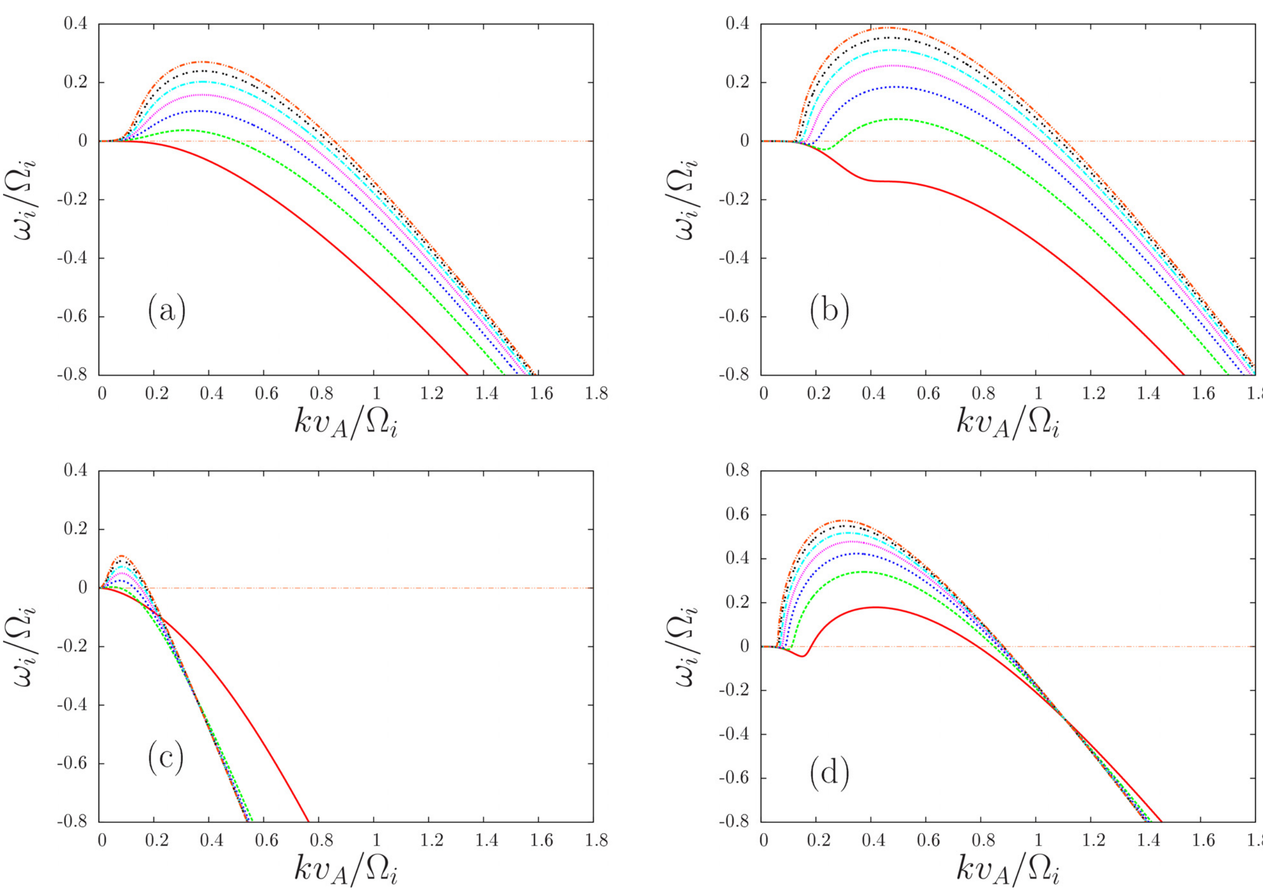

FIG. 1. Imaginary part of the frequency of waves in the ion-cyclotron mode *vs*. wave number. (a) Ions and electrons with a BKI distribution, with $\kappa_i = \kappa_e = 2.5$; (b) ions and electrons with a PBKI distribution, with $\kappa_{i\perp} = \kappa_{i\parallel} = \kappa_{e\perp} = \kappa_{e\parallel} = 2.5$; (c) ions and electrons with a BKII distribution, with $\kappa_i = \kappa_e = 2.5$; (d) ions and electrons with a PBKII distribution, with $\kappa_{i\perp} = \kappa_{i\parallel} = \kappa_{e\perp} = \kappa_{e\parallel} = 2.5$. For all panels, the curves represent the cases of $T_{\beta\perp}/T_{\beta\parallel} = 1.0$ (red), 2.0 (green), 3.0 (blue), 4.0 (magenta), 5.0 (cyan), 6.0 (black), and 7.0 (brown), with $\beta = e, i$. Other parameters are $\beta_{i\parallel} = 2.0$, $v_A/c = 1.0 \times 10^{-4}$.

