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Strongest gravitational waves from neutrino oscillations at supernova core bounce

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Abstract. Resonant active-to-active ($\nu_a \to \nu_a$), as well as active-to-sterile ($\nu_a \to \nu_s$) neutrino (ν) oscillations can take place during the core bounce of a supernova collapse. Besides, over this phase, weak magnetism increases the antineutrino ($\bar{\nu}$) mean free path, and thus its luminosity. Because the oscillation feeds mass-energy into the target ν species, the large mass-squared difference between the species ($\nu_a \to \nu_s$) implies a huge amount of energy to be given off as gravitational waves ($L_{GW} \sim 10^{49}\,\mathrm{erg\,s^{-1}}$), due to anisotropic but coherent ν flow over the oscillation length. This asymmetric ν-flux is driven by both the spin–magnetic and the *universal spin–rotation* coupling. The novel contribution of this paper stems from (1) the new computation of the anisotropy parameter $\alpha \sim 0.1$–0.01, and (2) the use of the tight constraints from neutrino experiments as SNO and KamLAND, and the cosmic probe WMAP, to compute the gravitational-wave emission during neutrino oscillations in supernovae core collapse and bounce. We show that the mass of the sterile neutrino ν_s that can be resonantly produced during the flavor conversions makes it a good candidate for dark matter as suggested by Fuller et al., Phys. Rev. D 68, 103002 (2003). The new spacetime strain thus estimated is still several orders of magnitude larger than those from ν diffusion (convection and cooling) or quadrupole moments of neutron star matter. This new feature turns these bursts into the more promising supernova gravitational-wave signals that may be detected by observatories as LIGO, VIRGO, etc., for distances far out to the VIRGO cluster of galaxies.

1 Introduction

That outflowing neutrinos (νs) from a supernova (SN) generate gravitational waves (GWs) was first pointed out by Epstein [12]. However, over the first $\sim 10\,\mathrm{ms}$ [25, 26] after the SN core bounce the central density gets so high that no radiation nor even νs can escape; they are thus frozen-in and strongly coupled to the neutron matter (N^0) as described by the Lagrangian (see [22] for this dynamics)

$$L^{\mathrm{int}}_{N^0 \leftrightarrow \nu} = \frac{G_F}{\sqrt{2}} \left[\bar{N}^0 \gamma_\mu (1-\gamma_5) N^0\right] \left\{\bar{\psi}\gamma^\mu (1-\gamma_5)\psi\right\} , \quad (1)$$

with the ν field (ψ) satisfying the time-dependent Dirac equation

$$\left[\mathrm{i}\gamma^0\partial_0 + \mathrm{i}\gamma^\alpha\partial_\alpha + \rho(t) v_\beta \gamma^\beta \left(\frac{1-\gamma_5}{2}\right) - m_\nu\right]\psi = 0. \quad (2)$$

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At this phase the whole proto-neutron star (PNS) dynamics is dominated by gravity alone and can be appropriately described by the general relativistic Oppenheimer–Volkoff equation for the $N^0 + \nu$ fluid [28]. As discussed by [25, 26], it is over this early transient that most ν flavor conversions are expected to resonantly take place and consequently the super strong GW burst from the oscillation process to be released. GWs from this decoupling have been suggested to likely be the ultimate process responsible for the neat kick given to a nascent pulsar during the SN collapse [28].

The contention of this paper is
(a) to pave, in the framework of general relativity (GR), the way to this fundamental astrophysical process of generation of GWs from ν oscillations in a PNS;
(b) to demonstrate, by taking into account experimental and observational constraints, that ν oscillations during the SN core bounce do produce GWs of the sort predicted by Einstein's GR theory, and more crucial yet,
(c) to stress that these bursts are the more likely SN GW-signals to be detected by interferometric observatories as

LIGO, VIRGO, GEO-600, etc. We speculate that such a signal perhaps might have been detected during the SN1987a event, despite the low sensitivity of the detectors at the time. Some claims in this direction were presented by Aglietta et al. [2] and in related papers.

2 The mechanism for generating GWs during neutrino oscillations

To start with, let us recall how the production of GWs during ν oscillations proceeds by considering the case of oscillations between active and sterile neutrinos in the supernova core. The essential point here is that oscillations into sterile neutrinos change dramatically the energy and momentum (linear and angular) configuration of the system: neutrinos plus neutron matter inside the PNS (check (2)). In particular, flavor conversions into sterile neutrinos drive a large mass and energy loss from the PNS because once they are produced they freely escape from the star. The reason: they do not interact with any ordinary matter around, i.e., they do couple to active ν species but neither to Z^0 nor to $W^{\pm}$ vector bosons. This means that oscillations into steriles, in dense matter, take place over longer oscillation lengths, compared to $\nu_{\rm a} \to \nu_{\rm a}$, and the steriles encounter infinite mean free paths thereafter. Physically, the potential, $V_{\rm s}(x)$, for sterile neutrinos in dense matter is zero. In addition, their probability of reconversion, still inside the star, into active species is quite small (see discussion below). This outflow translates into a noticeable modification of the PNS mass and energy quadrupole distribution, which, as discussed below, is dominated from the very beginning by rotational and magnetic field effects.

Since most steriles neutrinos escape along the directions defined by the dipole field and angular momentum vectors (see Fig. 2), the ν outflow is at least quadrupolar in nature. This produces a super strong gravitational-wave burst once the flavor conversions take place, the energy of which stems from the energy and momentum of the total number of neutrinos participating in the oscillation process[1]. Further, the gravitational-wave signal generated this way must exhibit a waveform with a *Christodolou's memory* [28].

The remaining configuration of the star must also reflect this loss. Hence, its own matter and energy distribution becomes also quadrupolar. Because this quadrupole configuration (the matter and energy still trapped inside the just-born neutron star) keeps changing over the time scale for which most of the oscillations take place, then GWs must be emitted from the star over that transient. At the end, the probability of conversion and the ν flux anisotropy parameter (α; see below) determine both how much energy partakes in the process and the degree of asymmetry during the emission. Both characteristics are determined next.

The case for oscillations among active species is a bit different, the key feature being that mass and energy is relocated coherently from one region to another inside the

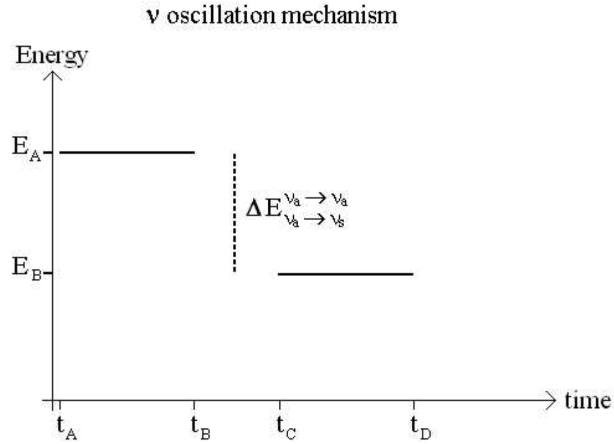

Fig. 1. The mechanism for ν conversions. Firstly, the state of a system, νs in the PNS, is defined by the energy E_A for the time interval $t_B - t_A$. After the flavor transitions that system is represented by a reduced energy E_B over time $t_D - t_C$. The total energy transferred to the new species is $\Delta E^{\nu_{\rm a}\leftrightarrow\nu_{\rm s}}_{\nu_{\rm a}\leftrightarrow\nu_{\rm a}} \equiv E_A - E_B$, and thus the ν luminosity over the transient is given by $L_\nu = \frac{E_A - E_B}{t_C - t_B}$

PNS, especially because of the *weak magnetism* of antineutrinos that allows them to have larger mean free paths (and thus oscillation lengths) [19]. In addition, oscillations of electron neutrinos into muon or tauon neutrinos leave these last species outside their own neutrinospheres, and hence they are in principle free to stream away. These neutral-current interacting ν species must be the very first constituents of the ν burst from any supernova since most ν_es are essentially trapped. This must also generate GWs during that sort of flavor conversions, although their specific strength (strain) must be a bit lower compared to conversions into sterile neutrinos where almost all the ν species may participate, and the large Δm^2 in the process.

