Eur. Phys. J. C 4, 115–127 (1998)
DOI 10.1007/s100529800762

THE EUROPEAN
PHYSICAL JOURNAL C

A chiral bag model with a soft surface : structure and solutions of the Fuzzy and the modified Fuzzy Bag Model

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Received: 10 March 1997 / Revised version: 14 October 1997 / Published online: 20 February 1998

Abstract. In the present work we propose a new bag model for hadrons, called the modified fuzzy bag model (MFBM). The distinguishing feature of this model is the suppression of the pion field, as it enters the bag, by means of a scalar potential for the pions, while still preserving chiral symmetry. The mechanism of pion suppression in the MFBM is similar to the mechanism of quark suppression in the fuzzy bag model (FBM). The standard chiral transformation for the pion field suffers a natural alteration in the MFBM, and as a result the model is chiral invariant. We present also a discussion of the FBM and study, in the quark sector, the implications of the soft surface of the bag on the expectation value of the mass operator. In the pion-quark sector, we study the effects of the suppression of the pion field on the form factor for the pion-nucleon interaction, on the pion-nucleon coupling constant $g_{\pi NN}$ and on the nucleon axial charge g_A. Calculations of the pion-nucleon form factor exhibit, in particular, an improvement over previous results. The pionic axial current induces, in the MFBM, a nonvanishing and orientation dependent contribution to axial charge. An analysis of the asymptotic behaviour of the axial charge shows that the role of the surface is to increase the difference of the contributions associated to different orientations.

1 Introduction

The highly nonlinear form and complexity of Quantum Chromodynamics (QCD) has motivated phenomenological models, which incorporate basic properties of QCD, to address problems of intermediate and low energy hadron physics. Most prominent among these models are different versions of the MIT bag model [1–5]. In these models, relativistic quarks are confined within a sharp surface, which represents a boundary between two different media — a perturbative vacuum inside the bag, and a nonperturbative vacuum outside the bag.

By coupling pions to quarks at the bag surface, bag models which preserve chiral symmetry can be constructed [6,7]. The pion is then regarded as an elementary field rather than a bag state. Two main versions of chiral bag models exist [8]: one where the pion is excluded from the bag's interior, and another where the pion can freely penetrate the bag, the latter being called the cloudy bag model [6].

With the inclusion of a pion field, the hadrons get dressed and their physical properties are changed. The corrections were thought to be finite and small, but they are infinite. As was demonstrated [9–11], the nucleon self-energy diverges in the MIT bag model when quarks are coupled to a pion field. This result does not depend on the form of the coupling and also does not depend on whether the pion field penetrates in or is excluded from the bag's interior. It is rather the sharp surface of the MIT bag, by allowing the creation of virtual pions with arbitrarily high momenta, which causes the divergence of the self-energy of the nucleon.

One way to solve this difficulty was proposed by Nogami et al. [11,12] and is called the fuzzy bag model (FBM). In this model, the surface of the bag is "fuzzy": it has a finite thickness. This is accomplished by the introduction of a volume- and a surface-filter distribution function. In the FBM, the parametrization of the boundary between the inside and the outside of the bag is more natural, being a smooth transition. In the chiral version of the model, a pion field which couples to quarks in the surface is introduced. It would then be desirable to keep the pion field away from the interior of the bag. This would maintain the interpretation that inside region of the bag is the perturbative vacuum of QCD, since there the chiral symmetry is realized in the Wigner mode and therefore the pions do not exist. The only mechanism known in the literature of preventing the pion field to penetrate into the bag is the multiplication of the whole pion field Lagrangian density by $\theta(r-R)$. This cannot be done in the FBM, because the surface is not sharp: it has no definite value for the radius.

In this paper, we introduce a modified version of the FBM, called the modified fuzzy bag model (MFBM), in which the pion field is allowed to propagate freely in the exterior of the bag, is suppressed in a smooth manner in the surface of the bag, and vanishes in the interior of the bag. By these means we can keep both the interpretation that inside region of the bag is the perturbative vacuum of QCD and also the finiteness of the self-energy of the nucleon. The suppression of the pion field in the MFBM is accomplished by means of a scalar potential for the pion field, but the model is still chiral invariant. By introducing such a mechanism, the πNN coupling constant, which tends to be underestimated in the cloudy bag model [6] and is even smaller in the FBM, is higher in the MFBM.

Here we also review the FBM and present different versions for the volume- and surface-filter distribution functions, which characterize the sharpness of bag surface. In particular, our version of the surface-filter function peaks at the surface of the bag, denoted by R, while in the FBM it peaks at $R+1fm$. So, our surface parametrization recovers the MIT bag model in the limit of a sharp surface, while the parametrization in [12] does not.

We organize our material as follows. In chapter two we review the fuzzy bag model. In Sect. 2.1 we present the basic formalism. In Sect. 2.2 we show that the original [12] parametrizations of the volume- and surface-filter function do not suppress the quark field appropriately in the interior of the bag, and in Sect. 2.3 we propose new forms of these functions, so that they recover the MIT bag model in the limit of a sharp surface. In Sect. 2.4 we solve the FBM with our parametrization for the volume- and surface-filter functions and assuming the bag constant B equal to zero. The results are applied in Sect. 2.4.1 in the calculation of the axial form factor of the nucleon. In Sect. 2.5 we study the FBM assuming $B \neq 0$. In chapter three we introduce the modified fuzzy bag model (MFBM). In Sect. 3.1 we discuss various aspects involving the inclusion or exclusion of pions inside the bag. In Sect. 3.2 we introduce and discuss the formalism of the model. In Sect. 3.3 we define an appropriate suppression function for the pions inside the bag and we determine the wavefunctions of the pion. In Sect. 3.4 we study the effects of the suppression of the pion field on the form factor for the pion-nucleon interaction, on the pion-nucleon coupling constant $g_{\pi NN}$ and, in Sect. 3.5, on the nucleon axial charge g_A. Finally, in chapter four we present some conclusions.

2 The Fuzzy Bag Model

2.1 Lagrangian of the FBM

In order to cure the divergence of the nucleon self-energy in the MIT bag model, Nogami and co-workers [11,12] proposed a model in which the surface of the bag is not sharply defined, the fuzzy bag model (FBM). The main idea of this model is to replace the step function $\theta(R-r)$ and the delta function $\delta(R-r)$ in the MIT-Lagrangian by continuous functions $F(r)$ and $G(r)$,

$$\mathcal{L}=\Big[\frac{i}{2}(\bar{q}\gamma^\mu\partial_\mu q-\partial_\mu\bar{q}\gamma^\mu q)-m_q\bar{q}q-B\Big]F(r)-\frac{1}{2}\bar{q}q\,G(r) \quad (1)$$

As will be seen later, the field $q(x)$ is related to the physical quark field, but is not identical to it. The functions $F(r)$ and $G(r)$ depend on a parameter n, such that

$$F(r) \underset{n\to\infty}{\longrightarrow} \theta(R-r)\,, \qquad G(r) \underset{n\to\infty}{\longrightarrow} \delta(R-r)\,, \quad (2)$$

and thus $F(r)$ and $G(r)$ are representations of the distributions $\theta(R-r)$ and $\delta(R-r)$. Just like $\partial_r\theta(R-r) = -\delta(R-r)$, we demand

$$\frac{dF(r)}{dr} = -G(r)\,. \quad (3)$$

For a given a supression function $F(r)$, relation (3) fixes the corresponding $G(r)$. The Euler-Lagrange equations applied to (1) give

$$i\gamma^\mu(\partial_\mu q)\,F + \frac{i}{2}\gamma^\mu(\partial_\mu F)\,q - m_q\,F\,q - \frac{1}{2}\,G\,q = 0\,. \quad (4)$$

With help of the vector $n_\mu = (0, -\hat{r})$, relation (3) can be written as $\partial_\mu F(r) = n_\mu G(r)$ and then (4) yields

$$i\gamma^\mu\partial_\mu q + [in_\mu\gamma^\mu\,V_c(r) - m_q - V_c(r)]\,q = 0\,, \quad (5)$$

where $V_c(r)$ is

$$V_c(r) = \frac{G(r)}{2F(r)} = -\frac{1}{2}\frac{d}{dr}\ln(F(r))\,. \quad (6)$$

It was shown [12], that the solutions of (5) diverge for $r\to\infty$ for any potential that shows confinement, i.e. when $V_c(r)\to\infty$ for $r\to\infty$. The field $q(x)$ consequently cannot represent the physical quark field. Since in the FBM the conserved vector current is $i\bar{q}\gamma_\mu qF$ instead of $i\bar{q}\gamma_\mu q$, the physical quark field is identified with

$$\psi(t,\mathbf{r}) = \sqrt{F(r)}\,q(t,\mathbf{r})\,. \quad (7)$$

The current assumes then the usual form, $i\bar{\psi}\gamma_\mu\psi$, and the field $\psi(x)$ behaves as $\psi\to 0$ for $r\to\infty$.