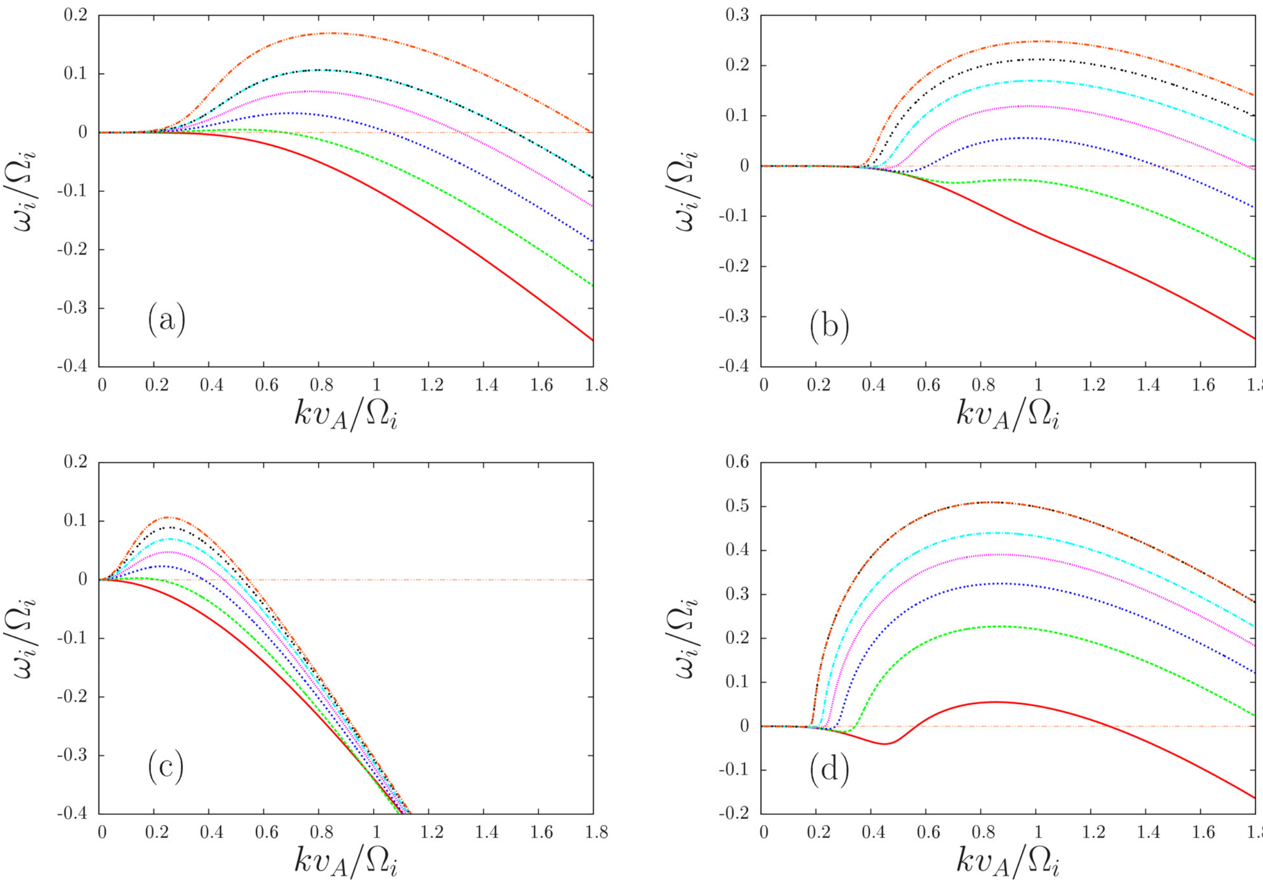

FIG. 2. Plots of z_i *vs.* q when $\beta_{i\parallel} = 0.2$. The distribution functions used in panels (a)–(d), values of the κ indexes and other parameters, and the color codes, are the same as in Fig. 1.

For the case of Fig. 1(a), we assume electrons and ions with BKI distributions, as given by Eq. (6), with $\kappa_e = \kappa_i = 2.5$. It is seen that there is instability for a range of wave numbers which start at $q \simeq 0.1$ for all temperature ratios above unity, and extends up to $q \simeq 0.5$ for $T_{\beta\perp}/T_{\beta\parallel} = 2.0$, and up to $q \simeq 0.9$ for $T_{\beta\perp}/T_{\beta\parallel} = 7.0$. The instability is the ion-cyclotron instability (IC). For Fig. 1(b), we assume electrons and ions with PBKI distributions, as given by Eq. (7), with $\kappa_{i\perp} = \kappa_{i\parallel} = \kappa_{e\perp} = \kappa_{e\parallel} = 2.5$. The comparison between Figs. 1(a) and 1(b) shows that the change from BKI to PBKI leads to enhancement of the growth rates of the IC instability, both in the wave number range and in the magnitude of the growth rate. One notices, however, that for a small temperature ratio the onset of instability in the case of PBKI distributions is moved to a larger value of wave number than seen in the case of BKI distributions. For instance, for $T_{\beta\perp}/T_{\beta\parallel} = 2.0$ the instability starts at $q \simeq 0.1$ in the case of BKI distributions, and at $q \simeq 0.3$ in the case of PBKI distributions.

Distributions of type II are considered for Figs. 1(c) and 1(d). In Fig. 1(c), electrons and ions are assumed with BKII distributions, as also given by Eq. (6), with $\kappa_e = \kappa_i = 2.5$. The comparison between Figs. 1(c) and 1(a) shows that distributions of type BKII lead to strong reduction of the growth rates of the IC instability, in comparison with those obtained with the BKI distributions. One also notices a reduction of the interval of wave numbers where the instability occurs. For Fig. 1(d), the distributions for electrons and ions are assumed to be PBKII distributions, as given by Eq. (7), with $\kappa_{i\perp} = \kappa_{i\parallel} = \kappa_{e\perp} = \kappa_{e\parallel} = 2.5$. Figure 1(d) shows that the maximum growth rates obtained using PBKII distributions are greater than those obtained using PBKI distributions, which appear in Fig. 1(b), although there is some reduction of the interval of unstable wave numbers. The difference between the cases of PBKII and BKII, seen in panels (d) and (c), respectively, is seen to be much more significant than the difference between PBKI and BKI, appearing in panels (b) and (a), respectively. In particular, Fig. 1(d) shows that the IC instability for a PBKII plasma occurs even when $T_{\beta\perp} = T_{\beta\parallel}$.

In Fig. 2 we consider a case which is similar to that considered in Fig. 1, except that $\beta_{i\parallel} = 0.2$ instead of 2.0. The results shown in Fig. 2 show the same general characteristics found in the case of Fig. 1, regarding the effect of the different forms of the particle velocity distributions. The difference is that now the maximum growth rate attained for a given value of the temperature ratio is smaller, and the extension of the unstable region along the wave number axis is larger, than in the case of Fig. 1.