Since the sterile neutrinos escape the core over a time scale of a few ms, the number of neutrinos escaping and their angular distribution is sensitive to the instantaneous distribution of neutrino production sites. Since thermalization cannot occur over such short times ($\Delta T^{\rm thermal} \sim 0.5\,{\rm s}$), and since the neutrino production rate is sensitive to the local temperature at the production site, the inhomogeneities during the collapse phase get reflected in the inhomogeneities in the escaping neutrino fluxes and their distributions. Because of both the ν spin–magnetic field ($\mathbf{B}$) and ν spin–angular momentum ($\mathbf{J}$) coupling the asymmetries in these distributions can give rise to quadrupole moments, which must generate gravitational waves as suggested by [28], and dipole moments which can explain the origin of pulsar kicks [15,23].

Fixed by the *probability of oscillation*, $P^{\nu_{\rm a}\to\nu_{\rm s}}_{\nu_{\rm a}\to\nu_{\rm a}}$, the *fraction* of neutrinos that can escape in the first few milliseconds is, however, *small*. Firstly, the neutrinos have to be produced roughly within one mean free path from their resonance surface. Secondly, since in the case of $\nu_{\rm a} \to \nu_{\rm s}$ oscillations $m_{\rm s}$ is the heaviest neutrino species, the sign of the effective potential $V(x)$ (see discussion below) and

[1] The attentive reader must regard that neutrinos carry away almost all of the binding energy of the just-born neutron star, i.e., $\Delta E_\nu \sim 3 \times 10^{53}$ erg.

the resonance condition indicates that only ν_es and the antineutrinos $\bar{\nu}_\mu$ and $\bar{\nu}_\tau$ can undergo resonant conversions. In Sect. 4 we address all these issues and determine this fundamental property of the mechanism for producing GWs from ν flavor conversions.

3 Anisotropic neutrino outflow: origin and computation

To provide a physical foundation for the procedure introduced here to determine the neutrino *asymmetry parameter*, α, which measures how large the deviation is of the ν flux from a spherical one, we recall next two fundamental effects that run into action once a PNS is forming after the supernova collapse. We stress that other physical processes such as convection, thermalization, etc., are not relevant over the time scale under consideration: $\Delta T_{\rm osc} \sim 3$–10 ms after the SN core bounce. These effects take a longer time (~ 30–100 ms) to start to dominate the physics of the PNS and therefore do not modify in a sensitive manner the picture described below.

At this point, a note of warning regarding the time scale we are using for the present calculations is of worth. This is specially so in the light of the very recent paper by Loveridge [24], where a very important extension of our original idea, introduced in [28], is provided in detail. In his computation of the gravitational radiation emission from an off-centered flavor-changing ν beam Loveridge used a time scale of $\Delta T^{\nu-{\rm burst}} \sim 10$ s, apparently based on the duration of the ν-burst from SN1987A.

The great novelty in Loveridge's paper is the prediction of a periodic GW signal from the flavor-changing ν beam eccentrically outflowing from the just-born pulsar. This GW signal would have characteristics that make it observable by both LIGO and LISA GW interferometers. Interestingly in itself, various arguments favoring evidence for regularly pulsed neutrino emission from SN1987a in the period range between (8.9–11.2) s were given by Harwit et al. [18] and by Saha and Chattopadhyay [35]. However, Fischer checked systematically for periodicities between (5–15) ms in the ν burst from SN1987A [13]. He disclaimed all those hypotheses by showing that the multitude of mediocre period fits seemed to be rather typical for events distributed randomly instead of periodically [13]. Thus, no significant periodicity exists in the arrival time of the neutrinos from SN1987A as detected by the Kamiokande II and IMB detectors. Nonetheless, any evidence for such a regular ν signal in a forthcoming (future) supernova might decidedly favor Loveridge's GW mechanism from ν oscillations in nascent pulsars.

Table 1. Time scales (in ms) called for in this paper

Supernova ν thermaliz. $\Delta T_\nu^{\rm thermal}$	Neutrino oscillations $\Delta T_{\rm osc}^\nu$	Supernova ν deleptoniz. $\Delta T_{\rm delept}^{\rm PNS}$	Supernova core-bounce $\Delta T_{\rm bounce}^{\rm PNS}$
~ 500	~ 3–10	~ 10	~ 20

As it stands, however, Loveridge's mechanism is decidedly different from ours in several respects. First, we do not invoke an off-centered rotating ν beam for producing the GW emission from the ν oscillations. In our mechanism the neutrino outflow is simultaneously acted upon by both the pulsar centered magnetic field and angular momentum vectors, as we describe below. Thus, the ν spin coupling to both vectors turns out to be the source of the, at least, quadrupolar ν outflow and GW emission during the flavor conversions. Secondly, as a consequence of this ν escape from the star the resulting GW signal, in principle, is not periodic, as opposed to Loveridge's. Indeed, the GW signal will look much like the one computed by Burrows and Hayes [7], including the appearance of a Christodoulou's memory effect in the waveform. Thirdly, as discussed next, the overall time scale for the process to take place in our mechanism is about three orders of magnitude shorter compared to the one assumed by Loveridge.

The ν oscillation time scale and the duration of the GW emission are both crucial features of both the mechanisms discussed above. In this regard, we believe that a very extensive set of references (here we just quote a few of them) showed that the ν signal from SN1987A exhibits a peculiar time profile [3, 9, 37]. According to the authors of these references, the ν burst observed from SN1987A is bunched into three clusters around (0.0–0.107) s, (1.541–1.728) s, (9.219–12.349) s. In particular, Cowsik [9] claimed that during the very early phase the lighter of the neutrinos arrived and by $\Delta T \sim 0.1$ s all of the neutrinos above the IMB threshold of 20 MeV had already gone past and thus were not seen by this ν detector. Therefore, if one stands on these pieces of evidence, it is clear that the largest portion of the total νs from SN1987A were emitted over a time scale smaller than 100 ms. This last time scale is an order of magnitude larger than the one we are favoring here, one which is taken from most of the theoretical analysis and numerical simulations of supernova core collapse and ν emission (see for instance [10–12]) in that a large portion of the released electron νs have undergone flavor conversions into sterile νs over a time scale much shorter than 100 ms, a reason why they went undetected.

In brief, although from the observational point of view, we may agree that the (~ 10 s) ν emission time scale assumed by Loveridge [24] appears to be a reasonable one, we think it is not a very realistic one to picture the ν flavor conversion mechanism inside the nascent neutron star, specially if one takes into account that the ν oscillation process implies a very short time scale: the resonance or *coherence length* time scale. As claimed above, this time scale must be related to the *oscillation length* (not the system's response time scale) over which most of the conversions must take place. Moreover, oscillations of massive neutrinos are damped when the propagation distance is greater than the coherence length

$$L_{\rm coh} = \lambda_{\rm osc} \left(\frac{E}{\Delta E} \right) . \tag{3}$$

Here E and ΔE are, respectively, the ν energy and energy spread determined by the production and detection conditions. In a supernova, the neutrinos non-forward scatter in continuous energy distributions so that $\Delta E \simeq E$, and hence the coherence length is nearly the oscillation length [32], which fixes the time scale $\Delta T_{\rm osc}$ we call for henceforth. Yet, in the early phases of a SN the neutrino flux is so large that the weak-interaction potential created by the neutrinos is compatible with that of the baryon matter around. Thus, neutrinos can be thought of as a dominant background medium that acts as a coherent superposition of flavor states that drives the conversions in a non-linear way. In other words, the oscillations become "synchronized", which means that all modes oscillate *collectively* with the *same frequency* [33,36]. Such a frequency should be related to the oscillation or coherence length and through it to the oscillation time scale. Thus, this last behavior adds to our argument in favor of a shorter time scale for the overall oscillations to take place. So, the $\Delta T_{\rm osc} \leq 10\,{\rm ms}$ is well-founded. Besides, a time scale as long as Loveridge's $\sim 10\,{\rm s}$ strongly disagrees with the Spruit and Phinney constraint on the overall time scale: $\sim 0.32\,{\rm s}$, for the kick driving mechanism [38].

Indeed, if ν thermalization, for instance, already took place, then the oscillations are severely precluded, since oscillations benefit of the existence of energy, matter density and entropy gradients inside the PNS [1,4], which are "washed out" once thermalization sets on.

