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ASSESSING THE CONTRIBUTION OF GARCH-TYPE MODELS WITH REALIZED MEASURES TO BM&FBOVESPA STOCKS ALLOCATION

Porto Alegre
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Dissertação submetida ao Programa de Pós-Graduação em Economia da Faculdade de Ciências Econômicas da UFRGS, como requisito parcial para obtenção do título de Mestre em Economia, com ênfase em Economia Aplicada.

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ABSTRACT

In this work we perform an extensive backtesting study targeting as a main goal to assess the performance of global minimum variance (GMV) portfolios built on volatility forecasting models that make use of high frequency (compared to daily) data. The study is based on a broad intradaily financial dataset comprising 41 assets listed on the BM&FBOVESPA from 2009 to 2017. We evaluate volatility forecasting models that are inspired by the ARCH literature, but also include realized measures. They are the GARCH-X, the High-Frequency Based Volatility (HEAVY) and the Realized GARCH models. Their perfomances are benchmarked against portfolios built on the sample covariance matrix, covariance matrix shrinkage methods, DCC-GARCH as well as the naive (equally weighted) portfolio and the Ibovespa index. Since the nature of this work is multivariate and in order to make possible the estimation of large covariance matrices, we resort to the Dynamic Conditional Correlation (DCC) specification. We use three different rebalancing schemes (daily, weekly and monthly) and four different sets of constraints on portfolio weights. The performance assessment relies on economic measures such as annualized portfolio returns, annualized volatility, Sharpe ratio, maximum drawdown, Value at Risk, Expected Shortfall and turnover. We also account for transaction costs. As a conclusion, for our dataset the use of intradaily returns (sampled every 5 and 10 minutes) does not enhance the performance of GMV portfolios.

Keywords: Volatility forecasting. Realized volatility. High frequency data. Dynamic Conditional Correlation.

RESUMO

Neste trabalho realizamos um amplo estudo de simulação com o objetivo principal de avaliar o desempenho de carteiras de mínima variância global construídas com base em modelos de previsão da volatilidade que utilizam dados de alta frequência (em comparação a dados diários). O estudo é baseado em um abrangente conjunto de dados financeiros, compreendendo 41 ações listadas na BM&FBOVESPA entre 2009 e 2017. Nós avaliamos modelos de previsão de volatilidade que são inspirados na literatura ARCH, mas que também incluem medidas realizadas. Eles são os modelos GARCH-X, HEAVY e *Realized* GARCH. Seu desempenho é comparado com o de carteiras construídas com base na matriz de covariância amostral, métodos de encolhimento e DCC-GARCH, bem como com a carteira igualmente ponderada e o índice Ibovespa. Uma vez que a natureza do trabalho é multivariada, e a fim de possibilitar a estimação de matrizes de covariância de grandes dimensões, recorremos à especificação DCC. Utilizamos três frequências de rebalanceamento (diária, semanal e mensal) e quatro conjuntos diferentes de restrições sobre os pesos das carteiras. A avaliação de desempenho baseia-se em medidas econômicas tais como retornos anualizados, volatilidade anualizada, razão de Sharpe, máximo *drawdown*, Valor em Risco, Valor em Risco condicional e *turnover*. Como conclusão, para o nosso conjunto de dados o uso de retornos intradiários (amostrados a cada 5 e 10 minutos) não melhora o desempenho das carteiras de mínima variância global.

Palavras-chave: Previsão de volatilidade. Volatilidade realizada. Dados de alta frequência. *Dynamic Conditional Correlation*.

SUMÁRIO

1 INTRODUCTION

In this paper we estimate conditional covariance matrices using GARCH-type models which incorporate high-frequency data. Our goal is to investigate whether this type of model is able to generate economically superior portfolio allocations relative to models that only use daily data.

Few studies in the literature are dedicated to exploring the potential benefits of using high frequency data on asset allocation problems. Among them Fleming, Kirby and Ostdiek (2003), Q. Liu (2009), Hautsch, Lada M. Kyj and Malec (2015), Garcia, Medeiros and F. E. d. L. e. A. d. Santos (2014), Borges, J. Caldeira and Ziegelmann (2015), and J. F. Caldeira et al. (2017) compare the economic performance of portfolios based on daily and intradaily data. In these articles, a variety of specifications for the covariance matrix of the assets are evaluated. The contribution of our paper is to provide an empirical test for different specifications: GARCH-X, as estimated by R. Engle (2002b), the HEAVY model of Shephard and Sheppard (2010) and the Realized GARCH model of Hansen, Huang and Shek (2012).

Determining optimal portfolio allocation for a set of risky assets depends heavily on the accuracy of the covariance matrix estimation. Alexander (2008) points out that there is a considerable degree of model risk inherent to the construction of a covariance matrix. Thus, very different results can be obtained using two different statistical models, even if they are based on exactly the same data.

Standard volatility models rely on squared daily returns as a proxy for *ex-post* volatility. Although they constitute an unbiased estimator for the latent volatility factor, they may yield very noisy measurements. As an alternative, the literature has been signaling the potential benefits of using realized measures built from high frequency data in the construction of more accurate measures of volatility (see T. G. Andersen and Bollerslev (1998)).

The most popular of the so called realized measures is the realized variance, which is obtained as a sum of intraday squared returns. In theory, the highest the sampling frequency, the most accurate the measure is (see, for instance, T. G. Andersen and Bollerslev (1998), Barndorff-Nielsen and Shephard (2001), T. G. Andersen, Bollerslev, et al. (2001), and Areal and Taylor (2002)). However, in practice microstructure effects introduce bias (see, for instance, Bandi and Russell (2008)). Hence, the optimal sampling frequency is usually not the highest available, but rather some intermediate rate, ideally high enough to produce a volatility estimate with negligible sampling variation, yet low enough to avoid bias (T. ANDERSEN et al., 2000). In this context, some studies try to give guidance on the choice of the sampling frequency. Among them, L. Y. Liu, Patton and Sheppard (2015) find that realized variance with returns sampled every 1 or 5 minutes perform well for individual equities and equity indexes. Actually, they conclude that it is difficult to significantly beat 5-minute realized variance. Moreover, to reduce the estimate variance, it is also possible to average across sparsely sampled realized volatility measures, which is known as the subsampling estimator of Zhang, Mykland and Ait-Sahalia (2005).

Our empirical application uses BM&FBOVESPA's stock returns sampled every minute. In order to make the estimation of large covariance matrices possible, GARCH-X, HEAVY and Realized GARCH models are combined with the Dynamic Conditional Correlation (DCC) specification of R. Engle (2002a) and R. F. Engle and Sheppard (2001)[1]. This seems to be a natural choice as it allows a direct extension for univariate models.

We optimize global minimum variance portfolios because they rely exclusively on estimates of the covariance matrix. In comparison to the optimization of mean-variance portfolios, this approach has the advantage of reducing the uncertainty from the asset returns mean's estimation, as it is more difficult to estimate means than covariance matrices (see Candelon, Hurlin and Tokpavi (2012)). The performances of our portfolios are then compared to those of a set of benchmark models based on daily returns. The competing methods are evaluated in terms of a number of economic measures. Transaction costs and different sets of restrictions for the portfolio weights are also taken into account.

In the main, we found similar out-of-sample results for portfolios using high frequency data and for portfolios based on DCC-GARCH model (using daily data). The difference between their levels of annualized volatility was negligible and Sharpe ratios were not statistically different. DCC-GARCH, as well as the naive and market portfolios, generated higher annualized returns. Notwithstanding, the models that use high frequency data seemed to present some superiority in maximum drawdown, turnover, and cumulative returns over a partial period when compared to DCC-GARCH under a monthly rebalancing frequency.

[1] For the multivariate HEAVY model, Noureldin, Shephard and Sheppard (2012) adopt a BEKK-type parameterization.

2 GLOBAL MINIMUM VARIANCE PORTFOLIOS

Given a universe of N infinitely divisible assets and a vector of weights $\boldsymbol{\omega_t} = (\omega_{1,t}, \dots, \omega_{N,t})'$, the portfolio risk is measured by the portfolio variance $\sigma^2_{\omega,t} = \boldsymbol{\omega_t'} \boldsymbol{\Sigma}_t \boldsymbol{\omega_t}$, where $\boldsymbol{\Sigma}_t$ denotes the positive semi-definite variance-covariance matrix of assets' returns at day t. The global minimum variance (GMV) portfolio optimization objective is to find a combination of assets that, in a given period of time, produces the least possible risk:

$$\arg\min_{\boldsymbol{\omega_t}} \sigma^2_{\omega,t} = \boldsymbol{\omega_t'} \boldsymbol{\Sigma}_t \boldsymbol{\omega_t}. \tag{2.1}$$

It is possible to impose different types of constraints on the optimization problem, for instance:

a) the full investment constraint determines that the weights must sum to one and means that the investor uses all his budget for the N assets:

$$\sum_{i=1}^{N} \omega_{i,t} = 1 \tag{2.2}$$

b) the box constraint specify upper and lower bounds on the weights of the assets:

$$0 \leq \omega_{i,t} \leq \zeta. \tag{2.3}$$

This set of constraints would imply that no short positions are allowed, since the lower bound is set to zero. No leveraged positions would be allowed as well, once the upper bound is never greater than one. It was found that such restrictions yield a favorable out-of-sample performance (see, for instance, Frost and Savarino (1988)) or are associated with a reduced portfolio risk (see, for instance, Eichhorn, Gupta and Stubbs (1998) and Jagannathan and Ma (2003)).

In the absence of inequality constraints, the analytical solution is given by

$$\boldsymbol{\omega}_t = \frac{\Sigma_t^{-1}\iota}{\iota'\Sigma_t^{-1}\iota} \tag{2.4}$$

ι being a $N \times 1$ vector of ones. When the box constraints are considered, the problem must be solved numerically.

The variance-covariance matrix of returns is not directly observable. Therefore, to find an estimate for the optimal vector of weights, we replace $\boldsymbol{\Sigma}_t$ with an estimate $\hat{\boldsymbol{\Sigma}}_t$. Portfolio optimization depends crucially on the accuracy of covariance matrix estimation, which, in turn, depends on the specification of the model. The literature suggests that estimates based on high frequency data yield more precise results than those based on daily returns. In this paper, we aim at assess if a specific class of models - which perform a joint modeling of daily squared returns and realized measures built from high frequency data - is able to generate portfolios with a higher performance relative to commonly used models for the purpose of GMV portfolio optimization. In chapter 3 we review the covariance matrix estimation methods employed in this study. In chapter 5 we describe the backtesting study and the performance evaluation methodology.

3 COVARIANCE MATRIX ESTIMATION AND PREDICTION

This chapter briefly presents the different models for estimating the assets covariance matrix that will be used in this work. It is divided into two sections: section 3.1 presents specifications based on low frequency daily data and section 3.2 presents specifications based on high frequency intradaily data.

3.1 COVARIANCE MATRIX ESTIMATION AND PREDICTION WITH DAILY RETURNS

In what follows, we let $p_{i,t}$ denote the day t price of stock i, $i = 1,\dots,N$, and we let $y_{i,t} = \log(p_{i,t}) - \log(p_{i,t-1})$ denote it's daily return rate. We write $\boldsymbol{y}_t = (y_{1,t},\dots,y_{N,t})$ for the vector of daily returns.

3.1.1 Sample Covariance Matrix

A traditional method for estimating the covariance matrix of N assets is to compute the sample covariance matrix (SCM) based on historical return data. The sample covariance between the return rates of assets i and j is estimated by:

$$\bar{q}_{ij} = \frac{1}{T}\sum_{t=1}^{T}(y_{i,t} - \bar{y}_i)(y_{j,t} - \bar{y}_j) \tag{3.1}$$

where $\bar{y}_i$ is the sample average of stock i's returns. The sample covariance matrix is the best estimator in terms of actual fit to the data, as long as, under normality, this is the maximum likelihood estimator, which is consistent and asymptotically efficient. Nevertheless, it can perform poorly in a small sample. Although this method is easy to compute and is essentially free of model assumptions, it has a drawback that is the high number of parameters that need to be estimated. In a covariance matrix of N assets, there are $N\dot{(}N-1)/2$ pairs of covariances, which means that the number of parameters increases very rapidly with the number of assets.

When the number of assets is large relatively to the number of historical return observations available, the sample covariance matrix ends up carrying a lot of estimation error. Moreover, portfolio optimization algorithms require the inverse of this matrix, which amplifies the estimation error. Besides that, portfolio optimization maximizes the estimation error insofar as it is based on the more extreme estimated values to give greater or less weight to the assets. These assets are, in turn, the most likely to have large estimation error.