The dynamical equation for the field $\psi(x)$ can be easily determined using the relation

$$\begin{aligned} i\gamma^\mu(F\partial_\mu q + \frac{1}{2}q\partial_\mu F) &= i\gamma^\mu(\sqrt{F}\,\partial_\mu q + q\,\partial_\mu\sqrt{F})\,\sqrt{F} \\ &= i\gamma^\mu\partial_\mu\psi\,\sqrt{F}\,, \end{aligned} \quad (8)$$

from which we obtain, uppon substituting in (4)

$$i\gamma^\mu\partial_\mu\psi - [m_q - V_c(r)]\,\psi = 0\,. \quad (9)$$

The Lagrangian density for the field $\psi(x)$ can be also easily determined. Starting from

$$\partial_\mu\psi = \partial_\mu(\sqrt{F}q) = \sqrt{F}\,\partial_\mu q + n_\mu\,\frac{G}{2\sqrt{F}}\,q\,. \quad (10)$$

we obtain

$$\frac{i}{2}\bar{\psi}\gamma^\mu\partial_\mu\psi = \frac{i}{2}(\bar{q}\gamma^\mu\partial_\mu q)F + \frac{i}{4}(\bar{q}n_\mu\gamma^\mu q)G\,, \quad (11)$$

which allow us to rewrite the Lagrangian density (1) in terms the field $\psi(x)$,

$$\mathcal{L}_{FBM} = \frac{i}{2}\left[\overline{\psi}\gamma^\mu\partial_\mu\psi - \partial_\mu\overline{\psi}\gamma^\mu\psi\right] - BF(r) - \left[m_q + V_c(r)\right]\overline{\psi}\psi \; . \qquad (12)$$

Thus, (9) and (12) equivalently define a bag model with a fuzzy surface. The FBM is very similar to relativistic potential models. Appart from the term $B\,F(r)$, the difference lies in that the scalar potential $V_c(r)$ is related by (6) to a suppression function $F(r)$, which in turn is constrained by the requirement (2).

2.2 Suppression function of the quarks – I

The functions $F(r)$ and $G(r)$ are to be chosen so that the potential $V_c(r)$ be simple. They also have to satisfy the distributional limits (2).

The supression function as defined by Nogami et al. [12], reads as

$$F_{Nog.}(r) = \begin{cases} 1 & , \; r < R \\ \exp(-\frac{\lambda}{n+1}(r-R)^{n+1}) & , \; r \geq R \; ; \end{cases} \qquad (13)$$

the corresponding $G_{Nog.}(r)$ can be calculated through (3), and the potential $V_{c,Nog.}(r)$ is given by (6) as

$$V_{c,Nog.} = \begin{cases} 0 & , \; r < R \\ \frac{1}{2}\lambda(r-R)^n & , \; r \geq R \; . \end{cases} \qquad (14)$$

For $R = 0\,fm$ and $n = 2$, $V_{c,Nog.}(r)$ reduces to the harmonic oscilator potential.

Unfortunately, the functions F and G so defined do not have the distributional limit (2), but rather satisfy the limits

$$F_{Nog.}(r) \underset{n\to\infty}{\longrightarrow} \theta(R+1-r)$$
$$G_{Nog.}(r) \underset{n\to\infty}{\longrightarrow} \delta(R+1-r) \; . \qquad (15)$$

We can see this by analysing the behaviour of $F_{Nog.}(r)$ in various intervals of the r-axis. In the region $r \leq R$, $F_{Nog.}(r)$ is defined to be 1 for all values of n. For $r > R$, one should get $F_{Nog.}(r) \to 0$, but, according to definition (13), one obtains in the interval $R < r < R+1$, since $r-R$ is positive and less than 1,

$$F_{Nog.}(r) = \exp\left(-\frac{\lambda(r-R)^{n+1}}{n+1}\right) \underset{n\to\infty}{\longrightarrow} e^0 = 1 \qquad (16)$$

for $r = R+1$ we have

$$F_{Nog.}(r) = \exp\left(-\frac{\lambda}{n+1}\right) \underset{n\to\infty}{\longrightarrow} e^0 = 1 \qquad (17)$$

and finally, for $r > R+1$ we have

$$F_{Nog.}(r) = \exp\left(-\frac{\lambda(r-R)^{n+1}}{n+1}\right) \underset{n\to\infty}{\longrightarrow} e^{-\infty} = 0 \qquad (18)$$

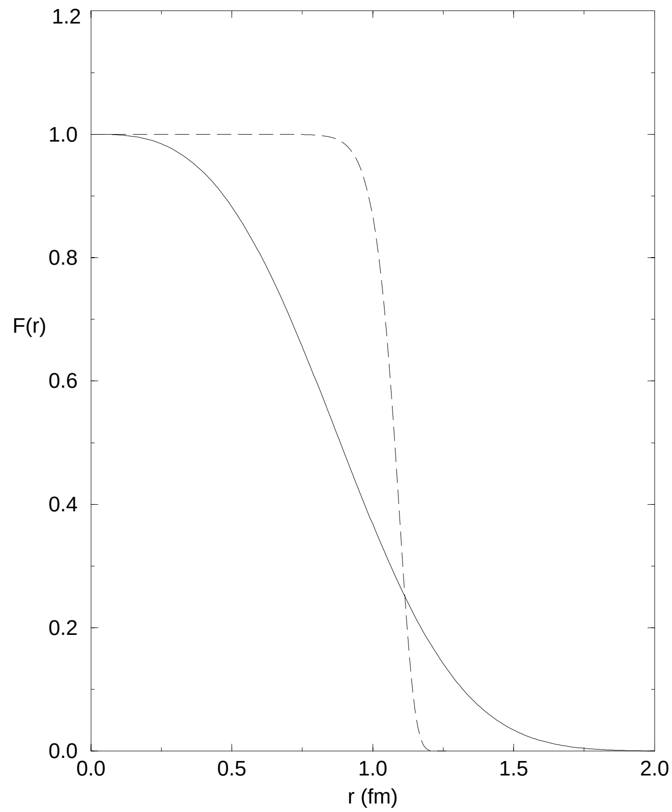

Fig. 1. Suppression function $F_{Nog.}(r)$ for $n = 2$ (*full line*) and $n = 20$ (*dashed line*) and $R = 0fm$ and $\lambda = 3fm^{-3}$

The behaviour of $F_{Nog.}(r)$ is then clear, and is also illustrated in Fig. 1.

The behaviour of $G_{Nog.}(r)$ is also easy to analyse. We first observe that the total area of $G_{Nog.}(r)$ is 1. Integrating (3) from 0 to $R+1$ we obtain

$$\int_0^{R+1} dr\, G_{Nog.}(r) = F_{Nog.}(0) - F_{Nog.}(R+1) \underset{n\to\infty}{\longrightarrow} 0 \; , \qquad (19)$$

and integrating from $R+1$ to $R+1+\epsilon$, with $\epsilon > 0$, we obtain

$$\int_{R+1}^{R+1+\epsilon} dr\, G_{Nog.}(r) = F_{Nog.}(R+1) - F_{Nog.}(R+1+\epsilon) \underset{n\to\infty}{\longrightarrow} 1 \; . \qquad (20)$$

So we see that $G_{Nog.}(r)$ is concentrated around $r = R+1$, as can also be seen in Fig. 2, and equations (15) are proved.

2.3 Suppression function of the quarks – II

In the following we define functions $F(r)$ and $G(r)$ with the correct distributional behavior (2). It can be shown that the MIT bag model Lagrangian can then be recovered from the FBM.

The correct distributional limit (2) can be achieved with a slight modification of definition (13) for $F(r)$ and

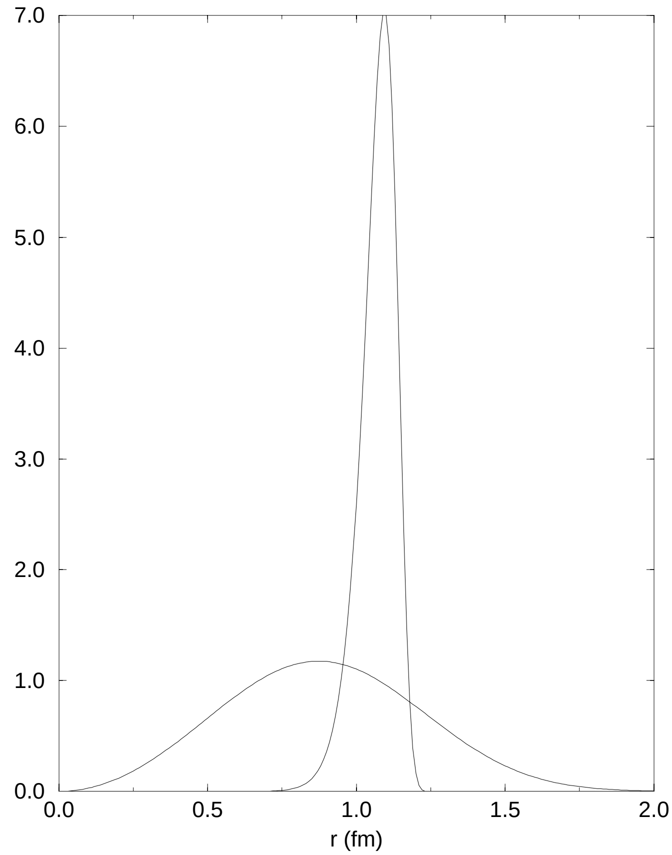

Fig. 2. Surface function $G_{Nog.}(r)$ for $n = 2$ (*full line*) and $n = 20$ (*dashed line*) for $R = 0fm$ and $\lambda = 3fm^{-3}$

of the corresponding one for $G(r)$. Introducing a constant C which depends on n and λ, we define:

$$F(r) = \begin{cases} 1 & ,r < R \\ (n+1)\exp\left[-\frac{\lambda}{n+1}(r-R+C)^{n+1}\right] & ,r \geq R \end{cases} \tag{21}$$

