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Resumo das Comunicações

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New Property for the Nonmodal Matrix*

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1 Analysis

The nonmodal matrix approach is an alternative form for the state transition matrix, which does not require the knowledge of the normal modes of the model. This solution have been used in vibration problems [3], and initialization [4] and integration [1] of limited area meteorological models .

Essentially, the nonmodal matrix is a solution of the following matrix differential system of order N:

$$\frac{dX}{dt} + A\,X = f(t)$$

which the solution is

$$X(t) = D(t)\,X_0 + \int_0^t D(t-\tau)\,f(\tau)\,d\tau$$

$$D(t) = \sum_{j=1}^{N} \nu_j(t)\,A^{N-j}\,;$$

$$\nu_j(t) = \sum_{k=0}^{j-1} \frac{b_k}{2\pi i} \oint_\Gamma \frac{\lambda^{N-k-1}\,e^{\lambda t}}{\Theta(\lambda)}\,d\lambda$$

where

$$\Theta(\lambda) = |\lambda I - A| = \sum_{k=0}^{N} b_k\,\lambda^{N-k} \quad (b_0 = 1)\,.$$

With the help of the residue theorem can be proven the following property for the nonmodal matrix:

Theorem: If $b_N = 0 \implies \nu_N(t) = 1$.

Proof:

$$\nu_N(t) = \frac{1}{2\pi i} \sum_{k=0}^{N-1} b_k \oint_\Gamma \frac{\lambda^{N-k-1}\,e^{\lambda t}}{\Theta(\lambda)}\,d\lambda =$$

$$= \frac{1}{2\pi i} \oint_\Gamma \Theta^{-1}(\lambda) \left(\sum_{k=0}^{N-1} b_k\,\lambda^{N-k-1} \right) e^{\lambda t} \left(\frac{\lambda}{\lambda}\right) d\lambda$$

$$= \frac{1}{2\pi i} \oint_\Gamma \frac{e^{\lambda t}}{\lambda}\,d\lambda$$

$$\overset{\text{(residue theorem)}}{=} \frac{1}{2\pi i} \lim_{\lambda \to 0} \left[2\pi i\,\frac{\lambda\,e^{\lambda t}}{\lambda} \right] = 1 \;\bullet$$

2 In Place of Conclusion

The property presented in the above theorem could be imagined as much restrictive. However, this characteristic can be found in some important applications, for example in shallow water equations used in many meteorological models [2].

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