THE NULL FIELD METHOD FOR ACOUSTIC
SCATTERING FROM ROUGH SURFACES

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E. Jakeman (J. Opt. Soc.Am;72,1982,1031-1041) has propo-
sed the classification of rough random surfaces into
Type I (fractal surfaces), Type II (smooth surfaces)
and a new intermediate Type III class with fractal nor-
mal. This may be formulated in the context of the boun-
dary integral method for locally Lipschitz surfaces (see
Verchota, J. Funct Anal, 54, 1984, 572-611; Costabel,
S.I.A.M. J. Math Anal; 19, 1988, 613-626). The standard
theory (Colton and Kress, Integral Equation Methods in
Scattering Theory, Wiley, N.Y., 1983) may be extended to
this case. Combining this existence theory with the null
field we may derive low order spectral approximation to
scattering by plane order incident waves by rough fluted
Type III cylinders.

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A P R E S E N T A Ç Ã O

Este volume reúne os resumos dos trabalhos apresentados nas sessões técnicas do **XV CONGRESSO NACIONAL DE MATEMÁTICA APLICADA E COMPUTACIONAL - CNMAC**. Os resumos são cópias dos originais encaminhados à **SBMAC**.

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A DIRETORIA

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0 Introduction

Scattering from fractal objects has received growing interest in recent years in a wide range of theoretical and experimental investigations involving acoustic, eletromagnetic, neutron scattering etc. At least two excellent review articles have been published [12], [38] and one recent book by Olgilvy, deals in some way with this topic while a variety of experimental results are reviewed in the papers by Courtens and Vacher [7] and Teixeira [34], and more classical techniques on rongh surfaces may be seen in [9]. In [12] there is reviewed a good deal of the work of Jaggard and his associates, published in papers by Jaggard and Kim [13], [14], [15]; Jaggard and Sun [16], [17], [18], [19] and Grebel and Jaggard [11]. Earlier and seminal contributions were given by Berry and Blackwell [3] and Berry [2]. In his papers [20], [21] Jakemann has proposed a classification of surfaces in three types:

(I) Fractal surfaces which are continuous but non-differentiable with power law spectrum.

(II) Smoothly varying surfaces differentiable to all orders with Gaussian-like spectral properties.

Type (I) generate only diffraction and interference effects. Type (II) generate geometric optic effects associated to rays or normals to scattered wave fronts. He proposed as well an intermediate class of random surfaces of type (III). where the surface height is continuons differentiable but the slope a fractal. The concept of ray is valid for this model but in the absence of higher order surface derivatives no geometrical catastrophes occur in the propagating wave fronts. A modification to (I) has been proposed with the notion of band limited "fractal" surfaces (see [12]. Both the type III case and band limited fractal case can be given an integral equations formulation. A type III case was analised by Macaskell in [26] in an attempt to establish the existence of enhanced backscatter. however, that analysis is not convincing.

In the present papers we try to deal with some type III cases and the band limited fractal situation in the same null field appromximation (see Colton and Kress [4] Thorem 3.44 p 104), Waterman [36],[37] and Martin [29]. The only reasonably rigorous treatment of such problems of which one has knowledge is that given in the papers by Jaggard and Sun [19], although some periodic cases are dealt with in [23]. Our discussion is limited to very large incident frequency and examines mean scattering cross-section at large distances from the scatterer.

1 The existence of scattered waves

We consider the scattering of a plane inciedent wave $u_i = \exp ik(x \sin \omega - y \cos \omega)$ by an cylinder D given by the equation $R(\phi) = 1 + \delta F(\phi, r), 0 \leq \psi < 2\pi$, under soft acoustical conditions, that is say under a Dirichlet condition. $F(\phi, \cdot)$ is a random variable and we work under the condition that $F(\phi, \cdot)$ is locally Lipschitz, which guaratees the existence of $F'(\phi, \cdot)$ almost everywhere. We also work under the hypotheses that F' is a band limited random Weierstrass function F:

$$F(\varphi) = C \sum_{n=0}^{m-1} \alpha^n \cos(k\beta^n \phi + \phi_n), \; < \alpha < 1,$$

