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The fuzzy bag and baryonic properties with center of mass and recoil corrections

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Abstract. The fuzzy bag is a hadronic model which has features both of the bag model (energy-momentum conservation, QCD vacuum energy) and of relativistic potential models (confinement achieved through a potential). It is also a chiral model, with the unique property that the pion field is suppressed in the interior of the bag by means of a scalar potential, and yet chiral symmetry is preserved. This scalar potential allows one to control how far the pion field can penetrate in the interior of the bag. We calculate the masses of the fundamental baryon octet taking into account the center of mass, one-gluon exchange and one-pion exchange corrections. We also calculate the nucleon axial charge, charge radii and magnetic moments including center of mass and recoil corrections. The agreement with experiment is excellent, and the results indicate that the pion field is suppressed only very close to the center of the bag.

1 Introduction

The fuzzy bag [1–3] is a hadronic model which has elements both of the MIT bag model and of relativistic potential models. It also has the unique property of preserving chiral symmetry, although the pion field is subject to a scalar potential.

In the quark sector, confinement is achieved through a potential, like in relativistic potential models. Due to energy-momentum conservation, the bag constant B acquires a radial dependence, $B \to B(r)$, which can be determined without any further assumptions. Unlike the MIT bag, which has a sharp surface, the fuzzy bag has a surface of finite extent. At the surface, a potential acts on the quarks, confining them to the bag, and another potential acts on the pions, hindering them from getting to the interior of the bag.

Prior models have suffered from the fact that the only way of excluding the pion field from the interior of the bag was through the use of a step function $\theta(r-R)$. But this is not consistent with relativistic potential models, in which there is no sharp surface at $r = R$. The mechanism through which chiral symmetry is realized in the fuzzy bag model makes it possible to exclude the pions from the interior of the bag and at the same time to avoid the unrealistic sharp surface of the MIT bag.

In the fuzzy bag model, the interior of the bag represents the perturbative QCD vacuum, where quarks are free and chiral symmetry is realized in the Wigner mode, so that pions do not exist there. The exterior of the bag represents the non-perturbative QCD vacuum, where quarks do not exist, due to confinement, and chiral symmetry is realized in the Goldstone mode, so that pions exist there and behave as free particles. The surface of the bag represents a transition region, in which the QCD mechanism of confinement is substituted by a scalar potential acting on the quarks, and the QCD mechanism which produces the change in the realization of chiral symmetry is substituted by a scalar potential acting on the pions.

In [4], the recoil corrections for many observables were calculated assuming a Lorentz scalar confining mechanism. However, we assume in our work that the quarks are subject to a scalar plus vector potential. It can be shown that the recoil corrections remain valid when a Lorentz vector interaction is also taken into account. The only requirement is that the energy-momentum tensor be conserved. More specifically, we have checked that the boost generator has the correct action on the quark wave functions and that the total energy and momentum of the bag behave as the components of a four-vector. The consistency of the recoil corrections is based on these two properties, but their actual forms are otherwise independent of the type of interaction.

In Sect. 2 we give a brief description of the MIT bag model coupled to the pion, and in Sect. 3 we introduce the fuzzy bag. In Sect. 4 we discuss how chiral symmetry is realized in our model. In Sect. 5 the confining potential for the quarks and the scalar potential for the pions are presented and solutions for the corresponding wave functions are obtained. In Sect. 6 we discuss energy-momentum conservation in the model and determine the radial dependence of the bag constant $B(r)$. In Sect. 7 we calculate the baryon masses taking into account the center of mass, one-gluon exchange and one-pion exchange corrections. In Sect. 8 we calculate, according to the fuzzy bag, the nucleon axial charge, charge radii and magnetic moments. In Sect. 9 we present our results and conclusions.

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2 The MIT bag coupled to pions

Many of the hadronic models existing in the literature are based on the static version of the MIT bag model [5] supplemented by a pion field, so as to restore chiral symmetry in the model. In most of these approaches the pion field is able to enter inside the bag, and this is allowed mainly because of technical advantages. As discussed in the introduction, from a more fundamental point of view it is desirable to consider the pions as Goldstone bosons and let them exist only in the exterior of the bag, which represents the non-perturbative QCD vacuum, where chiral symmetry is realized in the Goldstone mode. Furthermore, if pions are allowed in the interior of the bag, then the property of asymptotic freedom is violated by the pion–quark interaction. The Lagrangian density for a model which describes a baryon as an MIT bag with pions excluded from the interior can be written as

$$\begin{aligned}\mathcal{L} = &\left[\frac{\mathrm{i}}{2}(\overline{\psi}\,\gamma^\mu\,\partial_\mu\,\psi - \partial_\mu\,\overline{\psi}\,\gamma^\mu\,\psi) - m\,\overline{\psi}\,\psi - B\right]\theta(R-r)\\ &- \frac{1}{2}\,\overline{\psi}\,\psi\,\delta(R-r) + \frac{1}{2}\left[\partial_\mu\phi\cdot\partial^\mu\phi - m_\pi^2\,\phi^2\right]\theta(r-R)\\ &- \frac{\mathrm{i}}{2f_\pi}\,\overline{\psi}\,\gamma_5\,\tau\cdot\phi\,\psi\,\delta(R-r)\,. \end{aligned} \tag{1}$$

In the expression above, $\psi(x)$ is the quark field, m is the quark mass, R is the radius of the bag, $\theta(R-r)$ is a step function which has value 1 inside the bag and value 0 outside the bag, $\delta(R-r)$ is the delta function, $\phi(x)$ is the pion field and $\theta(r-R)$ is the step function, which has value 0 inside the bag and value 1 outside the bag.

The constant B represents the difference between the energy densities of the perturbative QCD vacuum inside the hadrons and the non-perturbative QCD vacuum outside the hadrons. By requiring the action to be invariant under arbitrary infinitesimal deformations of the bag's surface, one finds an equation which relates B with the quark wave functions,

$$B = -\frac{1}{2}\,\partial_r\left(\sum_q \overline{\psi}_q\psi_q\right)\Bigg|_R\,. \tag{2}$$

Contributions from the pion field are usually neglected in (2). The value for B obtained from (2) is the same as the one obtained from the more popular procedure of minimizing the bag's mass with respect to its radius R. It is also well known that (2) guarantees the conservation of energy and momentum (in the quark sector), and some authors impose energy-momentum conservation as an alternative criterion to derive (2).