A case with further reduction in the value of $\beta_{i\parallel}$ is shown in Fig. 3, which depicts results obtained considering $\beta_{i\parallel} = 0.05$, and all the other parameters the same as in Figs. 1 and 2. The comments to be made regarding the dependence on the velocity distributions, the magnitude of the growth rates, and the extension of the regions featuring instability, are similar to those made about the results obtained in Fig. 2.

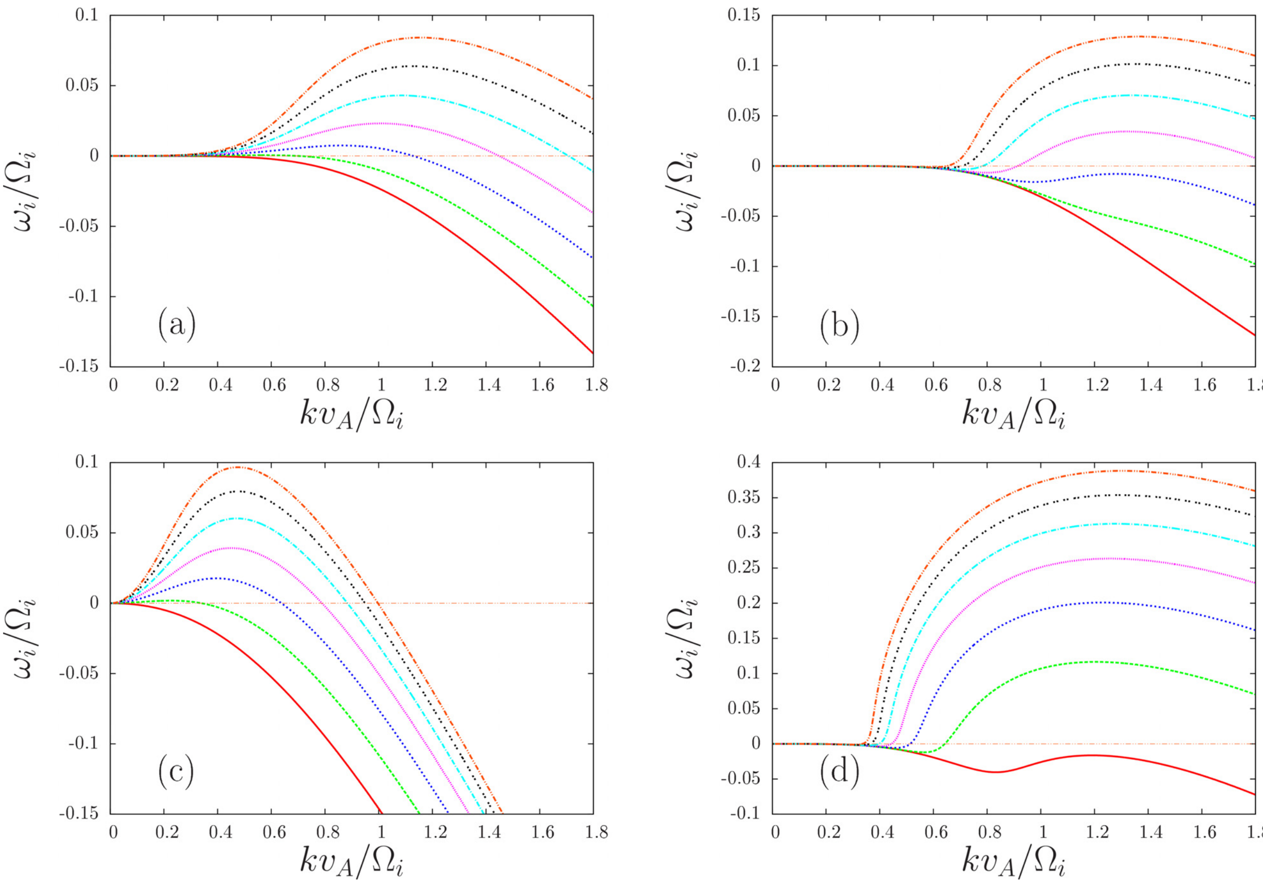

FIG. 3. Plots of z_i *vs.* q when $\beta_{i\parallel} = 0.05$. All other parameters and conventions are the same as in Figs. 1 and 2.

The effect of changes in the electron distribution function, associated to different forms of the ion distribution function, is studied in Figs. 4–7. In Fig. 4 we consider the case of ions described by anisotropic BKI distributions with $\kappa_i = 2.5$, plotting the cases of $T_{i\perp}/T_{i\parallel} = 1.0$, 2.0, 3.0, 4.0, 5.0, 6.0, and 7.0. Figure 4(a) depicts the results obtained in the case of electrons also described by BKI anisotropic distributions with $\kappa_e = 2.5$, with the same values of the temperature anisotropy as the ions, i.e., considering $T_{e\perp}/T_{e\parallel} = 1.0$, 2.0, 3.0, 4.0, 5.0, 6.0, and 7.0. Figure 4(a) is the same as Fig. 1(a). In Fig. 4(b) we show results obtained considering electrons described by an anisotropic Maxwellian distribution. It is seen that the results obtained cannot be distinguished from those appearing in Fig. 4(a). This result shows that in the case of ions with a BKI distribution, the change of the electron distribution, from a Maxwellian to a BKI distribution with small value of the κ_e index, does not affect the growth rates of the IC instability. Figure 4(c) shows results obtained with electrons described by an isotropic BKI distribution with $T_e = T_{i\parallel}$. The only difference between Figs. 4(c) and 4(a) is the anisotropy in the electron distribution, present in Fig. 4(a). The comparison between the results shown in Figs. 4(c) and 4(a) shows that, for a given value of the ion anisotropy and a given value of the electron kappa index, the occurrence of anisotropy of the electron temperatures leads to a significant decrease of both the magnitude of the maximum growth rate of the IC instability and of the size of the region in wave number space where the instability occurs. In Fig. 4(d), we show results obtained considering electrons described by an isotropic Maxwellian distribution, which are seen to be basically the same as those of Fig. 4(c). The conclusion which can be drawn here is that the shape of the electron distribution, when it is isotropic, is not relevant to modify the growth rates of the IC instability associated to BKI distributions for the ions. The general conclusion obtained from Fig. 4 is that in the case of ions described by BKI distributions the shape of the electron distribution, either isotropic or anisotropic, is not relevant for the instability, but the increase of anisotropy of the electron distribution, for a given shape, leads to decrease of the growth rates of the instability.