$k\beta^n$ integers, $n = 0, ..., M - 1$ and where the ϕ_n are independent uniformly distributed random variables.

$$C = \sqrt{\frac{2(1 - \alpha^2)}{1 - \alpha^{2M}}},$$

which sets the standard deviation equal to unity $\lambda d = 1 + \alpha$ is the so-called ronghness dimension and δ is chosen to satisfy $R(\phi) \geq R_0 > 0$, with some constant R_0. Obviously, for the band limited fractal, F is infinitely smooth, but as $M \rightarrow \infty$, it will be seen that all the differentiability is lost.

We assume that the total wave takes the form

$$u = u_i + u_s, \qquad (1)$$

so that u_s satisfies

$$(\Delta + k^2)u_s = 0, \; x \in D^c, \qquad (2)$$

$u_s = -u_i$, on $\partial\Omega$, together with the Sommerfeld radiation condition at infinity. Then the free space fundamental solution for the Helmholtz equation is given by

$$\Phi(x, y) = \frac{i}{4} H_0^{(1)}(k \mid x - y \mid).$$

Formally we define integral operators K and S by

$$(S\psi)(x) = 2 \int_{\partial D} \Phi(x, y)\psi(y)d\sigma$$

and

$$(K\psi)(x) = 2 \int_{\partial D} \frac{\partial}{\partial v(y)} \Phi(x, y)\psi(y)d\sigma.$$

Under the hypothesis that F is locally Lipschitz continuons one is up against the natural limit for integral operator methods. We recall that such methods were extended in recent years to C^1-boundaries by Fabes, Jodeit and Riviere in the paper [9] and in a far-reaching and delicate extension to Lipschitz domains by Verchota in his paper [35]. Further interesting results have been oftained by Costabel in [6] while a general discussion of the C^2-theory is given in the book by Colton and Kress [4].

Various authors have suggested that solutions of the exterior Dirichlet problem (2) should be sounght in the form of a combined double and single layer potential, *viz*:

$$u_s(x) = \int_{\partial D} (\frac{\partial \Phi}{\partial \nu(y)}(x - y) - i\eta\Phi(x, y))\psi(y)d\sigma, \ x \in D^c, \qquad (3)$$

with some $\eta > 0$.

It may be seen as in the paper by Verchota and the book of Colton and Kress that (3) provides a solution of (2) provided that the density ψ satisfies the boundary integral equation

$$\psi + K\psi - i\eta S\psi = -2u_s, \qquad (4)$$

considered in the spaces $L^2(\partial D)$ or $H^1(\partial D)$.

If the boundary regularity is relaxed to that of a Lipschitz domain it must be noted that K is not a compact operator and a special argument has to be given as in Verchota [35] and Colton and Kress [4].

That [4] should hold is established using the generalised relations given [35].

To be brief the combined double and single layer potential method functions as in the classical C^2-case, avoiding the problem of eigenvalues of the dual interior Neumann problem. We conclude that (3) and (4) establish the existence and uniqueness of a scattered wave satisfying (2).

Perhaps, it is relevant to observe that the natural conditioning number associated with (4) has been studied by Kress and Spassov in [24] and for the cylinder the optimal parameter to minimise the conditioning number has been determined for $k \in [0, 8]$. The conclusion of these numerical studies is

that with same local fluctuation the conditioning number $\eta(k)$ grows with k, $k \geq 1/2$.

Finally, let us note that an elementary representation theorem gives:

$$u_s(x) = \int_{\partial D}(u(y)\frac{\partial \Phi}{\partial \nu} - \frac{\partial u}{\partial \nu(y)}\Phi)d\sigma = \int_{\partial D}(-u_i(y)\frac{\partial \Phi}{\partial \nu} - \frac{\partial u}{\partial \nu(y)}\Phi)d\sigma,$$

$x \in D^c$.   (5)

A numerical solution of (4) for large wave lengths k is complicated (see for example [1] and for more recent results on random surfaces [28], [31], [8]). In order to simplify, this problem we prefer to use asymptotie results and the null field method, sometimes refered to as (Oswald - Oseen) extinction theorem (see for example [32], section 3.3)