3.1 Why would there be no room for ν-driven convection over $\Delta T_{\rm osc}$?

In order to back the dismissal in our discussion on neutrino oscillations of the effects of convection inside the proto-neutron star, we would like to take advantage of some arguments presented in the state-of-the-art treatment of the subject by Janka et al. [20], who provided a detailed analysis of convection inside the nascent neutron star. In particular, these authors showed that the growth time scale of convective instabilities ($\tau_{\rm cv}$) in the neutrino-heated region (adjacent layers outside the just-born neutron star, of relevance for successful supernova explosions) depends on the gradients of entropy and lepton number through the growth rate of Ledoux convection, $\sigma_{\rm L}$, as follows:

$$\tau_{\rm cv} \simeq \frac{\ln(100)}{\sigma_{\rm L}} \qquad (4)$$

$$\simeq 4.6\left[\frac{g}{\rho}\left(\frac{\partial\rho}{\partial s}\right)_{Y_e,P}\frac{{\rm d}s}{{\rm d}r} + \left(\frac{\partial\rho}{\partial Y_e}\right)_{s,P}\frac{{\rm d}Y_e}{{\rm d}r}\right]^{-1/2},$$

or equivalently

$$\tau_{\rm cv} \simeq 20\,{\rm ms}\left(\frac{R_{\rm s}}{R_{\rm g}} - 1\right)^{1/2}\frac{R_{\rm g,7}^{3/2}}{\sqrt{M_1}}, \qquad (5)$$

where $M_1 = 1M_\odot$, and $R_{\rm s}$ and $R_{\rm g}$ define the shock and gain radius, respectively, and $R_{\rm g,7}$ is a function of the gain radius, the temperature inside the star, and the neutrino luminosity and energy. Here the estimates were obtained for $g = GM/R_{\rm g}^2$, $(\partial\rho/\partial s)_P \sim -\rho/s$, and ${\rm d}s/{\rm d}r \sim -\frac{1}{2}\frac{s}{(R_{\rm s}-R_{\rm g})}$ (see [20] for further details).

Numerical simulations demonstrate that convection inside the proto-neutron star does start as early as a few tens of milliseconds after core bounce. It develops in both
(i) unstable near-surface regions, i.e., in layers around the neutrinosphere where the density is $\rho \leq 10^{12}\,{\rm g\,cm^{-3}}$, and (ii) deeper layers (of density $\rho \geq 10^{12}\,{\rm g\,cm^{-3}}$), where a negative lepton number gradient appears. Despite this piece of evidence, the time scale defined by (5) is relatively long compared to both the estimated time interval for the deleptonization process to take place: $\Delta T_{\rm delept} \sim 10\,{\rm ms}$, i.e., the time over which most electron neutrinos are produced [25,26], and the core bounce time scale: $\Delta T_{\rm bounce} \sim 20\,{\rm ms}$; where the large part of the neutrino luminosity associated with other flavors is produced through processes like bremsstrahlung, neutrino–neutrino and neutrino–nucleon scattering within less than 5 ms (see [20,25,26], and references therein). Indeed, from the convective regions below the neutrinosphere neutron fingers dig into the star and reach its center in about 1 s. Then they propagate outwards to englobe almost the whole exploding star. Under the physical conditions dominant over the first 10–20 ms after core bounce one expect most neutrino oscillations of all flavors to take place at that time. Thence, convective effects are not relevant during a time scale that short. As such it cannot modify in a significant fashion our analysis regarding the mechanism for the generation of gravitational waves from neutrino oscillations, here highlighted.

Moreover, the authors of [20] stressed the "disastrous" rôle of rotation for convection. A high rotation velocity of the just-born neutron star reduces dramatically the effects of convection because of the suppression of the neutrino–nucleon interaction due to nucleon correlations in the nuclear medium composing the proto-neutron star. Physically, rotation leads to a suppression of convective motions near the rotation axis because of a stabilizing stratification of the star matter's specific angular momentum. In passing, we stress that a similar effect is also expected from the action of a background magnetic field. Both effects, rotation and magnetic field, then appear to be more crucial for the physics of neutrino interactions inside the newly-born neutron star, and for the production of GWs during the oscillation transient. Thus we address both of these next.

3.2 ν–rotation interaction

That gravity couples to neutrinos is well known since Dirac. The very first work, as far as we are aware of, to show that the particle spin and PNS rotation couples gravitationally, a *universal* feature [39], was that of Unruh [41]. It was shown that a consistent minimal-coupling generalization to a curved background (a Kerr spacetime in that case) of the ν equations is possible, and that it leads to equations *separable* for the radial and angular components, though coupled. For a massless ν field (compare to (2)), and standard Minkowski space Dirac matrices γ^A, the Dirac equation derived by Unruh reads

$$\gamma^A \left(\frac{\partial}{\partial x^A} - \Gamma_A \right) \psi = 0 \,, \quad (6)$$

where the Dirac γ^A matrices relate to the Kerr spacetime metric through

$$\gamma^A \gamma^B + \gamma^B \gamma^A = 2 g^{AB}_{\mathrm{Kerr}} \,. \quad (7)$$

Equation (7) shows that the γ^A matrices satisfy the Clifford algebra. Further, the spin-affine connections Γ_A in (6) are uniquely determined by the relations

$$\Gamma_A \gamma^B - \gamma^B \Gamma_A = \frac{\partial \gamma^B}{\partial x^A} + \Gamma_{\alpha A}{}^B \gamma^\alpha \quad \text{and} \quad \mathrm{tr}(\Gamma_A) = 0 \,. \quad (8)$$

From the ν-number current, $J^A(x) = \bar{\psi}(x)\gamma^A\psi(x)$, it was shown that the ν-number density is always positive and is given by $(-g)^{1/2} J^t(x) = (-g)^{1/2} \bar{\psi}(x)\gamma^t\psi(x)$ (see [41] for details). The complete analysis of the coupling shows that the ν field in this background is not *superradiant*, as opposed to the case of the classical fields studied previously.

Vilenkin [42] extended the above analysis and showed that, upon admitting *helicity* (L) to be a *good* quantum number, the angular distribution $F_{jm}(\theta)$ of the thermal fermion gas of νs $(L = +1)$ and $\bar{\nu}$s $(L = -1)$ in the mode (j, m), with the function $F_{jm}(\theta)$ satisfying the normalization condition $2\pi \int_0^\pi F_{jm}(\theta) \sin\theta \mathrm{d}\theta = 1$, leads to an *asymmetric* ν emission (see illustration in Fig. 2) from a Kerr black hole (BH), of specific angular momentum $a \equiv J/M$, described by

$$\frac{\mathrm{d}N}{\mathrm{d}t\mathrm{d}\omega\mathrm{d}\theta} = \frac{1}{8\pi^2} M^2 \omega^2 \frac{\sum_\pm (1 \pm La\cos\theta)}{\left\{ \mathrm{e}^{\left[\frac{2\pi}{k}\left(\omega \pm \frac{1}{2}\Omega_{\mathrm{BH}}\right)\right]} + 1 \right\}} \,. \quad (9)$$

Equation (9) shows that more $\bar{\nu}$s are emitted in the direction parallel to the BH's spin, whilst more νs escape in the antiparallel direction.[2] Further, for other weak-interacting particles emitted from the BH, *parity* is *not conserved* [42]. More fundamental yet, the work by Vilenkin demonstrates that the same physics must be valid for any other rotating star. In other words, the ν spin coupling to rotation, in a gravitational background, is a *universal* feature, regardless which the spacetime source can be. From here onwards we shall take advantage of this feature for the case of neutrino emission from a rotating NS, and suggest that the basic quadrupole nature of the ν emission from the PNS stems in part from this spacetime effect. The other fundamental effect we address next.

3.3 ν–B field interaction

That the electromagnetic properties of νs are modified due to its interaction with a background matter distribution is a well-known fact. This is reflected in the additional contribution to its self-energy stemming from the ν spin-to-magnetic field coupling $(\mathbf{B} \cdot \mathbf{k}_\nu)$. In the case of $\nu_\tau \to \nu_e$ oscillations, for instance, the geometry of the ν sphere is dramatically deformed by $\mathbf{B}$ (see Fig. 2), an effect that strongly depends upon the relative pointing directions of both $\mathbf{B}$ and ν momentum $\mathbf{k}_\nu$. The magnetic coupling distorts the ν_τ surface ("sphere") in such a way that it is no more concentric with the ν_e sphere (see Figs. 1 and 2 in [21]). Therefore, ν_τs escaping parallel pointing to the $\mathbf{B}$ field have a *lower* temperature than those flowing away in the opposite (anti-parallel) direction.