3.1.2 Shrinkage methods

The method of shrinkage for estimating the covariance matrix takes advantage of the strengths of the sample covariance matrix while seeks to mitigate its deficiencies, seeking to balance the trade off between bias and variance. The ideia behind this approach consists in obtaining a weighted average between the sample covariance matrix, $\bar{\boldsymbol{Q}}$, and an estimator $\boldsymbol{F}$

which imposes some structure on the covariance matrix of stock returns:

$$\hat{\lambda}\boldsymbol{F} + (1-\hat{\lambda})\bar{\boldsymbol{Q}}, \quad \hat{\lambda} \in (0,1) \tag{3.2}$$

This is a bayesian approach in which the matrix $\boldsymbol{F}$ works as the prior information we have about the true covariance matrix and λ represents the shrinkage intensity. Since the true covariance matrix is unobservable, our choice for $\boldsymbol{F}$ will, in general, result in a biased estimator due to misspecification. But, provided $\boldsymbol{F}$ involves only a small number of free parameters, it will be less variable than the sample covariance matrix, and the resulting shrinkage estimator will be relatively more efficient. It will also be invertible and well conditioned, which means that inverting it does not amplify estimation error.

In this paper, we use the following priors considered by Olivier Ledoit and Wolf (2003), Olivier Ledoit and Wolf (2004a), and Olivier Ledoit and Wolf (2004b):

a) the one factor model of William F. Sharpe (1963), where the factor is equal to the cross-sectional average of all the random variables. Henceforth;

b) the identity matrix;

c) a constant correlation matrix based on the assumption that all the pairwise correlations are identical and equal to the average of all the sample correlations, and that the variances are equal to the sample variances. Let $\boldsymbol{Q} = (q_{ij})$ denote the sample covariance matrix. The sample correlations between the returns on stocks i and j are given by $r_{ij} = q_{ij} \cdot \left(\sqrt{q_{ii}q_{jj}}\right)^{-1}$ and its average is given by $\bar{r} = 2\left((N-1)N\right)^{-1}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} r_{ij}$. The constant correlation matrix $\boldsymbol{F} = (f_{ij})$ is defined by means of the sample variances and the average sample correlation:

$$f_{ii} = q_{ii} \qquad \text{and} \qquad f_{ij} = \bar{r}\sqrt{q_{ii}q_{jj}}. \tag{3.3}$$

Henceforth, we will refer to these models as shrinkage-1fac, shrinkage-I and shrinkage-CC, respectively. The optimal weight $\hat{\lambda}$ in Eq. 3.2 is estimated as in Olivier Ledoit and Wolf (2003).

3.1.3 DCC-GARCH

The generalized autoregressive conditional heteroskedastic (GARCH) models of Bollerslev (1986) treat volatility as a time-dependent, persistent process. These models also account for frequently observed characteristics of financial returns series such as volatility clustering and leptokurticity of the marginal distributions.

In this paper, we assume an ARMA(1,1)-GARCH(1,1) specification for each of the series of stock returns, $y_{i,t}$, $i = 1, \ldots, N$, which we define by

$$y_{i,t} - \mu_{i,t} = \varepsilon_{i,t} = h_{i,t}^{1/2} z_{i,t} \tag{3.4}$$

$$\mu_{i,t} = \mu_i + \phi_i y_{i,t-1} + \theta_i \varepsilon_{i,t-1} \tag{3.5}$$

$$h_{i,t} = \omega_{G,i} + \alpha_{G,i} \varepsilon_{i,t-1}^2 + \beta_{G,i} h_{i,t-1} \tag{3.6}$$

$$\omega_{G,i} > 0, \alpha_{G,i}, \beta_{G,i} \geq 0$$

where $\mu_{i,t}$ and $h_{i,t}$ are the conditional mean and variance of $y_{i,t}$, and we assume that $z_{i,t}$ is independent and identically skewed Student's t distributed, that is, $z_{i,t} \sim skew\ t(\gamma_i, \nu_i)$, where γ_i represents the shape parameter and ν_i represents degrees of freedom.

Numerous different specifications have been proposed[1] to generalize univariate GARCH models to the multivariate domain (thus obtaining an estimator for the covariance matrix). In this article, we use the Dynamic Conditional Correlation (DCC) model of R. Engle (2002a) and R. F. Engle and Sheppard (2001). In this specification, univariate ARMA-GARCH models are estimated for each of the return series, and then, standardized residuals resulting from the first step are used to construct a time varying correlation matrix with a GARCH-like dynamic.

Consider the vector $\boldsymbol{y}_t$ of financial returns and the vector $\boldsymbol{\mu}_t$ of conditional expectations of $\boldsymbol{y}_t$ given the information set $\boldsymbol{I}_{t-1}$. We can write:

$$\boldsymbol{y}_t | \boldsymbol{I}_{t-1} = \boldsymbol{\mu}_t + \boldsymbol{\varepsilon}_t \tag{3.7}$$

$$\boldsymbol{\varepsilon}_t = \boldsymbol{H}_t^{1/2} \boldsymbol{z}_t \tag{3.8}$$

where $\boldsymbol{\varepsilon}_t$ is the vector of residuals of the process, $\boldsymbol{z}_t$ is an i.i.d. random vector with $E(\boldsymbol{z}_t) = 0$ and $Var(\boldsymbol{z}_t) = \boldsymbol{I}_N$ (the identity matrix of order N) and $\boldsymbol{H}_t^{1/2}$ is an $N \times N$ positive definite matrix such that $\boldsymbol{H}_t$ is the conditional covariance matrix of $\boldsymbol{y}_t$. Following Ghalanos (2015b), given the information set $\boldsymbol{I}_{t-1}$, $\boldsymbol{H}_t$ may be defined as:

$$Var(\boldsymbol{y}_t | \boldsymbol{I}_{t-1}) = Var_{t-1}(\boldsymbol{y}_t) = Var_{t-1}(\boldsymbol{\varepsilon}_t) \tag{3.9}$$

$$= \boldsymbol{H}_t^{1/2} Var_{t-1}(\boldsymbol{z}_t) \left(\boldsymbol{H}_t^{1/2} \right)' \tag{3.10}$$

$$= \boldsymbol{H}_t. \tag{3.11}$$

The DCC model makes use of the decomposition of the covariance matrix, $\boldsymbol{H}_t$, into standard deviations and correlations, so that the univariate and multivariate dynamics may be separated, easing the estimating process:

$$\boldsymbol{H}_t \equiv \boldsymbol{D}_t \boldsymbol{R}_t \boldsymbol{D}_t \tag{3.12}$$

[1] Literature reviews of multivariate GARCH models can be found in Bauwens, Laurent and Rombouts (2006) and Silvennoinen and Teräsvirta (2009).

where $\boldsymbol{D}_t = diag(\sqrt{h_{11,t}},\ldots,\sqrt{h_{NN,t}})$, $\sqrt{h_{ii,t}}$ are the conditional standard deviations from the univariate ARMA-GARCH models, and $\boldsymbol{R}_t$ is the time varying correlation matrix. The dynamic correlation structure is:

$$\boldsymbol{Q}_t = (1-\alpha-\beta)\bar{\boldsymbol{Q}} + \alpha(z_{t-1}z'_{t-1}) + \beta\boldsymbol{Q}_{t-1} \tag{3.13}$$

$$\alpha,\beta \geq 0,\ \alpha+\beta < 1$$

$$\boldsymbol{R}_t = (\boldsymbol{I}_N \odot \boldsymbol{Q}_t)^{-1/2}\boldsymbol{Q}_t(\boldsymbol{I}_N \odot \boldsymbol{Q}_t)^{-1/2} \tag{3.14}$$

where $\bar{\boldsymbol{Q}}$ is the unconditional covariance of the standardized residuals, $\boldsymbol{z}_t$, $\boldsymbol{I}_N$ is the identity matrix of order N and $\odot$ denotes the elementwise product of two conformable matrices. The typical element of $\boldsymbol{R}_t$ will be of the form

$$r_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t}q_{jj,t}}}. \tag{3.15}$$

The DCC model allows for two stage quasi maximum likelihood (QML) estimation. We assume that the assets returns are conditionally multivariate Student t distributed. So, in the first stage, conditional variances are estimated through univariate ARMA-GARCH models together with the shape parameter. Then, the conditional correlation parameters are estimated using the standardized residuals resulting from the first step. We also use the DCC specification to estimate covariance matrices based on high-frequency data. In this case, the conditional variances of the first stage are estimated through the models presented in section 3.2.

3.2 COVARIANCE MATRIX ESTIMATION AND PREDICTION WITH INTRA-DAILY RETURNS

Squared daily innovations are a widely used proxy for the unobserved variance but, although not biased, they may yield very noisy estimates and they may offer a weak signal on the current level of volatility. The implication is that models based solely on these proxies, as the GARCH models, are poorly suited for situations where volatility changes rapidly to a new level.

An alternative approach is to use realized measures built from high frequency data. Realized measures are estimators of asset price quadratic variation that have proven to be more precise compared to squared returns (see T. G. Andersen and Bollerslev (1998)). The most commonly used realized measure is the realized variance, computed as the sum of squared intradaily returns.

In theory, as the sampling frequency increases from daily to an infinitesimal interval, this measure converges to a genuine measurement of the latent volatility factor. In practice, employing very high sample frequencies is hampered not only by data limitations, but also because market microstructure effects induces serial autocorrelation in the observed returns, which biases the realized variance estimate. For this reason, the price process is often sampled sparsely in order to find a balance between increased accuracy and bias (a popular choice being 5-minute sampling).

Several robust estimators were proposed as a way of mitigating the effects of microstructure noise from very high frequency sampling and/or to improve efficiency of relatively sparse-sampled estimators. Among them, the subsample and average estimator suggested by Zhang, Mykland and Ait-Sahalia (2005) is based on splitting the full grid of observations within a day into K sub-grids, calculating a realized measure for each of the sub-grids, and finally averaging the estimators derived from the sub-grids.

Suppose that the intradaily log returns are sampled every Δ minutes. If we are interested in computing the realized variance using a lower sampling frequency, say, $K\Delta$ minutes, we may split our original sample into K non-overlapping sub-grids of size m: $\boldsymbol{y}_{i,t} = (y_{i,t,(k-1+jK)}), k = 1,\ldots,K, j = 0,\ldots,m$. The method of Zhang, Mykland and Ait-Sahalia (2005) consists of calculating the realized volatility for each of the sub-grids:

$$RV_t^k = \sum_{j=1}^{m} y_{i,t,(k-1+jK)}^2, \quad k = 1,\ldots,K, \tag{3.16}$$

and then taking the average of those K estimators:

$$RV_t^{avg} = \frac{1}{K}\sum_{k=1}^{K} RV_t^k \tag{3.17}$$

The increasing availability of intradaily data has given rise to research dedicated to predicting volatility through time-series models of daily realized variance. A strand of the literature focuses on incorporating the rich information present in these kind of data into a GARCH modeling framework, performing a joint modeling of daily returns and realized measures. Examples of such models in the univariate case are the GARCH-X of R. Engle (2002b), the High-Frequency-Based Volatility (HEAVY) model of Shephard and Sheppard (2010) and the Realized GARCH of Hansen, Huang and Shek (2012).

In the following sections, we present an overview of these models. They are brought to a multivariate framework via the DCC model. This choice is due to its parsimony in relation to the number of parameters that need to be estimated, which allow for relatively easy estimation of high dimensional covariance matrices and also allow for easy interpretation of the model parameters.

3.2.1 GARCH-X

The GARCH-X refers to a model similar to the standard GARCH, but the equation for the conditional variance includes an exogenous variable x_t, which can be a realized measure, for instance the lagged realized variance as in R. Engle (2002b). In an ARMA(1,1)-GARCH-X(1,1) specification we would substitute equation (3.6) for:

$$h_{i,t} = \omega_{X,i} + \alpha_{X,i}\varepsilon_{t-1}^2 + \beta_{X,i}h_{i,t-1} + \gamma_{X,i}x_{i,t-1} \tag{3.18}$$
$$\omega_{X,i} > 0, \alpha_{X,i}, \beta_{X,i}, \gamma_{X,i} \geq 0$$

Within the GARCH-X framework, no effort is paid to explain the variation in the realized measure, so these models are referred by Hansen, Huang and Shek (2012) as partial models that have nothing to say about returns and volatility beyond a single period into the future. The parameters of this model are estimated by means of the maximum likelihood estimator.