$G(r)$ is given by (3) and the constant C is determined so that $F(r)$ is continuous, that is $F(R) = 1$. We explicitly find (assuming $n > 0$)

$$C = \left[\left(\frac{n+1}{\lambda}\right)\ln(n+1)\right]^{\frac{1}{n+1}} . \tag{22}$$

It is easy to show that the redefinition of $F(r)$ and $G(r)$ cures the inconsistencies. We see that for $n \to \infty$, C approaches 1 from the right, that is, $C > 1$ and $C \to 1$. This means that for $r > R$, we allways have $r - R + C > 1$, and thus

$$F(r) = (n+1)\exp\left(-\frac{\lambda(r-R+C)^{n+1}}{n+1}\right) \underset{n\to\infty}{\longrightarrow} e^{-\infty} = 0 \tag{23}$$

It is clear then that the behaviour of $F(r)$ is given by (2).

The behaviour from $G(r)$ follows from the behaviour of $F(r)$:

$$\int_{R-\epsilon}^{R+\epsilon} dr\, G(r) = F(R-\epsilon) - F(R+\epsilon) \underset{n\to\infty}{\longrightarrow} 1 - 0 = 1. \tag{24}$$

So the behaviour of $G(r)$ is also given by (2).

In addition, the confining potential is given by

$$V_c = \frac{G}{2F} = \begin{cases} 0 & ,r < R \\ \frac{1}{2}\lambda(r-R+C)^n & ,r \geq R \end{cases} \tag{25}$$

and behaves like an infinite potential well in the limit $n \to \infty$.

2.4 Wavefunctions for quarks in the FBM ($B = 0$)

As we already mentioned, the FBM is very similar to relativistic potential models. For a given form of $V_c(r)$, and by setting the bag constant $B = 0$, the treatment is in fact identical. In order to reproduce the masses of various hadrons, a constant scalar term $V_0/2$ and a vector potential $W_\mu(r)$ have to be added to the Lagrangian density (12), which then reads as

$$\mathcal{L}_{FBM} = \frac{i}{2}[\overline{\psi}\gamma^\mu\partial_\mu\psi - \partial_\mu\overline{\psi}\gamma^\mu\psi] - \overline{\psi}[m_q + V_0/2 + V_c(r) + \gamma^\mu W_\mu]\psi . \tag{26}$$

The vector potential is conveniently defined as $W_0 = V_0/2 + V_c(r)$ and $W_i = 0$. In that way the quarks interact with a potential of the type

$$(1+\gamma^0)\left[\frac{V_0}{2} + V_c(r)\right] \equiv (1+\gamma^0)U(r) . \tag{27}$$

Setting $R = C$ and $n = 2$, we recover the harmonic oscillator potential

$$U(r) = \frac{V_0}{2} + \frac{\lambda}{2}r^2 . \tag{28}$$

The Dirac equation for the quarks is then

$$i\gamma^\mu\partial_\mu\psi - \left[m_q + (1+\gamma^0)\left(\frac{V_0}{2} + V_c(r)\right)\right]\psi = 0 . \tag{29}$$

A potential of the form (27) was first considered by Ferreira and Zaguri [13] and subsequently studied intensively by several authors [14–16]. In the present work we mainly focus on the static properties of hadrons and on effects due to the pion field. To compare our results with other calculations we use in this section the parameters found in [16] whose values are $E = 611.842MeV$, $m_q = 78.75MeV$, $V_0 = -137.5MeV$ and $\lambda = 2.273fm^{-3}$. The ground state quark wavefunctions are given by

$$g(r) = \frac{N_q}{r_0}\, e^{-r^2/2r_0^2} \tag{30}$$

$$f(r) = -\frac{N_q\, r}{r_0^3(E+m_q)}\, e^{-r^2/2r_0^2} , \tag{31}$$

where N_q is the normalization constant, and

$$r_0 = [\lambda(E+m_q)]^{-\frac{1}{4}} . \tag{32}$$

2.4.1 Axial form factor

The axial form factor for the nucleon, defined via its quark content, is given in the Breit reference frame for $k^2 << 4\,M_N^2$, by [17,16]

$$\langle N|\sigma_N\,\tau_N|N\rangle\,G_A(k^2) = \int d^3r\,e^{i\mathbf{k}\cdot\mathbf{r}}\,\langle N|\sum_q \overline{\psi}_q\gamma\gamma_5\tau_q\psi_q|N\rangle\,. \qquad (33)$$

Above, the matrices σ_N and τ_N and σ_q e τ_q act on the nucleon and the quark degrees of freedom, respectively. The sum over quarks is carried out over all degrees of freedom (color and flavor indices are here suppressed for convenience). For the nucleon, with all the quarks in the ground state, the integral can be solved analytically and yields for the axial form factor

$$G_A(k^2) = \frac{5(5E+7m_q+V_0)}{9(3E+m_q-V_0)}\,e^{-k^2r_0^2/4}\left[1-\frac{3}{2}\frac{k^2}{(E+m_q)(5E+7m_q+V_0)}\right]\,. \qquad (34)$$

For $k^2=0$, we obtain for the axial charge

$$g_A = G_A(0) = \frac{5(5E+7m_q+V_0)}{9(3E+m_q-V_0)} = 0.944\,; \qquad (35)$$

including center of mass corrections into the calculation of g_A [16], this value increases to

$$g_A = 1.182\,, \qquad (36)$$

and the experimental value of g_A [18] is

$$g_A = 1.2573 \pm 0.0028\,. \qquad (37)$$

2.5 Wavefunctions for quarks in the FBM ($B \neq 0$)

Taking $B \neq 0$, we consider again a potential of the type $(1+\gamma_0)V(r)$, and the following Lagrangian density holds

$$\mathcal{L}_{FBM} = \frac{i}{2}\left[\overline{\psi}\gamma^\mu\partial_\mu\psi - \partial_\mu\overline{\psi}\gamma^\mu\psi\right] - \overline{\psi}[m_q + (1+\gamma^0)V(r)]\,\psi - BF(r)\,, \qquad (38)$$

with $V(r)$ now given by $V(r) = \frac{1}{2}V_0 + W(r)$ and

$$W(r) = \begin{cases} 0 & ,\, r < R \\ \frac{1}{2}\lambda\,[r-R+C]^2 - \frac{\lambda}{2}\,C^2 & ,\, r \geq R \end{cases}\,, \qquad (39)$$

where the last term in (39) makes the potential $V(r)$ continuous and C is given by (22). The Dirac equation is

$$i\gamma^\mu\partial_\mu\psi - m_q\psi - (1+\gamma^0)V(r)\psi = 0\,; \qquad (40)$$

and the parameters of the model are m_q, B, V_0, λ and n.

2.5.1 Solutions

Using the definition

$$\psi(\mathbf{r}) = \begin{pmatrix} g_k(r)\mathcal{Y}_{jl}^{j_z}(\hat{r}) \\ i f_k(r)\mathcal{Y}_{jl'}^{j_z}(\hat{r}) \end{pmatrix}\,, \qquad (41)$$

for the quark spinor, the upper component can be written in the form

$$g(r) = \frac{u(r)}{r}\,, \qquad (42)$$

and lower component is, for the ground state ($l=0$), given by

$$f(r) = \frac{1}{(E+m_q)}\frac{dg}{dr}\,. \qquad (43)$$

For $u(r)$ we obtain the differential equation ($l=0$)

$$\frac{d^2u}{dr^2} + [a - 2(E+m_q)W(r)]\,u = 0\,. \qquad (44)$$

where $a = (E+m_q)(E-m_q-V_0)$. In the region $r < R$ the solution is

$$u(r) = N_1\,\sin(\sqrt{a}r)\,. \qquad (45)$$

For $r > R$, the solution is

$$u(r) = N_2\,e^{-x^2/2}\,{}_1F_1\left(\frac{1-br_0^2}{4},\frac{1}{2},x^2\right) + N_3\,x\,e^{-x^2/2}\,{}_1F_1\left(\frac{3-br_0^2}{4},\frac{3}{2},x^2\right) \qquad (46)$$

with the definitions

$$b = a + \lambda C^2(E+m_q) \qquad (47)$$
$$x = (r-R+C)/r_0\,. \qquad (48)$$

The continuity of $g(r)$ and $f(r)$ at $r=R$ determines the ratios N_2/N_1 and N_3/N_1. The solution (46) diverges unless

$$\frac{N_3}{N_2} = -2\frac{\Gamma((3-br_0^2)/4)}{\Gamma((1-br_0^2)/4)}\,, \qquad (49)$$

as can be checked by the asymptotic expantion of (46). The eigenvalue condition is then

$$\sqrt{a}\,\cot(\sqrt{a}R) + \frac{C}{r_0} = \left\{\frac{C(1-br_0^2)}{r_0}\,{}_1F_1\!\left(\frac{5-br_0^2}{4},\frac{3}{2},\frac{C^2}{r_0^2}\right) + \frac{C^2(3-br_0^2)}{3r_0^2}\frac{N_3}{N_2}\,{}_1F_1\left(\frac{7-br_0^2}{4},\frac{5}{2},\frac{C^2}{r_0^2}\right) + \frac{N_3}{N_2}\,{}_1F_1\left(\frac{3-br_0^2}{4},\frac{3}{2},\frac{C^2}{r_0^2}\right)\right\}\Big/\left(\frac{u(R)}{N_2}\,e^{C^2/2r_0^2}\right) \qquad (50)$$