The presence of the step and delta functions in (1) implies an abrupt transition from the interior to the exterior of the bag. The quark wave functions in the MIT bag do not vanish at the surface, they are simply cut out by the step function. A more serious drawback of the sharp surface of the MIT bag is that it makes the quark self-energy diverge when the pion–quark interaction is turned on and all quark states are considered [6–8]. This divergence is not renormalizable, which means that the model is, even on the phenomenological level, formally inconsistent.

One way of getting rid of the above mentioned divergence is to substitute the MIT bag for a relativistic potential model. But then another problem is encountered, since the bag radius R appears explicitly in the function $\theta(r-R)$, which is responsible for excluding the pion field from the interior of the bag. So, in the quest for a satisfactory hadronic model, it seems that fixing one part of the model spoils the other part. One possible way out of this dilemma [2,3] is the fuzzy bag model.

3 The fuzzy bag

As commented in the preceding section, for a hadronic model to be consistent when quarks are coupled to pions, the bag surface should not be sharp, but rather must have a finite thickness. The hadronic model which satisfies this requirement and is closest to the MIT model is the fuzzy bag model. In order to obtain the fuzzy bag, the sharp functions $\theta(R-r)$, $\delta(R-r)$ and $\theta(r-R)$ in the Lagrangian density (1) are substituted by smoothed-out versions, denoted respectively by $F(r)$, $G(r)$ and $F_\pi(r)$,

$$\begin{aligned}\mathcal{L} = &\left[\frac{\mathrm{i}}{2}(\overline{\psi}\,\gamma^\mu\,\partial_\mu\,\psi - \partial_\mu\,\overline{\psi}\,\gamma^\mu\,\psi) - m\,\overline{\psi}\,\psi - B\right]F(r)\\ &- \frac{1}{2}\,\overline{\psi}\,\psi\,G(r) + \frac{1}{2}\left[\partial_\mu\phi\cdot\partial^\mu\phi - m_\pi^2\,\phi^2\right]F_\pi(r)\\ &- \frac{\mathrm{i}}{2f_\pi}\,\overline{\psi}\,\gamma_5\,\tau\cdot\phi\,\psi\,G(r)\,. \end{aligned} \tag{3}$$

The functions $F(r)$ and $G(r)$ should be representations of the distributions $\theta(R-r)$ and $\delta(R-r)$, so that $G(r)$ is related to $F(r)$ through

$$G(r) = -\frac{\mathrm{d}F(r)}{\mathrm{d}r}\,. \tag{4}$$

Similarly, $F_\pi(r)$ should be a representation of $\theta(r-R)$. The form of the functions $F(r)$ and $F_\pi(r)$ is displayed in Fig. 1. The Lagrangian density (3) is not very convenient to describe the fuzzy bag. By defining the physical quark and pion fields as

$$\begin{aligned} q(x) &= \sqrt{F(r)}\,\psi(x)\,,\\ \pi(x) &= \sqrt{F_\pi(r)}\,\phi(x)\,, \end{aligned} \tag{5}$$

and and rewriting the Lagrangian density (3) in terms of $q(x)$ and $\pi(x)$, one obtains

$$\begin{aligned}\mathcal{L} = &\frac{\mathrm{i}}{2}[\overline{q}\,\gamma^\mu\,\partial_\mu\,q - \partial_\mu\,\overline{q}\,\gamma^\mu\,q] - [m + V_\mathrm{c}(r)]\,\overline{q}\,q - BF(r)\\ &+ \frac{1}{2}\,\partial_\mu\pi\cdot\partial^\mu\pi - \frac{1}{2}\,[m_\pi^2 + V_\pi(r)]\,\pi^2\\ &- \frac{\mathrm{i}\,V_\mathrm{c}(\mathrm{r})}{f_\pi\sqrt{F_\pi(r)}}\,\overline{q}\,\gamma_5\,\tau\cdot\pi\,q\,, \end{aligned} \tag{6}$$

where the scalar potentials $V_\mathrm{c}(r)$ and $V_\pi(r)$ are given by

$$V_\mathrm{c}(r) = -\frac{1}{2\,F(r)}\,\frac{\mathrm{d}F(r)}{\mathrm{d}r} \tag{7}$$

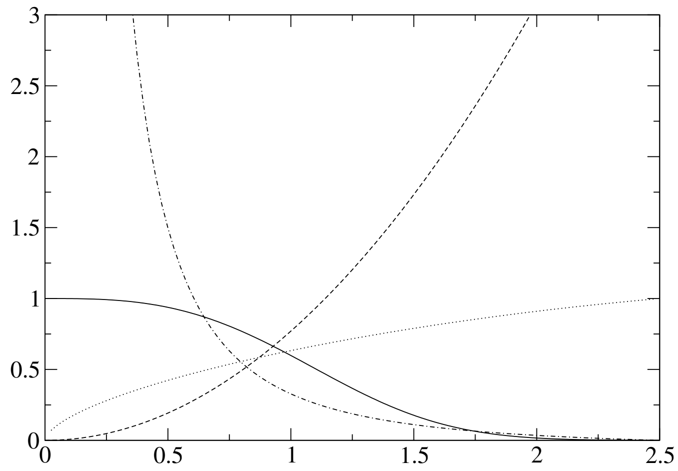

Fig. 1. The behavior of $F(r)$ (—), $V_c(r)$ (dash), $F_\pi(r)$ (dots) and $V_\pi(r)$ (dash-dot)

$$V_\pi(r) = \frac{1}{2F_\pi}\frac{\mathrm{d}^2F_\pi}{\mathrm{d}r^2} - \left(\frac{1}{2F_\pi}\frac{\mathrm{d}F_\pi}{\mathrm{d}r}\right)^2 + \frac{1}{rF_\pi}\frac{\mathrm{d}F_\pi}{\mathrm{d}r} \ . \quad (8)$$

Notice that while $V_c(r)$ should confine the quarks inside the bag, the scalar potential $V_\pi(r)$ should hinder the pion field from entering the bag. It can easily be checked that $V_\pi(r)$ always has this property, independently of the specific form of $F_\pi(r)$: if we assume that $F_\pi(r)$ behaves near the origin as $F_\pi(r) \approx ar^{n_\pi}$, we find that the dominant contribution to the pion potential is always repulsive,

$$V_\pi(r) \approx \left(\frac{n_\pi^2}{4} + \frac{n_\pi}{2}\right)\frac{1}{r^2} \ . \quad (9)$$

Notice also that expression (6) closely resembles a relativistic potential model. But by taking the limit in which $F(r) \to \theta(R-r)$ and $F_\pi(r) \to \theta(r-R)$, the MIT bag model is recovered. So the fuzzy bag can be thought of as a bridge between relativistic potential models and the MIT bag model.