At the same time, the ν spin coupling to rotation, (9) [42], also drives an effective momentum (and thus energy flux) asymmetry along the angular momentum direction $\mathbf{J}$, as shown in Fig. 2. For a relative orientation $\theta(\mathbf{B} \leftrightarrow \mathbf{J}) \neq 0$ between $\mathbf{B}$ and $\mathbf{J}$, i.e., a canonical pulsar, this combined action on the escaping νs of a rotating background spacetime plus magnetic field makes their ν sphere a decidedly distorted surface. More precisely, the volumetric region obtained by rotating around the hatched regions in Fig. 2 becomes at least a quadrupolar outflowing energy distribution. This is the source of the strong GW bursts in this mechanism when the oscillations ensue. It also justifies the *large value* of the anisotropy parameter α used in (14) below (see also [7]).

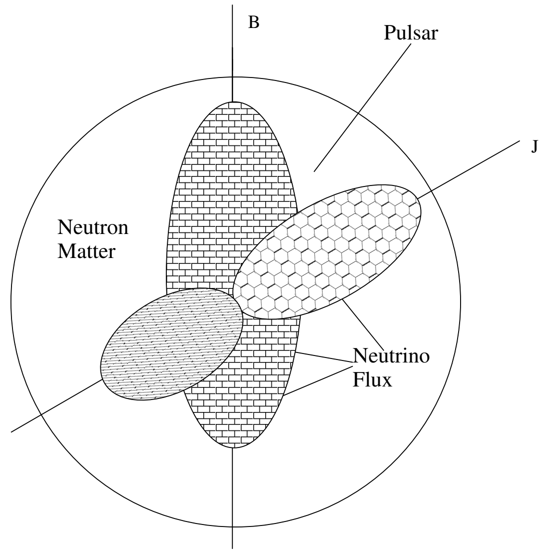

Fig. 2. A schematic representation of the ν-flux distribution (hatched regions) in a nascent pulsar. The at least quadrupolar distribution is evident and stems from the neutrino spin–magnetic and spin–rotation couplings

Table 2. Parameters of the PNS used in this paper

Radius	Mass	Density	ν sphere	Ang. Mom.
R_{PNS}	M_{PNS}	ρ_{PNS}	R^{ν}_{PNS}	a
$\sim 80\,\mathrm{km}$	$\sim 1.3\,M_\odot$	$3 \times 10^{11}\ \frac{\mathrm{g}}{\mathrm{cm}^3}$	~ 35–$60\,\mathrm{km}$	~ 0.9–1
		$2 \times 10^{14}\ \frac{\mathrm{g}}{\mathrm{cm}^3}$		

[2] Notice that this behavior is also manifest in the case of the magnetic field ν spin coupling discussed below. Therefore, these effects affect the oscillation probability, as we discuss later on.

3.4 The anisotropy parameter

Based on the concommitant action of both effects, the ν spin coupling to both the magnetic field and rotation described previously, one can determine the ν flow anisotropy in a novel, self-consistent fashion by defining α as the ratio of the total volume filled by the distorted ν spheres to that of the proto-neutron star (PNS), as one can infer from Fig. 2. The ν_e sphere radius of a non-magnetic non-rotating star is obtained from the condition

$$\tau_{\nu_e}(R_{\nu_e}) = \int_{R_{\nu_e}}^{\infty} \mathcal{K}_{\nu_e}\ \rho(r)\ \mathrm{d}r = \frac{2}{3}\ , \qquad (10)$$

where τ_{ν_e} is the optical depth, $\mathcal{K}_{\nu_e}$ the scattering opacity for electron neutrinos, and ρ the matter density. Following [8] one can take hereafter $R_{\nu_e} \equiv R_{\mathrm{PNS}}^{\nu_e} \equiv R_\nu \sim 35\,\mathrm{km}$, which is of the order of magnitude of the oscillation length λ_ν of a typical supernova ν, constituent as well of the atmospheric νs for which $\Delta m^2 \sim 10^{-3}\,\mathrm{eV}^2$ has been estimated by the Superkamiokande ν detector [14]:

$$\lambda_\nu \sim 31\,\mathrm{km}\ \left[\frac{E_{\nu_e}}{10\,\mathrm{MeV}}\right]\left(\frac{10^{-3}\,\mathrm{eV}^2}{\Delta m^2}\right)\ . \qquad (11)$$

Therefore, resonant conversions between active species may take place at the position r from the center defined by

$$r = R_{\nu_e} + \delta_0 \cos\phi\ , \qquad (12)$$

with ϕ the angle between the ν spin and $\mathbf{B}$, i.e., $\cos\phi = \frac{(\mathbf{k}\cdot\mathbf{B})}{|\mathbf{k}||\mathbf{B}|}$, and

$$\delta_0 = \frac{e\mu_e B}{2\pi^2(\mathrm{d}N_e/\mathrm{d}r)} \sim 1\text{–}10\,\mathrm{km}\ , \qquad (13)$$

for $B \sim 10^{14-15}$ G, respectively. Here e, $N_e = Y_e N_n$ and μ_e represent the electron charge, density and chemical potential, respectively. This defines in Fig. 2 an ellipsoidal figure of equilibrium with semi-axes

$$a = R_\nu + \delta_0, \qquad \text{and} \qquad b = R_\nu \qquad (14)$$

and volume (after rotating around $\mathbf{B}$)

$$V_{\mathrm{ellips.}} = \frac{4}{3}\pi R_\nu^2(R_\nu + \delta_0)\ . \qquad (15)$$

Meanwhile, the ν spin coupling to rotation described by (9) generates an asymmetric lemniscate-like plane curve (see Fig. 2)

$$r = R_\nu\ (1 \pm L\ a\ \cos\ \theta)\ , \qquad (16)$$

which upon a 2π rotation around the star angular momentum axis generates a volume

$$V_{\mathrm{lemnisc.}} = \frac{1}{4}(2R_\nu)^2 \times 2\pi\bar{y} + \frac{1}{4}(R_\nu)^2 \times 2\pi\bar{y}\ , \qquad (17)$$

where the quantity $\bar{y}$ (and $\bar{x}$) is defined as the location of the centroid of one of the lobes of that plane figure with respect to its coordenate center (x, y), and is obtained from the standard definition $\bar{y} = \frac{\int y\mathrm{d}A}{\int \mathrm{d}A}$. After a long, but straightforward, calculation one obtains

$$\bar{y} = \frac{\sqrt{\pi}}{\Gamma^2(1/4)}\ R_\nu\ , \qquad \bar{x} = \frac{4\sqrt{\pi}}{\Gamma^2(1/4)}\ R_\nu\ . \qquad (18)$$

Thence, for a PNS as the one modeled by [8], with parameters as given in Table 2 one obtains

$$\alpha_{\min} = \frac{V_{\mathrm{ellips.}} + V_{\mathrm{lemnisc.}}}{V_{\mathrm{PNS}}} \sim 0.11\text{–}0.01\ , \qquad (19)$$

a figure clearly compatible with the corresponding one in [7]. As such, this is essentially a new result of this paper. The attentive reader must notice in passing that the definition in (19) does take into account all of the physics of the neutrino oscillations: luminosity, density gradients and angular propagation, since (12) and (16) do gather the relevant information regarding the spatial configuration of the ν luminosity in as much as is done in the standard definition of α [7, 29]:

$$\alpha(t) \equiv \frac{1}{L_\nu(t)}\ \int_{4\pi} \mathrm{d}\Omega'\ \Psi(\theta', \phi')\ \frac{L_\nu(\Omega', t)}{\mathrm{d}\Omega'}\ . \qquad (20)$$

Indeed, one can get the “flavor” of the relationship between these two definitions by noticing that the quantity $L_\nu(\Omega', t)$ in the integrand of (20) can be expressed as $L_\nu(\Omega', t) \equiv L_\nu(t)F(\Omega')$, where the function $F(\Omega')$ now contains all the information regarding the angular distribution of the neutrino emission. Hence $L_\nu(t)$ can be factorized out of the integral and dropped from (20). This converts (20) in a relationship among (*solid*) angular quantities, which clearly can be reduced to a volumetric one, similar to the one introduced in (19), upon a transformation using the definition of *solid angle* in the form of *Lambert's law*: $\mathrm{d}\Omega = \frac{\mathrm{d}A\cos\theta}{R^2}$, and applied to the sphere representing the PNS. Here θ, measured from a coordinate system centered at the PNS (source coordinate system in [29], plays the role of the angle between the direction towards the observer and the direction Ω' of the radiation emission in (20), and subsequent equations, in [29]. Therefore, the novel result presented here is physically consistent with the standard one for the asymmetry parameter α.