2.5.2 Nucleon mass

Dropping the Dirac-sea contributions, i.e. neglecting the sum over states with negative energy, we obtain for the expectation value of the mass operator M, in the state where $\mathcal{N}$ quarks occupy the state with energy E_κ,

$$M = <\mathcal{N}|\hat{M}|\mathcal{N}> = \mathcal{N}E_k + \frac{4\pi}{3}BR^3 + 4\pi B\int_R^\infty dr\, r^2 F(r)\,. \quad (51)$$

A relation between the bag constant B and the equilibrium radius of the bag R can be obtained by minimizing M with respect to R,

$$\frac{dM}{dR} = \mathcal{N}\frac{dE_\kappa}{dR} + 4\pi BR^2\left[1 - F(R)\right] = 0\,, \quad (52)$$

We then obtain

$$B = \frac{-\mathcal{N}}{4\pi R^2[1 - F(R)]}\frac{dE_\kappa}{dR}\,. \quad (53)$$

The nucleon mass can be fitted, for example with $m_q = 150.7MeV$, $V_0 = -180MeV$, $\lambda = 0.3fm^{-3}$, $B = 20MeV/fm^3$ and $R = 0.79$. A more extensive fit, taking into account renormalization effects due to the pion cloud, is in progress.

3 Modified Fuzzy Bag Model (MFBM)

In this section we present the modified fuzzy bag model (MFBM), whose main new feature is a smooth supression of the pion field inside the bag by means of a scalar potential, in such a form that chiral symmetry is still preserved.

3.1 Inclusion or exclusion of pions in the bag

Chiral invariance is a stringent restriction to the form of the pion-quark interaction. However, it does not tell whether to include or exclude the pion field from the bag's interior. In the following two subsections we present arguments with respect to inclusion or exclusion of the pions from the interior of the bag and also discuss this question in the context of the MFBM.

3.1.1 Pions inside the bag

In the work of A. Chodos e C. B. Thorn [19], in which a phenomenological pion field was introduced in the MIT bag model, the question of the inclusion or exclusion of the pion field in the interior of the bag is discussed in detail. In the pure bag model, which contains only quark fields, the strong interaction between hadron bags was explained by introducing a bag fission mechanism analogous to the fission of strings [1] (in the original concept, the bag is understood as a 3 + 1 dimensional string). In the theory of strings there exists an alternative method for the calculation of scattering amplitudes: the emission of a string can be mocked up in the tree approximation by an elementary field which locally couples to the surface of the string. Guided by this model, A. Chodos and C. B. Thorn proposed the inclusion of a phenomenological pion field which is noninteracting inside the bag and couples to the quarks only at the (sharp) surface of the bag.

3.1.2 Pions excluded from the bag

There are strong arguments leading to the conclusion that the phenomenological pion field has to be excluded from the interior of the bag. The basic motivation has its origin in the vacuum structure of QCD and in the identification of the pion as the Goldstone boson of the theory. The key argument is the implementation of chiral symmetry in the Goldstone mode, originally studied by Y. Nambu and G. Jona-Lasinio [20] and extended later by T. D. Lee e G. C. Wick [21] with the Goldstone mode implemented in a defined region of space and the Wigner mode in the complementary region. With this concept C. G. Callan, R. F. Dashen and D. J. Gross [22] studied the vacuum of QCD in a semiphenomenological way and concluded that the interaction of the vacuum with a single quark has perturbative character in the vicinity of the quark (Wigner mode), and that beyond a certain distance the vacuum spontaneuously breaks chiral symmetry (Goldstone mode). Following these arguments, the pion field, which is identified with the Goldstone boson, should exist only in the bag's exterior, i.e. in the region where chiral symmetry is broken (c.f.[23, 24]).

3.1.3 Pions gently suppressed in the interior of the bag

In the phenomenological hadron models found in the literature, the concept of a two-phase vacuum can only be implemented by multiplying the whole pion field Lagrangian density by $\theta(r - R)$. This implies that the hadron must have a definite, sharp surface, as in the MIT bag model. But, as was pointed out in the Introduction, the sharp surface of the bag gives rise to unphysical features, like the divergence of the self-energy of the nucleon. It would then be desirable to find another mechanism for excluding the pion field from the interior of the bag. No such attempts are found in the literature, and the reason is very simple: the exclusion must be realized by means of a scalar potential, but scalar potentials break chiral symmetry.

In this work, we present an alternative mechanism. It is similar to the suppression of quarks in the fuzzy bag model. Through the suppression mechanism, the pion field behaves differently in different regions of space: in the bag's exterior the pion field is free, in the surface of the bag the pion field decreases smoothly, and in the interior of the bag the pion field is zero. These three regions can be set in correspondence to the vacuum structure of QCD, respectivelly the nonperturbative vacuum ("exterior of the bag"), where chiral symmetry is spontaneously broken and the Golstone bosons (pions) live, the perturbative vacuum ("interior of the bag"), where chiral symmetry is realized in the Wigner mode, and a transition region between the two vacua ("surface"). We emphasize that the concepts developed in this chapter are indeed of more general character and can be extended to other relativistic bag and potential models.

3.2 The formalism of the MFBM

3.2.1 Suppression of the pion field

In this section we introduce the contribution of the pion field in the MFBM, together with the structure of the Lagrangian density and the field equations. The starting point is the Lagrangian density for the excluded pion field in the MIT bag model,

$$\mathcal{L} = \frac{1}{2}\left[\partial_\mu \phi \cdot \partial^\mu \phi - m_\pi^2 \phi^2\right] \, \theta(r-R) \; . \tag{54}$$

Replacing $\theta(r-R)$ by a suppression function $F_\pi(r)$, we obtain

$$\mathcal{L} = \frac{1}{2}\left[\partial_\mu \phi \cdot \partial^\mu \phi - m_\pi^2 \phi^2\right] \, F_\pi(r) \; . \tag{55}$$

Similarly to (2), the function $F_\pi(r)$ depends on a parameter n_π in such a way that

$$F_\pi(r) \underset{n_\pi \to \infty}{\longrightarrow} \theta(r-R) \; . \tag{56}$$

From the Lagrangian density (55) we obtain the dynamical equation for the field $\phi(x)$,

$$\left(\partial_\mu \partial^\mu \phi + m_\pi^2 \phi\right) \, F_\pi(r) + \partial_\mu F_\pi(r) \, \partial^\mu \phi = 0 \; . \tag{57}$$

3.2.2 Redefinition of the pion field

The spatial part of the isovector current of the suppressed pion field is

$$j^0(x) = \epsilon_{3ij} \, F_\pi(r) \, \phi_i \, \partial^0 \phi_j \; , \tag{58}$$

and by defining the physical pion field through the relation

$$\pi(x) = \sqrt{F_\pi(r)} \, \phi(x) \; , \tag{59}$$

we recognize that it can be written in the usual form,

$$j^0(x) = \epsilon_{3ij} \, \pi_i \, \partial^0 \pi_j \; . \tag{60}$$

3.2.3 Dynamical equation and Lagrangian density for the pion field

Applying the transformation (59) to (57), we get the expression

$$\left[\partial_\mu \partial^\mu \frac{\pi}{\sqrt{F_\pi}} + m_\pi^2 \frac{\pi}{\sqrt{F_\pi}}\right] F_\pi + \partial_\mu F_\pi \, \partial^\mu \frac{\pi}{\sqrt{F_\pi}} = 0 \; . \tag{61}$$