3.2.2 HEAVY

The HEAVY models, proposed by Shephard and Sheppard (2010), are made up of a system with two equations. In its most basic linear specification, the model can be represented as (in addition to equations (3.4) and (3.5)):

$$Var(y_{i,t}|\mathscr{F}^{HF}_{t-1}) = h_{i,t} = \omega_{1,i} + \beta_{1,i}h_{i,t-1} + \gamma_{1,i}x_{i,t-1}, \tag{3.19}$$
$$\omega_{1,i}, \gamma_{1,i} \geq 0,\ \beta_{1,i} \in [0,1)$$
$$E(x_{i,t}|\mathscr{F}^{HF}_{t-1}) = \chi i,t = \omega_{2,i} + \beta_{2,i}\chi_{i,t-1} + \gamma_{2,i}x_{i,t-1}, \tag{3.20}$$
$$\omega_{2,i}, \beta_{2,i}, \gamma_{2,i} \geq 0,\ \beta_{2,i} + \gamma_{2,i} \in [0,1)$$

where $\mathscr{F}^{HF}_{t-1}$ denote the information set generated by high frequency data up to time $t-1$.

equation (3.19) models the close-to-close conditional variance, while 3.20 models the conditional expectation of the open-to-close variation. These equations could be extended to include the variable ε^2_{t-1}, in such a way that the equation for the conditional variance would be the same as in the GARCH-X model. Despite of this, the authors found the coefficients on these variables to be non statistically significant in most cases.

The inclusion of equation (3.20) allows for multistep-ahead forecasting. For one-step ahead forecasts of volatility, we only need equation (3.19). The models also allow for both mean reversion and momentum effects, and they adjust quickly to structural breaks in the level of the volatility process. The estimation of each of the equations is performed separately by means of quasi-likelihood. This is convenient, as existing GARCH type code can simply be used in this context.

3.2.3 Realized GARCH

The Realized GARCH model of Hansen, Huang and Shek (2012) is close in structure to the HEAVY model, but it treats the dynamics of the realized measure differently. While the HEAVY model postulates GARCH-type dynamics for both the conditional variance and the realized measure, the Realized GARCH model relates the realized measure to the latent volatility and includes asymmetric reaction to shocks.

The Realized GARCH model in a simple log-linear specification of order (1,1) is given by (in addition to equations (3.4) and (3.5)):

$$\log h_{i,t} = \omega_{R,i} + \beta_{R,i}\log h_{i,t-1} + \gamma_{R,i}\log x_{i,t-1}, \tag{3.21}$$
$$\log x_{i,t} = \xi_i + \phi_i \log h_{i,t} + \tau(z_{i,t}) + u_{i,t} \tag{3.22}$$

where $h_{i,t}$ is the conditional variance of asset i returns, $x_{i,t}$ is a realized measure of volatility, $z_{i,t}$ is independent and identically skewed Student's t distributed and $u_{i,t} \sim$ i.i.d (0, $\sigma^2_{u,i}$), with $z_{i,t}$ and $u_{i,t}$ being mutually independent. The leverage effect is captured by the $\tau(\cdot)$ function. A simple yet versatile specification for this function is

$$\tau(z_{i,t}) = \tau_{1,i} z_{i,t} + \tau_{2,i}(z^2_{i,t} - 1). \tag{3.23}$$

Equation (3.22) provides a simple way to model the joint dependence between $y_{i,t}$ and $x_{i,t}$ through the presence of $z_{i,t}$. A logarithmic specification for this equation seems natural because equation (3.4) implies that $\log y^2_{i,t} = \log h_{i,t} + \log z^2_{i,t}$ and a realized measure is in many ways similar to the squared return, $y^2_{i,t}$, albeit a more accurate measure of h_t. A logarithmic form for 3.4 makes it convenient to specify the GARCH equation with a logarithmic form, because this induces a convenient ARMA structure. Besides that, an obvious advantage of using a logarithmic specification is that it automatically ensures a positive variance. The parameters are estimated by means of quasi maximum likelihood.

4 EMPIRICAL LITERATURE REVIEW

This section reviews previous findings of empirical applications comparing the use of daily and intradaily data in asset allocation problems. Fleming, Kirby and Ostdiek (2003) pioneer the analysis of the potential benefits of incorporating intradaily data in the context of portfolio allocation. In their study, conditional covariance matrices using rolling estimators of the form analyzed by Foster and Nelson (1996) and Andreou and Ghysels (2002) are constructed using 5-minute and daily returns on S&P500 index, treasury bonds and gold. These two sets of estimates are used in a volatility-timing strategy for portfolio optimization with daily rebalancing and the authors find that a mean-variance efficient investor would be willing to pay 50 to 200 basis points per annum for being able to use daily covariance matrix forecasts based on intradaily instead of daily returns.

A different focus is used by Pooter, Martens and Dijk (2008): the authors are concerned with determining the optimal sampling frequency as judged by the performance of the corresponding portfolios. The portfolios' weights are determined based on forecasts of the daily conditional covariance matrix constructed using the realized covariance matrix with the sampling frequency of intradaily returns ranging from 1 to 130 minutes. They find that, for global minimum risk portfolios, the optimal sampling frequency for the S&P100 constituents ranges between 30 and 65 minutes. They also conclude that selecting the appropriate sampling frequency appears to be much more important than choosing between different bias and variance reduction techniques for the realized covariance matrices.

A question that motivates the study of Q. Liu (2009) is how and when one can benefit from using high frequency data in what it comes to portfolio optimization. Using the framework of a professional investment manager who wishes to track the S&P 500 with the 30 Dow Jones Industrial Average stocks, the author finds that the benefits depend upon the rebalancing frequency and estimation window. If the portfolio is rebalanced monthly and the manager has access to at least the previous 12 months of data, daily returns have the potential to perform as well as high frequency data. However, if the manager rebalances daily or has less than a 6-month estimation window, intradaily returns perform better. The analysis is based on forecasts of the conditional covariance matrix based on 5-minute and daily returns.

An empirical application of much higher dimension was presented by Hautsch, Lada M. Kyj and Malec (2015). The authors construct global minimum variance portfolios based on the 400 constituents of the S&P500 with the longest continuous trading history during the sample period between January 2006 and December 2009. High frequency based covariance matrix predictions are obtained by applying a blocked realized kernel estimator as in Hautsch, Lada M Kyj and Oomen (2012) with different smoothing windows, various regularization methods and two forecasting models. They use the highest frequency possible for the returns and find that high frequency based predictions yield a significantly lower portfolio volatility than methods employing daily returns, as multivariate GARCH, rolling-window sample covariance matrix and RiskMetrics approaches.

Studies dealing with high frequency data for the Brazilian stock market are scarce in the literature - a survey can be found in Perlin and Ramos (2016). Even scarcer are applications in volatility or in portfolio selection and, as far as we concern, none analyzed the performance of GARCH-type models that carry out the joint modeling of returns and realized measures in portfolio allocation. Garcia, Medeiros and F. E. d. L. e. A. d. Santos (2014) evaluate the economic gains from mean-variance portfolio optimization based on a covariance matrix estimated by means of a multivariate version of the HAR-RV model. Their database encompassed the twenty most liquid stocks from BM&FBOVESPA and covered the period from February 2006 to January 2011. They find that economic gains are attained when using a high target return (15% - 17.5%), unconditional mean is used as reference for expected returns through the whole sample (controlling for estimation risk) and no restriction to short selling is imposed. Borges, J. Caldeira and Ziegelmann (2015) perform an application similar to that in Q. Liu (2009) and Pooter, Martens and Dijk (2008), but they estimate additional models for the covariance matrix (both with daily and intradaily data) as the scalar variance targeting VECH and the Multivariate Realized Kernel of Barndorff-Nielsen, Hansen, et al. (2011), and they focus on the Brazilian stock market, using the 30 most liquid stocks traded on BM&FBOVESPA in the period from February 2009 to December 2011. Their results point to a superior performance of the scalar vt-VECH model based on high frequency data in terms of lower portfolio risk and turnover and higher Sharpe ratio. More specifically, returns sampled each 5 minutes generated portfolios with lower turnovers while returns sampled each 90 and 120 minutes generated portfolios with lower risk and higher Sharpe ratios. Besides that, as in Pooter, Martens and Dijk (2008), using an appropriate sampling frequency seemed to be more relevant than using models which are robust to microstructure effects. Akin to this study is J. F. Caldeira et al. (2017), who find that the 5-min sampling interval seems to be more appropriate to generate portfolios with lower portfolio risk and turnover.

5 BACKTESTING STUDY

This chapter details our empirical application. Section 5.1 describes the construction of our dataset. Section 5.2 explains how the portfolio optimization process is performed using a moving window. Section 5.3 presents the metrics used to compare portfolios performances and finally section 5.4 presents the results of our study.

5.1 DATA AND CLEANING

In our empirical analysis we have used high frequency data on transaction prices for assets listed on the BM&FBOVESPA - securities, commodities and futures exchange. The sample period ran from December 18, 2009 to February 17, 2017, delivering 1761 distinct days.

We have used functions from the "GetHFData" R package by Perlin and Ramos (2016) in different stages of the data cleaning process. In the first step, we discarded any record with a timestamp outside the regular marketing opening hours. Throughout our sample, the official BM&FBOVESPA trading day have changed ten times, being the opening time at 10 a.m. or at 11 a.m. and the closing time between 16:55 and 17:55. On holy Wednesday the stock exchange operates half day. Those days were discarded in order to avoid outliers. The same was done on dates in which there were delays due to technical failure or matches of the Brazil national team in the 2014 FIFA World Cup, resulting in the elimination of 13 observations. Canceled trades were also deleted.

In order to construct homogeneous time series from the raw data, prices were sampled every 1, 5 and 10 minutes using the last tick approach, which means that we set up a time grid and selected the last tick within each interval. This resulted in 422, 84 and 43 daily observations for the 1, 5 and 10 minute frequency respectively, on average, since it depended on the length of the trading day. If the time grid was empty, we used the previous point interpolation, as proposed by Dacorogna et al. (2001).

From the available assets, we selected those 41 that had non-zero returns in more than eighty percent of the time intervals at a sampling frequency of 5 minutes. The ticker symbols, names and sector for each of the assets are provided in the appendix (table A.1).

We adjusted our historical price data to remove gaps caused by stock splits and reverse stock splits in order to prevent misleading signals. Other adjustments were made to remove smaller gaps caused by different types of corporate actions, such as dividends, bonus issues of shares, rights issues of shares, interest on equity and spinoffs. Such adjustments ensure that all the resulting price movements are caused solely by market forces.

Based on the cleaned and adjusted prices data, we computed daily realized variances with subsampling as outlined in section 3.2. We used 5 minutes and 10 minutes returns subsampled at 1 minute frequency. The daily closing price was used to compute squared close to close log returns. They were not part of our intradaily data series, since they are defined by an auction, after the regular trading hours.

As mentioned earlier, when realized variance is implemented in practice, the price process is often sampled sparsely to strike a balance between increased accuracy from using higher frequency data and the adverse effects of microstructure noise (L. Y. LIU; PATTON; SHEPPARD, 2015). Beyond this reason, our choice of sampling frequency is related to the liquidity level of the shares listed in BM&FBOVESPA. Table 1 shows that, at one minute frequency, no stock has more than 80 percent of non-zero returns. At 5 minutes frequency this amount increases to 41 assets and, at the 10 minutes frequency, the number of stocks increases to 60.

Table 1 – Number of stocks listed at BM&BOVESPA with 50% to 90% of non-zero returns for different sampling frequencies

Non-zero returns	1 minute	5 minutes	10 minutes
50 %	58	109	124
60 %	35	86	105
70 %	12	64	85
80 %	0	41	60
90 %	0	3	20

Source: Own elaboration from research data (2018).
Note: The total number of stocks traded in the sample period was 1,635.

Tables A.2 and A.3 provide summary statistics (mean, minimum, maximum, standard deviation, skewness and excess kurtosis) for the daily - close-to-close - and intradaily - open-to-close - returns. They indicate that the assets reproduce stylized facts in the financial time series literature, such as average return near zero and heavy tails.

5.2 SETUP

The objective of our empirical application is to study the out-of-sample perfomance of global minimum variance portfolios constructed from a handful of covariance matrix estimation methods and to assess their sensitivity with respect to the use of high frequency data. For this purpose, we split our original sample into in-sample and out-of sample period. The former consisted of approximately 5 years of data (1,254 days), starting on December 21, 2009, and the latter consisted of approximately 2 years of data (506 days), starting on February 2, 2015 and ending on February 17, 2017.

