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Piezoelectric actuator design considering spillover effects

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Abstract

In this work, the topology optimization method using the solid isotropic material with penalization (SIMP) approach is employed to find the optimum design of piezoelectric actuators taking into account the excitation of the residual vibration modes. The structure governing equations are written into the state-space representation and the controllability Gramian eigenvalues are used to measure how the residual modes are susceptible to be excited by the control system. The proposed optimization formulation aims to find the distribution of piezoelectric material which maximizes the controllability for a given vibration mode while the undesirable effects of the feedback control on the residual modes are limited by including a spillover constraint term written as a p-norm of the residual controllability Gramian eigenvalues. The optimization of the shape and placement of the embedded piezoelectric actuators are carried out using a Sequential Linear Programming (SLP) algorithm. Numerical examples are presented for the control of the first bending vibration mode of a cantilever beam varying the set of residual modes considered in the spillover norm.

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Keywords: Topology optimization method; active vibration control; piezoelectric actuators; spillover instability.

1. Introduction

Control systems with state feedback and full state observer are usually designed considering truncated models in which only few representative low frequency modes are taken into account to represent the dynamic behavior of a flexible structure [8]. However, the control input can excite one or more residual modes. This interaction between the residual modes and the controller is called control spillover and can cause severe performance degradation [1]. Several works presented sensor and/or actuator design methodologies considering a limit to the spillover effects. Bruant et al. [3] employed Genetic Algorithm (GA) to find the optimal location of actuators by the maximization of an objective function written in terms of the normalized mechanical energy transmitted for the vibration modes. Hanis and Hromcik [7] studied the optimal sensors placement by means of the effective independence (EFI) method considering spillover. Biglar and Mirdamadi [2] studied the location and orientation of piezoelectric sensors/actuators attached to plate structures using GA to find the optimal configuration based on spatial controllability/observability considering residual modes. Cinquemani et al. [4] defined a fitness function written in terms of the non-diagonal elements of the damping matrix introduced by the control, which depend only on the placement of the actuators and sensors, aiming

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to ensure both controllability and observability and attenuate the spillover effects. Gonçalves et al. [6] employed the concept of topology optimization, based on the SIMP approach, to find the shape and location of piezoelectric actuators which maximize the control performance considering a limit to the spillover effects. In the present work, this topology optimization formulation is studied focusing on the structural responses of the optimum actuator designs with and without spillover. The numerical results are obtained by the application of a Linear Quadratic Regulator (LQR) scheme.

2. Topology Actuator design considering the spillover effects

Considering the stationary solutions for a stable system, the controllability Gramian $\mathbf{W}_c$ is obtained by the Lyapunov equation:

$$\mathbf{A}\mathbf{W}_c + \mathbf{W}_c\mathbf{A}^{\mathrm{T}} + \mathbf{B}\mathbf{B}^{\mathrm{T}} = \mathbf{0} \tag{1}$$

where $\mathbf{A}$ and $\mathbf{B}$ are the system and input matrices, respectively. Thus, the controllability Gramian eigenvalues can be obtained by:

$$(\mathbf{W}_c - \lambda_i\mathbf{I})\,\mathbf{Y}_j = \mathbf{0}, \tag{2}$$

$$\mathbf{Y}_j^{\mathrm{T}}\mathbf{Y}_j = 1, \qquad j = 1, 2, ..., m \tag{3}$$

where $\mathbf{I}$ is the identity matrix; λ_j is the j-th eigenvalue of the controllability Gramian $\mathbf{W}_c$; and $\mathbf{Y}_j$ is its respective eigenvector.