Upon acting with the derivatives, combining various terms and dividing by $\sqrt{F_\pi}$, we obtain the dynamical equation for the pion field in the MFBM,

$$\partial_\mu \partial^\mu \pi + \left[m_\pi^2 + v_\pi(r)\right] \pi = 0 \; , \tag{62}$$

where the scalar potential $v_\pi(r)$ is defined as:

$$\begin{aligned} v_\pi(r) &\equiv \frac{1}{4F_\pi^2} \, \partial_\mu F_\pi \, \partial^\mu F_\pi - \frac{1}{2F_\pi} \, \partial_\mu \partial^\mu F_\pi \\ &= \frac{1}{2F_\pi} \frac{d^2 F_\pi}{dr^2} - \left(\frac{1}{2F_\pi} \frac{dF_\pi}{dr}\right)^2 + \frac{1}{rF_\pi} \frac{dF_\pi}{dr} \; . \end{aligned} \tag{63}$$

A Lagrangian density for the pion field can be easily obtained from the Lagrangian density (55). Using the identity

$$\begin{aligned} \left(\partial_\mu \phi \cdot \partial^\mu \phi\right) F_\pi &= F_\pi \, \partial_\mu \left(\frac{\pi}{\sqrt{F_\pi}}\right) \cdot \partial^\mu \left(\frac{\pi}{\sqrt{F_\pi}}\right) \\ &= \partial_\mu \pi \cdot \partial^\mu \pi - \frac{1}{F_\pi} \, \pi \cdot \partial^\mu \pi \, \partial_\mu F_\pi \\ &\quad + \frac{1}{4F_\pi^2} \, \pi^2 \, \partial_\mu F_\pi \, \partial^\mu F_\pi \; , \end{aligned} \tag{64}$$

we find

$$\begin{aligned} \mathcal{L} = \; & \frac{1}{2} \partial_\mu \pi \cdot \partial^\mu \pi - \frac{1}{2F_\pi} \pi \cdot \partial^\mu \pi \, \partial_\mu F_\pi \\ & + \frac{1}{8F_\pi^2} \, \pi^2 \, \partial_\mu F_\pi \, \partial^\mu F_\pi - \frac{1}{2} \, m_\pi^2 \, \pi^2 \; , \end{aligned} \tag{65}$$

which yields (62) via the Euler-Lagrange equations.

3.2.4 Chiral invariance

For the free massless pion field the (infinitesimal) chiral transformation is given by

$$\phi'(x) = \phi(x) + f_\pi \theta \; , \tag{66}$$

and leaves invariant both the Lagrangian density

$$\mathcal{L} = \frac{1}{2} \, \partial_\mu \phi \cdot \partial^\mu \phi \; , \tag{67}$$

as well as the massless form of (55)

$$\mathcal{L} = \frac{1}{2} \, \partial_\mu \phi \cdot \partial^\mu \phi \, F_\pi(r) \; . \tag{68}$$

The chiral transformation for the suppressed pion field $\pi(x)$ is, as a consequence of (59), given by

$$\pi'(x) = \pi(x) + f_\pi \, \sqrt{F_\pi(r)} \, \theta \; , \tag{69}$$

and leaves the Lagrangian density (65) invariant. We show this explicitly by calculating the variation of the Lagrangian density (65), under the chiral transformation (69).

Setting $m_\pi = 0$, we get:

$$\begin{aligned}\delta\mathcal{L} &= \frac{\partial\mathcal{L}}{\partial\pi}\cdot\delta\pi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\pi)}\cdot\delta(\partial_\mu\pi)\,,\\ &= \left(-\frac{1}{2F_\pi}\,\partial_\mu F_\pi\,\partial^\mu\pi + \frac{1}{4F_\pi^2}\,\pi\,\partial_\mu F_\pi\,\partial^\mu F_\pi\right)\cdot\delta\pi\\ &\quad + \left(\partial^\mu\pi - \frac{1}{2F_\pi}\,\pi\,\partial^\mu F_\pi\right)\cdot\delta(\partial_\mu\pi)\,. \end{aligned}\tag{70}$$

The variation of the field $\pi(x)$ is determined from (69) as $\delta\pi = f_\pi\sqrt{F_\pi}\,\theta$. Using the relation $\delta(\partial_\mu\pi) = \partial_\mu(\delta\pi)$, we obtain

$$\delta\mathcal{L} = f_\pi\sqrt{F_\pi}\left(\partial^\mu\pi - \frac{1}{2F_\pi}\,\pi\,\partial^\mu F_\pi\right)\cdot\partial_\mu\theta\ . \tag{71}$$

Finally, using the Gell-Man Levy theorem, we determine the symmetry current related to the transformation and its divergence:

$$\jmath^\mu_{A,\pi}(x) = \frac{\partial(\delta\mathcal{L})}{\partial(\partial_\mu\theta)} = f_\pi\sqrt{F_\pi}\partial^\mu\pi - f_\pi\pi\partial^\mu\sqrt{F_\pi} \tag{72}$$

$$\partial_\mu\jmath^\mu_{A,\pi}(x) = \frac{\partial(\delta\mathcal{L})}{\partial\theta} = 0\,. \tag{73}$$

As expected, the axial current is equal to $\jmath^\mu_{A,\pi}(x) = f_\pi\,F_\pi\,\partial^\mu\phi$. It is equal to the axial current for the field $\phi(x)$, and is conserved.

3.2.5 A simpler Lagrangian density

In view of the field equation (62), the Lagrangian density (65) seems to be too complicated. In fact, it can also be written as

$$\mathcal{L} = \frac{1}{2}\partial_\mu\pi\cdot\partial^\mu\pi - \frac{1}{2}\left[m_\pi^2 + v_\pi(r)\right]\pi^2 \tag{74}$$

plus a total divergence term,

$$-\partial_\mu\left(\frac{\pi^2}{4F_\pi}\,\partial^\mu F_\pi\right)\,, \tag{75}$$

which can be integrated out. It can also be checked that (74) is invariant under the chiral transformation (69) only up to a total derivative term. Of course, the current (72) stays conserved. From now on we adopt (74) as the Lagrangian density for the suppressed pion field.

3.2.6 Interaction Lagrangian density

The Lagrangian density of the MFBM in the quark sector is identical to the one of the FBM (26). In the pionic sector, it is given by (74). The interaction term is obtained using the interaction piece of the nonlinear sigma model, which in our case is

$$\mathcal{L}_I = -\left[V_0/2 + V_c(r)\right]\overline{\psi}\left(\frac{i}{f_\pi}\gamma_5\tau\cdot\phi\right)\psi\,, \tag{76}$$

Using then the transformation (59), and the definition (28) for $U(r)$ we obtain the interaction Lagrangian density of the MFBM

$$\mathcal{L}_I = -U(r)\,\overline{\psi}\left(\frac{i}{f_\pi\sqrt{F_\pi(r)}}\gamma_5\tau\cdot\pi\right)\psi\,. \tag{77}$$

Putting all pieces together, the complete Lagrangian density finally is given by

$$\begin{aligned}\mathcal{L}_{MFBM} &= \frac{i}{2}\left[\overline{\psi}\gamma^\mu\partial_\mu\psi - \partial_\mu\overline{\psi}\gamma^\mu\psi\right] - \overline{\psi}\gamma^\mu W_\mu\psi - BF(r)\\ &\quad + \frac{1}{2}\,\partial_\mu\pi\cdot\partial^\mu\pi - \frac{1}{2}\left[m_\pi^2 + v_\pi(r)\right]\pi^2\\ &\quad - U(r)\,\overline{\psi}\left(1 + \frac{i}{f_\pi\sqrt{F_\pi(r)}}\gamma_5\tau\cdot\pi\right)\psi\,. \end{aligned}\tag{78}$$

From the Euler-Lagrange equations we obtain the dynamical equations for $\psi(x)$ and $\pi(x)$:

$$i\gamma^\mu\partial_\mu\psi - \gamma^\mu W_\mu\psi - U(r)\left(1 + \frac{i\gamma_5\tau\cdot\pi}{f_\pi\sqrt{F_\pi(r)}}\right)\psi = 0 \tag{79}$$

$$\partial_\mu\partial^\mu\pi + \left(m_\pi^2 + v_\pi(r)\right)\pi = -\frac{iU(r)\,\overline{\psi}\,\gamma_5\,\tau\,\psi}{f_\pi\sqrt{F_\pi(r)}}\ . \tag{80}$$

3.3 Wavefunction of the pion

3.3.1 The suppression function $F_\pi(r)$

In this section we present different parametrizations of the suppression function $F_\pi(r)$ and obtain the (numerical) solutions of the pion field equation.

Besides (56), the suppression function $F_\pi(r)$ should satisfy the following conditions:

(i) – The values of R in (2) and (56) have to be identical, so that in the limits $n \to \infty$ and $n_\pi \to \infty$, the quark field is confined in a volume of radius R, and the pion field is excluded from a volume of the same radius.

(ii) – The suppression function $F_\pi(r)$ has to be defined in such a way that the pion field is free in the exterior of the bag, suppressed in the surface of the bag and zero inside the bag.

(iii) – The region of suppression of the pion field has to coincide with the region where $v_\pi(r)$ is significantly different from zero.