As is done in relativistic potential models, we add in the Lagrangian density (6) a constant term $V_0/2$ to the scalar potential and introduce a vector potential $\gamma^0\, V(r)$, where

$$V(r) = \frac{1}{2}\, V_0 + V_c(r) \ , \quad (10)$$

so that we obtain

$$\begin{aligned}\mathcal{L} = &\frac{\mathrm{i}}{2}[\overline{q}\,\gamma^\mu\,\partial_\mu\, q - \partial_\mu\,\overline{q}\,\gamma^\mu\, q] - \overline{q}\,[m + (1+\gamma^0)V(r)]\, q \\ &- B(r)F(r) + \frac{1}{2}\,\partial_\mu\pi\cdot\partial^\mu\pi - \frac{1}{2}\,[m_\pi^2 + V_\pi(r)]\,\pi^2 \\ &- \frac{\mathrm{i}\,\mathrm{V(r)}}{f_\pi\sqrt{F_\pi(r)}}\,\overline{q}\,\gamma_5\,\tau\cdot\pi\, q \ . \end{aligned} \quad (11)$$

In the above Lagrangian density we have allowed for the MIT bag constant B to have a radial dependence, $B \to B(r)$. This will be justified in Sect. 6, where we consider the energy-momentum conservation. By analogy with the MIT bag, we can interpret the product $B(r)F(r)$ as the vacuum energy density in our model, that is, it represents the difference between the energy densities of the perturbative QCD vacuum inside and the non-perturbative QCD vacuum outside the hadrons [3]. The form of $B(r)$ will be determined in Sect. 6. We also observe that the choice of the scalar plus vector potential reduces the spin–orbit splitting, and this seems to be a feature of QCD [9].

4 Chiral invariance

Our model is unique in the literature, in that the pion field is subject to a scalar potential but still preserves chiral symmetry. This happens because

(i) we started with the Lagrangian density (1), which is chiral symmetric up to first order in the pion field (quark and pion masses assumed to vanish); and

(ii) the final Lagrangian density (11) was obtained by appling the field transformation (5), which does not do any harm to chiral symmetry.

In order to understand in more detail how chiral symmetry is preserved in the fuzzy bag, let us consider an infinitesimal chiral transformation of $\phi(x)$,

$$\phi'(x) = \phi(x) + f_\pi\,\theta \ . \quad (12)$$

For the physical pion field $\pi(x)$, the corresponding chiral transformation is given by

$$\pi'(x) = \pi(x) + f_\pi\sqrt{F_\pi(r)}\,\theta \ . \quad (13)$$

Under this infinitesimal transformation the change in the field is $\delta\pi = f_\pi\sqrt{F_\pi}\,\theta$. The corresponding change in the Lagrangian density can be calculated as

$$\delta\mathcal{L} = f_\pi\sqrt{F_\pi}\left(\partial^\mu\pi - \frac{1}{2F_\pi}\,\pi\,\partial^\mu F_\pi\right)\cdot\partial_\mu\theta \ . \quad (14)$$

We recall the Gell-Mann–Levy equations, which relate the change in the Lagrangian density to a current and its divergence,

$$\begin{aligned}\mathbf{j}^\mu_{\mathrm{A}\pi}(x) &= \frac{\partial\mathcal{L}}{\partial(\partial_\mu\theta)} \ , \\ \partial_\mu\,\mathbf{j}^\mu_{\mathrm{A}\pi}(x) &= \frac{\partial\mathcal{L}}{\partial\theta} \ . \end{aligned} \quad (15)$$

By using the above equations, we see that the axial-vector current for the physical pion field is conserved and is given by

$$\mathbf{j}^\mu_{\mathrm{A}\pi}(x) = f_\pi\,\sqrt{F_\pi}\left(\partial^\mu\pi - \frac{1}{2F_\pi}\,\pi\,\partial^\mu F_\pi\right) \ . \quad (16)$$

Now it is clear why the scalar potential $V_\pi(r)$ is necessary for the chiral transformation to be a symmetry of the Lagrangian (11). Because of the presence of $F_\pi(r)$ in (13), the chiral transformation for the physical pion field is always space-dependent, even if the infinitesimal parameter θ is a constant. When the chiral transformation is applied to the Lagrangian density, there will be terms coming from $\frac{1}{2}\,\partial_\mu\pi\cdot\partial^\mu\pi$ which are compensated by the scalar potential $V_\pi(r)$.

5 Solutions for the quark and pion wave functions

We now choose a definite form for the quark and pion potentials. For simplicity, we choose harmonic confinement,

$$V_{\mathrm{c}}(r) = \frac{\lambda}{2}\, r^2 \,, \tag{17}$$

and then it can be seen from (7) that the quark suppression function is given by

$$F(r) = \mathrm{e}^{-\lambda\, r^3/3} \,. \tag{18}$$

We have seen that the pion potential $V_\pi(r)$ behaves as (9) near the origin, and we also expect that it vanishes as $r \to \infty$. We choose then the potential

$$V_\pi(r) = \begin{cases} \left(\frac{n_\pi^2}{4} + \frac{n_\pi}{2}\right)\left(\frac{1}{r^2} - \frac{1}{R_\pi^2}\right) , & r < R_\pi \,, \\ 0 \,, & r > R_\pi \,. \end{cases} \tag{19}$$

The pion suppression function can be obtained by writing (8) as a differential equation for $F_\pi(r)$,

$$\frac{\mathrm{d}^2\left(r\sqrt{F_\pi}\right)}{\mathrm{d}r^2} - V_\pi(r)\left(r\sqrt{F_\pi}\right) = 0 \,. \tag{20}$$

With $\mu = (n_\pi + 1)/2$ and $\alpha = \sqrt{n_\pi^2/4 + n_\pi/2}$, we find

$$F_\pi(r) = \begin{cases} \frac{R_\pi}{r}\left[\frac{J_\mu(\alpha r/R_\pi)}{J_\mu(\alpha)}\right]^2 , & r < R_\pi \,, \\ 1 \,, & r > R_\pi \,. \end{cases} \tag{21}$$

In Fig. 1 we have displayed the behavior of the suppression functions $F(r)$ and $F_\pi(r)$ and of the potentials $V_{\mathrm{c}}(r)$ and $V_\pi(r)$.