The optimization formulation aims to maximize the controllability for the i-th vibration mode and, at the same time, to limit the control spillover effects by means of a p-norm constraint term which takes into account the undesirable modes eigenvalues:

$$\begin{aligned} \max_{\rho} :\quad & f = \lambda_i \\ \text{s.t.} :\quad & g_1 = \frac{\lambda_{sum}{}^{\frac{1}{p}}}{\lambda_i} \leq C_\lambda \\ & g_2 = \frac{V_2}{V} \leq C_V \end{aligned} \tag{4}$$

where $\boldsymbol{\rho}$ is the design variables vector; λ_i is the eigenvalue related to the control of the i-th vibration mode; V and V_2 are respectively the total and actuator volumes; C_λ and C_V are constraint parameters for residual eigenvalues norm and actuator material volume, respectively; and λ_{sum} is given by the summation of the residual eigenvalues:

$$\lambda_{sum} = \sum_{j=i+1}^{m} \alpha_j \lambda_j{}^p \tag{5}$$

where p is a norm exponent ($p \geq 1$) and α_j is a weighting factor. More details about this formulation and the sensitivities (gradients) of the objective function f and constraints g_1 and g_2 with respect to the design variables $\boldsymbol{\rho}$ can be found in [6].

3. Numerical Results

The optimization formulation is examined by means of a numerical example considering a cantilever beam in plane stress state, as presented in Fig. 1. The structure is discretized into finite elements and each element can have piezoelectric, base material, or an intermediate material properties following a multi-phase interpolation. The passive base material is defined as an isotropic elastic material with aluminum constitutive properties while piezoelectric ceramic (properties for PZT-5A can be found in [9]) polarized in z-direction is considered for the actuator.

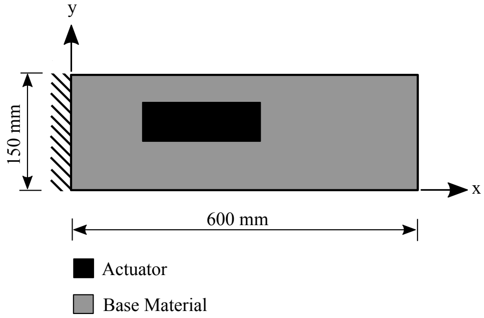

Fig. 1. Cantilever beam with its boundary conditions and a generic design for the actuator.

Five cases were analyzed, following the parameters presented in Tab. 1. A set with five bending vibration modes were considered in the control system. Extensional modes can lead the optimization process to poor local minima [5] and, therefore, these modes were not taken into account. The volume limit C_V was defined as 0.1 for all cases. Case A was carried out with $C_\lambda = 1000$ in order to force the constraint g_1 to be inactive through the optimization process.

Table 1. Description of the analyzed cases.

Case	C_λ	Residual modes
A	1000	2,3,4,5
B	0.10	2
C	0.10	2,3
D	0.10	2,3,4
E	0.10	2,3,4,5

The actuator topologies were obtained by the maximization of λ_1 with spillover constraint considering a set with residual modes, according to Tab. 1. Fig.2 presents the optimal distribution of piezoelectric material for case A.

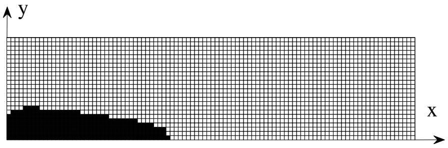

Fig. 2. Actuator topology obtained by the maximization of λ_1 for case A.

Fig.3 presents the optimal distribution of piezoelectric material for cases B, C, D and E. One can observe there is no significant influence on the optimization solution when only the second vibration mode is considered in g_1.

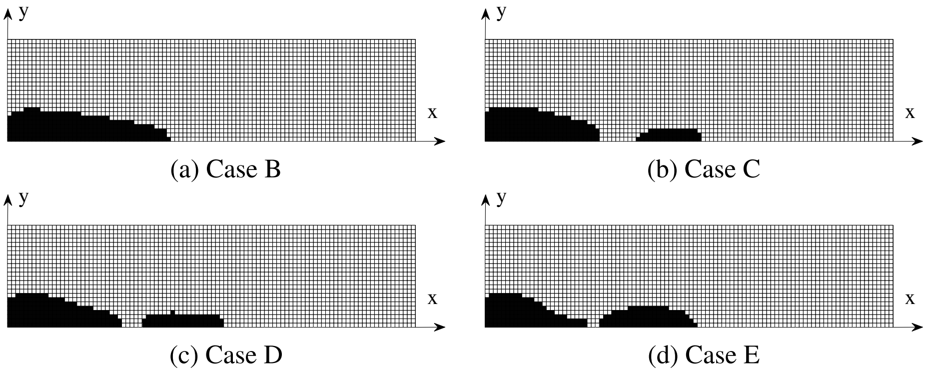

Fig. 3. Actuator topologies obtained by the maximization of λ_1.