In Fig. 5 we consider the case of ions described by anisotropic BKII distributions with $\kappa_i = 2.5$, plotting the cases of $T_{i\perp}/T_{i\parallel} = 1.0$, 2.0, 3.0, 4.0, 5.0, 6.0, and 7.0. Figure 5(a) shows the results obtained assuming that the electrons are also described by BKII anisotropic distributions with $\kappa_e = 2.5$, with the same values of the temperature anisotropy as the ions, i.e., considering $T_{e\perp}/T_{e\parallel} = 1.0$, 2.0, 3.0, 4.0, 5.0, 6.0, and 7.0. Figure 5(a) is the same as Fig. 1(c). In Fig. 5(b) the results are obtained with electrons described by an anisotropic Maxwellian distribution. It is seen that the magnitude of the maximum growth rate is significantly higher than that depicted in Fig. 5(a), and the size of the region in wave number space where there is instability is also larger. The inference is that the increase in the non thermal character in the electron distribution, which here is associated to the change from Maxwellian to BKII distribution, contributes to decrease the IC instability in the case of ions described by a

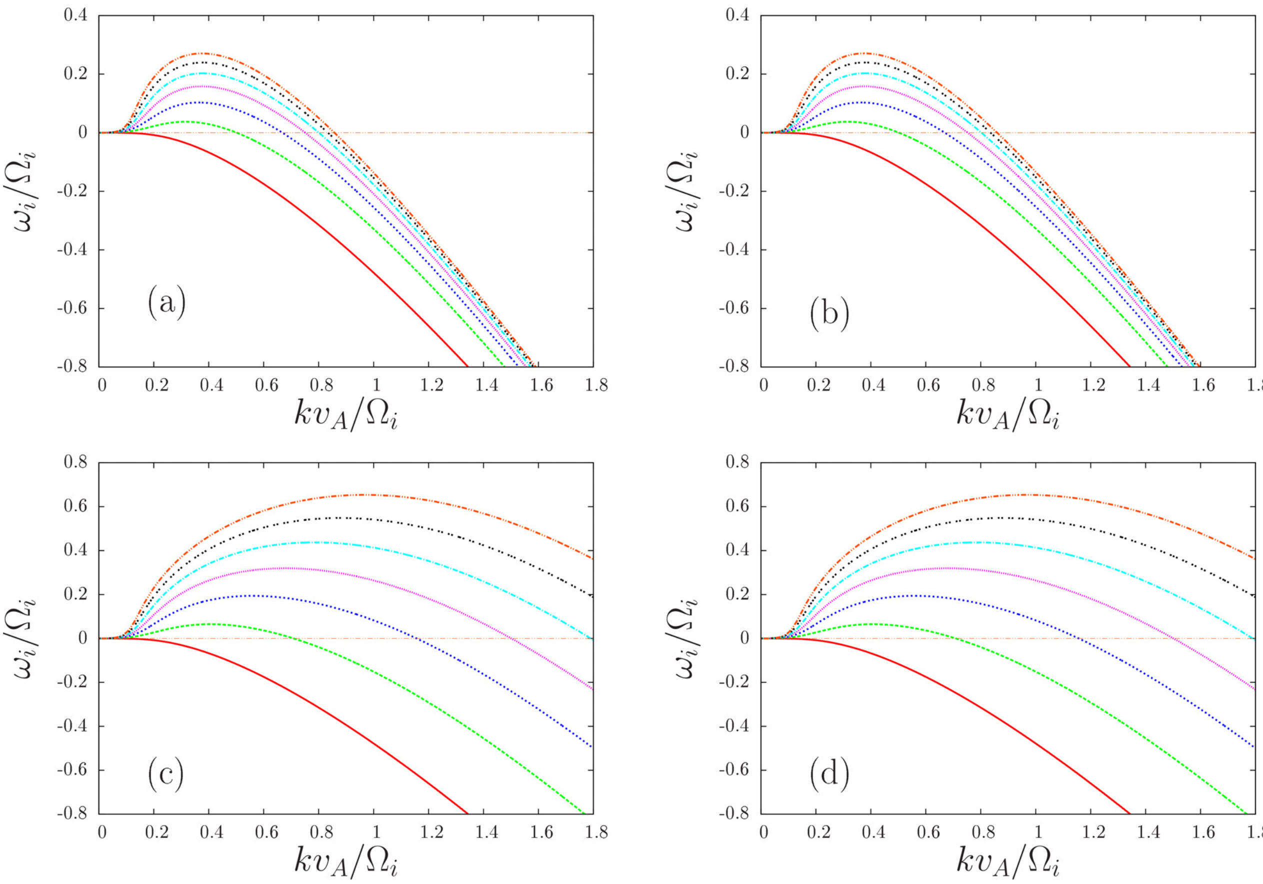

FIG. 4. Imaginary part of the frequency of waves in the ion-cyclotron mode *vs.* wave number. The ions are described by a BKI distribution, with $\kappa_i = 2.5$ and $T_{i\perp}/T_{i\parallel} = 1.0$, 2.0, 3.0, 4.0, 5.0, 6.0, and 7.0. (a) Electrons with a BKI distribution, with $\kappa_e = 2.5$; (b) electrons with a bi-Maxwellian distribution; for panels (a) and (b), the curves represent the cases of $T_{e\perp}/T_{e\parallel} = 1.0$, 2.0, 3.0, 4.0, 5.0, 6.0, and 7.0, with the same color codes as in Fig. 1. (c) Electrons with a BKI distribution with isotropic temperatures, with $\kappa_e = 2.5$; (d) electrons with a Maxwellian distribution; other parameters are $\beta_{i\parallel} = 2.0$, $v_A/c = 1.0 \times 10^{-4}$.