5.1 The quark wave function

Let us now determine the dynamical equation for the quark wave function. From the Lagrangian density (11), neglecting the pion–quark interaction, we obtain

$$\mathrm{i}\,\gamma^\mu\,\partial_\mu\, q - [m + (1+\gamma^0)\, V(r)]\, q = 0 \,. \tag{22}$$

The wave function $q(x)$ can be written in a separable, two-component form,

$$q(x) = \mathrm{e}^{-\mathrm{i}Et} \begin{pmatrix} g(r) \\ -\mathrm{i}\, f(r)\, \sigma \cdot \hat{r} \end{pmatrix} \chi \,, \tag{23}$$

and the radial wave functions $g(r)$ and $f(r)$ can be written in terms of the reduced wave function $u(r)$,

$$g(r) = \frac{u(r)}{r} \,, \qquad f(r) = \frac{1}{E+m}\,\frac{\mathrm{d}g(r)}{\mathrm{d}r} \,. \tag{24}$$

One finds that the differential equation for $u(r)$ is

$$\frac{\mathrm{d}^2\, u(r)}{\mathrm{d}r^2} + (E+m)[E - m - V_0 - \lambda r^2]\, u(r) = 0 \,. \tag{25}$$

This equation can be solved exactly, yielding the solution

$$u(r) = \frac{Nr}{r_0}\, \mathrm{e}^{-r^2/2r_0^2} \,, \tag{26}$$

and the eigenvalue equation for the energy,

$$\sqrt{E+m}\,(E - m - V_0) = 3\sqrt{\lambda} \,, \tag{27}$$

where r_0 determines the fall-off of the quark wave function,

$$r_0^{-4} = \lambda(E+m) \,, \tag{28}$$

and we have defined the variables E' and m', which in many cases are more practical to use than E and m,

$$E' = E - V_0/2 \,, \qquad m' = m + V_0/2 \,. \tag{29}$$

The normalization condition for the quark wave function is

$$\int \mathrm{d}^3 r\, q^\dagger(\mathbf{r})\, q(\mathbf{r}) = 1 \,. \tag{30}$$

After inserting the expression (23) for the quark spinor and doing several integrations by parts, the normalization condition, for a general form of $u(r)$ and $V(r)$, can be written as

$$2\int_0^\infty \mathrm{d}r\, u^2(r)\,[E - V(r)] = E + m \,. \tag{31}$$

With the specific forms of $u(r)$ and $V(r)$ found in this section, we obtain

$$N^2 = \frac{8\,(E'+m')}{\sqrt{\pi}\, r_0\, (3E'+m')} \,. \tag{32}$$

5.2 The pion wave function

We have seen that, even neglecting the pion–quark interaction, the pion field is not free in the fuzzy bag: it is always subject to a scalar potential $V_\pi(r)$, which represents some effects of the QCD vacuum, as was discussed in Sects. 1 and 3. As a consequence, the homogeneous equation for the pion field is

$$\partial_\mu\, \partial^\mu\, \pi + \left[m_\pi^2 + V_\pi(r)\right] \pi = 0 \,. \tag{33}$$

Writing the pion field as

$$\pi(x) = \hat{\alpha}\, \mathrm{e}^{-\mathrm{i}\,\omega_k\, t}\, \frac{h_k(r)}{r}\, Y_\ell^m(\theta,\phi) \,, \tag{34}$$

where $\hat{\alpha}$ is an unitary vector in isospin space, $h_k(r)$ is the reduced radial wave function, and $\omega_k^2 = k^2 + m_\pi^2$, it can be seen that for $\ell = 1$ we get

$$h''(r) + \left(k^2 - \frac{2}{r^2} - V_\pi(r)\right) h(r) = 0 \,. \tag{35}$$

By defining the constants

$$\nu = \sqrt{\frac{n_\pi^2}{4} + \frac{n_\pi}{2} + \frac{9}{4}}\,,$$
$$\beta = \sqrt{k^2 + \left(\frac{n_\pi^2}{4} + \frac{n_\pi}{2}\right)\frac{1}{R_\pi^2}}\,, \qquad (36)$$

the solution can be written as

$$h_k(r) = \begin{cases} N_\pi \sqrt{\beta r}\, J_\nu(\beta r)\,, & r < R_\pi\,, \\ N_\pi \sqrt{\frac{2}{\pi}}\, kr[A\, j_1(kr) + B\, y_1(kr)]\,, & r > R_\pi\,, \end{cases} \qquad (37)$$

where A and B are determined by continuity conditions and N_π is the normalization factor. The normalization condition for the pion radial wave function is

$$\int_0^\infty \mathrm{d}r\, h_{k_1}(r)\, h_{k_2}(r) = \delta(k_1 - k_2)\,, \qquad (38)$$

and it implies that the normalization factor N_π is given by $N_\pi = 1/\sqrt{A^2 + B^2}$.

6 Energy-momentum conservation

The question of energy-momentum conservation in the quark sector can be addressed by considering the contribution of the quarks to the energy-momentum tensor. This tensor should be written in terms of the field $\psi(x)$ which appears in the original Lagrangian density (3), but it can be shown that the same functional form arises when $T^{\mu\nu}$ is expressed in terms of the physical quark field $q(x)$,

$$T^{\mu\nu} = \frac{\partial\mathcal{L}}{\partial(\partial_\mu q)}\,\partial^\nu q + \partial^\nu \bar{q}\,\frac{\partial\mathcal{L}}{\partial(\partial_\mu \bar{q})} - g^{\mu\nu}\,\mathcal{L}\,. \qquad (39)$$

This is the usual form of the energy-momentum tensor, but notice that $q(x)$ is a suppressed field, so the result (39) is not at all obvious. The criterion of energy-momentum conservation implies that the divergence of $T^{\mu\nu}$ must vanish. After some algebra, we arrive at the condition

$$\partial_\mu\,[B(r)\,F(r)] + \sum_i \bar{q}_i\,(1+\gamma^0)\,q_i\,\partial_\mu V_\mathrm{c}(r) = 0\,, \qquad (40)$$

where the sum over valence quarks in the hadron is written explicitly. If B were simply a constant, the above equation would not be satisfied. This is because the suppression function $F(r)$ determines the confining potential $V_\mathrm{c}(r)$, which on its turn determines the quark wave functions. So these three quantities are tied together, and the only way to satisfy (40) is to let B depend on the radial variable, $B \to B(r)$. In this way we justify the radial dependence of B which was introduced in the fuzzy bag Lagrangian (11).