At a first glance condition (iii) looks surprising; from conditions (i) and (ii) it seems that the third condition is automatically satisfied. It is however easy to show the opposite. Assume,

$$F_\pi(r) = e^{-a\left(\frac{R}{r}\right)^{n_\pi}}\,, \tag{81}$$

where $a > 0$. Then $F_\pi(r)$ generates the potential

$$v_\pi(r) = \frac{a^2}{4}\,\frac{n_\pi^2\,R^{2n_\pi}}{r^{2n_\pi+2}} - \frac{a}{2}\,\frac{n_\pi(n_\pi-1)\,R^{n_\pi}}{r^{n_\pi+2}}\,, \tag{82}$$

which is large only close to the origin, where $F_\pi(r)$ is practically zero. Thus definition (81) is not appropriate.

As the general conditions (i) – (iii) constrain $F_\pi(r)$ only qualitatively, it is tempting to choose a parametrization such that the differential equation for $\pi(x)$ has an analytical solution. However, in all cases the corresponding $v_\pi(r)$ are not suitable to parametrize even qualitatively the physics at the bag surface.

With the findings above, we have to study numerical solutions for the pion field. In this case a possible form for the suppression function is

$$F_\pi(r) = \begin{cases} 0 & , r < R - R_0 \\ 1 - \exp\left[\frac{-\lambda_\pi}{n_\pi+1}(r - R + R_0)^{n_\pi+1}\right] & , r \geq R - R_0 \end{cases} \tag{83}$$

with $R_0 = 1 fm$. This definition satisfies conditions (i)-(iii) and in addition preserves the similarity to the suppression function of the quarks (21). For simplicity, we set $R = R_0$ and obtain

$$F_\pi(r) = 1 - \exp\left(-\frac{\lambda_\pi}{n_\pi + 1} r^{n_\pi+1}\right) , \tag{84}$$

and, from the second equation in (63),

$$v_\pi(r) = \lambda_\pi r^{n_\pi}\left[\frac{n_\pi}{2r} + \frac{1}{r} - \frac{\lambda_\pi}{2} r^{n_\pi}\right]\left[\exp\left(\frac{\lambda_\pi r^{n_\pi+1}}{n_\pi+1}\right) - 1\right]^{-1} - \frac{\lambda_\pi^2 r^{2n_\pi}}{4}\left[\exp\left(\frac{\lambda_\pi r^{n_\pi+1}}{n_\pi+1}\right) - 1\right]^{-2} . \tag{85}$$

The typical behavior of $F_\pi(r)$ and $v_\pi(r)$ for specific values of n_π and λ_π is shown in Fig. 3.

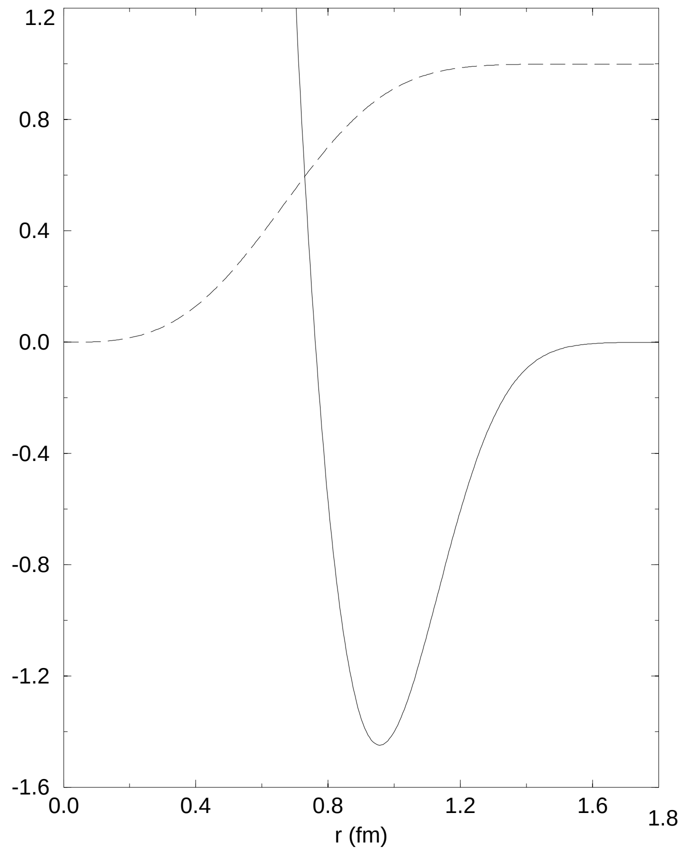

Fig. 3. Radial dependence of $F_\pi(r)$ (*dashed line*) and $v_\pi(r)$ (*full line*), in units of fm^{-1}, for $n_\pi = 2.12$ and $\lambda_\pi = 7.56$

3.3.2 Solution of the homogeneous differential equation

In the MFBM, even if the source term in the dynamical equation for the pion field (80) is set equal to zero, the pion field is not free because of the potential $v_\pi(r)$. The homogeneous differential equation for the pion field is

$$\partial_\mu \partial^\mu \pi + \left(m_\pi^2 + v_\pi(r)\right)\pi = 0 . \tag{86}$$

The solution of (86) is separable in the form

$$\pi(t, \mathbf{r}) = \hat{\alpha}\, e^{-i\,\omega_k\, t}\, Y_{lm}(\theta, \phi)\, \frac{y(r)}{r} , \tag{87}$$

where $\hat{\alpha}$ is a unit vector in isospin space, $\omega_k^2 = k^2 + m_\pi^2$, and $Y_{lm}(\theta, \phi)$ is a spherical harmonic. For our purposes it is sufficient to know the solution for orbital angular momentum $l = 1$. The equation for $y(r)$ is then given by

$$y''(r) + \left(k^2 - \frac{2}{r^2} - v_\pi(r)\right) y(r) = 0 . \tag{88}$$

In Fig. 4 we show the radial dependence of the effective potential $V_{eff} = 2/r^2 + v_\pi(r)$.

The asymptotic solutions of (88) are easily derived. In the limit of r going to infinity, the potential $v_\pi(r)$ tends rapidly to zero, and the solution tends to

$$y(r) \underset{r\to\infty}{\longrightarrow} A\sqrt{\frac{2}{\pi}}\cos(kr + \delta) , \tag{89}$$

where A and δ are constants. Close to the origin, the potential behaves like

$$v_\pi(r) \underset{r\to 0}{\longrightarrow} \frac{(n_\pi + 1)\,(n_\pi + 3)}{4}\,\frac{1}{r^2} . \tag{90}$$

Setting

$$l'\,(l' + 1) = 2 + \frac{(n_\pi + 1)\,(n_\pi + 3)}{4} , \tag{91}$$

the solution is

$$y(r) \underset{r\to 0}{\longrightarrow} B\,k\,r\,j_{l'}(kr) , \tag{92}$$

where B is a constant. The results (89) and (92) were used for checking the numerical solution. As a final step we have to orthonormalize the radial solutions, which means that they should satisfy

$$\int_0^\infty dr\, y_{k_1}(r)\, y_{k_2}(r) = \delta(k_1 - k_2) . \tag{93}$$

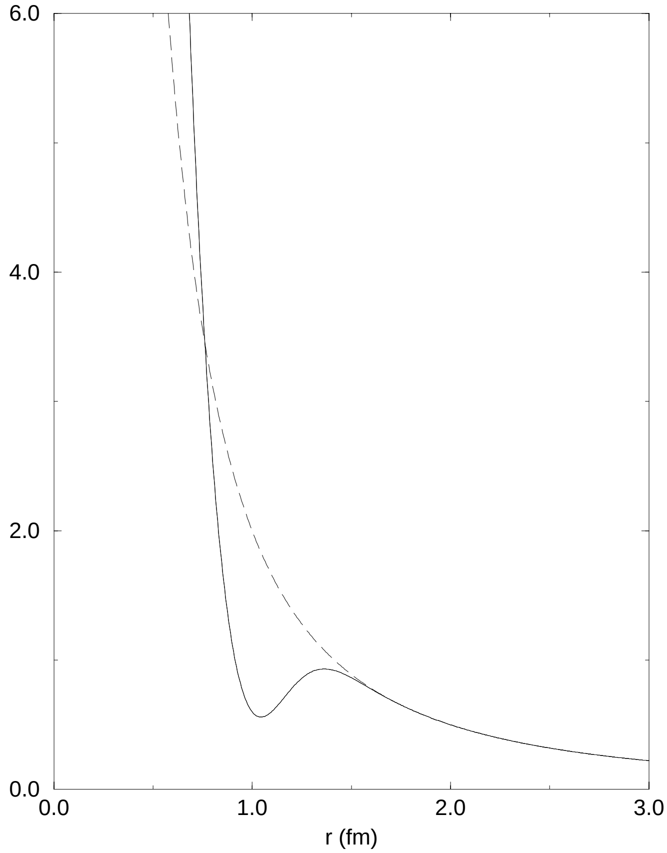

Fig. 4. Radial dependence of $V_{eff} = 2/r^2 + v_\pi(r)$ (*full line*) and $2/r^2$ (*dashed line*), for $n_\pi = 2.12$ and $\lambda_\pi = 7.56$