BKII distribution. This result is in contrast with that obtained in the case of BKI distributions, which appeared in Figs. 4(a) and 4(b). Figure 5(c) shows results obtained with electrons described by an isotropic BKII distribution, with $T_e = T_{i\parallel}$. The difference between Figs. 5(c) and 5(a) is the anisotropy in the electron distribution, present in Fig. 5(a). The results shown in Figs. 5(c) and 5(a) show that, for a given value of the ion temperature ratio and a given value of the electron kappa index, the occurrence of anisotropy of electron temperatures leads to significant decrease of the magnitude of the maximum growth rate of the IC instability, and also a significant decrease of the size of the wave number region where occurs the instability. In this regard, the behaviour is similar to that obtained in the case of BKI distributions, shown in Fig. 4. In Fig. 5(d), we show results obtained with electrons described by an isotropic Maxwellian distribution. The results obtained are seen to be the same as those of Fig. 5(c), indicating that in the case of ions with BKII distributions, the shape of the electron distribution, when it is isotropic, is not relevant for the instability. In this regard, the result is analogous to that obtained in the case of BKI distributions, shown in Fig. 4. The results obtained in Fig. 5 can be summarized as follows: in the case of ions described by anisotropic BKII distributions and isotropic electron distributions, the shape of the electron distribution is not relevant for the IC instability. When the electron distribution becomes anisotropic, the IC instability becomes greatly reduced, and the reductive effect is much more pronounced in the case of anisotropic BKII distributions for electrons than in the case of bi-Maxwellian distributions.

In Fig. 6, we consider the case of ions described by anisotropic PBKI distributions with $\kappa_{i\parallel} = \kappa_{i\perp} = 2.5$, plotting the cases of $T_{i\perp}/T_{i\parallel} = 1.0$, 2.0, 3.0, 4.0, 5.0, 6.0, and 7.0. The four panels of Fig. 6 show that, for the case of ions with PBKI distribution, the growth rates obtained for the IC instability have a dependence on the electron distribution function which is analogous to that obtained in the case of BKI distribution, shown in Fig. 4. Figure 6(a) shows the results obtained assuming that the electrons are also described by PBKI anisotropic distributions, with $\kappa_{e\parallel} = \kappa_{e\perp} = 2.5$, with the same values of temperature anisotropy considered for the ions, i.e., considering $T_{e\perp}/T_{e\parallel} = 1.0$, 2.0, 3.0, 4.0, 5.0, 6.0, and 7.0. Figure 6(a) is the same as Fig. 1(b). In Fig. 6(b) we show results obtained with electrons described by an anisotropic Maxwellian distribution. The results obtained cannot be distinguished from those appearing in Fig. 6(a). Figure 6(c) displays the results obtained with electrons described by a PBKI distribution with isotropic temperatures, with $T_e = T_{i\parallel}$. The comparison between the results shown in Fig. 6(c) and those shown in Fig. 6(a) shows that for a given