Fig.4 present the convergence of the objective function for cases B, C, D and E. These values were normalized by the converged objective function obtained for case A. It can be seen the converged objective function for cases A and B are similar while for the other cases there is significant reduction.

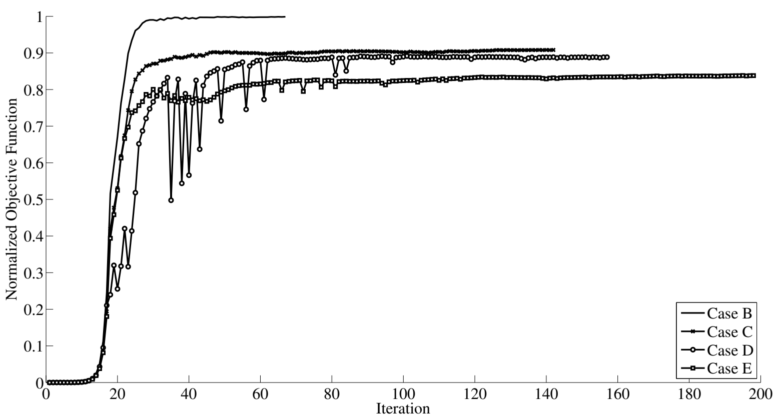

Fig. 4. Normalized objective function.

The simulation of the feedback control system is not necessary during the optimization process. Only the knowledge of the controllability Gramian and, consequently, the parameters used in the state-space representation [5]. However, a state feedback control law is used to analyze the performance of optimized actuator designs. This step is performed by means of an LQR controller, in which an optimum feedback gain matrix is chosen in a way that minimizes a quadratic cost function subject to the dynamics of the system [8]. This cost function is given by:

$$J = \frac{1}{2}\int_0^{t_f} (\mathbf{x}^{\mathrm{T}}\mathbf{Q}\mathbf{x} + \boldsymbol{\phi}^{\mathrm{T}}\mathbf{R}\boldsymbol{\phi})\, dt \tag{6}$$

where $\mathbf{Q}$ is a matrix of weights for the state variables and $\mathbf{R}$ is a matrix of weights for the control inputs. The performance of the control system depends on the values adopted for the components of these matrices. In this study, these values were defined as [10]:

$$\mathbf{Q} = \begin{bmatrix} q\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \qquad \mathbf{R} = \mathbf{I} \tag{7}$$

where $\mathbf{I}$ and $\mathbf{0}$ identity and zero matrices, respectively, and $q = 10^{17}$. Assuming complete state feedback, the gain matrix which minimize the cost function is:

$$\mathbf{G} = \mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P} \tag{8}$$

where **P** is the solution of algebraic Riccati equation:

$$\mathbf{A}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{A} - \mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P} + \mathbf{Q} = \mathbf{0} \tag{9}$$

Thus, the dynamics of the system is governed by the closed loop system matrix $\mathbf{A}_c = \mathbf{A} - \mathbf{B}\mathbf{G}$. Fig.5 presents both open and closed loop frequency responses to a modal load of the first vibration mode. Only the first mode peak appears for the open-loop system. The LQR controller reduces this peak magnitude while excite the residual modes.

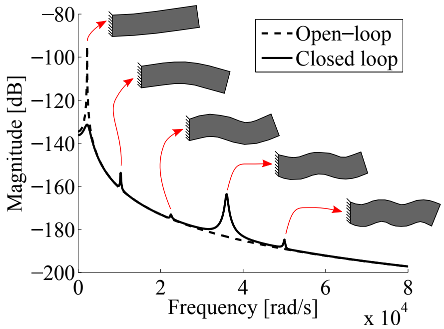

Fig. 5. Frequency response to a modal load for case A and vibration modes representaton.