Using the first five years of data, we estimated the covariance matrix and obtained one-step-ahead forecasts according to each of the methodologies described in sections 3.1 and 3.2. We then solved the portfolio optimization problem described in section 1 by plugging in eq. (2.1) each of the forecasted covariance matrices, thus obtaining a set of optimal weights corresponding to each of the discussed methods. Finally, we repeated this process rolling the estimation window 1 day ahead until the end of the data set was reached. By the end of this process, we had 506 out-of-sample observations for each portfolio, which were used for the purpose of performance evaluation. In addition to the optimal portfolios, we also computed the equally weighted, or naive, or even "$1/N$" portfolio, which assign equal weights for all assets. In an universe of N assets, each one would be assigned with a weight equal to $\omega_i = 1/N \; \forall \; i$.

The backtest was carried out with daily, weekly and monthly rebalancing frequency. Initially, we only applied the "full-investment" and the "long-only" constraints to the optimization problem. In section 5.6 we conduct some robustness checks, adding box constraints on the optimal vector of weights.

The analysis was performed through software R. To calculate realized measures, we used the "highfrequency" package by Boudt, Cornelissen and Payseur (2014). To estimate and forecast covariance matrices, we used the "RiskPortfolios" and the "rmgarch" packages by Ardia, Boudt and Gagnon-Fleury (2017) and Ghalanos (2015a), respectively. For portfolio optimization, we used the "PortfolioAnalytics" package by B. G. Peterson and Carl (2015). Finally, for performance evaluation and inference, we used "PerformanceAnalytics" and "PeerPerformance" packages by B. G. Peterson and Carl (2014) and Ardia and Boudt (2017), respectively.

5.3 PERFORMANCE EVALUATION

The economic evaluation of covariance forecasts was made by means of the out-of-sample performance of the portfolios formed using these forecasts as input. Once we get the optimal portfolio weights through time, we can multiply them by the stocks' returns in order to obtain the time series of portfolio returns: $y^p_{t+1} = \sum_{i=1}^{N} \omega_{i,t} y_{i,t+1}$. Letting T denote the sample size and τ the size of the estimation window, then $T - \tau$ will be the size of out-of-sample data. For each of the optimal portfolios, annualized average returns and standard deviation, Sharpe ratio, modified Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) and average turnover were computed. Below we present these measures in some detail.

5.3.1 Annualized average return

According to the equation below, we calculate the geometric average of daily portfolio returns, y^p_t, over the out-of-sample period, $T - \tau$, and we adopt the business/252 day count convention is to scale the average return to an anual basis.

$$\hat{\mu}^p_{ann} = \left(\prod_{t=\tau+1}^{T} \left(1 + y^p_t\right) \right)^{\frac{252}{T-\tau}} - 1. \tag{5.1}$$

5.3.2 Annualized standard deviation

The annualized standard deviation is computed as

$$\hat{\sigma}^p_{ann} = \sqrt{\frac{\sum_{t=\tau+1}^{T} \left(y_t^p - \hat{\mu}^p\right)^2}{T-\tau-1}} \cdot \sqrt{252} \tag{5.2}$$

where $\hat{\mu}^p$ denotes the unconditional mean of daily portfolio returns, y_t^p, along the out-of-sample period, $T-\tau$.

5.3.3 Sharpe ratio

The Sharpe ratio (William F SHARPE, 1966) represents the risk/reward tradeoff of a portfolio and it is useful to compare portfolios with different returns and levels of risk. It can be described as the return per unit of risk:

$$SR = \frac{\hat{\mu}^p}{\hat{\sigma}^p} \tag{5.3}$$

We use the method of Oliver Ledoit and Wolf (2008) to test the null hypothesis of equal Sharpe ratios between each of the GMV portfolios and the market portfolio (Ibovespa). The method is based on constructing a two-sided bootstrap confidence interval with confidence level $1-\alpha$ for the difference between two Sharpe ratios. If this interval does not contain zero, then the null hypothesis is rejected at the significance level α. To generate bootstrap data, we use the studentized circular block bootstrap of Politis and Romano (1992), resampling 1000 times blocks of 5 pairs of Sharpe ratios.

5.3.4 Maximum drawdown

Any time the cumulative returns fall below its maximum, it is a drawdown. The maximum drawdown is the worst cumulative loss ever sustained by the portfolio. It is measured as a percentage of the maximum cumulative return:

$$\frac{(\text{Peak value - Trough value})}{\text{Peak value}} \tag{5.4}$$

5.3.5 Value-at-Risk and Conditional Value-at-Risk

VaR and CVaR (also known as Expected Shortfall (ES) or Expected Tail Loss (ETL)) are industry standards for measuring downside risk. VaR is the negative value of the portfolio return such that lower returns will only occur with at most a probability level α (which we choose to be 1%):

$$VaR_\alpha(X) = min\{z|F_X(z) \geq \alpha\} \text{ for } \alpha \in]0,1[\tag{5.5}$$

A variety of estimation methods were proposed in the literature, such as those based on Monte Carlo Simulation, the empirical or the Gaussian distribution function. If a normal distribution is assumed for the returns, then the VaR can be computed as:

$$VaR_\alpha = -\mu - \sigma\Phi^{-1}(\alpha) \tag{5.6}$$

where $\Phi^{-1}(\cdot)$ is the quantile function of the standard normal distribution. We follow Zangari (1996), who proposed a parametric method for VaR estimation that corrects the Gaussian VaR for skewness and excess kurtosis in the return series. It relies on adjusting the Gaussian quantile function for higher moments using the Cornish-Fisher expansion (CORNISH; FISHER, 1938):

$$mVaR_\alpha = VaR_\alpha - \sigma\left(\frac{(q_\alpha^2 - 1)S}{6} + \frac{(q_\alpha^3 - 3q_\alpha)K}{24} - \frac{(2q_\alpha^3 - 5q_\alpha)S^2}{36}\right) \tag{5.7}$$

where S denotes skewness, K denotes excess kurtosis and q_α denotes the quantile of a standard normal random variable with level α. In the case of a normally distributed random variable, $mVaR_\alpha = VaR_\alpha$.

The CVaR attempts to measure the magnitude of the average loss exceeding the VaR. For $\alpha \in]0,1[$:

$$CVaR_\alpha(X) = \int_{-\infty}^{+\infty} z dF_X^\alpha(z) \tag{5.8}$$

$$F_X^\alpha(z) = \begin{cases} 0 & \text{when } z < VaR_\alpha(X) \\ \frac{F_X(z)-\alpha}{1-\alpha} & \text{when } z \geq VaR_\alpha(X) \end{cases} \tag{5.9}$$

We follow Boudt, B. Peterson and Croux (2008), who derived a definition for modified CVaR that, like modified VaR, uses asymptotic expansions to adjust the Gaussian distribution function for the non-normality in the observed return series.

5.3.6 Turnover

The turnover can be interpreted as the average percentage of wealth traded on each period. The average turnover is measured as:

$$TO = \frac{1}{T-\tau-1}\sum_{t=\tau}^{T-1}\sum_{i=1}^{N}(|\omega_{i,t+1} - \omega_{i,t+}|) \tag{5.10}$$

where $\omega_{i,t+1}$ are the weights after rebalancing and $\omega_{i,t+}$ are the weights just before rebalancing.

The turnover may be considered as a proxy for portfolio transaction costs. Each time the portfolio is rebalanced, i.e., shares are traded, there are costs such as brokerage, emoluments, taxation and bid-ask spread. In this study, we assume that the costs are proportional to the amount traded. The portfolio return net of transaction costs on time t is calculated as:

$$y_t^{p,net} = (1 - c\cdot\text{turnover}_t)(1 + y_t^p) - 1 \tag{5.11}$$

where c is the proportional fee. In order to assess the impact of transaction costs on the performance of the optimal portfolios, we consider four alternative scenarios for the value of c: 0, 15, 30 and 45 basis points (bp). It is close to the scenarios of 0, 20 and 50 basis points as defined in Ferreira and A. A. P. Santos (2017).

5.4 RESULTS

Table 3 reports the out-of-sample performance of global minimum variance portfolios constructed from the covariance matrix estimation and forecasting methods presented in sections 3.1 and 3.2 for daily, weekly and monthly rebalancing frequencies. The "full-investment" and "long-only" constraints were applied to these portfolios, which means the weights could range from zero to one. For the sake of comparison, table 3 reports also the performance of the Ibovespa index, which is designed to gauge the stock market's average performance tracking changes in the prices of the more actively traded and better representative stocks of the Brazilian stock market.

Table 3 – Out-of-sample performance of global minimum variance portfolios with $0 < w_{i,t} < 1$

(continues)

	$\hat{\mu}^p_{ann}$	$\hat{\sigma}^p_{ann}$	SR	MD	VaR	CVaR	TO
Ibovespa	16.09%	24.61%	0.0459	36.66%	-3.57%	-4.41%	-
Daily rebalancing							
Naive	17.70%	25.68%	0.0480	42.13%	-3.78%	-4.70%	1.67%
SCM	1.95%	17.52%	0.0125	25.23%	-2.59%	-3.08%	1.65%
Shrinkage-1fac	1.95%	17.52%	0.0125	25.01%	-2.59%	-3.08%	1.65%
Shrinkage-I	2.03%	17.52%	0.0127	25.31%	-2.60%	-3.09%	1.66%
Shrinkage-CC	1.80%	17.57%	0.0119	25.42%	-2.60%	-3.09%	1.64%
GARCH	13.69%	15.45%	0.0572	19.11%	-2.43%	-3.35%	15.14%
R-GARCH 5m	8.07%	15.13%	0.0371	21.19%	-2.37%	-3.16%	19.92%
R-GARCH 10m	7.10%	15.23%	0.0332	22.58%	-2.40%	-3.22%	21.28%
GARCH-X 5m	2.50%	15.91%	0.0148	29.20%	-2.59%	-3.54%	24.93%
GARCH-X 10m	5.53%	15.95%	0.0263	28.35%	-2.60%	-3.55%	23.89%
HEAVY 5m	3.83%	16.08%	0.0198	26.63%	-2.62%	-3.46%	34.26%
HEAVY 10m	4.57%	16.14%	0.0225	23.75%	-2.63%	-3.47%	34.49%
Weekly rebalancing							
Naive	17.69%	25.69%	0.0480	42.28%	-3.76%	-4.65%	0.76%
SCM	1.80%	17.47%	0.0119	25.58%	-2.59%	-3.07%	0.78%
Shrinkage-1fac	1.82%	17.47%	0.0120	25.37%	-2.59%	-3.07%	0.78%
Shrinkage-I	1.85%	17.47%	0.0121	25.67%	-2.59%	-3.08%	0.79%
Shrinkage-CC	1.64%	17.53%	0.0114	25.78%	-2.60%	-3.08%	0.78%
GARCH	12.16%	15.50%	0.0515	19.40%	-2.34%	-3.07%	7.62%
R-GARCH 5m	4.83%	15.40%	0.0241	22.26%	-2.41%	-3.12%	7.74%
R-GARCH 10m	3.04%	15.62%	0.0170	23.17%	-2.52%	-3.33%	8.53%
GARCH-X 5m	2.75%	16.15%	0.0157	28.16%	-2.62%	-3.60%	10.07%
GARCH-X 10m	4.15%	16.17%	0.0209	28.33%	-2.58%	-3.62%	9.70%
HEAVY 5m	4.48%	15.84%	0.0224	25.40%	-2.48%	-3.18%	7.39%
HEAVY 10m	3.20%	15.90%	0.0175	23.48%	-2.54%	-3.30%	8.39%
Monthly rebalancing							
Naive	18.17%	25.91%	0.0480	42.21%	-3.87%	-4.93%	0.38%
SCM	1.75%	17.58%	0.0118	25.87%	-2.60%	-3.10%	0.38%
Shrinkage-1fac	1.76%	17.58%	0.0118	25.67%	-2.60%	-3.09%	0.37%
Shrinkage-I	1.80%	17.58%	0.0119	25.97%	-2.61%	-3.10%	0.38%
Shrinkage-CC	1.57%	17.64%	0.0111	26.08%	-2.61%	-3.11%	0.37%
GARCH	7.22%	15.74%	0.0329	26.54%	-2.43%	-3.09%	2.97%
R-GARCH 5m	3.87%	15.65%	0.0202	23.95%	-2.43%	-3.07%	2.87%
R-GARCH 10m	2.30%	15.73%	0.0140	24.16%	-2.48%	-3.18%	3.04%

Table 3 – Out-of-sample performance of global minimum variance portfolios with $0 < w_{i,t} < 1$

(conclusion)

	$\hat{\mu}^p_{ann}$	$\hat{\sigma}^p_{ann}$	SR	MD	VaR	CVaR	TO
GARCH-X 5m	3.97%	16.66%	0.0199	32.09%	-2.81%	-3.97%	3.53%
GARCH-X 10m	5.76%	16.51%	0.0266	30.57%	-2.72%	-3.89%	3.40%
HEAVY 5m	5.24%	16.01%	0.0251	25.12%	-2.49%	-3.19%	2.46%
HEAVY 10m	5.39%	16.37%	0.0253	22.17%	-2.66%	-3.39%	2.76%

Source: Own elaboration from research data (2018).