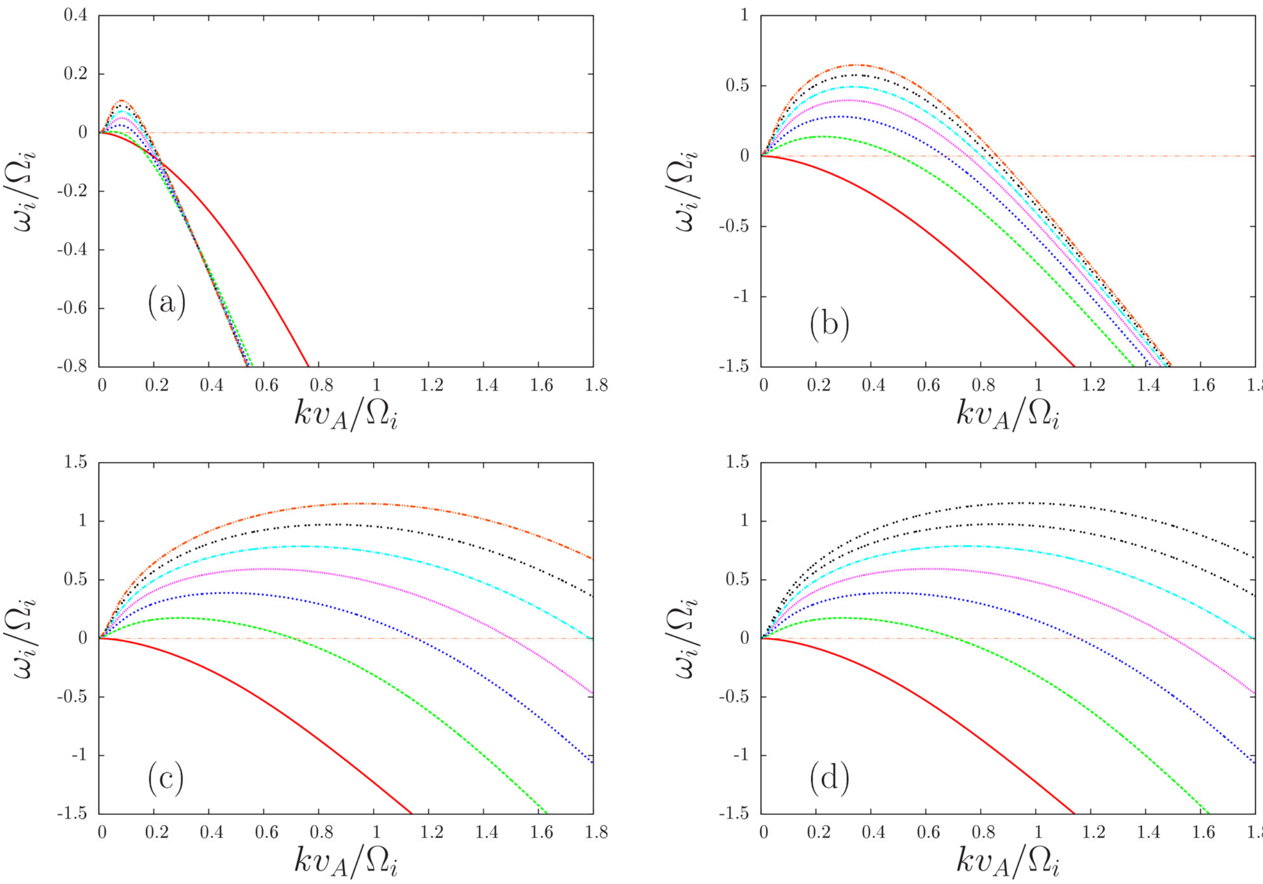

FIG. 5. Imaginary part of the frequency of waves in the ion-cyclotron mode *vs.* wave number. The ions are described by a BKII distribution, with $\kappa_i = 2.5$ and $T_{i\perp}/T_{i\parallel} = 1.0$, 2.0, 3.0, 4.0, 5.0, 6.0, and 7.0. (a) Electrons with a BKII distribution, with $\kappa_e = 2.5$; (b) electrons with a bi-Maxwellian distribution; (c) electrons with a BKII distribution with isotropic temperatures, with $\kappa_e = 2.5$; (d) electrons with a Maxwellian distribution. All other parameters and conventions are the same as in Fig. 4.

value of the ion temperature ratio and given values of the electron kappa indexes, the occurrence of temperature anisotropy in the electron distribution leads to a significant decrease in the prevalence of the IC instability. The influence of the electron temperature anisotropy is seen in the decrease of the magnitude of the maximum growth rate, and also in the decrease in the size of the wave number region where the instability occurs. In Fig. 6(d), we show results obtained with electrons described by an isotropic Maxwellian distribution. It is seen that the results obtained are the same as those appearing in Fig. 6(b). These results lead to the conclusion that the shape of the electron distribution, when it features isotropy of temperatures, is not relevant for the IC instability associated to PBKI distributions for the ions. This conclusion is similar to that which has been obtained in the case of ions described by BKI distributions. For anisotropic electron distributions, the occurrence of the IC instability is reduced, with a reduction which is similar in the cases of bi-Maxwellian or PBKI distribution.

In Fig. 7, we show the normalized values of the imaginary part of the wave frequency which have been obtained in the case of ions described by anisotropic PBKII distributions with $\kappa_{i\parallel} = \kappa_{i\perp} = 2.5$, plotting the cases of $T_{i\perp}/T_{i\parallel} = 1.0$, 2.0, 3.0, 4.0, 5.0, 6.0, and 7.0. In Fig. 7(a), we can see the results obtained in the case of electrons also described by a PBKII distribution, with $\kappa_{e\parallel} = \kappa_{e\perp} = 2.5$, considering the same values of temperature anisotropy which have been considered for the ions, i.e., $T_{e\perp}/T_{e\parallel} = 1.0$, 2.0, 3.0, 4.0, 5.0, 6.0, and 7.0. Figure 7(a) is the same as Fig. 1(d). Figure 7(b) shows the results obtained with electrons described by an anisotropic Maxwellian distribution, also with the same values of the temperature ratio as in the ion distribution. Similarly to what was seen in the case of ions with BKII distribution in Fig. 5, the results appearing in Fig. 7(b) show that the magnitude of the maximum growth rate and the size of the region in wave number space where there is instability are significantly larger than those obtained in the case of electrons with PBKII distribution, seen in Fig. 7(a). The increase in the non thermal character in the electron distribution, due to the change from a Maxwellian to a PBKII distribution, contributes to decrease the prevalence of the IC instability, in the case of ions described by a PBKII distribution. This effect due to non thermal electron distribution is analogous to the effect observed in the case of BKII distributions. This result is in contrast with that obtained in the case of BKI and PBKI distributions, which appeared in Figs. 4(a), 4(b), 6(a), and 6(b). Figure 7(c) shows results obtained with electrons described by a PBKII distribution with isotropy of temperatures, with $T_e = T_{i\parallel}$. Figures 7(c) and 7(a) show that in the case of PBKII distributions for the ions the occurrence of temperature anisotropy in the electron distribution leads to a significant decrease in the magnitude of the growth rates