Fig.5 presents similar frequency responses for the cases B, C, D and E, where can be observed the influence of the spillover constraint on the structure response.

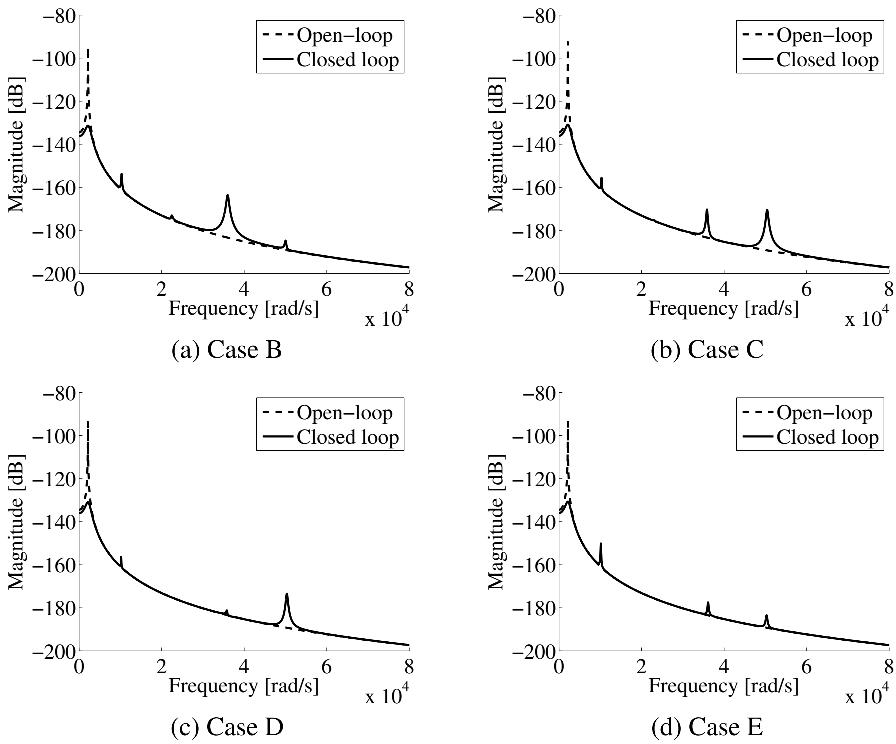

Fig. 6. Frequency responses to a modal load for cases B, C, D and E.

These results are summarized in Fig. 7, where the changes in the peaks magnitude are presented in a bar chart.

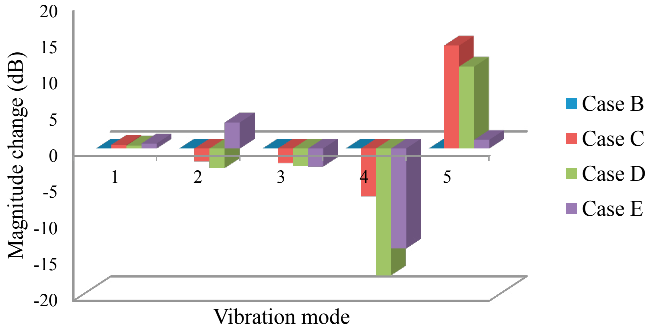

Fig. 7. Changes in the peaks magnitude.

4. Conclusions

In this work, the piezoelectric actuator topology design was studied taking into account the undesirable effects of the control system on the residual vibration modes. For the studied cases, we observed that the optimal designs for controlled structures obtained with and without active spillover constraint g_1 presented similar control performances while attenuating the first mode vibration. The excitation of the residual modes by the system control was limited, according to the p-norm containing the respective controllability Gramian eigenvalues λ_j, for the cases with active spillover constraint. The weighting factors were assumed as being the same for all modes. However, they can be suitable chosen in order to tune the spillover constraint considering a particular application or the importance of the individual residual modes.

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