Notes: This table reports the out-of-sample performance of global minimum variance portfolios whose weights were submitted to the constraint $0 < \omega_{i,t} < 1$. It refers to the period from 02-02-2015 to 02-17-2017. The rows report the results for different estimators of the covariance matrix. R-GARCH refers to the Realized GARCH model. The expression 5m or 10m next to the model name indicates the sample frequency of the intradaily returns. The columns report different performance measures: $\hat{\mu}^p_{ann}$ represents annualized returns, $\hat{\sigma}^p_{ann}$ are the annualized volatilities, SR are the Sharpe ratios, MD are the maximum drawdowns, VaR and CVaR were calculated with a confidence interval of 99% and TO are the turnovers. All these measures were calculated as in section 5.3. Transaction costs are assumed to be zero.

All of the portfolios optimized in this study had the common objective of minimization of risk, measured by the standard deviation of returns. Therefore, it is natural to start the performance analysis by risk measures. For all rebalancing frequencies the lowest levels of annualized volatility were achieved by the portfolios based on the Realized GARCH model, in special when we used 5-minute sampling frequency for the realized measure (15.13%, 15.4% and 15.65% for daily, weekly and monthly rebalancing, respectively). The second best model for covariance matrix forecast was the DCC-GARCH (15.45%, 15.5% and 15.74%), followed by the remainder conditional variance models based on high frequency data (16.02% on average) and then by the static covariance matrix specifications (17.53% on average). Nevertheless, the results are pretty similar. The equally weighted and the market portfolios stand out with a volatility level of around 7 to 10 percentage points higher. These results are similar to those of Borges, J. Caldeira and Ziegelmann (2015): the lowest standard deviation was achieved by a portfolio based on high-frequency data, however, some of the portfolios based on realized measures were outperformed by those based on daily data. This suggests that the results depend on the estimator of the covariance matrix and the intraday frequency used.

In terms of maximum drawdown, for daily and weekly rebalancing the smallest fall occurred for the portfolio based on the GARCH model (19.11% and 19.4%), followed by the Realized GARCH (from 21.19% to 23.17%) and by the HEAVY model with 10-minute sampling frequency of returns (23.75 and 23.48%). For monthly rebalancing, the best performers were the HEAVY (22.17% and 25.12%) and Realized GARCH models (23.95% and 24.16%). The naive and the market portfolio had the worst performances in all rebalancing frequencies, with falls about twice as large as those os the best performers.

Regarding the VaR measure, when the portfolios are rebalanced daily, the Realized GARCH model with returns sampled every 5 minutes presents the lowest worst expected loss with 1% chance (-2.37%). For weekly rebalancing frequency, it is surpassed by the DCC-GARCH and, finally, for the monthly rebalancing frequency, both display the same VaR. Except for the market and equally weighted portfolios - which present the highest VaR, in the order of -3.57%

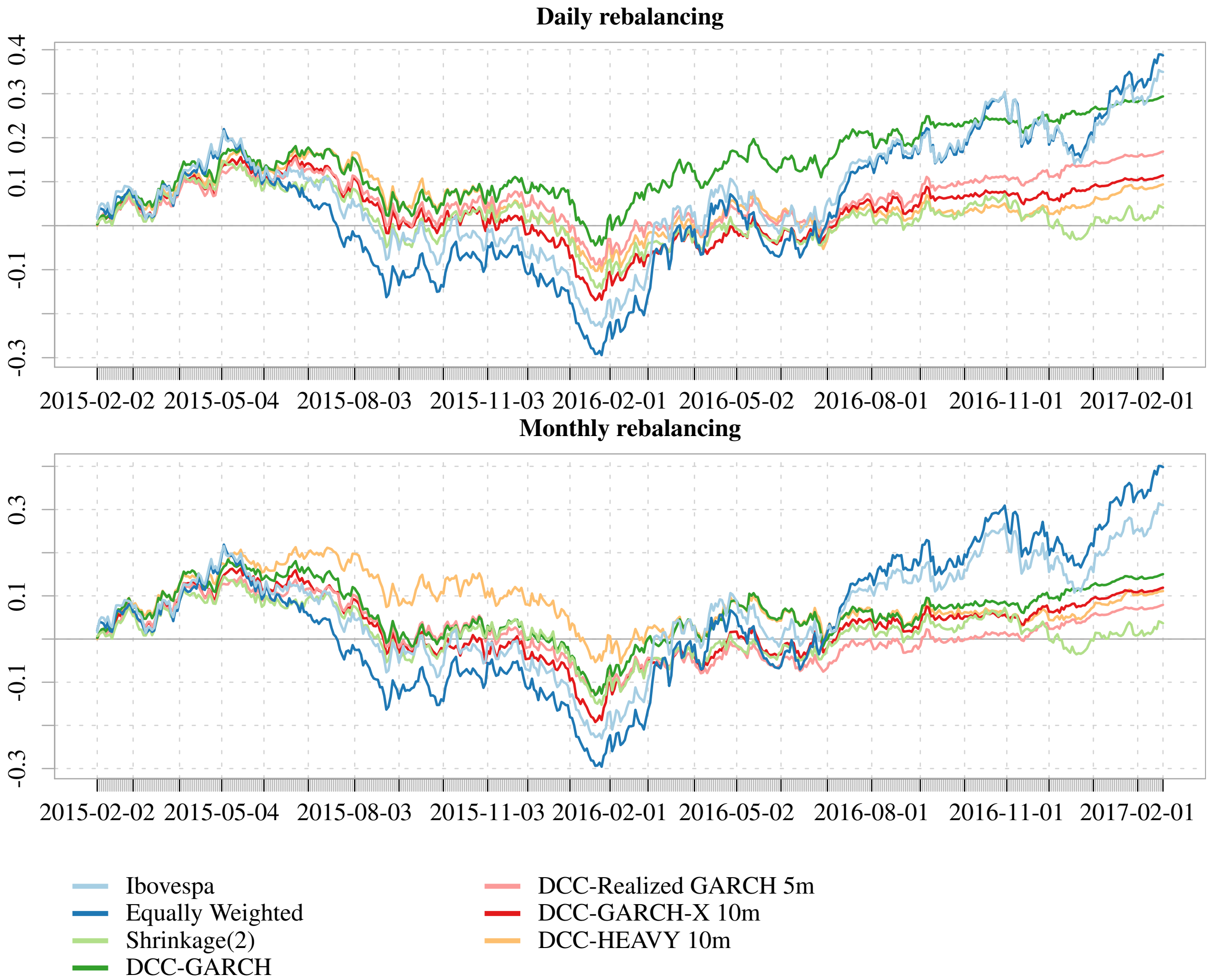
to -3.87% - the difference between the results for the rest of models never exceeds 0.4 percentage points.

Regarding the CVaR measure, when the frequency of rebalancing is monthly, the Realized GARCH with returns sampled every 5 minutes generates the portfolio with the lowest expected average loss exceeding the VaR (-3.07%). Nevertheless, for daily and weekly rebalancing, portfolios based on static covariance matrix specifications are the best performers, together with the DCC-GARCH for weekly rebalancing.

When an investor subjects his capital to risk, it is because he or she expects to be rewarded with higher returns. Therefore, we continue to analyze the portfolios performances by evaluating the gross returns and the returns adjusted to the risk. We can observe that the portfolios with the highest annualized gross returns are also those whose returns are the most volatile: the equally weighted and the market portfolios. For the monthly frequency, the average return of the naive portfolio is more than twice as high (18.17%) as the portfolio based on DCC-GARCH model (7.22%). Still, the DCC-GARCH presents the third highest annualized return and, since its returns are among those with the lowest standard deviation, it is the model with the highest Sharpe ratio for the daily (0.0572) and weekly (0.0515) rebalancing frequencies. All portfolios based on high frequency data achieve higher annualized returns and lower volatility when compared to portfolios based on static covariance matrices specification, so that their Sharpe ratios are higher. But even so, they are not able to outperform the portfolio based on DCC-GARCH and the equally weighted and market portfolios. In fact, we tested the statistical significance of the differences between Sharpe's ratios through the bootstrap referred to in section 5.3, and the p-values indicated that there is no statistically significant difference between the results.

Figure 1 shows cumulative returns over the out-of-sample period for daily and monthly rebalancing frequencies. For the sake of clarity, we have omitted some data that we consider of little relevance: (a) the results for weekly rebalancing were similar to those of daily rebalancing; (b) the portfolios based on SCM and those based on shrinkage methods showed very similar dynamics and we chose to keep only one of them, which had slightly better perfomance; (c) for portfolios based on realized measures we present the results only for the sample frequency of returns that performed better (5-minute or 10-minute). The graphs show that, at the end of the period, the naive portfolio as well as the market portfolio outperformed the others, displaying cumulative returns of approximately 16 to 34 percentage points higher than the remainder portfolios, except for that based on the DCC-GARCH. Despite that, the naive portfolio and the Ibovespa were among the worst performers in the 9 months between mid-June 2015 and mid-March 2016. On the other hand, DCC-GARCH performed consistently well throughout the period when the rebalancing frequencies were daily or weekly. This behavior is related to the lower volatility of the latter relative to the formers. For the monthly rebalancing frequency, the cumulative returns of the portfolio based on DCC-GARCH deteriorate, whilst the portfolio based on the DCC-HEAVY model with returns sampled at 10-minute frequency achieved the best performance in the one year period between May 2015 and April 2016.

Figure 1 – Cumulative returns over the out-of-sample period



Source: Own elaboration from research data (2018).

The choice of a dynamic approach (DCC) instead of a static approach (SCM and shrinkage methods) for the covariance matrix estimation, regardless of the use of high frequency data, had a significant impact on the turnover. For daily rebalancing, the static models generated portfolios with turnover from 1.64% to 1.67%. Among the dynamic models, the lowest turnover was 15.14% - this result is for the DCC-GARCH. The results for models based on intradaily data ranges from 19.92%, for the Realized GARCH 5m, to 34.39%, for the HEAVY 10m. Reducing the rebalancing frequency results in a substantial shrinkage in turnover. For some models, it cames at the cost of deterioration of portfolio performance in terms of Sharpe ratio, but for others this actually results in an improvement. For instance, the Sharpe ratio of the DCC-GARCH undergoes a reduction from 0.052 to 0.0515 and 0.0329 with the successive reductions in the frequency of rebalancing. The same behavior is verified for the Realized GARCH model. On the other hand, the Sharpe ratio of the DCC-HEAVY 10m model increases from 0.0198 to 0.0224 and 0.0251, respectively.

5.5 SHARPE RATIO NET OF TRANSACTION COSTS

So far we have assumed zero transaction costs, but, of course, this hypothesis is not realistic. Therefore, in this section we analyze the impact of transaction costs on portfolios performance measured by the Sharpe ratio considering three proportional transaction costs scenarios: 15, 30 and 45 basis points.

Table 4 shows that for daily rebalancing, all of the portfolios based on high frequency data are significantly outperformed by the market portfolio when the proportional transaction costs are 30 bp or greater. For the most conservative scenario (15 bp), GARCH-X with returns sampled every 5 minutes and the HEAVY model generate Sharpe ratios statistically inferior than Ibovespa. For the weekly rebalancing frequency with transaction costs of 30 bp or less and for monthly rebalancing frequency, no covariance matrix estimation model is able to generate portfolios with Sharpe ratios statistically different from that generated by the market portfolio.