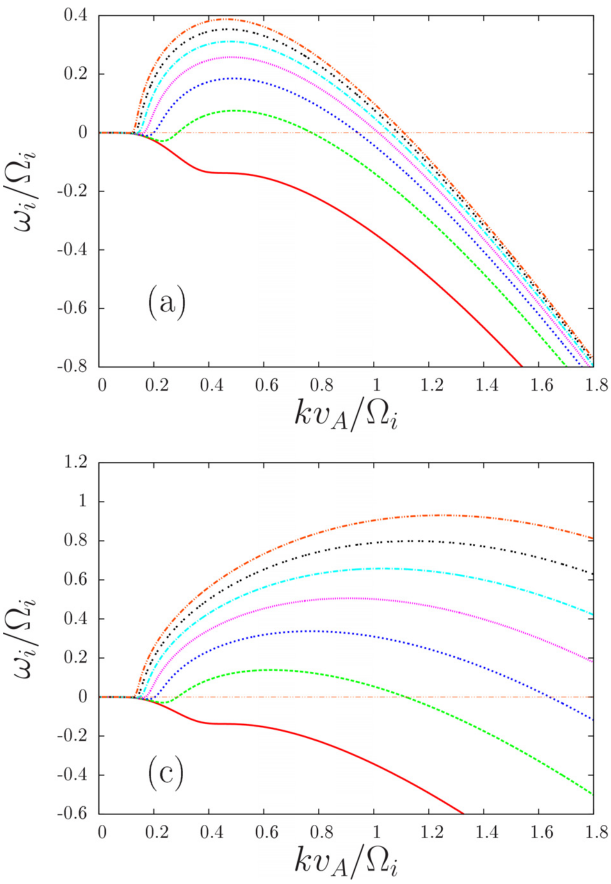

FIG. 6. Imaginary part of the frequency of waves in the ion-cyclotron mode *vs.* wave number. The ions are described by a PBKI distribution, with $\kappa_{i\perp} = \kappa_{i\parallel} = 2.5$ and $T_{i\perp}/T_{i\parallel} = 1.0, 2.0, 3.0, 4.0, 5.0, 6.0$, and 7.0. (a) Electrons with a PBKI distribution, with $\kappa_{e\perp} = \kappa_{e\parallel} = 2.5$; (b) electrons with a bi-Maxwellian distribution; (c) electrons with a PBKI distribution with isothermal temperatures, with $\kappa_{e\perp} = \kappa_{e\parallel} = 2.5$; (d) electrons with a Maxwellian distribution; all other parameters and conventions are the same as in Fig. 4.

of the IC instability, and to decrease in the size of the wave number region where occurs the instability. In this regard, the behaviour is similar to that obtained in the case of PBKI ion distributions, shown in Fig. 6, and also similar to the behaviour obtained considering BKI ion distributions, shown in Fig. 4. Finally, Fig. 7(d) shows results obtained with electrons described by an isotropic Maxwellian distribution. In comparison with the results show in Fig. 7(c), relative to the case of electrons with isotropic PBKII distribution, Fig. 7(d) shows a sizable increase in the range of wave numbers which present instability, as well as an increase in the magnitude of the growth rates of the instability. This behavior differs from that seen in the case of ions with BKI and BKII distributions, in Figs. 4(c), 4(d), 5(c), and 5(d), respectively. The behavior of the dispersion relation also differs from the behavior obtained in the case of PBKI distributions, shown in Figs. 6(c) and 6(d), regarding the dependence on the shape of the electron distribution.

IV. FINAL REMARKS

We have presented results of a numerical analysis of the dispersion relation for ion-cyclotron waves propagating along the ambient magnetic field. The analysis has been made considering different forms of isotropic and anisotropic kappa distributions for ions and electrons. We have concentrated the analysis on small values of the kappa index, because in this case the differences between kappa distributions and bi-Maxwellian distributions, and between different types of kappa distributions, should be more prominent. The focus has been on the influence of the particle distributions on the growth rate of the ion-cyclotron instability.

For the analysis of the instability, we have introduced generalized forms of anisotropic kappa distributions, which can be characterized as bi-kappa distributions or as product-bi-kappa distributions. For a suitable choice of parameters, these general distributions correspond to anisotropic forms of well known distributions of widespread use in the literature. In the case of bi-kappa distributions, these two different forms were called BKI and BKII distributions, and in the case of product-bi-kappa distributions the two different forms were called PBKI and PBKII distributions.

The results obtained have shown that for small value of the ion kappa index there is considerable effect of the form of the ion distribution on the magnitude and range of the ion-cyclotron instability. This significant influence of the type of kappa distribution has been seen for a wide range of values of the plasma beta, and is of significance for the interpretation of measurements made in the solar wind, where non thermal distributions with extended power-law tails, isotropic, and anisotropic, are commonly observed.