2   The null field method

We are interested in the averaged scattering cross-section at large distance $\sigma(k, \theta)$:

$$\sigma(k, \theta) = \lim_{r \to \infty} E\Big(2\pi r \frac{|u_s(x, \cdot)|^2}{|u_i(x, \cdot)|^2}\Big).$$

Recalling the representation

$$0 = \int_{\partial D}\Big(u_i\frac{\partial \Phi}{\partial \nu} - \frac{\partial u_i}{\partial v}\Phi\Big)\partial\sigma, \ x \in D^c,$$

together with (5) we see that

$$u_s(x) = -\frac{i}{4}\int_{\partial D}(\frac{\partial u}{\partial \nu})(y)H_o^{(1)}(k\mid x - y\mid)d\sigma(y).$$

Also by the asymptotic behaviour of the Hankel function we obtain

$$u_s(x) = \frac{1}{4}\exp i(kr - \frac{\pi}{4})\sqrt{\frac{2}{\pi kr}}F(k, \theta) + 0(r^{-3/2})$$

where

$$F(k, \theta) = -\int_{\partial D}\Big(\frac{\partial u}{\partial \nu}\Big)(y)\exp(-ik\rho\cos(\theta - \phi))d\sigma(y).   (6)$$

Then it follows that

$$\sigma(k,\theta) = (4k)^{-1}E \mid F(k,\theta) \mid^2 . \qquad (7)$$

We recall the Graf addition formula, valid for $r < \rho$ (see [27] section 3.9 p 107):

$$H_0^{(1)}(k \mid x - y \mid) = \sum_{-\infty}^{\infty} H_n^{(1)}(k\rho)J_n(kr)e^{in(\theta-\phi)}. \qquad (8)$$

Using the representation formula for radiative solution, we see that

$$\int_{\partial D}(u_s(y)\frac{\partial\Phi}{\partial\nu} - \frac{\partial u_s(4)}{\partial\nu}\Phi)d\sigma(y) = 0, \; x \in D. \qquad (9)$$

From (8) and (9), we obtain the moment expression (see Waterman [36], [37], Martin [29], Colton and Kress [4]).

$$-\int_{\partial D}u_i\frac{\partial}{\partial r}(\exp(-in\phi)H_n^{(1)}(k\rho)d\sigma(y) =$$

$$= \int_{\partial D}\frac{\partial u_s}{\partial\nu}(y)H_n^{(1)}(k\rho)\exp(-in\phi)d\rho. \qquad (10)$$

From (6) we obtain

$$F(k,\theta) = +\int_{\partial D}\frac{\partial}{\partial\nu}(\exp -ik\rho\cos(\phi - \omega))\exp(-ik\rho\cos(\theta - \phi)d\sigma(y)$$

$$-\int_{\partial D}\frac{\partial u_s}{\partial\nu}(y)\exp(-ik\rho\cos(\theta - \phi))d\sigma(y).$$

Now the normal derivative $\frac{\partial}{\partial\nu}$ is given by

$$\frac{\partial}{\partial\nu} = (R(\phi)^2 + R'(\phi)^2)^{\frac{-1}{2}}(R(\Phi)\frac{\partial}{\partial\rho}, -\frac{R'(\phi)}{R(\phi)}\frac{\partial}{\partial\phi}).$$

It follows that

$$F(k,\theta) = \int_0^{2\pi}(-ikR(\phi)\cos(\phi - \omega) - ikR'(\phi)\sin(\phi - \omega))\exp -ikR(\phi),$$

$$\cos(\phi - \omega_s)x\exp -ikR(\phi), \cos(\theta - \phi)d\phi$$

$$-\int_{\partial D}\frac{\partial u_s}{\partial\nu}(y)\exp(-ih\rho\cos(\theta - \phi))d\sigma(y)$$

$$= -ik \int_0^{2\pi} [R(\phi)\sin(\phi - \omega) - R'(\phi)\cos(\phi - \omega)]$$

$$\exp -ikR(\phi)(\sin(\phi - \omega) + \cos(\theta - \phi))d\phi$$

$$- \int_{\partial D} \frac{\partial u_s}{\partial \nu}(y) \exp(-ik\rho \cos(\theta - \phi))d\sigma(y). \qquad (11)$$