4 Enlarged ν and GW luminosity from oscillations in dense matter

The outflow of νs from a SN core bounce is a well-known source of GWs [7, 12, 28, 29]. Numerical simulations [29] showed that, in general, the fraction of the total binding energy emitted as GWs by pure ν convection is $E_{\mathrm{GW}}^{\nu} \sim [10^{-10}\text{–}10^{-13}]\ M_\odot c^2$, for a ν total luminosity of $L_\nu \sim 10^{53}\,\mathrm{erg\,s}^{-1}$.

Unlike GWs produced by ν convection [29], in the production of GWs via ν oscillations [28] ($\nu_{\mathrm{a}} \leftrightarrow \nu_{\mathrm{a}}$ or $\nu_{\mathrm{a}} \leftrightarrow \nu_{\mathrm{s}}$) there exist two main reasons for expecting a major enhancement in the GW luminosity during the transition:

(a) the conversion itself, which makes the overall luminosity (L_{ν_x}) of a given ν_x species grow by a large factor: $\Delta L_{\nu_x} \leq 10\% L_\nu^{\rm total}$; see below. The enhancement stems from the mass-energy being given to, or drained from, the new ν species into which oscillations take place. This augment gets reflected in the species mass-squared difference, Δm^2, and their relative abundances: one species is number depleted while the other gets its number enhanced. But, even if the energy increase, or given up, is small,
(b) the abrupt resonant conversion over the transition time (see Table 1)

$$\Delta T_{\rm osc} \equiv \frac{\lambda_{\rm osc}}{\bar{V}_{\nu\text{-Diff.}}} \sim [10^{-4}\text{–}10^{-3}]\,{\rm s} \tag{21}$$

also magnifies transiently L_{ν_x}. Here $\lambda_{\rm osc}$ defines the oscillation length (computed below), and $\bar{V}_{\nu\text{-Diff.}} \sim 10^9\,{\rm cm\,s^{-1}}$ the convective ν diffusion velocity [29]. In Sect. 4 we estimate the transition probability $P_{\nu_{\rm a}\to\nu_{\rm a}}(|\mathbf{x}-\mathbf{x}_0|)$, $P_{\nu_{\rm a}\to\nu_{\rm s}}(|\mathbf{x}-\mathbf{x}_0|)$, the quantity that measures how many νs can indeed oscillate. This probability also fixes the total amount of energy participating in the generation of the GWs through this mechanism, as shown in Sect. 5.

If flavor transitions can indeed take place during supernova (SNe) core collapse and bounce, then they must leave some imprints in the SNe neutrino spectrum. The main observational consequences of neutrino conversions inside SNe include
(a) the partial or total disappearance of the neutronization peak; the moment at which most ν_es are produced,
(b) the interchange of the original spectrum and the appearance of a hard ν_e spectrum, together with
(c) distortions of the ν_e energy spectrum and
(d) alterations of the $\bar{\nu}_e$ spectrum [10]. As discussed below in Sect. 4.4, observations of the neutrino burst from SN1987a have allowed one to put some bounds on both the $\nu_{\rm a} \to \nu_{\rm a}$ and the $\nu_{\rm a} \to \nu_{\rm s}$ classes of flavor conversions.

4.1 Resonance, adiabaticity and the role of weak magnetism

As argued by [28], ν oscillations in vacuum produce no GWs. In the case of active-to-active ν oscillations (essentially the same physical argument holds also for active-to-sterile ν oscillations), the main reason for this negative result is that this class of conversions do not increase in a significant figure the total number of particles escaping from the proto-neutron star. In the case of active-to-active ν oscillations in dense matter, the process generates no GWs since the oscillations develop with the neutrinos having very short mean free paths, so that they move outwards can be envisioned as a standard diffusion process.

However, if one takes into account the novel result by Horowitz [19], the situation may change dramatically. According to this author, because of the active *antineutrino* species' *weak magnetism* their effective luminosity can be enlarged as much as 15% compared to the typical one they achieve when this effect is not taken into consideration during their propagation in dense matter. This result can be interpreted by stating that the number of oscillating (and potentially escaping) antineutrinos may be augmented by a large factor because now they do encounter longer mean free paths. Below we take advantage of this peculiar behavior of ν outflow in supernovae in computing the overall probability of the transition between active species and the GWs emitted in the process.

Wolfenstein [43] and Mikheev and Smirnov [27] pointed out that the neutrino oscillation pattern in vacuum can get noticeably modified by the passage of neutrinos through matter because of the effect of *coherent forward scattering*. Therefore, the interaction with matter, as pictured by (1), may help in allowing more νs to escape if resonant conversions into active [26] and/or sterile νs [28] occur inside the ν sphere of the active νs.

The description of the two-neutrino oscillations process in matter follows from the Schrödinger-like (because the dynamics is described as a function of the space variable x instead of the standard time t) differential equation [4,16]

$$\begin{aligned} {\rm i}\frac{{\rm d}}{{\rm d}x}\begin{pmatrix} a_\alpha \\ a_\beta \end{pmatrix} &= H_\nu^{\rm mat}\begin{pmatrix} a_\alpha \\ a_\beta \end{pmatrix} \\ &= \frac{1}{4E}\Big\{\Big[m_1^2+m_2^2+2\sqrt{2}G_{\rm F}(N(\nu_\alpha)+N(\nu_\beta))\Big] \\ &\quad\times\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ &\quad+\begin{pmatrix} A-\Delta m^2\cos 2\theta & \Delta m^2\cos 2\theta \\ \Delta m^2\cos 2\theta & -A+\Delta m^2\cos 2\theta \end{pmatrix}\begin{pmatrix} a_\alpha \\ a_\beta \end{pmatrix}\Big\}\,, \end{aligned} \tag{22}$$

where

$$A \equiv 2\sqrt{2}G_{\rm F}E\,(N(\nu_\alpha)-N(\nu_\beta))\,, \tag{23}$$

and $N(\nu_\alpha) \equiv \delta_{\alpha e}N_e - \frac{1}{2}N_n$, $\alpha,\beta = e,\mu,\tau,s$, $N(\nu_{\rm s}) = 0$, and $\Delta m^2 \equiv m_2^2 - m_1^2$. The eigenfunctions of the matter effective Hamiltonian follow from the relation

$$H_\nu^{\rm mat}\psi_{mj} = E_j\psi_{mj}\;; \tag{24}$$

where

$$\psi_{m1} = \begin{pmatrix} \cos\theta_m \\ -\sin\theta_m \end{pmatrix}, \qquad \psi_{m2} = \begin{pmatrix} \sin\theta_m \\ \cos\theta_m \end{pmatrix}. \tag{25}$$

The eigenvalues of E_j and the matter mixing angle θ_m are thus given by

$$\begin{aligned} E_{1,2} = &\Big[m_1^2+m_2^2+2\sqrt{2}G_{\rm F}(N(\nu_\alpha)+N(\nu_\beta))\Big] \\ &\mp\sqrt{(A-\Delta m^2\cos 2\theta)^2+(\Delta m^2\sin 2\theta)^2}\,, \end{aligned} \tag{26}$$

Table 3. Matter densities relevant for two-ν oscillations. Notice that $\nu_\mu \to \nu_\tau$ oscillations take place as in vacuum. The coupling Fermi constant is $G_{\rm F}^2 = 5.29\times10^{-44}\,{\rm cm^2\,MeV^{-2}}$, or equivalently $G_{\rm F} = 10^{-49}\,{\rm erg\,cm^{-3}}$

	$\nu_e \to \nu_{\mu,\tau}$	$\nu_e \to \nu_{\rm s}$	$\nu_\mu \to \nu_\tau$	$\nu_{\mu,\tau} \to \nu_{\rm s}$
$\frac{A}{2\sqrt{2}G_{\rm F}E}$	N_e	$N_e - \frac{1}{2}N_n$	0	$-\frac{1}{2}N_n$

and

$$\tan 2\theta_m = \frac{\tan 2\theta}{1 - \frac{A}{\Delta m^2 \cos 2\theta}} \; , \quad (27)$$

where θ is the vacuum mixing angle. By defining $U_m(x) = (\psi_{m1}, \ldots, \psi_{mn})$ as the mixing matrix of $n\nu$, and $\delta_j \equiv -i \int_{x_0}^{x_1} \mathrm{d}x' \psi_{mj}(x')^\dagger \psi_{mj}(x')$ as the adiabatic phases, one can compute the *oscillation amplitude* as follows:

$$\mathcal{A}^{\text{adiab}}_{\nu_\alpha \to \nu_\beta} = \sum_j U_m(x_1)_{\beta j} \exp\left(-\mathrm{i}\left[\delta_j + \int_{x_0}^{x_1} \mathrm{d}x' E_j(x')\right]\right) \times U_m^\star(x_0)_{\alpha j} \; . \quad (28)$$

Finally, by averaging over neutrino energies, i.e., by setting $\left\langle \exp\left(-\mathrm{i}\int_{x_0}^{x_1} \mathrm{d}x' \Delta E(x')\right)\right\rangle_{\text{av}} = 0$ [4], the outcoming transition probability among the active flavor-changing species thus reads

$$P_{\nu_a \to \nu_a}(|\mathbf{x}_1 - \mathbf{x}_0|) = 1 - \frac{1}{2}\left[1 + \cos 2\theta_m(x_0) \cos 2\theta_m(x_1)\right] , \quad (29)$$

where x_0 and x_1 correspond to the production (emission) and detection sites, respectively.