For the ground state, the expression $\bar{q}(x)(1+\gamma^0)q(x) = g^2(r)/2\pi$ is independent of the angular variables, and thus (40) turns into a trivial first order differential equation for $B(r)$,

$$\frac{\mathrm{d}}{\mathrm{d}r}\,[B(r)\,F(r)] = -\frac{1}{2\pi}\sum_q g_q^2(r)\,\frac{\mathrm{d}V_\mathrm{c}(r)}{\mathrm{d}r}\,. \qquad (41)$$

Integrating the above equation from zero up to some finite value of r and setting $B_0 = B(0)$, we obtain

$$B(r)\,F(r) = B_0 - \frac{1}{2\pi}\sum_q \int_0^r \mathrm{d}r'\, g_q^2(r')\,\frac{\mathrm{d}V_\mathrm{c}(r')}{\mathrm{d}r'}\,. \qquad (42)$$

Remembering that the product $B(r)F(r)$ represents the difference between the energy densities of the perturbative and the non-perturbative QCD vacua, it is natural to require $B(r)F(r) \to 0$ as $r \to \infty$. In this way we are able to determine B_0. Upon using the reduced wave function $u(r)$ instead of $g(r)$, the vacuum energy density can be finally written as

$$B(r)\,F(r) = \frac{1}{2\pi}\sum_q \int_r^\infty \mathrm{d}r'\,\frac{u_q^2(r')}{r'^2}\,\frac{\mathrm{d}V_\mathrm{c}(r')}{\mathrm{d}r'}\,. \qquad (43)$$

This formula should be compared to (2), which gives the MIT value for the bag constant. For the confining potential of (17) and using the solution for $u(r)$ in (26), we find

$$B(r)\,F(r) = \frac{\lambda}{4\pi}\sum_q N_q^2\,\mathrm{e}^{-r^2/r_{0q}^2}\,. \qquad (44)$$

Notice that there is a steady decrease of $B(r)\,F(r)$ as r increases. This represents a smooth transition between the perturbative and the non-perturbative vacuum of QCD.

7 Baryon masses

Without any corrections, the mass of a baryon is given simply by the sum of the energy carried by the quarks E_B and the vacuum energy E_vac,

$$M_B^0 = E_B + E_\mathrm{vac} = \sum_q E_q + \int \mathrm{d}^3 r\, B(r)\,F(r)\,. \qquad (45)$$

By performing integration by parts, we obtain for quarks in the ground state

$$E_\mathrm{vac} = \frac{2}{3}\sum_q \int_0^\infty \mathrm{d}r\, r\, u_q^2(r)\,\frac{\mathrm{d}V_\mathrm{c}(r)}{\mathrm{d}r}\,. \qquad (46)$$

In the specific case of the confining potential (17) and the solution for $u(r)$ in (26), we find

$$E_\mathrm{vac} = 2\,\frac{[\lambda\,(E'_q + m'_q)]^{1/2}}{3E'_q + m'_q}\,. \qquad (47)$$

The value M_B^0 is only a "zeroth order" approximation to the mass of a baryon. In the following we consider the 8 ground state baryons and investigate some standard corrections to M_B^0.

7.1 Center of mass correction

For the center of mass correction, we use the prescription of [10],

$$\Delta E_{\mathrm{cm}} = \left[\left(\sum_q E_q\right)^2 - \sum_q \langle\, \mathbf{p}_q^2 \,\rangle\right]^{1/2} , \qquad (48)$$

with $\langle\, \mathbf{p}_q^2 \,\rangle$ denoting the expectation value of the linear momentum of a quark taken with respect to his wave function,

$$\langle\, \mathbf{p}^2 \,\rangle = \int \mathrm{d}^3 r\, q^\dagger(\mathbf{r})\, \mathbf{p}^2\, q(\mathbf{r}) \, . \qquad (49)$$

By inserting expression (23) for the quark wave function, performing various integration tricks and using the normalization condition (31), we arrive at the formula

$$\langle\, \mathbf{p}^2 \,\rangle = E^2 - m^2 - 4\int_0^\infty \mathrm{d}r\, u^2(r)\, V(r)\, [E - V(r)] \, . \quad (50)$$

From this expression it is clear how Einstein's formula for a free particle is modified by the potential $V(r)$. We emphasize that (31) and (50) are not restricted to the ground state, but are valid for any eigenstate of the form (23), as long as the potential is of the type $(1+\gamma^0)V(r)$. Substituting the wave function given in (26), we get

$$\langle\, \mathbf{p}^2 \,\rangle = \frac{(11E' + m')\,(E'^2 - m'^2)}{6\,(3E' + m')} \, . \qquad (51)$$

7.2 One-gluon exchange

The non-perturbative part of the quark–gluon interaction is modeled by the confining potential $V_{\mathrm{c}}(r)$, which was introduced in Sect. 3. The perturbative part is taken into account by adding to the fuzzy bag Lagrangian (11) a pure glue term and a quark–gluon interaction term, as prescribed by QCD. Since we are considering only one-gluon exchange, there are no contributions from gluon self-interactions. The calculations proceed then much in the same way as in electrodynamics. The color-electric and color-magnetic fields are generated by the quark color-vector currents, which are given by

$$j^\mu_{aq}(x) = \overline{q}_q(x)\gamma^\mu \lambda_a q_q(x) \, . \qquad (52)$$

It can be checked that the $j^\mu_{aq}(x)$ are time-independent, and this implies that the color-electromagnetic fields are static. They obey the following Maxwell equations:

$$\begin{aligned} \nabla \cdot \mathbf{E}^a(\mathbf{r}) &= -g\,\rho^a(\mathbf{r}) \, , \\ \nabla \times \mathbf{E}^a(\mathbf{r}) &= 0 \, , \\ \nabla \times \mathbf{B}^a(\mathbf{r}) &= -g\,\mathbf{j}^a(\mathbf{r}) \, , \\ \nabla \cdot \mathbf{B}^a(\mathbf{r}) &= 0 \, , \end{aligned} \qquad (53)$$

where $\rho^a(\mathbf{r})$ and $\mathbf{j}^a(\mathbf{r})$ are the time and the space components of $j^\mu_a(\mathbf{r})$. The total quark current $j^\mu_a(\mathbf{r})$ is the sum of the individual currents written in (52). Due to the linearity of (53), one can write the color-electromagnetic fields as superpositions of the fields generated by each valence quark and then solve (53) for each quark flavor. The energy shift due to the quark–gluon interaction is then written as

$$\begin{aligned} \Delta E_g &= \Delta E_g^{\mathrm{E}} + \Delta E_g^{\mathrm{M}} \, , \\ \Delta E_g^{\mathrm{E}} &= \frac{g^2}{8\pi} \sum_{q,q'} \sum_a \int \mathrm{d}^3 r\, \mathrm{d}^3 r'\, \frac{j^0_{aq}(\mathbf{r})\, j^0_{aq'}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} \, , \\ \Delta E_g^{\mathrm{M}} &= -\frac{g^2}{8\pi} \sum_{q\neq q'} \sum_a \int \mathrm{d}^3 r\, \mathrm{d}^3 r'\, \frac{\mathbf{j}_{aq}(\mathbf{r}) \cdot \mathbf{j}_{aq'}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} \, . \end{aligned} \qquad (54)$$