Table 4 – Sharpe ratios based on portfolio returns net of transaction costs of 15, 30 and 45 basis points (bp)

(continues)

	15 bp	30 bp	45 bp
Daily rebalancing			
Naive	0.0442	0.0427	0.0411
SCM	0.0090	0.0068	0.0045
Shrinkage-1fac	0.0090	0.0068	0.0045
Shrinkage-I	0.0092	0.0070	0.0047
Shrinkage-CC	0.0084	0.0061	0.0039
GARCH	0.0334	0.0101	−0.0132
R-GARCH 5m	0.0052	−0.0261*	−0.0575***
R-GARCH 10m	−0.0006	−0.0339**	−0.0672***
GARCH-X 5m	−0.0231**	−0.0604***	−0.0977***
GARCH-X 10m	−0.0098	−0.0455**	−0.0812***
HEAVY 5m	−0.0309**	−0.0815***	−0.1319***
HEAVY 10m	−0.0283**	−0.0790***	−0.1294***
Weekly rebalancing			
Naive	0.0451	0.0443	0.0436
SCM	0.0097	0.0086	0.0075
Shrinkage-1fac	0.0097	0.0087	0.0076
Shrinkage-I	0.0098	0.0087	0.0077
Shrinkage-CC	0.0090	0.0079	0.0069
GARCH	0.0392	0.0275	0.0158
R-GARCH 5m	0.0116	−0.0004	−0.0122
R-GARCH 10m	0.0034	−0.0095	−0.0223*
GARCH-X 5m	0.0003	−0.0145	−0.0291*
GARCH-X 10m	0.0062	−0.0080	−0.0221*
HEAVY 5m	0.0113	0.0003	−0.0108
HEAVY 10m	0.0049	−0.0076	−0.0200*
Monthly rebalancing			
Naive	0.0462	0.0458	0.0455
SCM	0.0101	0.0096	0.009
Shrinkage-1fac	0.0101	0.0096	0.0091

Table 4 – Sharpe ratios based on portfolio returns net of transaction costs of 15, 30 and 45 basis points (bp)

(conclusion)

	15 bp	30 bp	45 bp
Shrinkage-I	0.0102	0.0097	0.0092
Shrinkage-CC	0.0093	0.0088	0.0083
GARCH	0.0279	0.0234	0.0189
R-GARCH 5m	0.0153	0.0109	0.0065
R-GARCH 10m	0.0089	0.0043	−0.0003
GARCH-X 5m	0.0144	0.0093	0.0043
GARCH-X 10m	0.0212	0.0163	0.0114
HEAVY 5m	0.0215	0.0178	0.0142
HEAVY 10m	0.0214	0.0173	0.0133

Source: Own elaboration from research data (2018).
Notes: The Sharpe ratio for Ibovespa in the same period was 0.044. This value is slightly different from that shown in table 3 because an observation was lost during the net return calculation. The asterisks indicate that the values are statistically different from those obtained with the Ibovespa at the significance level of 10% (*), 5% (**) and 1% (*). R-GARCH stands for the Realized GARCH model.

5.6 PERFORMANCE WITH BOX CONSTRAINTS

A potential problem with the GMV portfolios is that they are likely to be very concentrated in stocks with low volatility so that the portfolios' risk is driven by few stocks. Indeed, we found several occurrences of individual assets with allocations as high as 99% in the optimal vectors of weights of the various portfolios. Therefore, we repeat the optimization imposing maximum allocation limits of 10%, 15% and 30% per share and, in this section, we analyze the results. Tables containing all the results were inserted in the appendix (tables A.4 to A.6).

The analysis of returns reveals that models using high frequency data are more sensitive to the constraints that restrict allocations to 10% and 15% and they shift positions from one another. Nevertheless, they keep being outperformed by the naive, market and DCC-GARCH-based portfolios and, in general, outperforming the portfolios based on constant covariance matrices.

The annualized volatility remains virtually unchanged for models based on static covariance matrices and undergoes a small increase for dynamic ones, especially when the weights are limited to 10% or 15%, the largest variation being 2 percentage points (from 15.23 to 17.25 for the model Realized GARCH 10m with daily rebalancing).

The results for Sharpe's ratios do not change considerably with the imposition of restrictions. It should be noted, however, that the equally weighted portfolio outperforms the DCC-GARCH when we impose a maximum weight of 15% on stocks or when rebalancing is monthly. However, the difference between Sharpe's ratios of all models is not statistically significant at the 5% level.

In terms of maximum drawdown, in general, imposing a restriction does not change the results found previously, and the magnitude of the falls is little affected, with the exception of the Realized GARCH 5m model with daily rebalancing. The maximum drawdown suffered by this model decreases from 21.19% to 14.5% when weights are restricted to a maximum of 15%.

In most of cases, the Realized GARCH model generates portfolios with the most conservative VaR, revealing a discrete prevalence of this model against the DCC-GARCH for portfolios with box constraints. The same goes for CVaR, as Realized GARCH generated the lowest expected average loss over VaR.

The imposition of box constraints, in addition to ensuring greater portfolio diversification, reduces the turnover of portfolios based on dynamic covariance matrices, especially when a 10% limit on the shares' weight is imposed. A smaller reduction occurs when a limit of 15% is imposed. The weight limitation at 30% only causes a reduction in turnover for the HEAVY model, or when the rebalancing is monthly.

6 CONCLUSION

The goal of our empirical application was to assess the potential benefits of using high frequency data for global minimum variance portfolio allocation. We used GARCH-type models that comprise realized measures for estimating the assets variances and covariances. Prior research has documented mixed economic results for comparable applications using different covariance matrix specifications.

Our results suggest that, in general, in the absence of transaction costs, portfolios generated with daily and intraday data present similar economic performance. Their volatility levels are very close and there is no statistical difference between their Sharpe ratios.

Considering annualized returns, DCC-GARCH, the naive, and the market portfolios performed better. On the other hand, the models based on high frequency data presented some superiority in maximum drawdown, turnover, and cumulative returns over a partial period when they were compared to DCC-GARCH and the frequency of rebalancing was monthly.

With regard to Sharpe ratio, when transaction costs of 30 basis points or greater were considered, and the rebalancing frequency was daily, portfolios based on high frequency data were significantly outperformed by the market portfolio. Some of them were outperformed even with a transaction cost of 15 basis points. Decreasing the frequency of rebalancing mitigated the effects of transaction costs, but, at most, generated Sharpe ratios of a statistically similar magnitude to that of Ibovespa.

The literature on realized measures points out to their potential of generating more accurate measures of volatility. Our results did not suggest significant economic gains for portfolios that use high frequency data. Some possible explanations may arise from Q. Liu (2009) and Pooter, Martens and Dijk (2008) and Borges, J. Caldeira and Ziegelmann (2015), as they find that: (a) the benefits of using high frequency data may be associated with the length of estimation horizon in the sense that it may be more advantageous to use them when the available data spans a short period of, say, less than 6 months; (b) selecting the appropriate sampling frequency appears to be much more important than choosing between different bias and variance reduction techniques for the realized covariance matrices estimates and, depending on the data, economic superior performance may result from sampling frequency schemes considerably lower than the popular 5 minutes.

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APPENDIX A – SUPPLEMENTARY TABLES

Table A.1 – Assets symbol, company and sector

Symbol	Company	Sector
BBAS3	Banco do Brasil S.A.	Financial
BBDC3	Banco Bradesco S.A.	Financial
BBDC4	Banco Bradesco S.A.	Financial
BOVA11	(Exchange-Traded Fund - ETF)	
BRAP4	Bradespar S.A.	Basic materials
BRFS3	BRF S.A.	Consumer non cyclical
BRKM5	Braskem S.A.	Basic materials
BRML3	BR Malls Participações S.A.	Financial
BTOW3	B2W - Companhia Digital	Consumer cyclical
CCRO3	CCR S.A.	Capital goods and services
CESP6	CESP	Utilities
CIEL3	Cielo S.A.	Financial
CMIG4	CEMIG	Utilities
CPFE3	CPFL Energia S.A.	Utilities
CPLE6	COPEL	Utilities
CSAN3	Cosan S.A. Indústria e Comércio	Oil, gas and biofuels
CSMG3	COPASA MG	Utilities
CSNA3	Companhia Siderúrgica Nacional	Basic materials
CYRE3	Cyrela Brazil Realty S.A.	Consumer cyclical
ELET6	Centrais Elétricas Brasileiras S.A.	Utilities
EMBR3	Embraer S.A.	Capital goods and services
FIBR3	Fibria Celulose S.A.	Basic materials
GGBR4	Gerdau S.A.	Basic materials
GOAU4	Metalúrgica Gerdau S.A.	Basic materials
HGTX3	Cia. Hering	Consumer cyclical
HYPE3	Hypermarcas S.A.	Consumer non cyclical
ITUB4	Itaú Unibanco Holding S.A.	Financial
LAME4	Lojas Americanas S.A.	Consumer cyclical
LIGT3	Light S.A.	Utilities
LREN3	Lojas Renner S.A.	Consumer cyclical
MULT3	Multiplan Empreendimentos Imobiliários S.A.	Financial
NATU3	Natura Cosméticos S.A.	Consumer non cyclical
PETR3	Petrobras	Oil, gas and biofuels
PETR4	Petrobras	Oil, gas and biofuels
PSSA3	Porto Seguro S.A.	Financial
RENT3	Localiza Rent a Car S.A.	Consumer cyclical
SANB11	Banco Santander (Brasil) S.A.	Financial
SBSP3	SABESP	Utilities
TRPL4	CTEEP	Utilities
VALE3	Vale S.A.	Basic materials
VALE5	Vale S.A.	Basic materials

Source: Own elaboration based on BM&FBOVESPA (n.d.)

Table A.2 – Summary statistics for the daily close-to-close returns over the period December 21, 2009–February 17, 2017.

Symbol	Mean	Max	Min	SD	Skewness	Kurtosis	Symbol	Mean	Max	Min	SD	Skewness	Kurtosis
BBAS3	3.54E-04	0.1343	-0.2379	0.0255	-0.1035	5.9707	FIBR3	-7.72E-05	0.1054	-0.1128	0.0244	-0.0112	1.0377
BBDC3	4.61E-04	0.1141	-0.0936	0.0192	0.1491	2.1895	GGBR4	-3.54E-04	0.1492	-0.1237	0.0272	0.2344	2.1272
BBDC4	3.72E-04	0.1225	-0.0970	0.0197	0.1003	2.5892	GOAU4	-9.17E-04	0.1627	-0.2096	0.0304	0.0068	4.1867
BOVA11	-9.26E-06	0.0678	-0.0902	0.0149	-0.0147	1.5625	HGTX3	4.35E-04	0.1088	-0.2537	0.0234	-0.6875	8.7591
BRAP4	-6.16E-05	0.1297	-0.1089	0.0264	0.0887	1.6343	HYPE3	2.19E-04	0.1918	-0.1538	0.0221	0.0412	5.9709
BRFS3	4.64E-04	0.0932	-0.1004	0.0170	-0.0697	2.7304	ITUB4	4.03E-04	0.1037	-0.1022	0.0193	0.1187	1.9716
BRKM5	6.55E-04	0.1175	-0.2204	0.0254	-0.2797	5.2923	LAME4	4.74E-04	0.0923	-0.0884	0.0209	-0.0051	0.8350
BRML3	3.97E-04	0.0817	-0.0855	0.0223	0.0728	0.8769	LIGT3	2.29E-04	0.1038	-0.1246	0.0233	0.0769	2.3383
BTOW3	-8.23E-04	0.2457	-0.1675	0.0352	0.4286	2.9033	LREN3	8.25E-04	0.1062	-0.0849	0.0211	0.1896	1.1318
CCRO3	5.36E-04	0.0957	-0.0916	0.0202	-0.2050	1.4494	MULT3	4.67E-04	0.0705	-0.0694	0.0180	0.0332	0.8220
CESP6	1.69E-04	0.1724	-0.3220	0.0230	-1.2812	23.4476	NATU3	-1.08E-05	0.1044	-0.1251	0.0208	0.1218	2.2377
CIEL3	8.42E-04	0.1060	-0.1495	0.0184	-0.2353	4.1758	PETR3	-4.32E-04	0.1497	-0.1203	0.0289	0.2751	2.3607
CMIG4	7.58E-05	0.1374	-0.2364	0.0257	-1.1283	11.6367	PETR4	-3.67E-04	0.1509	-0.1316	0.0286	0.1205	2.6323
CPFE3	3.81E-04	0.0859	-0.0794	0.0172	0.0731	1.6953	PSSA3	3.96E-04	0.0953	-0.0857	0.0185	0.0693	1.9759
CPLE6	1.16E-04	0.0931	-0.1823	0.0215	-0.4025	4.8472	RENT3	5.36E-04	0.0803	-0.0797	0.0213	0.0486	0.6647
CSAN3	5.37E-04	0.1017	-0.1013	0.0202	0.0168	1.5851	SANB11	5.66E-04	0.1465	-0.1022	0.0211	0.1795	3.4019
CSMG3	3.63E-04	0.1466	-0.1620	0.0248	-0.1761	4.2876	SBSP3	7.40E-04	0.1010	-0.1239	0.0210	-0.2219	2.1760
CSNA3	-2.30E-04	0.1875	-0.2295	0.0339	0.3036	4.3759	TRPL4	4.37E-04	0.0974	-0.2752	0.0182	-1.9605	31.6883
CYRE3	-2.10E-04	0.1061	-0.0843	0.0236	0.0158	0.9034	VALE3	-2.07E-05	0.1377	-0.1567	0.0267	0.0958	2.7907
ELET6	2.01E-04	0.2116	-0.2242	0.0258	0.0399	8.6609	VALE5	5.33E-05	0.1075	-0.1284	0.0250	0.0566	2.1297
EMBR3	4.52E-04	0.0955	-0.1678	0.0204	-0.6308	6.0749							

Source: Own elaboration from research data (2018).