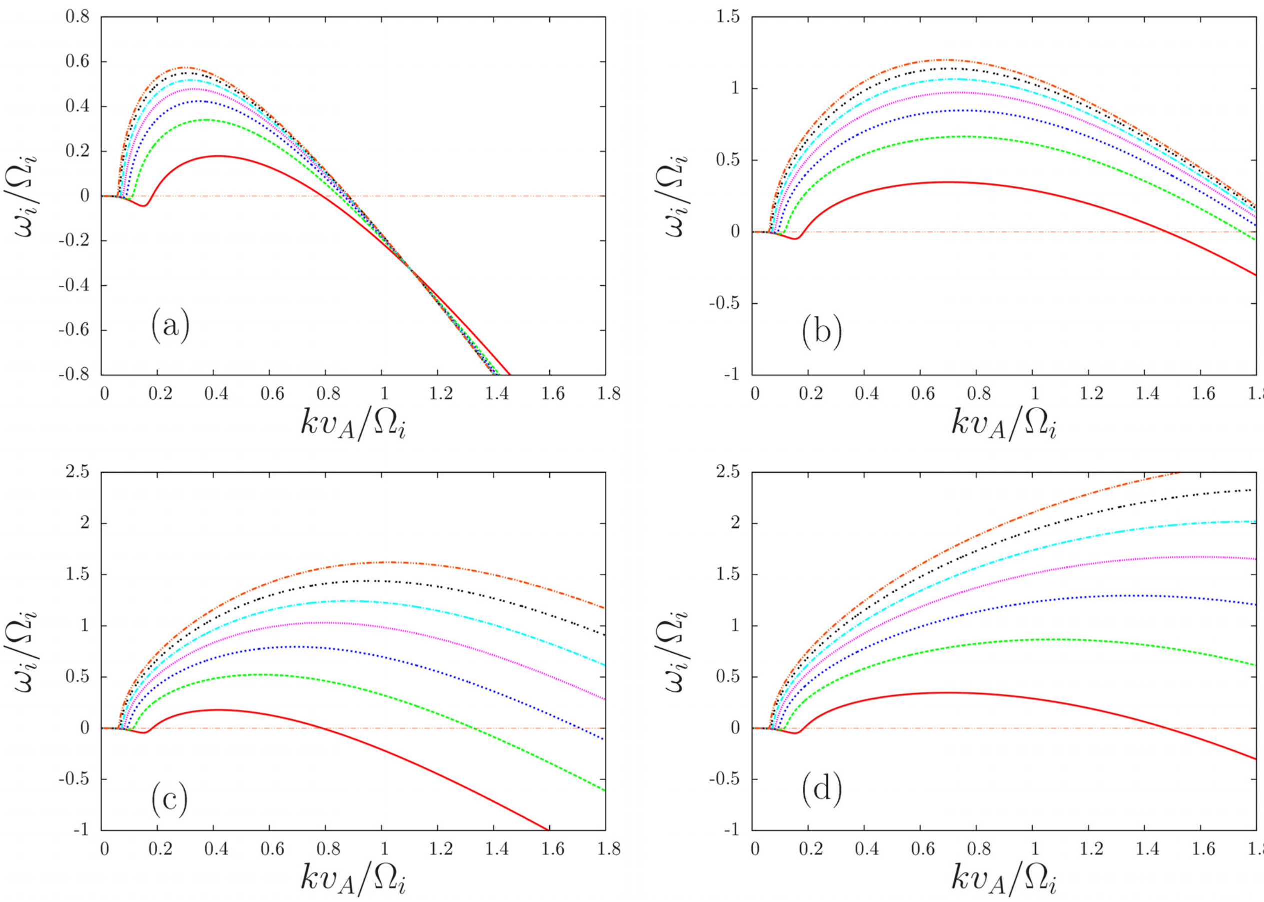

FIG. 7. Imaginary part of the frequency of waves in the ion-cyclotron mode *vs.* wave number. The ions are described by a PBKII distribution, with $\kappa_{i\perp} = \kappa_{i\parallel} = 2.5$ and $T_{i\perp}/T_{i\parallel} = 1.0$, 2.0, 3.0, 4.0, 5.0, 6.0, and 7.0. (a) Electrons with a PBKII distribution, with $\kappa_{e\perp} = \kappa_{e\parallel} = 2.5$; (b) electrons with a bi-Maxwellian distribution; (c) electrons with a PBKII distribution with isothermal temperatures, with $\kappa_{e\perp} = \kappa_{e\parallel} = 2.5$; (d) electrons with a Maxwellian distribution; all other parameters and conventions are the same as in Fig. 4.

The effect of the shape of the electron distribution has also been investigated. The results obtained have shown that in the case of BKI and PBKI ion distributions the change of the electron distribution, between the bi-Maxwellian form and the BKI or PBKI forms, leads to negligible modification in the growth rates of the ion-cyclotron instability.

It has also been seen that the occurrence of anisotropy in the electron distribution, with $T_{e\perp} > T_{e\parallel}$, contributes to decrease of the magnitude of the growth rates and the range of the IC instability, whether the electron distribution is a BKI, or PBKI, or bi-Maxwellian distribution.

These results corroborate and extend the findings of the previous analysis. For instance Ref. 20 discussed the effect of the electron anisotropy in the case of ions and electrons described by bi-Maxwellian distributions. It has been found that in the case $T_{e\perp}/T_{e\parallel} > 1$ the electrons contribute to decrease the growth rates of the IC instability, with effect which increases with the increase of the electron anisotropy.[20] Reference 20 also discussed the case of $T_{e\perp}/T_{e\parallel} < 1$, showing that in this case the anisotropy of the electron temperatures contribute to increase the growth rates of the IC instability.

Another example of previous work which is corroborated and extended by our results is that of Ref. 24, which utilizes anisotropic Maxwellian distribution for the ions and BK distribution for the electrons. According to the results obtained in Ref. 24, for $T_{e\perp}/T_{e\parallel} > 1$ the increase of non thermal features due to the decrease of the electron kappa index leading to smaller values of the IC growth rates. On the other hand, in the case of $T_{e\perp}/T_{e\parallel} < 1$, the decrease of the kappa index leads to increased values of the IC growth rates.[24]

In the case of BKII ion distributions, the change of the electron distribution between the bi-Maxwellian form and the BKII form, for isotropic electron temperatures, also leads to a negligible modification in the growth rates of the ion-cyclotron instability. On the other hand, if there is anisotropy in the electron temperatures, the increase in the non thermal character of the electron distribution, from the Maxwellian form to the BKII form, leads to very significant decrease of the magnitude of the growth-rate of the ion-cyclotron instability, and to decrease in the range of wave numbers where the instability occurs.

In the case of PBKII ion distributions, the pattern is different. For isotropic electron temperatures, we have obtained results showing that the change of the electron distribution, from a Maxwellian to a PBKII distribution, leads to decrease in the magnitude of the IC growth rates and to decrease of the region of wave number where the instability occurs. These results are in accordance with results found in Ref. 23, related to cases where ions and electrons were described by

PBK distributions of type II, with isotropic electron temperatures.

In addition, in the present paper we have also considered the case of anisotropic electron temperatures. It has been seen that, for both forms of electron distribution, PBKII and bi-Maxwellian, the occurrence of anisotropy in the electron temperatures causes reduction of growth rates and of range of the IC instability.

We may conclude that the results obtained show that the effect of the velocity distribution of electrons on the growth rates of the ion-cyclotron instability do not depend only on the existence or not of a power law feature in the electron distribution. It has been shown that the growth rates of the instability may also depend on details of the power-law electron distribution. These results present novel features, which have not been reported in previous analysis of the ion-cyclotron instability. The dependence of the growth rates of the ion-cyclotron instability on the detailed characteristics of the electron distribution therefore should be taken into account when kappa distributions are utilized for fitting of observed particle distributions and for the interpretation of observed features regarding waves and instabilities.

The results presented here can be relevant, for instance, for the explanation of the values of temperature anisotropy observed in the solar wind. The observed velocity distributions are affected by interactions of the particles with electrostatic/electromagnetic fields also present in the interplanetary environment. Since the intensity levels of the waves are dependent on the growth rates of the instabilities, and these, in turn, depend on the particle's velocity distributions, good understanding of the dependency of the growth rates on details of the velocity distribution is a requirement for the analysis of the wave intensity levels.

A comprehensive study of the wave-particle interactions in the solar wind, employing different combinations of particle distributions, can help to elucidate the observed temperature anisotropies, and the observed instability thresholds. Despite many analysis already made on these subjects, the situation is not yet completely understood. Study along this line is currently under development.

ACKNOWLEDGMENTS

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