By using the quark wave function (23) and the solution (26), one can write the one-gluon exchange correction in the form

$$\begin{aligned} \Delta E_g^{\mathrm{E}} &= \alpha_{\mathrm{c}}\, (a_{uu}\, I^{\mathrm{E}}_{uu} + a_{us}\, I^{\mathrm{E}}_{us} + a_{\mathrm{ss}}\, I^{\mathrm{E}}_{ss}) \, , \\ \Delta E_g^{\mathrm{M}} &= \alpha_{\mathrm{c}}\, (b_{uu}\, I^{\mathrm{M}}_{uu} + b_{us}\, I^{\mathrm{M}}_{us} + b_{\mathrm{ss}}\, I^{\mathrm{M}}_{ss}) \, , \end{aligned} \qquad (55)$$

where

$$\begin{aligned} I^{\mathrm{E}}_{qq'} &= \frac{16}{3\sqrt{\pi}}\, \frac{1}{R_{qq'}} \left[1 - \frac{\alpha_q + \alpha_{q'}}{R^2_{qq'}} + \frac{3\alpha_q\alpha_{q'}}{R^4_{qq'}}\right] , \\ I^{\mathrm{M}}_{qq'} &= \frac{256}{9\sqrt{\pi}}\, \frac{1}{R^3_{qq'}}\, \frac{1}{(3\,E'_q + m'_q)}\, \frac{1}{(3\,E'_{q'} + m'_{q'})} \, , \end{aligned} \qquad (56)$$

and we defined

$$\begin{aligned} \alpha_q &= \frac{1}{(E'_q + m'_q)(3E'_q + m'_q)} \, , \\ R^2_{qq'} &= \frac{3}{(E'^2_q - m'^2_q)} + \frac{3}{(E'^2_{q'} - m'^2_{q'})} \, . \end{aligned} \qquad (57)$$

The constants $a_{qq'}$ are given by

$$\begin{aligned} a_{qq'} &= \langle B|\sum_a \lambda^a_q \lambda^a_{q'} |B\rangle \, , \\ b_{qq'} &= -\frac{3}{16}\, \langle B|\sum_a \lambda^a_q \lambda^a_{q'}\; \sigma_q \cdot \sigma_{q'} |B\rangle \, , \end{aligned} \qquad (58)$$

and their values for the fundamental baryon octet can be found for example in [11]. We observe that the indices q and q' now refer to the quark flavors, and furthermore that a d quark counts as a u quark, since we assume $m_d = m_u$.

7.3 One-pion exchange

The pion–quark interaction induces renormalization effects on the hadronic wave functions, and this should be taken into account in a fit of the hadronic properties. The calculations are done in a framework similar to the Chew and Low model of the nucleon–pion interaction. This means that anti-baryons are not taken into account and that hadron recoil is neglected. The starting point of

Table 1. Renormalization coefficients for one-pion exchange

Baryon	N	Δ	Λ	Σ	Σ^*	Ξ	Ξ^*	Ω^-
a_B	$\frac{513}{25}$	$\frac{297}{25}$	$\frac{324}{25}$	$\frac{180}{25}$	$\frac{180}{25}$	$\frac{81}{25}$	$\frac{81}{25}$	0

the calculation is the Hamiltonian for the pion–bag system, composed by the bag energy, the pion field energy and the pion–bag interaction energy,

$$\mathcal{H} = \sum_B M_B^0\, B^\dagger\, B + \frac{1}{2}\int \mathrm{d}^3 r\ \left(\partial_t\,\pi\cdot\partial_t\,\pi + \nabla\pi\cdot\nabla\pi + \left[m_\pi^2 + V_\pi(r)\right]\pi^2\right) + \frac{\mathrm{i}}{f_\pi}\int \mathrm{d}^3 r\,\frac{V(r)}{\sqrt{F_\pi(r)}}\sum_q \overline{q}_q\,\gamma_5\,\tau_q\cdot\pi\,q_q\ . \tag{59}$$

Here $B^\dagger$ and B are creation and destruction operators for bag states and M_B^0 is the bare (unperturbed) hadron mass calculated in (46). Since the pion field is subject to a scalar central potential, it is convenient to expand the quantized pion field in angular momentum modes. With the exception of this point, the calculation proceeds in a standard way. Details can be looked up in the literature [12,13]; we quote here just the final results. The perturbed, physical baryon states $|\tilde{B}\rangle$ determined up to one order of the interaction are given by

$$|\tilde{B}\rangle = Z_B^{1/2}|B\rangle - Z_B^{1/2}\sum_{j\ell m}\sum_{B'}\int_0^\infty \mathrm{d}k\frac{v_{j\ell m}^{BB'}(k)}{\omega_k} \times|B',\pi_{j\ell m}(k)\rangle\ . \tag{60}$$

The probability Z_B of finding the bare baryon component in the physical baryon state is found to be

$$Z_B^{-1} = 1 + a_B\,f_{NN\pi}^2\,Z_\pi\ , \tag{61}$$

and the pion contribution to the mass is

$$\Delta E_\pi = -\,a_B\,f_{NN\pi}^2\,\delta_\pi\ , \tag{62}$$

with the values of a_B given in Table 1. The quantities Z_π and δ_π are defined as

$$Z_\pi = \int_0^\infty \mathrm{d}k\,k^2\,\frac{v^2(k)}{\omega_k^3}\ ,\qquad \delta_\pi = \int_0^\infty \mathrm{d}k\,k^2\,\frac{v^2(k)}{\omega_k^2}\ , \tag{63}$$

where $v(k)$ is the interaction vertex,

$$v(k) = \frac{20}{3\sqrt{3\pi}\,m_\pi\,g_\mathrm{A}}\int_0^\infty \mathrm{d}r\,r\,\frac{V(r)}{\sqrt{F_\pi(r)}}\,h_k(r)\,g(r)f(r)\ . \tag{64}$$