The Hankel approximation yields for $n << k$

$$H_n^{(1)}(kR(\phi)) = (n,0)\frac{\exp i(kR(\phi) - \frac{n\pi}{2} - \frac{\pi}{4})}{\sqrt{\frac{\pi}{2}kR(\phi)}}\{1 + 0(k^{-1})\}.$$

Where $(n,0) = 4n^2 - 1$, since $R(\phi) \geq R_0 > 0$.
Let $\psi = \frac{\partial u_s}{\partial \nu}$ and set

$$Q(\varphi) = \frac{\psi(\phi)\sqrt{R(\phi)^2 + R'(\phi)^2}}{\sqrt{R(\phi)}} \exp(ikR(\phi))$$

and

$$c_n = \frac{1}{\sqrt{2\pi}} \int_\sigma^{2\pi} R(\varphi)\exp(-in\phi)d\phi.$$

Then (10) yields

$$c_n \frac{\exp(-i(\frac{n\pi}{2} + \frac{\pi}{4}))}{\sqrt{\frac{\pi}{2}k}} = -\frac{1}{\sqrt{2\pi}} \int_0^{2\pi} u_i(y)\frac{\partial}{\partial \nu}(\exp(-in\phi)H_n^{(1)}(kR(\phi)))$$

$$\times \sqrt{R(\phi)^2 + R'(\phi)^2}d\phi + 0(k^{-1}).$$

A short calculation yields that

$$c_n = +\frac{1}{\sqrt{2\pi}} \int_0^{2\pi} \exp(-ikR(\phi)\sin(\phi - \omega))\times$$

$$\times \left\{ \frac{n\left(1 - i\frac{R'(\phi)}{R(\phi)}\right)}{\sqrt{R(\phi)}} + ik\sqrt{R(\phi)}\frac{(n+1,0)}{(n,0)} \right\}\exp(-in\phi) + 0(k^{-1}) \qquad (12)$$

It follows that a lower order spectral approximation to R is given by

$$R_N = \sum_{|n|\leq N} \frac{c_n \exp(in\exp(in\phi)}{\sqrt{2\pi}}, N << k, \qquad (13)$$

where the c_n are given by (12).

Then the approximation to the F of (11) is gives by F^N where

$$F^N = -ik \int_0^{2\pi} \left(R(\phi)\sin(\phi - \omega) - R'(\phi)\cos(\phi - \omega)\right)$$

$$\exp{-ikR(\phi)(\sin(\phi - \omega) + \cos(\theta - \phi))}$$

$$-\int^{2\pi} R_n(\phi)\sqrt{R(\phi)}\exp(-ikR(\phi))\exp[-ikR(\phi)\cos(\theta - \phi)]d\phi + 0(k^{-1}) \quad (14)$$

The lowest order approximation is given by

$$c_0 = \frac{-3ik}{\sqrt{2\pi}} \int_0^{2\pi} \exp(-ikR(\hat{\phi})\sin(\hat{\phi} - \omega))\sqrt{R(\hat{\phi})}, d\phi$$

So that

$$F^0 = -ik \int_0^{2\pi} [R(\phi)\sin(\phi - \omega) - R'(\phi)\cos(\phi - \omega)]$$

$$\exp{-ikR(\phi)(\sin(\phi - \omega) + \cos(\theta - \phi))}d\phi$$

$$\frac{-3ik}{2\pi} \int_0^{2\pi}\int_0^{2\pi} \sqrt{R(\phi)}R(\hat{\phi})\exp(-ikR(\phi)))$$

$$\exp(-ikR(\hat{\phi})\sin(\hat{\phi} - \omega))\exp(-ikR(\phi)\cos(\theta - \phi))d\phi\hat{\phi}.$$

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