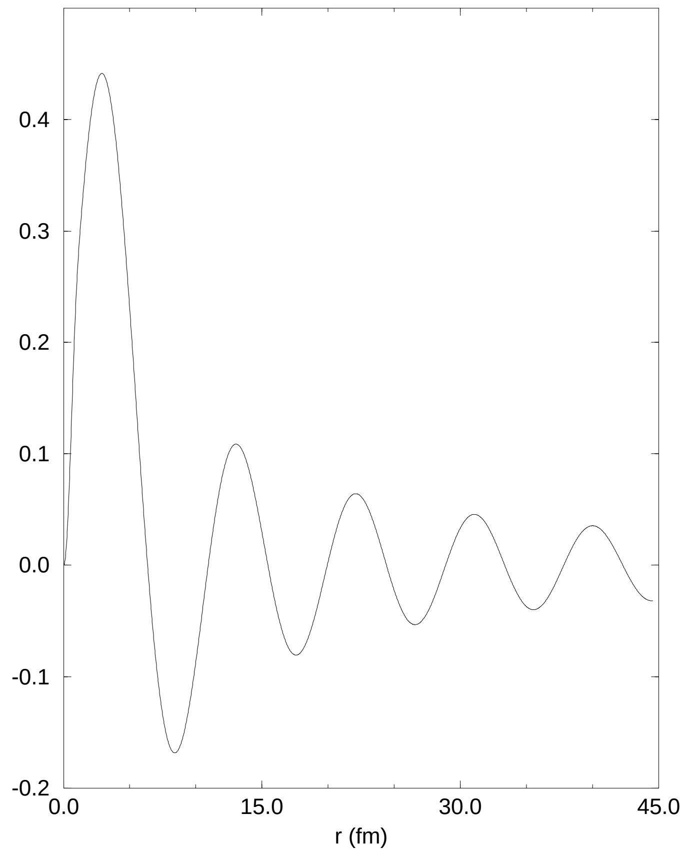

Fig. 5. Homogeneous solution $y(r)/r$ of the pion field for the parameters $n_\pi = 2.12$ and $\lambda_\pi = 7.56$

Using the differential equation (88), we obtain

$$(k_1^2 - k_2^2)\, y_{k_1}\, y_{k_2} = y_{k_1}\, y''_{k_2} - y_{k_2}\, y''_{k_1}\ , \tag{94}$$

which yields for the normalization integral

$$\begin{aligned}\int_0^\infty dr\, y_{k_1}(r)\, y_{k_2}(r) &= \int_0^\infty dr \frac{y_{k_1}(r)\, y''_{k_2}(r) - y_{k_2}(r)\, y''_{k_1}(r)}{(k_1^2 - k_2^2)} \\ &= \left. \frac{y_{k_1}(r)\, y'_{k_2}(r) - y_{k_2}(r)\, y'_{k_1}(r)}{(k_1^2 - k_2^2)} \right|_0^\infty \end{aligned} \tag{95}$$

At the origin, the function $y(r)$ is zero; and for large values of r the asymptotic expression for y and y' can be used. The normalization integral is then equal to

$$\begin{aligned}\lim_{r\to\infty} \frac{A^2}{\pi} \left[\frac{\sin(k_1 r - k_2 r)}{k_1 - k_2} - \frac{\sin(k_1 r + k_2 r)}{k_1 + k_2} \right] = \\ A^2 \left[\delta(k_1 - k_2) - \delta(k_1 + k_2) \right]\ ,\end{aligned} \tag{96}$$

where, as $k > 0$, only the first term contributes. From (96) we see that $A = 1$. In Fig. 5, we present the curve of $y(r)/r$ for $k = m_\pi = 0.70 fm^{-1}$, $n_\pi = 2.12$ and $\lambda_\pi = 7.56$.

3.3.3 Particular solution

We now study (80) including the source term. For a stationary pion field it simplifies to

$$\nabla^2 \pi - \left(m_\pi^2 + v_\pi(r) \right) \pi = \frac{i\, U(r)}{f_\pi \sqrt{F_\pi(r)}} \sum_q \overline{\psi}_q\, \gamma_5\, \tau_q\, \psi_q \tag{97}$$

It is convenient to represent the $\pi(\mathbf{r})$ in the hedgehog form [15]

$$\pi(\mathbf{r}) = \sum_q \sigma_q \cdot \hat{r}\, \tau_q\, h(r)\ . \tag{98}$$

Using the identity [15]

$$\begin{aligned}\nabla \pi &= \nabla \left[\sigma_q \cdot \mathbf{r}\, h(r) \right] \\ &= h'(r)\, \hat{r}\, (\sigma_q \cdot \mathbf{r}) + \frac{h(r)}{r} \left[\sigma_q - \hat{r}\, (\sigma_q \cdot \mathbf{r}) \right]\ ,\end{aligned} \tag{99}$$

the Laplacian can be written as

$$\nabla^2 \pi = \left(h'' + \frac{2h'}{r} - \frac{2h}{r^2} \right) (\sigma_q \cdot \mathbf{r})\ . \tag{100}$$

In the right hand side of (97) we substitute the quark fields

$$\psi(\mathbf{r}) = \begin{pmatrix} g_\kappa(r) \\ i(\sigma_q \cdot \hat{r}) f_\kappa(r) \end{pmatrix} \mathcal{Y}^s_{\frac{1}{2}0}(\hat{r})\ , \tag{101}$$

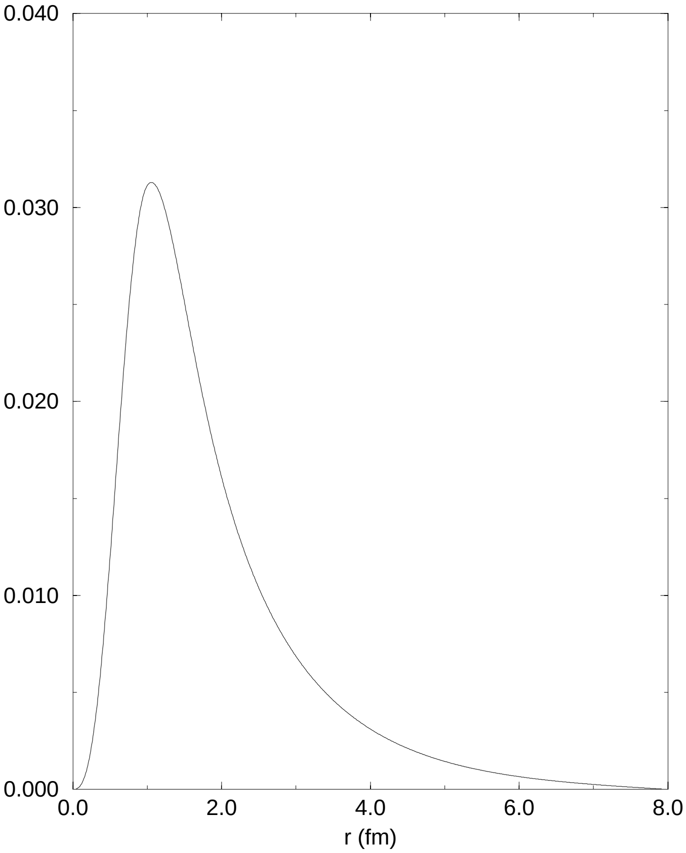

Fig. 6. Inhomogeneous solution $h(r)$ of the pion field for the parameters $n_\pi = 2.12$ and $\lambda_\pi = 7.56$

to obtain

$$\overline{\psi}_q \gamma_5 \psi_q = \frac{i}{2\pi}\, g(r)\, f(r)\, \left[\mathcal{Y}^s_{\frac{1}{2}0}(\hat{r})\right]^\dagger (\sigma_q \cdot \hat{r})\, \mathcal{Y}^s_{\frac{1}{2}0}(\hat{r}) \; . \tag{102}$$

Projecting (97) onto the states of the nucleon and using (100) and (102), we obtain the differential equation for the radial part of the pion field, $h(r)$,

$$h'' + \frac{2h'}{r} - \left(\frac{2}{r^2} + m_\pi^2 + v_\pi(r)\right) h = \frac{-U(r)g(r)f(r)}{2\pi f_\pi \sqrt{F_\pi(r)}} \tag{103}$$

A numerical solution for $h(r)$ is shown in Fig. 6.