4.2 Active-to-active ν oscillations

As stated above, in order to produce an effect neutrinos must be able to escape the core without thermalizing with the stellar material. For active neutrino species of energies $\approx 10\,\text{MeV}$, this is not possible as long as the matter density is $\geq 10^{10}\,\text{g}\,\text{cm}^{-3}$. Since the production rate of neutrinos is a steeply increasing function of the matter density (production rate $\propto \rho^n$, where ρ is the matter density and $n > 1$), the overwhelming majority of the neutrinos of all species produced are trapped. This way, there seems to be no contribution to the GW amplitude for neutrino conversions taking place within the active neutrino flavors. In the first paper of this series [28], this difficulty was overcome only by addressing neutrino conversions into sterile species. Nonetheless, if there were indeed *weak magnetism effects* [19], one can rethink conversions within active species. In his new result on *weak magnetism for antineutrinos in core collapse supernovae*, Horowitz [19] showed that the antineutrinos' ($\bar{\nu}_x$s) luminosity could be noticeably increased because of their longer mean free paths, and this means that the total energy flux can be augmented in $\sim 15\%$ for νs of temperature $\sim 10\,\text{MeV}$. One can verify that longer mean free paths allows for a larger oscillation probability, and hence the contribution to the generation of GWs during flavor conversions within active species becomes non-negligible compared to the earlier case [28] where weak magnetism effects were not taken into account.

For active-to-active ν flavor conversions, for instance: $\nu_e \to \nu_\mu, \nu_\tau$; as implied by SNO results, the resonance must take place at a distance x_{res} from the PNS center and amid the active ν-spheres, whenever the following relation is satisfied (see Table 3):

$$\Delta m^2 \cos 2\theta_m = 2\sqrt{2}\;\; G_{\text{F}}\;\; N_e(x_{\text{res}})\;\; k_{\nu_e} \; . \quad (30)$$

Notice that we neglected the magnetic field contribution. Here $k_{\nu_e} = E_{\nu_e}/c$ is the ν_e momentum, and $N_e(x_{\text{res}}) = N_{e^-} - N_{e^+} \sim 10^{39-40}\,\text{cm}^{-3}$ is the electron number density. Thus the right-hand part of this equation reduces to

$$2\sqrt{2} G_{\text{F}} E_{\nu_e} N_e(x_{\text{res}}) = 6.9 \times 10^4\,\text{eV}^2 \left[\frac{\rho}{10^{11}\,\text{g}\,\text{cm}^{-3}}\right] \times \left(\frac{E_{\nu_e}}{10\,\text{MeV}}\right) , \quad (31)$$

for densities of order $\rho_{x=x_{\text{res}}} \sim 10^{11}\,\text{g}\,\text{cm}^{-3}$. One must recall that ν_es are produced at the PNS outermost regions [26], where the electron to baryon ratio is $Y_e \sim 0.1$. Hence, the resonance condition is satisfied for a mass-squared difference of about $\Delta m^2 \sim 10^4\,\text{eV}^2$, which implies a neutrino mass of about $m_\nu \sim 10^2\,\text{eV}$. Besides, neutrino flavor conversions in the resonance region can be strong if the adiabaticity condition is fulfilled [26], i.e., whenever [4,16]

$$\frac{\Delta m^2 \sin^2 2\theta_m}{2E_\nu \cos 2\theta_m} \left(\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}x}\right)^{-1}_{x=x_{\text{res}}} \gg 1 \; , \quad (32)$$

where x_{res} is the position of the resonance layer. Recalling that the typical scale of density variations in the PNS core is $h_{N_e} \sim (\mathrm{d}N_e/\mathrm{d}r)^{-1} \sim 6\,\text{km}$, this adiabatic behavior could be achieved as far as the density and magnetic field remain constant over the oscillation length:

$$\lambda_{\text{osc}} \equiv \left(\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}x}\right)^{-1}_{|x_{\text{res}}} \sim \frac{1}{\left(\frac{1}{2\pi}\frac{\Delta m^2}{2k_\nu}\sin(2\theta_m)\right)} \sim \frac{1\,\text{cm}}{\sin(2\theta_m)} \; , \quad (33)$$

of order h_{N_e}, which can be satisfied for $\Delta m^2 \sim 10^4\,\text{eV}^2$ as long as [23]

$$\sin^2 2\theta_m > 10^{-8} \; . \quad (34)$$

Although these ν oscillations could be adiabatic for a wide range of mixing angles and thus a large number of νs could actually oscillate, a ν_x mass such as this is incompatible with both the viable solutions to the solar neutrino problem (SNP) and the most recent cosmological constraints on the total mass of all stable neutrino species that could have left their imprint in the cosmic microwave background radiation (CMBR), as inferred from the observations performed by the satellite WMAP: $m_\nu \sim 1\,\text{eV}$. Therefore, we dismiss this possibility, since there appears to be no evidence for neutrino masses in this parameter range ($m_{\nu_x} \sim 10^2\,\text{eV}$) inside the PNS core.

On the other hand, if one takes into account the KamLAND results [11], one can see that resonant conversions with $\Delta m^2 = m^2_{\nu_2} - m^2_{\nu_1} \sim 10^{-4}\,\text{eV}^2$ would take place in supernova regions where the density is about $\rho \sim 10$–$30\,\text{g}\,\text{cm}^{-3}$, which corresponds to the outermost layers of the exploding star. The KamLAND results definitively demonstrated that

(i) a large mixing angle (LMA) solution of the solar ν problem is favored: $\sin^2 2\theta \sim 0.8$,

(ii) for the mass-squared difference we have $\Delta m^2 \sim 5.5 \times 10^{-5}\,\text{eV}^2$ (we use the approximate value $\Delta m^2 \sim 10^{-4}\,\text{eV}^2$ for the estimates).

Although a large number of ν species can in effect participate in the transitions there, i.e, the neutrino luminosity can still be a very large quantity, these regions are of no interest for the gravitational-wave emission from neutrino oscillations, since the overall energy density at that distance from the star center is relatively small. This does not mean that no GWs are emitted from transitions there, it does mean that their strain is very small so as to be detectable. As discussed by [10], observations of ν oscillations in this parameter range would provide useful information regarding the SNP, the hierarchy of neutrino masses and the mixing $|U_{e3}|^2$. Note in passing that oscillations in this range would imply a mass for the ν_2 species $m_{\nu_2} \sim 10^{-2}\,\mathrm{eV}\left(\frac{\rho}{10\,\mathrm{g\,cm^{-3}}}\right)$, in the case when $m_{\nu_2} > m_{\nu_1}$. This m_{ν_2} is compatible with current limits from WMAP [17,34].

Finally, let us consider oscillations in the parameter range estimated from CMBR by WMAP observations. In this case, resonant ν transitions would take place in regions where the density is as high as $\rho \sim 10^{7-8}\,\mathrm{g\,cm^{-3}}$, that is, at the supernova mantle or PNS upper layers. At these densities the oscillation length can still be $\lambda_{\rm osc} \sim 1$–$5\,\mathrm{km}$, and thus the conversions can be considered as adiabatic. Thus the resonance condition can be satisfied for $\Delta m^2 \sim 1\,\mathrm{eV}^2$ as long as $\sin^2 2\theta_m \leq 10^{-3}$.