8 Nucleon properties

8.1 The axial coupling constant of the nucleon

The axial coupling constant g_A is defined as

$$\langle N|\,\sigma_N^i\,\frac{\tau_N}{2}\,|N\rangle\,g_\mathrm{A} = \langle N|\int \mathrm{d}^3 r\left[\sum_q \mathbf{j}_{\mathrm{A}q}^i(\mathbf{r}) + \mathbf{j}_{\mathrm{A}\pi}^i(\mathbf{r})\right]|N\rangle \tag{65}$$

and receives contributions from the quarks as well as from the pions. The axial-vector current carried by the quarks is given by

$$\mathbf{j}_{\mathrm{A}q}^\mu(\mathbf{r}) = \overline{q}_q\gamma^\mu\gamma_5\frac{\tau_q}{2}\,q_q\ , \tag{66}$$

and, for quarks in the ground state, the quark contribution to g_A can be written in the form

$$g_{\mathrm{A}q}^0 = \frac{20}{9}\int_0^\infty \mathrm{d}r\,u^2(r) - \frac{5}{9}\ . \tag{67}$$

We should also consider center of mass and recoil corrections to the quark contribution to the axial charge. According to [10], the center of mass correction is given by

$$g_{\mathrm{A}q} = g_{\mathrm{A}q}^0\left(1 + \frac{\langle\,\mathbf{p}^2\,\rangle}{M_N^2}\right)\ , \tag{68}$$

while the recoil correction vanishes [4]. With the explicit solution (26), we find

$$g_{\mathrm{A}q}^0 = \frac{5\,(5E_u' + 7m_u')}{9\,(3E_u' + m_u')}\ , \tag{69}$$

and the value of $\langle\,\mathbf{p}^2\,\rangle$ is calculated from (51).

The pion contribution to g_A is obtained by inserting in (65) the pion axial-vector current given in (16). In contrast to most hadronic models in the literature, the pion contribution does not vanish in the fuzzy bag model. We find

$$\langle N|\sigma_N^i\,\frac{\tau_N}{2}|N\rangle\,g_{\mathrm{A}\pi} = -f_\pi\,\langle N|\int \mathrm{d}^3 r\,\frac{\pi\,\partial^i F_\pi}{\sqrt{F_\pi}}\,|N\rangle\ . \tag{70}$$

The pion field here is generated by the quarks in the nucleon. It is time-independent, and, by writing it in the form

$$\pi(\mathbf{r}) = \langle N|\sum_q(\sigma_q\cdot\hat{r})\,\tau_q|N\rangle\,\frac{h(r)}{r}\ , \tag{71}$$

we obtain

$$g_{\mathrm{A}\pi} = -\frac{40\pi f_\pi}{9}\int_0^\infty \mathrm{d}r\,r\,h(r)\,\frac{\partial_r F_\pi}{\sqrt{F_\pi}}\ . \tag{72}$$

The radial component of the pion wave function obeys the equation

$$h''(r) - \left(m_\pi^2 + \frac{2}{r^2} + V_\pi(r)\right)h(r) = \frac{rV(r)g(r)f(r)}{2\pi f_\pi\sqrt{F_\pi(r)}}\ , \tag{73}$$

which is derived from the non-homogeneous equation for the pion field, obtainable from the Lagrangian density (6).

8.2 The nucleon charge radius

The charge radius of the nucleon is defined as a derivative of the nucleon charge form-factor. By using the physical nucleon states obtained in Sect. 7.3 and using phenomenological parameters for the pion contribution [14,13,15], we find for the proton and the neutron respectively that

$$\begin{aligned}\langle r^2\rangle_p &= Z_N\left[1+\frac{459}{25}\,Z_\pi f_{NN\pi}^2\right]\left(\langle r_q^2\rangle-\frac{3}{4M_N^2}\right)\\&+\frac{54}{25}\,Z_N Z_\pi f_{NN\pi}^2\left(\langle r^2\rangle_\pi+\frac{3}{2}\,\Lambda^2\right)\,,\\\langle r^2\rangle_n &= Z_N\left[\frac{54}{25}\,Z_\pi f_{NN\pi}^2\right]\left(\langle r_q^2\rangle-\frac{3}{4M_N^2}\right)\\&-\frac{54}{25}\,Z_N Z_\pi f_{NN\pi}^2\left(\langle r^2\rangle_\pi+\frac{3}{2}\,\Lambda^2\right)\,,\end{aligned}\tag{74}$$

where $\langle r^2\rangle_\pi^{1/2}=0.78\,\mathrm{fm}$ is the experimental value of the root-mean-squared radius of the pion and $\Lambda^2=2\,\mathrm{fm}^2$. The mean-squared radius of a quark wave function, $\langle r_q^2\rangle$, is estimated with center of mass [10] and recoil [4] corrections as

$$\begin{aligned}\langle r_q^2\rangle &= \left(\langle r_q^2\rangle_0-\frac{3e_N}{4\langle\mathbf{p}^2\rangle}\right)\left(1+\frac{\langle\,\mathbf{p}^2\,\rangle}{M_N^2}\right)\\&\times\left(1-\frac{2E_q}{M_N}+\frac{3E_q^2}{M_N^2}\right)\,,\end{aligned}\tag{75}$$

where e_N is the electric charge, and the static value of the mean-squared radius is

$$\begin{aligned}\langle r_q^2\rangle_0 &= \int \mathrm{d}^3r\,q^\dagger(\mathbf{r})\,r^2\,q(\mathbf{r})\\&=\frac{2}{E+m}\int_0^\infty \mathrm{d}r\,r^2\,u^2(r)[E-V(r)]\\&+\frac{3}{(E+m)^2}\int_0^\infty \mathrm{d}r\,u^2(r)\,.\end{aligned}\tag{76}$$

Substituting the solution (26) for $u(r)$ we get

$$\langle r_q^2\rangle_0=\frac{3}{2}\frac{(11E'+m')}{(3E'+m')(E'^2-m'^2)}\,.\tag{77}$$