Table A.3 – Summary statistics for the 5-minute and 10-minute open-to-close returns over the period December 21, 2009–February 17, 2017.

(continues)

	5-min returns						10-min returns					
Symbol	Mean	Max	Min	SD	Skewness	Kurtosis	Mean	Max	Min	SD	Skewness	Kurtosis
BBAS3	-3.55E-06	0.0343	-0.0397	0.0022	-0.1086	11.0400	-2.21E-06	0.0479	-0.0489	0.0032	-0.1625	15.4579
BBDC3	1.70E-06	0.0387	-0.0344	0.0021	0.0140	8.2438	7.07E-06	0.0387	-0.0443	0.0030	-0.0621	10.0643
BBDC4	-1.35E-06	0.0334	-0.0324	0.0019	0.0458	8.8074	2.18E-07	0.0373	-0.0511	0.0028	-0.0677	12.4010
BOVA11	-5.85E-06	0.0275	-0.0229	0.0013	0.0240	7.8218	-7.01E-06	0.0289	-0.0386	0.0020	-0.0330	15.6656
BRAP4	-2.36E-06	0.0599	-0.0412	0.0024	0.0987	11.5190	-1.73E-06	0.0623	-0.0654	0.0035	-0.0819	17.1338
BRFS3	-2.53E-06	0.0315	-0.0293	0.0019	0.0530	8.4925	4.43E-06	0.0396	-0.0420	0.0026	0.2287	11.1104
BRKM5	-4.94E-06	0.0761	-0.0333	0.0026	0.1654	12.4832	1.44E-08	0.0451	-0.0639	0.0037	-0.0251	9.4819
BRML3	6.55E-08	0.0324	-0.0418	0.0025	0.0268	8.4813	8.39E-06	0.0442	-0.0479	0.0035	0.0870	9.4768
BTOW3	-1.51E-05	0.0538	-0.0645	0.0037	0.0572	9.3961	-1.78E-05	0.0763	-0.0741	0.0052	0.1937	12.0102
CCRO3	-4.64E-06	0.0384	-0.0376	0.0023	-0.0372	8.4266	1.46E-07	0.0469	-0.0560	0.0032	-0.0357	10.5199
CESP6	-4.52E-06	0.0701	-0.0461	0.0026	0.0735	11.9270	-3.16E-06	0.0940	-0.0523	0.0037	0.2805	15.4654
CIEL3	-6.32E-06	0.0336	-0.0340	0.0020	-0.1395	9.0049	6.93E-06	0.0483	-0.0410	0.0028	0.1133	12.3086
CMIG4	-1.30E-05	0.0367	-0.0472	0.0024	-0.1188	12.4613	-1.30E-05	0.0564	-0.0574	0.0034	0.0170	15.0444
CPFE3	3.04E-06	0.0362	-0.0398	0.0020	0.0888	9.7120	8.25E-06	0.0387	-0.0398	0.0027	0.1543	9.8628
CPLE6	4.55E-06	0.0347	-0.0324	0.0023	0.0604	9.3050	8.50E-06	0.0715	-0.0533	0.0032	0.1023	15.1704
CSAN3	1.04E-07	0.0291	-0.0375	0.0023	-0.0148	7.6474	3.85E-06	0.0377	-0.0618	0.0033	0.0129	9.9039
CSMG3	-1.42E-05	0.0433	-0.0373	0.0030	-0.0238	11.1401	-9.32E-06	0.0491	-0.0539	0.0040	0.0805	10.1719
CSNA3	-2.37E-05	0.0431	-0.0679	0.0031	0.0829	10.0819	-3.71E-05	0.0644	-0.0679	0.0045	0.1068	13.1070
CYRE3	-2.04E-05	0.0913	-0.0498	0.0026	0.2850	19.3291	-2.34E-05	0.1045	-0.0698	0.0037	0.2575	22.0741
ELET6	-7.73E-06	0.0626	-0.0466	0.0026	0.1589	14.4407	-6.05E-06	0.0619	-0.0692	0.0037	0.1347	16.3269
EMBR3	-2.16E-06	0.0588	-0.0372	0.0022	0.0500	16.2728	6.90E-06	0.0568	-0.0530	0.0031	0.1672	16.1695
FIBR3	-1.30E-05	0.0437	-0.0307	0.0024	0.0394	8.2704	-1.78E-05	0.0437	-0.0596	0.0034	-0.0478	10.5517
GGBR4	-2.49E-05	0.0536	-0.0383	0.0027	0.0863	10.3419	-3.59E-05	0.0560	-0.0393	0.0038	0.2770	11.0797
GOAU4	-2.92E-05	0.0611	-0.0618	0.0032	0.0913	13.4074	-4.51E-05	0.0611	-0.0625	0.0044	0.1481	13.5093
HGTX3	3.41E-06	0.0495	-0.0549	0.0027	-0.0702	16.2226	7.25E-06	0.0495	-0.0632	0.0037	-0.0971	13.8213
HYPE3	-6.98E-06	0.0307	-0.0399	0.0025	-0.1596	9.2904	-6.38E-06	0.0413	-0.0415	0.0034	0.0176	8.9732
ITUB4	-1.71E-06	0.0302	-0.0277	0.0018	0.0174	7.8248	1.75E-06	0.0397	-0.0459	0.0027	0.1302	13.3420
LAME4	6.79E-06	0.0328	-0.0365	0.0024	0.0225	5.7160	1.72E-05	0.0414	-0.0488	0.0033	-0.0244	6.8667
LIGT3	-7.07E-06	0.0477	-0.0384	0.0026	0.0315	9.5308	-2.16E-06	0.0637	-0.0500	0.0037	0.1101	10.6925
LREN3	-7.43E-06	0.1120	-0.1168	0.0023	-0.0156	99.3162	-1.03E-06	0.1120	-0.1168	0.0032	0.0227	55.5759
MULT3	-5.44E-06	0.0428	-0.0333	0.0025	0.1050	12.1773	-5.07E-06	0.0428	-0.0373	0.0033	0.0684	9.2435
NATU3	-7.67E-06	0.0646	-0.0637	0.0022	-0.0415	24.8119	-7.21E-06	0.0649	-0.0638	0.0031	-0.1156	21.3577

Table A.3 – Summary statistics for the 5-minute and 10-minute open-to-close returns over the period December 21, 2009–February 17, 2017.

(conclusion)

	5-min returns						10-min returns					
Symbol	Mean	Max	Min	SD	Skewness	Kurtosis	Mean	Max	Min	SD	Skewness	Kurtosis
PETR3	-1.92E-05	0.0536	-0.0603	0.0026	-0.0394	11.9101	-3.05E-05	0.0593	-0.0852	0.0038	-0.0613	18.5366
PETR4	-1.93E-05	0.0580	-0.0500	0.0024	-0.0179	15.2028	-3.25E-05	0.0611	-0.0791	0.0035	-0.0268	21.5849
PSSA3	-1.54E-05	0.0521	-0.0548	0.0025	-0.0271	15.4510	-1.82E-05	0.0648	-0.0544	0.0034	0.0588	13.7183
RENT3	-7.48E-06	0.0521	-0.0372	0.0025	0.0455	11.6203	1.44E-06	0.0831	-0.0385	0.0034	0.2945	13.2341
SANB11	2.95E-06	0.0368	-0.0551	0.0024	0.1159	10.9112	2.24E-06	0.0473	-0.0551	0.0033	-0.1384	13.1180
SBSP3	1.18E-05	0.0425	-0.0327	0.0024	0.0624	9.9514	2.47E-05	0.0545	-0.0543	0.0033	0.0496	11.8939
TRPL4	6.22E-06	0.0421	-0.0737	0.0021	-0.3924	26.1302	1.73E-05	0.0515	-0.0855	0.0029	-0.6742	35.4840
VALE3	-1.13E-05	0.0514	-0.0404	0.0023	0.1177	14.8580	-1.29E-05	0.0627	-0.0539	0.0034	0.1221	17.1774
VALE5	-8.66E-06	0.0642	-0.0343	0.0021	0.2732	19.0547	-9.55E-06	0.0636	-0.0455	0.0030	0.3000	20.1582

Source: Own elaboration from research data (2018).

Table A.4 – Out-of-sample performance of global minimum variance portfolios with $0 < w_{i,t} < 0.1$

	$\hat{\mu}^{p}_{ann}$	$\hat{\sigma}^{p}_{ann}$	SR	MD	VaR	CVaR	TO
Ibovespa	14.46%	24.57%	0.0423	36.66%	-3.58%	-4.43%	-
			Daily rebalancing				
Naive	17.70%	25.68%	0.0480	42.13%	-3.78%	-4.70%	1.67%
SCM	2.42%	17.58%	0.0141	25.92%	-2.63%	-3.15%	1.68%
Shrinkage-1fac	2.48%	17.57%	0.0143	25.76%	-2.63%	-3.15%	1.67%
Shrinkage-I	2.43%	17.58%	0.0141	25.95%	-2.63%	-3.16%	1.68%
Shrinkage-CC	2.35%	17.60%	0.0139	25.96%	-2.63%	-3.15%	1.66%
GARCH	13.23%	16.94%	0.0515	19.61%	-2.52%	-3.08%	13.02%
R-GARCH 5m	6.85%	16.30%	0.0307	22.05%	-2.45%	-2.97%	15.97%
R-GARCH 10m	6.94%	17.00%	0.0302	21.98%	-2.46%	-2.91%	14.55%
GARCH-X 5m	7.21%	17.68%	0.0304	27.54%	-2.70%	-3.37%	22.87%
GARCH-X 10m	9.83%	17.64%	0.0390	27.21%	-2.69%	-3.35%	20.70%
HEAVY 5m	5.64%	16.75%	0.0259	26.68%	-2.51%	-3.06%	24.70%
HEAVY 10m	7.07%	16.85%	0.0308	27.06%	-2.56%	-3.17%	26.44%
			Weekly rebalancing				
Naive	17.69%	25.69%	0.0480	42.28%	-3.76%	-4.65%	0.76%
SCM	2.30%	17.53%	0.0137	26.28%	-2.62%	-3.14%	0.81%
Shrinkage-1fac	2.32%	17.52%	0.0138	26.11%	-2.62%	-3.14%	0.80%
Shrinkage-I	2.31%	17.53%	0.0137	26.30%	-2.62%	-3.14%	0.80%
Shrinkage-CC	2.24%	17.55%	0.0135	26.31%	-2.62%	-3.14%	0.79%
GARCH	11.34%	16.87%	0.0454	21.76%	-2.48%	-2.98%	6.41%
R-GARCH 5m	6.69%	16.26%	0.0302	23.06%	-2.44%	-2.96%	6.24%
R-GARCH 10m	5.77%	16.41%	0.0267	23.23%	-2.47%	-3.01%	6.49%
GARCH-X 5m	8.04%	17.68%	0.0331	27.57%	-2.68%	-3.40%	8.53%
GARCH-X 10m	7.28%	17.78%	0.0305	28.19%	-2.67%	-3.40%	7.89%
HEAVY 5m	3.59%	16.65%	0.0186	27.79%	-2.53%	-3.10%	5.73%
HEAVY 10m	4.77%	16.73%	0.0228	25.51%	-2.58%	-3.20%	6.30%
			Monthly rebalancing				
Naive	18.17%	25.91%	0.0480	42.21%	-3.87%	-4.93%	0.38%
SCM	2.17%	17.65%	0.0132	26.62%	-2.64%	-3.17%	0.41%
Shrinkage-1fac	2.22%	17.63%	0.0134	26.45%	-2.63%	-3.17%	0.40%
Shrinkage-I	2.19%	17.64%	0.0133	26.64%	-2.64%	-3.18%	0.40%
Shrinkage-CC	2.15%	17.67%	0.0131	26.65%	-2.64%	-3.17%	0.40%
GARCH	9.44%	16.92%	0.0389	26.58%	-2.50%	-2.98%	2.41%
R-GARCH 5m	3.66%	16.54%	0.0189	24.26%	-2.49%	-3.00%	1.97%
R-GARCH 10m	3.34%	16.56%	0.0177	24.45%	-2.51%	-3.07%	2.03%
GARCH-X 5m	6.17%	17.54%	0.0270	27.80%	-2.72%	-3.49%	2.57%
GARCH-X 10m	7.36%	17.54%	0.0310	27.82%	-2.72%	-3.48%	2.45%
HEAVY 5m	1.83%	16.76%	0.0121	27.66%	-2.53%	-3.05%	1.80%
HEAVY 10m	6.74%	16.67%	0.0299	23.53%	-2.53%	-3.12%	1.86%

Source: Own elaboration from research data (2018).