Hence, by plugging this constraint into (27), and recalling that
(i) at least 6 ν species can participate in the flavor transitions,
(ii) most νs are emitted having $\mathbf{k}_\nu$ parallel to $\mathbf{B}$, which implies a further reduction factor of 2, and also
(iii) most νs are emitted having $\mathbf{k}_\nu$ parallel to $\mathbf{J}$, implying an additional reduction factor of 2, one can show from (29) that the *fraction* of $\nu_{\rm a}$ species that can eventually exchange flavor during the first few milliseconds after core bounce turns out to be [28]

$$P_{\nu_{\rm a}\to\nu_{\rm a}}(|\mathbf{x}-\mathbf{x}_{\rm res}|) \sim 0.1\% \,. \tag{35}$$

Thus, the total energy involved in the oscillation process we are considering could be estimated to be $E_\nu^{\rm tot} \simeq P_{\nu_{\rm a}\to\nu_{\rm a}}(|\mathbf{x}-\mathbf{x}_{\rm res}|) \times N_\nu\ k_\nu\ c\ (1+0.15)$, with N_ν the total number of neutrinos undergoing flavor conversions during the time scale $\Delta T_{\rm osc}$. This value leads to a bit stronger GW burst, as we show in Sect. 5 below.

Note in passing that the KamLAND $\bar{\nu}_e$ experiment suggests $P_{\bar{\nu}_e\to\bar{\nu}_{\mu,\tau}} \sim 40\%$ [11], while for LSND we have $P_{\bar{\nu}_\mu\to\bar{\nu}_e} \sim 0.26\%$. This result from LSND has so far not been ruled out by any terrestrial experiment, and there is a high expectation that it could be verified by MiniBoone at Fermilab.

4.3 Active-to-sterile ν oscillations

On the other hand, if the Liquid Scintillator Neutrino Detector (LSND) were a true indication of oscillations ($\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$) [17,34] with $\Delta m^2_{\rm LSND} \sim 10^{-1}\,\mathrm{eV}^2$, then active-to-sterile $\nu_{\rm a} \to \nu_{\rm s}$ oscillations could take place during a supernova core bounce [28]. Such oscillations would be resonant whenever the resonance condition is satisfied. As seen from Table 3 this could happen for

$$\sqrt{2}G_{\rm F}\left[N_e(x) - \frac{1}{2}N_n^0(x)\right] \equiv V(x) = \frac{\Delta m^2}{2E_\nu}\cos 2\theta_m \,. \tag{36}$$

Table 3 also shows that for $\nu_{\mu,\tau} \leftrightarrow \nu_{\rm s}$ the N_e term is absent, while in the case of $\bar{\nu}$s, the potential $V(x)$ changes by an overall sign. Numerically, for $\nu_e \leftrightarrow \nu_{\rm s}$ oscillations

$$V(x) = 7.5\times 10^2\left(\frac{\mathrm{eV}^2}{\mathrm{MeV}}\right)\left[\frac{\rho}{10^{10}\,\mathrm{g\,cm^{-3}}}\right]\left[\frac{3Y_e}{2} - \frac{1}{2}\right] . \tag{37}$$

For $\nu_{\mu,\tau} \leftrightarrow \nu_{\rm s}$ oscillations, the last term in parentheses becomes $\left(\frac{Y_e}{2} - \frac{1}{2}\right) \sim 1$.

ν conversions in the resonance region prove to be enhanced if the adiabaticity condition is fulfilled [26]. This is the same as requiring the oscillation probability in (29) to become $P_{\nu_{\rm a}\to\nu_{\rm s}} = \cos^2 2\theta_m \sim 1$. Moreover, after the resonance region the newly created sterile νs have very a small probability ($\langle P_{\nu_{\rm s}\to\nu_{\rm a}}\rangle = \frac{1}{2}\sin^2 2\theta_m$) of oscillating back to active νs, which could be potentially trapped. It is easy to check that the resonance condition in (36) is satisfied whenever

$$1\,\mathrm{eV}^2 \leq \Delta m^2_{\nu_a\to\nu_4} \leq 10^4\,\mathrm{eV}^2 \,. \tag{38}$$

Meanwhile, the adiabaticity condition, (32), holds if

$$\Delta m^2\frac{\sin^2 2\theta_m}{\cos 2\theta_m} \gg 10^{-3}\,\mathrm{eV}^2\left(\frac{E_\nu}{10\,\mathrm{MeV}}\right), \tag{39}$$

since at the PNS core $\lambda_{\rm osc} \sim 1\,\mathrm{km}$ (see (33)). This is easily satisfied for $\Delta m^2 \leq 10^4\,\mathrm{eV}^2$ as long as $\sin^2 2\theta_m \leq 10^{-7}$. This limit on the mixing angle is in agreement with Superkamiokande strong constraints on $\nu_e \to \nu_{\rm s}$ mixing [30]. Thence, we find that a substantial fraction ($P_{\nu_{\rm a}\to\nu_{\rm s}} \sim 1\%$) of νs may get converted to sterile νs and escape the core of the star, if the sterile ν mass (m_{ν_4}) is such that

$$1\,\mathrm{eV}^2 \leq \Delta m^2_{\nu_a\to\nu_4} \leq 10^4\,\mathrm{eV}^2 \,. \tag{40}$$

By looking at the lower limit for Δm^2 in (40) one can say that such a mass difference cannot solve the observed solar ν problem and/or be compatible with the atmospheric ν observations, but the possibility of having three light active νs of mass $m_{\nu_{1,2,3}} \sim [10^{-2}$–$10^{-3}]\,\mathrm{eV}$ explaining these anomalies and a "heavy" sterile $\nu_{\rm s}$ of mass $m_{\nu_4} \leq 1\,\mathrm{eV}$ as required by the LSND experiment remains as a viable alternative. On the other hand, if the sterile neutrino could be as massive as $m_{\nu_4} \leq 1\,\mathrm{keV}$ this mass will make it a very promising candidate as a constituent of the universe's dark matter. This last possibility was recently readdressed by Fuller et al. [15], who estimated sterile neutrino masses in the range (1–20) keV and small mixing angle $\sin^2 2\theta \sim 10^{-4}$ with the electron neutrino as a potential explanation of pulsar kicks. The same argument has also been considered by [28]; see also the references therein.

As quoted above, for both classes of ν conversions the number of νs escaping and their angular distribution is

Table 4. Parameter range for GW emission from νs oscillations in SNe. The symbol DM stands for $\nu_{\rm s}$ as a dark matter candidate

ν sphere $R^{\nu_e}_{\rm PNS}$	ν luminosity $L(\nu\text{-Diff.})$	Oscil. length $\lambda^{\nu}_{\rm osc}$	ν velocity $\bar{V}_{\nu\text{-Diff.}}$	$\Delta m^2(\nu_{\rm a} \leftrightarrow \nu_{\rm a})$ (SNO, KamLAND)	$\Delta m^2(\nu_{\rm a} \leftrightarrow \nu_{\rm s})$ (WMAP)	$\Delta m^2(\nu_{\rm a} \leftrightarrow \nu_{\rm s})$ ($\nu_{\rm s}$ DM)
$\sim 35\,{\rm km}$	$\sim 10^{53}\,{\rm erg\,s^{-1}}$	$\sim 1\,{\rm km}$	$\sim 10^9\,{\rm cm\,s^{-1}}$	$\sim 10^{-4}\,{\rm eV^2}$	$\sim 1\,{\rm eV^2}$	$\sim 10^4\,{\rm eV^2}$

sensitive to the instantaneous distribution of production sites. These inhomogeneities can give rise to quadrupole moments that generates GWs [28,29] and dipole moments that could drive the runaway pulsar kicks [15,23]. Noting that at least 6 ν species can participate in both types of oscillations and that the interaction with both the magnetic field and angular momentum of the PNS brings with it an overall reduction factor of 4 in the oscillation probability, one can show that the *fraction* of νs that can actually undergo flavor transitions in the first few milliseconds is [28]

$$P^{\nu_{\rm a}\to\nu_{\rm a}}_{\nu_{\rm a}\to\nu_{\rm s}}(|\mathbf{x}-\mathbf{x}_{\rm res}) \leq 1\% \tag{41}$$

of the total number of νs: $M_\odot \times m_p^{-1} \sim 10^{57}\nu$, which corresponds to a total energy exchanged during the transition of

$$\Delta E^{\nu_{\rm a}\to\nu_{\rm a}}_{\nu_{\rm a}\longrightarrow\nu_{\rm s}} \sim 3\times 10^{51}\,{\rm erg}. \tag{42}$$

Equations (42) and (21) determine the total luminosity of the neutrinos participating in the resonant flavor conversions. These figures are called for in the definition used in (43) below, as the basis to compute the characteristic amplitude of the GWs emitted during the transitions discussed above.