3.4 The πNN-vertex in the MFBM

As a first application of the MFBM we study the form factor of the pion nucleon vertex $G_{\pi NN}(k^2)$. The pion-nucleon form factor in the MFBM is given by [25]

$$G_{\pi NN}(k^2) = \frac{-10\sqrt{2\pi} M_N}{3 f_\pi k^2} \int dr r^2 \, \frac{U(r)}{\sqrt{F_\pi(r)}} g(r) f(r) \frac{y(r)}{r} \tag{104}$$

If we make $F_\pi(r) = 1$, then we get $v_\pi(r) = 0$ and $y_k(r)$ tends to the normalized radial wavefunction for the pion field $y_k(r) \to \sqrt{\frac{2}{\pi}}\, k\, r\, j_1(kr)$. The usual form of $G_{\pi NN}(k^2)$ is then recovered

$$G_{\pi NN}(k^2) \to -\frac{20\, M_N}{3\, f_\pi\, k} \int dr\, r^2\, U(r)\, g(r)\, f(r)\, j_1(kr) \tag{105}$$

The experimental value of the coupling constant of the pion nucleon interaction $g^2_{\pi NN}/4\pi = 14.1$ is calculated through $g_{\pi NN} = G_{\pi NN}(m^2_\pi)$. In the MFBM it can be fitted exactly, just by choosing adequate values for n_π and λ_π. In the present work we set the value $n_\pi = 2.12$, and $g_{\pi NN}$ is then exactly given when $\lambda_\pi = 7.56$. Comparing the value of $g_{\pi NN}$ obtained with the MFBM

$$g_{\pi NN} = 13.31 \; , \tag{106}$$

with the value calculated in [16], $g_{\pi NN} = 11.16$, it becomes apparent that, due to the fuzzy surface, the correction for $g_{\pi NN}$ is approximately 16 %.

3.5 The axial charge of the nucleon in the MFBM

The pionic contribution for the axial charge is defined by

$$\langle N|\sigma^i_N \frac{\tau_N}{2}|N\rangle\, g_{A,\pi} = \langle N| \int d^3 r\, j^i_{A,\pi} |N\rangle \; , \tag{107}$$

where the iso-vector axial current carried by the pions is given by (72). Integrating the right hand side of (107) by parts and noting that $\pi \to 0$ when $r \to \infty$ and that $\pi \to 0$ and $F_\pi(r) \to 0$ when $r \to 0$, we obtain

$$\langle N|\sigma^i_N \frac{\tau_N}{2}|N\rangle\, g_{A,\pi} = -2 f_\pi \langle N| \int d^3 r\, \pi \partial^i \sqrt{F_\pi} |N\rangle \tag{108}$$

Substituting the particular solution for the pion field (98) in the above equation, we see that the angular integral, given by

$$\int d\theta\, d\phi\, \sin(\theta)(\sigma_q \cdot \hat{r})\hat{r}^i \tag{109}$$

depends on the space direction i. For $i = 1$ and $i = 2$ (x and y directions), we have

$$\frac{4\pi}{3}\sigma^i_N \tag{110}$$

and for $i = 3$ (z direction) we have

$$\frac{2\pi}{3}\sigma^i_N \tag{111}$$

As a result, the pionic contribution to the axial charge is not isotropic [25]. Denoting by $g^x_{A,\pi}$, $g^y_{A,\pi}$ and $g^z_{A,\pi}$ the contributions in different directions, with our values of n_π and λ_π we get

$$\begin{aligned} g^x_{A,\pi} = g^y_{A,\pi} &= -0.09534 \\ g^z_{A,\pi} &= -0.04767 \; . \end{aligned} \tag{112}$$

So, in the MFBM the pion contribution to the axial charge does not vanish. This is a peculiarity of the MFBM. In most models, the axial current of the pion is given by

$$\jmath^{\mu}_{A,\pi}(r) = f_\pi \, \partial^\mu \pi \ , \tag{113}$$

and it has been shown [15], that if in addition the pion field is continuous, then the pionic contribution to g_A is zero. It has also been observed [26], that if the axial current is the one from the linear sigma model

$$\jmath^{\mu}_{A,\pi}(r) = f_\pi \left(\sigma \, \partial^\mu \phi - \phi \, \partial^\mu \sigma\right) \ , \tag{114}$$

then the pionic contribution to the axial current does not vanish. In the MFBM the axial current (72) is similar to (114), and the pionic contribution to g_A is nonzero, too. Thus we would like to emphasize that a pionic axial current of the type (114) induces a nonvanishing and orientation dependent contribution to g_A. Up to now there are no comparable results, which could experimentally confirm this peculiarity. In the limit $\mathbf{q} \to 0$, $g_{A,\pi}$ becomes isotropic, but still different from zero.

3.5.1 Effect of the bag surface on g_A

Using a diferent parametrization for the volume filter distribution

$$F_\pi(r) = \begin{cases} 0 & , \ r < R_0 \\ (r - R_0)/(R_1 - R_0) & , R_0 \le r < R_1 \\ 1 & , r \ge R_1 \end{cases} \tag{115}$$

we have analyzed the asymptotic behaviour of the axial charge. This definition has the advantage of simplicity and, moreover, it parametrizes the different regions of the bag in a very intuitive way: in the region for which $r < R_0$ we have the interior of the bag, where the pion fields are supposed to be zero; in the region $R_0 \le r < R_1$ we have the surface of the bag (v_π is different from zero in this region) and, for $r \ge R_1$ we have the exterior of the bag, where the pions are supposed to be free. In Fig. 7 we have plotted the components g^A_{xy} and g^A_z for different values of R_0 and R_1. The results indicate that the presence of the surface increases the difference of the contributions associated to different orientations.

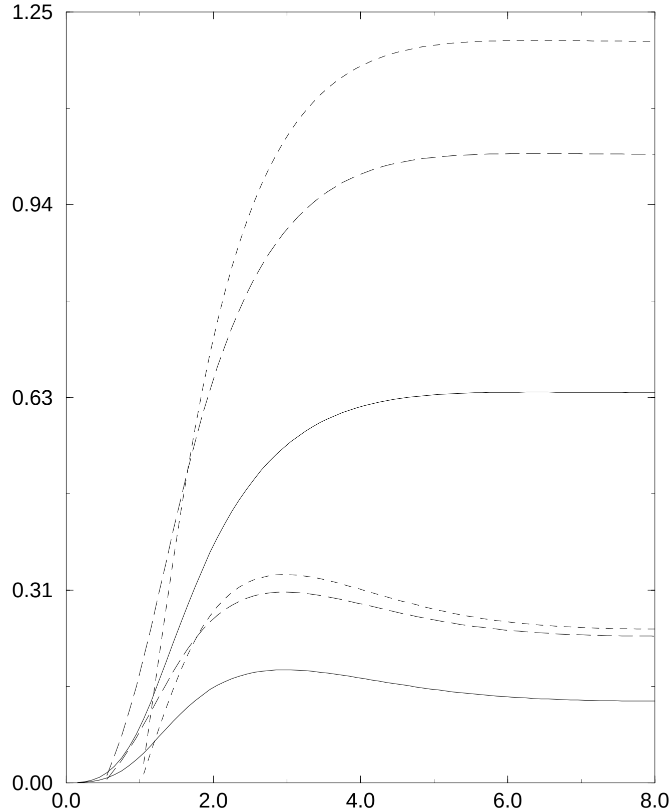

Fig. 7. Behaviour of $g^{xy}_{A,\pi}$ (*curves above*) and $g^{z}_{A,\pi}$ (*curves below*) for $R_0 = 0.1 fm$ (—), $R_0 = 0.5 fm$ (– –) and $R_0 = 1.0 fm$ (- - - -) as a function of R_1

4 Conclusions

In the present work we proposed a model (MFBM) in which the pion field is excluded from the bag interior. The suppression of the pion field is effectively realized by means of a scalar potential. Scalar potentials usually violate chiral symmetry, and this is why no bag models with a scalar suppression of the pion field are found in the literature. In the MFBM, the chiral transformation of the pion field is “modulated” by the suppression function, and as a result chiral symmetry is preserved.

Through the suppression mechanism, the pion field behaves differently in different regions of space: in the bag's exterior the pion field is free, in the surface of the bag the pion field decreases smoothly, and in the interior of the bag the pion field is zero. These three regions can be set in correspondence to the vacuum structure of QCD, respectivelly the nonperturbative vacuum ("exterior of the bag"), where chiral symmetry is spontaneously broken and the Golstone bosons (pions) live, the perturbative vacuum ("interior of the bag"), where chiral symmetry is realized in the Wigner mode, and a transition region between the two vacua ("surface").

In the MFBM the pion-nucleon coupling constant can be fitted exactly. Also, the pionic contribution to the axial charge of the nucleon does not vanish and is furthermore non-isotropic. The flexibility in the parameters n_π and λ_π of the suppression function $F_\pi(r)$, would easily allow to fit the experimental values of $g_{\pi NN}$ and g_A. In the actual stage of the MFBM it is too early to perform such a fitting, because of renormalization effects due to the pion-quark interaction, which so far were not incorporated in the model. Work on the renormalization of the MFBM and a detailed test of its properties is presently in progress. This will hopefully provide hints on the proper form of the suppression function of the pion and maybe also an interpretation of it in terms of more fundamental quantities from QCD. The results will be presented in a subsequent paper.

Acknowledgements. This work has been funded by Deutsche Forschungsgemeinschaft (DFG, Germany), by Hans-Seidel--Stiftung (Germany), by Fundação de Amparo à Pesquisa do Estado do Rio Grande do Sul (FAPERGS, Brazil), by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq, Brazil) and by Pró-Reitoria de Pesquisa e Pós-Graduação da Universidade Federal do Rio Grande do Sul (PROPESP/ UFRGS, Brazil).

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