8.3 The nucleon magnetic moment

The nucleon magnetic moments are obtained by taking the limit $k\to 0$ of the magnetic form-factors. By using the physical nucleon states obtained in Sect. 7.3 and using phenomenological parameters for the pion contribution as in the preceding subsection, we find for the proton and the neutron respectively

$$\begin{aligned}\mu_p &= 2M_N Z_N\left[1+\frac{87}{5}\,f_{NN\pi}^2 Z_\pi\right]\mu_q\\&+\frac{528}{25}\,M_N Z_N f_{NN\pi}^2 Z_2\,,\\\mu_n &= 2M_N Z_N\left[1+18f_{NN\pi}^2 Z_\pi\right]\mu_q\\&-\frac{528}{25}\,M_N Z_N f_{NN\pi}^2 Z_2\,,\end{aligned}\tag{78}$$

where Z_2 is defined as

$$Z_2=\int_0^\infty \mathrm{d}k\,k^2\,\frac{v^2(k)}{\omega_k^4}\,.\tag{79}$$

The magnetic moment of the quarks is estimated with center of mass [10] and recoil [4] corrections as

$$\begin{aligned}\mu_q &= c_N\mu_q^0\left(1+\frac{3\langle\,\mathbf{p}^2\rangle}{2M_N^2}\right)\left(1-\frac{E_q}{M_N}\right)\\&+\frac{e_N\langle\,\mathbf{p}^2\rangle}{2M_N^2}+c_N\Delta\mu_q\,,\end{aligned}\tag{80}$$

where $c_N=1$ for the proton and $c_N=-2/3$ for the neutron, and

$$\Delta\mu_q=\int_0^\infty \mathrm{d}r\,r^2\left[g^2(r)-\frac{1}{3}\,f^2(r)\right]\,.\tag{81}$$

The static value of the magnetic moment can be written in a simple form as

$$\mu_q^0=\frac{2M_N}{E+m}\int_0^\infty \mathrm{d}r\,u^2(r)\,.\tag{82}$$

Substituting the solution (26) for $u(r)$ we get

$$\begin{aligned}\mu_q^0 &= \frac{4M_N}{3E'+m'}\,,\\\Delta\mu_q &= \frac{5E'+7m'}{3(3E'+m')}\,.\end{aligned}\tag{83}$$

9 Results and concluding remarks

We adjusted the parameters of the model so that the masses of the fundamental baryon octet and the nucleon observables g_A, $\langle r^2\rangle_p$, $\langle r^2\rangle_n$, μ_p and μ_n were as close as possible to their experimental values. The optimal set of parameters was found to be

$$\begin{aligned}&m_u=80\,\mathrm{MeV}\,,\quad m_s=255\,\mathrm{MeV}\,,\quad V_0=-158\,\mathrm{MeV}\,,\\&\lambda=1.54\,\mathrm{fm}^{-3}\,,\quad R_\pi=2.16\,\mathrm{fm}\,,\quad n_\pi=0.14\,,\\&\alpha_\mathrm{c}=0.55\,,\quad f_{NN\pi}=0.671\,.\end{aligned}\tag{84}$$

The masses of the 8 ground state baryons are given by

$$M=E_B+E_\mathrm{vac}+\Delta E_\mathrm{cm}+\Delta E_g^\mathrm{E}+\Delta E_g^\mathrm{M}+\Delta E_\pi\,.\tag{85}$$

In Table 2 we show the values obtained in comparison with experiment and also discriminate the different contributions to the masses. The vacuum energy is seen to represent about 30% of the baryon masses, while the center of mass, one-gluon and one-pion corrections are sufficiently low and can be considered as additive, as in (85).

Table 2. Baryon masses in comparison with experiment. Also shown are the different contributions to the masses. All quantities are in MeV

Baryon	N	Δ	Λ	Σ	Σ^*	Ξ	Ξ^*	Ω^-
exp.	939	1232	1116	1193	1385	1318	1533	1672
theory	934	1244	1127	1220	1391	1352	1523	1639
E_B	1187	1187	1270	1270	1270	1353	1353	1435
$E_{\rm vac}$	316	316	312	312	312	307	307	302
$\Delta E_{\rm cm}$	−189	−189	−184	−184	−184	−181	−181	−177
$\Delta E_g^{\rm E}$	0	0	5	5	5	5	5	0
$\Delta E_g^{\rm M}$	−95	95	−95	−82	89	−88	83	78
ΔE_π	−285	−165	−180	−100	−100	−45	−45	0

Table 3. Nucleon properties in comparison with experiment

	$g_{\rm A}$	$\langle r^2\rangle_p$	$\langle r^2\rangle_n$	μ_p	μ_n
exp.	1.2573	0.743	−0.119	2.79	−1.91
theory	1.0610	0.760	−0.100	2.95	−1.75

The results for the nucleon properties are presented in Table 3. As commented in Sect. 8.1, the nucleon axial charge $g_{\rm A}$ receives contributions from both quarks and pions. The quark part is obtained from (67), while the pion part is calculated numerically from (72). Finding $g_{{\rm A}\pi}$ also involves finding the solution of (73) for the radial part of the pion wave function. We have found that $g_{{\rm A}\pi}$ is always negative and of small magnitude. For the specific set of parameters (84), we obtained

$$g_{{\rm A}\pi} = -0.0119 \, . \tag{86}$$

From Table 3, one sees that the mean-squared radii and magnetic moments of the proton and of the neutron are in excellent agreement with the experimental values. The relative errors for $\langle r^2\rangle_p$, $\langle r^2\rangle_n$, μ_p and μ_n are respectively 2%, 16%, 6% and 8%. The greater relative error in $\langle r^2\rangle_n$ is expected, since its magnitude is small in comparison with $\langle r^2\rangle_p$. The center of mass correction for $g_{\rm A}$ and the mean-squared radii was 16%, while for the magnetic moments it was 23%. The recoil correction, which has opposite sign to the center of mass correction, was 31% for the mean-squared radii and 58% for the magnetic moments. We have also found that the renormalized pion–nucleon coupling constant is $f^{\rm (R)}_{NN\pi} = 0.307$, which is close to the experimental value 0.283.

The parameter n_π determines both the strength of the pion potential (19) and the rise of the pion suppression function (see discussion before (9)). The value we obtained, $n_\pi = 0.14$, seems to indicate that the pion field is suppressed only very near the center of the bag.

We have obtained a very good fit of baryon masses and nucleon properties. Future plans are to consider two-pion exchange and to address the magnetic moments of the baryon octet.

We have also obtained simple expressions for the normalization condition, (31), the average squared momentum of a quark, (50), and the quark contributions to the axial charge (67), to the mean-squared radius, (76), and to the magnetic moment, (82). These formulas can also be used in other relativistic potential models. In particular, we mention (50), which expresses the expectation value of the squared momentum in a way that makes clear how Einstein's formula for a free particle is modified by a potential $V(r)$ of the form (10).

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