The attentive reader must notice, however, that the lower limit in (38) stems from using the constraint derived from the observations of the CMBR performed by the satellite WMAP, which suggested that the total mass of all neutrino species should not be larger than 1 eV. In a hierarchy where the heaviest ν is the sterile $\nu_{\rm s}$, this limit leads to a maximum mass: $m^{\rm max}_{\nu_{\rm s}} \sim 1\,{\rm eV}$. The use of this mass-squared difference constraint implies that the density at the PNS region where the oscillations can take place is $\rho^{\rm WMAP}_{m_\nu} \sim 10^8\,{\rm g\,cm^{-3}}$. If the WMAP constraint on m_ν stands also for sterile neutrinos, this would preclude the mechanism for giving to nascent pulsars the natal velocities (kicks) from active neutrino flavor conversions into sterile neutrinos, as claimed by [15,23], since in such a case the transitions would take place in regions far outside the PNS core from where no influence could be received back once the oscillations develop. Nonetheless, the GW emission would still take place, as we stressed above.

4.4 Experimental bounds on supernova ν oscillations

Studies of SN physics have also focused on the potential rôle of oscillations [26] between active and sterile νs. In particular, there are limits on the $\nu_e \leftrightarrow \nu_{\rm s}$ conversion rate inside the SN core from the detected ν_e flux from SN1987a [30,31]. According to these authors, the time spread and the number of detected ν_e events constrain the $\nu_e \leftrightarrow \nu_{\rm s}$ oscillations by $10^6 \leq \Delta m^2 \leq 10^8\,{\rm eV^2}$ for $10^{-3} \leq \sin^2 2\theta_{\nu_e\to\nu_{\rm s}} \leq 10^{-7}$. More stringent constraints stem from arguing that if there were too many "escaping νs", the SN explosion itself would not take place [30,31]. Such bounds are, however, model dependent. One should keep in mind that the mechanism through which the explosion takes place is, in fact, not well established [5]. In effect, it has recently been claimed that even after including all the best physics we know today about, i.e., state-of-the-art supernova physics, the numerical models of exploding stars do not explode as they should [5]. There seems to be a *piece* of *missing* physics in those formulations. Thence, there seems to be no hope of achieving $P^{\nu_{\rm a}\to\nu_{\rm a}}_{\nu_{\rm a}\to\nu_{\rm s}}$ larger than $\sim 10\%$ during ν oscillations in supernovae.

5 GW energetics from ν luminosity and detectability

If ν oscillations do take place in the SN core, then the most likely detectable GW signal should be produced over the time interval for which the conditions for flavor conversions to occur are kept, i.e., $\sim (10^{-1}\text{–}10)\,{\rm ms}$. This time scale implies GW frequencies in the band: $f_{\rm GW} \sim [10\text{–}0.1]\,{\rm kHz}$, centered at 1 kHz, because of the maximum ν production around 1–3 ms after core bounce [26]. This frequency range includes the optimal bandwidth for detection by ground-based observatories. For a 1 ms conversion time span the ν luminosity reads

$$L_\nu \equiv \frac{\Delta E_{\nu_{\rm a}\longrightarrow\nu_{\rm s}}}{\Delta T_{\rm osc}} \sim \frac{3\times 10^{51}\,{\rm erg}}{1\times 10^{-3}\,{\rm s}} = 3\times 10^{54}\,\frac{\rm erg}{\rm s}\,. \tag{43}$$

Hence, the GW luminosity, $L_{\rm GW}$, as a function of the ν luminosity can be obtained from the equation

$$\frac{c^3}{16\pi G}\left|\frac{{\rm d}h}{{\rm d}t}\right|^2 = \frac{1}{4\pi R^2}L_{\rm GW}\,, \tag{44}$$

which relates the GW flux to the GW amplitude, h_ν. In the case of GW emission from escaping νs this amplitude is computed from the expression [7,12,29]

$$h_{ij} = \frac{2G}{c^4 R}\int_{\infty}^{t-R/c} {\rm d}t' \;\; L_\nu(t') \;\; \alpha(t') \;\; e_i\otimes e_j\,, \tag{45}$$

where $e_i \otimes e_j$ represents the GW polarization tensor.

To attain order of magnitude estimates one can transform (45) to get the amplitude of the GWs burst produced by the non-spherical outgoing front of oscillation-produced $\nu_{\rm s}$ as [7,28,29]

$$h_\nu = \frac{2G}{c^4 R}\,(\Delta t\times L_\nu\times\alpha) \tag{46}$$

Table 5. GW estimates from ν oscillations and diffusion

GW freq. $f_{\mathrm{GW_s}}$	GW energy $E_{\mathrm{GW}}\left(\begin{smallmatrix}\nu_a\leftrightarrow\nu_s\\ \nu_a\leftrightarrow\nu_a\end{smallmatrix}\right)$	L_ν asymm. α	GW energy $E_{\mathrm{GW}}(\nu\text{-Diff.})$
[10–0.1] kHz	$[10^{-6}\text{–}10^{-8}]\, M_\odot\, c^2$	[0.1–0.01]	$[10^{-10}\text{–}10^{-13}] M_\odot\, c^2$

$$\simeq |A| \left[\frac{55\,\mathrm{kpc}}{R}\right]\left(\frac{\Delta T}{10^{-3}\,\mathrm{s}}\right)\left[\frac{L_\nu}{3\times 10^{54}\,\frac{\mathrm{erg}}{\mathrm{s}}}\right]\left(\frac{\alpha}{0.1}\right) ,$$

where $|A| = 4\times 10^{-23}\,\mathrm{Hz}^{-1/2}$ is the amplitude.[3] Equivalently, that amplitude can be reparametrized as

$$h_\nu \simeq |A| \left[\frac{55\,\mathrm{kpc}}{R}\right]\left(\frac{\Delta E_{\mathrm{GW}}}{10^{-7}\, M_\odot c^2}\right)^{1/2}\left[\frac{10^3\,\mathrm{Hz}}{f_{\mathrm{GW}}}\right]^{1/2} . \tag{47}$$

A GW signal this strong will likely be detected by the first generation of GW interferometers as LIGO, VIRGO, etc. Its imprint in the GW waveform may resemble a spike of high amplitude and timewidth of $\sim$ ms followed by a Christodoulou's memory [28]. From (47) the GW luminosity turns out to be

$$L_{\mathrm{GW}} \sim 10^{50\text{–}48}\ \frac{\mathrm{erg}}{\mathrm{s}}\ \left[\frac{L_\nu}{3\times 10^{54}\,\frac{\mathrm{erg}}{\mathrm{s}}}\right]^2\left(\frac{\alpha}{0.1\text{–}0.01}\right)^2 , \tag{48}$$

while the GW energy radiated in the process yields

$$E_{\mathrm{GW}} \equiv L_{\mathrm{GW}} \times \Delta T_{\mathrm{osc}} \sim 10^{47\text{–}45}\,\mathrm{erg} . \tag{49}$$

This is about $10^{5\text{–}3} \times L_\nu^{\mathrm{Diff.}}$ the luminosity from ν diffusion inside the PNS, as estimated earlier [28].

6 Conclusion

One can see that if ν flavor conversions indeed take place during SN core bounce inasmuch as they take place in our Sun and Earth [40], then GWs should be released during the transition. The GW signal from the process is expected to irradiate much more energy than current mechanisms figured out to drive the NS dynamics at birth do. A luminosity this large (see (44)) would turn these bursts the strongest GW signal to be detected from any SN that may come to occur, in the future, on distances up to the VIRGO cluster, $R \sim [10\text{–}20]$ Mpc. It is stressed that this signal will still be the stronger one from a given SN, even in the worst case in which the probability of ν conversion is three orders of magnitude smaller than the one estimated in the present paper. In proviso, we argue that a GW signal that strong could have been detected during SN1987a from the Tarantula Nebula in the Milky Way's satellite galaxy Large Magellanic Cloud, despite the low sensitivity of the detectors at the time. In such a case, the GW burst must have been correlated in time with the earliest arriving neutrino burst constituted of some active species given off during the very early oscillation transient where some ν_es went into ν_μs, ν_τs or ν_ss. Thenceforth, it could be of worth to reanalyze the data collected for that event taking a careful follow up of their arrival times, if appropriate timing was available at that moment.

[3] One must realize at this point that the high value of the anisotropy parameter here used is consistently supported by the discussion regarding the neutrino coupling to rotation and magnetic field presented above.

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