Notes: This table reports the out-of-sample performance of global minimum variance portfolios whose weights were submitted to the constraint $0 < \omega_{i,t} < 0.1$. It refers to the period from 02-02-2015 to 02-17-2017. The rows report the results for different estimators of the covariance matrix. R-GARCH refers to the Realized GARCH model. The expression 5m or 10m next to the model name indicates the sample frequency of the intradaily returns. The columns report different performance measures: $\hat{\mu}^{p}_{ann}$ represents annualized returns, $\hat{\sigma}^{p}_{ann}$ are the annualized volatilities, SR are the Sharpe ratios, MD are the maximum drawdowns, VaR and CVaR were calculated with a confidence interval of 99% and TO are the turnovers. All these measures were calculated as in section 5.3. Transaction costs are assumed to be zero.

Table A.5 – Out-of-sample performance of global minimum variance portfolios with $0 < w_{i,t} < 0.15$

	$\hat{\mu}^{p}_{ann}$	$\hat{\sigma}^{p}_{ann}$	SR	MD	VaR	CVaR	TO
Ibovespa	14.46%	24.57%	0.0423	36.66%	-3.58%	-4.43%	-
			Daily rebalancing				
Naive	17.70%	25.68%	0.0480	42.13%	-3.78%	-4.70%	1.67%
SCM	1.87%	17.49%	0.0480	25.19%	-2.59%	-3.07%	1.66%
Shrinkage-1fac	1.88%	17.48%	0.0122	24.98%	-2.59%	-3.07%	1.65%
Shrinkage-I	1.96%	17.49%	0.0122	25.25%	-2.59%	-3.08%	1.66%
Shrinkage-CC	1.71%	17.54%	0.0125	25.39%	-2.60%	-3.08%	1.64%
GARCH	11.26%	16.39%	0.0116	17.68%	-2.46%	-3.08%	14.80%
R-GARCH 5m	2.36%	16.01%	0.0142	14.46%	-2.13%	-2.51%	7.12%
R-GARCH 10m	0.72%	17.25%	0.0080	21.06%	-2.45%	-2.84%	13.14%
GARCH-X 5m	5.76%	17.11%	0.0260	27.49%	-2.65%	-3.36%	25.13%
GARCH-X 10m	8.74%	17.32%	0.0359	26.94%	-2.67%	-3.36%	22.66%
HEAVY 5m	5.28%	16.41%	0.0249	25.22%	-2.46%	-3.01%	26.20%
HEAVY 10m	6.89%	16.36%	0.0308	23.40%	-2.47%	-3.06%	26.50%
			Weekly rebalancing				
Naive	17.69%	25.69%	0.0480	42.28%	-3.76%	-4.65%	0.76%
SCM	1.71%	17.44%	0.0116	25.55%	-2.59%	-3.07%	0.78%
Shrinkage-1fac	1.74%	17.44%	0.0117	25.35%	-2.59%	-3.06%	0.77%
Shrinkage-I	1.78%	17.45%	0.0118	25.63%	-2.59%	-3.08%	0.78%
Shrinkage-CC	1.55%	17.50%	0.0110	25.76%	-2.59%	-3.08%	0.77%
GARCH	9.67%	16.38%	0.0406	19.06%	-2.40%	-2.89%	7.31%
R-GARCH 5m	5.38%	15.88%	0.0258	22.08%	-2.38%	-2.89%	7.02%
R-GARCH 10m	4.47%	15.91%	0.0223	21.88%	-2.38%	-2.91%	7.54%
GARCH-X 5m	5.61%	17.27%	0.0253	26.43%	-2.66%	-3.40%	9.84%
GARCH-X 10m	6.61%	17.28%	0.0288	27.23%	-2.63%	-3.39%	9.26%
HEAVY 5m	5.45%	16.25%	0.0257	24.97%	-2.45%	-2.99%	5.89%
HEAVY 10m	6.36%	16.25%	0.0290	21.23%	-2.47%	-3.02%	6.19%
			Monthly rebalancing				
Naive	18.17%	25.91%	0.0480	42.21%	-3.87%	-4.93%	0.38%
SCM	1.68%	17.55%	0.0115	25.86%	-2.60%	-3.09%	0.38%
Shrinkage-1fac	1.71%	17.55%	0.0116	25.66%	-2.60%	-3.09%	0.38%
Shrinkage-I	1.75%	17.56%	0.0117	25.94%	-2.60%	-3.10%	0.38%
Shrinkage-CC	1.51%	17.61%	0.0109	26.07%	-2.61%	-3.10%	0.38%
GARCH	5.90%	16.33%	0.0272	25.30%	-2.39%	-2.83%	2.74%
R-GARCH 5m	2.06%	16.08%	0.0130	23.35%	-2.41%	-2.86%	2.25%
R-GARCH 10m	0.89%	16.12%	0.0085	23.19%	-2.43%	-2.91%	2.47%
GARCH-X 5m	4.54%	17.13%	0.0217	28.25%	-2.69%	-3.52%	3.03%
GARCH-X 10m	6.01%	17.15%	0.0268	27.74%	-2.67%	-3.49%	2.95%
HEAVY 5m	4.37%	16.30%	0.0216	23.65%	-2.44%	-2.93%	1.76%
HEAVY 10m	7.93%	16.15%	0.0348	19.77%	-2.43%	-2.93%	1.96%

Source: Own elaboration from research data (2018).

Notes: This table reports the out-of-sample performance of global minimum variance portfolios whose weights were submitted to the constraint $0 < \omega_{i,t} < 0.15$. It refers to the period from 02-02-2015 to 02-17-2017. The rows report the results for different estimators of the covariance matrix. R-GARCH refers to the Realized GARCH model. The expression 5m or 10m next to the model name indicates the sample frequency of the intradaily returns. The columns report different performance measures: $\hat{\mu}^{p}_{ann}$ represents annualized returns, $\hat{\sigma}^{p}_{ann}$ are the annualized volatilities, SR are the Sharpe ratios, MD are the maximum drawdowns, VaR and CVaR were calculated with a confidence interval of 99% and TO are the turnovers. All these measures were calculated as in section 5.3. Transaction costs are assumed to be zero.

Table A.6 – Out-of-sample performance of global minimum variance portfolios with $0 < w_{i,t} < 0.3$

	$\hat{\mu}^p_{ann}$	$\hat{\sigma}^p_{ann}$	SR	MD	VaR	CVaR	TO
Ibovespa	14.46%	24.57%	0.0423	36.66%	-3.58%	-4.43%	-
			Daily rebalancing				
Naive	17.70%	25.68%	0.0480	42.13%	-3.78%	-4.70%	1.67%
SCM	1.95%	17.52%	0.0125	25.23%	-2.59%	-3.08%	1.65%
Shrinkage-1fac	1.95%	17.52%	0.0125	25.01%	-2.59%	-3.08%	1.65%
Shrinkage-I	2.03%	17.52%	0.0127	25.31%	-2.60%	-3.09%	1.66%
Shrinkage-CC	1.80%	17.57%	0.0119	25.42%	-2.60%	-3.09%	1.64%
GARCH	12.34%	15.89%	0.0511	19.22%	-2.44%	-3.19%	16.30%
R-GARCH 5m	7.84%	15.51%	0.0355	21.14%	-2.37%	-3.03%	20.69%
R-GARCH 10m	6.22%	15.65%	0.0292	22.59%	-2.41%	-3.09%	22.44%
GARCH-X 5m	2.96%	16.39%	0.0164	28.63%	-2.59%	-3.37%	26.83%
GARCH-X 10m	5.01%	16.62%	0.0238	28.19%	-2.62%	-3.41%	24.94%
HEAVY 5m	4.10%	16.35%	0.0206	26.68%	-2.54%	-3.20%	31.49%
HEAVY 10m	5.04%	16.48%	0.0240	23.54%	-2.61%	-3.36%	30.58%
			Weekly rebalancing				
Naive	17.69%	25.69%	0.0480	42.28%	-3.76%	-4.65%	0.76%
SCM	1.80%	17.47%	0.0119	25.58%	-2.59%	-3.07%	0.78%
Shrinkage-1fac	1.82%	17.47%	0.0120	25.37%	-2.59%	-3.07%	0.78%
Shrinkage-I	1.85%	17.47%	0.0121	25.67%	-2.59%	-3.08%	0.79%
Shrinkage-CC	1.64%	17.53%	0.0114	25.78%	-2.60%	-3.08%	0.78%
GARCH	10.75%	15.95%	0.0454	19.67%	-2.36%	-2.94%	7.97%
R-GARCH 5m	4.88%	15.73%	0.0240	22.22%	-2.42%	-3.02%	8.07%
R-GARCH 10m	2.94%	15.90%	0.0165	23.27%	-2.48%	-3.15%	8.89%
GARCH-X 5m	4.03%	16.61%	0.0202	27.89%	-2.62%	-3.44%	10.46%
GARCH-X 10m	3.73%	16.81%	0.0190	28.49%	-2.60%	-3.43%	9.82%
HEAVY 5m	4.31%	16.16%	0.0215	25.73%	-2.50%	-3.12%	6.97%
HEAVY 10m	3.20%	16.35%	0.0173	23.18%	-2.58%	-3.23%	7.44%
			Monthly rebalancing				
Naive	18.17%	25.91%	0.0480	42.21%	-3.87%	-4.93%	0.38%
SCM	1.75%	17.58%	0.0118	25.87%	-2.60%	-3.10%	0.38%
Shrinkage-1fac	1.76%	17.58%	0.0118	25.67%	-2.60%	-3.09%	0.37%
Shrinkage-I	1.80%	17.58%	0.0119	25.97%	-2.61%	-3.10%	0.38%
Shrinkage-CC	1.57%	17.64%	0.0111	26.08%	-2.61%	-3.11%	0.37%
GARCH	5.34%	16.12%	0.0254	26.54%	-2.45%	-3.02%	2.98%
R-GARCH 5m	3.46%	15.99%	0.0184	23.72%	-2.44%	-2.97%	2.70%
R-GARCH 10m	1.93%	16.08%	0.0126	24.02%	-2.49%	-3.08%	2.88%
GARCH-X 5m	4.06%	16.93%	0.0201	29.62%	-2.73%	-3.78%	3.38%
GARCH-X 10m	4.79%	16.86%	0.0228	29.23%	-2.69%	-3.72%	3.30%
HEAVY 5m	4.29%	16.38%	0.0213	24.92%	-2.51%	-3.10%	2.17%
HEAVY 10m	4.59%	16.73%	0.0222	23.26%	-2.70%	-3.39%	2.32%

Source: Own elaboration from research data (2018).

Notes: This table reports the out-of-sample performance of global minimum variance portfolios whose weights were submitted to the constraint $0 < \omega_{i,t} < 0.3$. It refers to the period from 02-02-2015 to 02-17-2017. The rows report the results for different estimators of the covariance matrix. R-GARCH refers to the Realized GARCH model. The expression 5m or 10m next to the model name indicates the sample frequency of the intradaily returns. The columns report different performance measures: $\hat{\mu}^p_{ann}$ represents annualized returns, $\hat{\sigma}^p_{ann}$ are the annualized volatilities, SR are the Sharpe ratios, MD are the maximum drawdowns, VaR and CVaR were calculated with a confidence interval of 99% and TO are the turnovers. All these measures were calculated as in section 5.3. Transaction costs are assumed to be zero.