UNIVERSIDADE FEDERAL DO RIO GRANDE DO SUL
FACULDADE DE CIÊNCIAS ECONÔMICAS
PROGRAMA DE PÓS-GRADUAÇÃO EM ECONOMIA

LÍVIA SILVA PARANHOS

COMPUTING OPTIMAL AND REALISED MONETARY POLICY RULES FOR BRAZIL: A MARKOV-SWITCHING DSGE APPROACH

Porto Alegre
2017

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Dissertação submetida ao Programa de Pós-Graduação em Economia da Faculdade de Ciências Econômicas da UFRGS, como requisito parcial para obtenção do título de Mestre em Economia, com ênfase em Economia Aplicada.

Orientador: Prof. Dr. Marcelo Savino Portugal

Porto Alegre

2017

CIP - Catalogação na Publicação

Paranhos, Livia Silva
Computing optimal and realised monetary policy rules for Brazil: a markov-switching DSGE approach / Livia Silva Paranhos. -- 2017.
45 f.

Orientador: Marcelo Savino Portugal.

Dissertação (Mestrado) -- Universidade Federal do Rio Grande do Sul, Faculdade de Ciências Econômicas, Programa de Pós-Graduação em Economia, Porto Alegre, BR-RS, 2017.

1. Markov-switching DSGE. 2. Optimal monetary policy. 3. Bayesian estimation. I. Portugal, Marcelo Savino, orient. II. Título.

Elaborada pelo Sistema de Geração Automática de Ficha Catalográfica da UFRGS com os dados fornecidos pelo(a) autor(a).

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Trabalho aprovado em: Porto Alegre, _____/____/_____:

BANCA EXAMINADORA:

____

Prof. Dr. Marcelo Portugal - Orientador
UFRGS

____

Prof. Dr. Edilean Kleber da Silva Aragón
UFPB

____

Prof. Dr. Flávio Augusto Ziegelmann
UFRGS

____

Prof. Dr. João Frois Caldeira
UFRGS

Acknowledgements

To the Graduate Program in Economics (PPGE) of the Federal University of Rio Grande do Sul - UFRGS.

To the *Conselho Nacional de Desenvolvimento Científico e Tecnológico - CNPq*, for the funding support during my studies.

To the technical support from Aptech Systems for their unfailing and continuous assistance.

I am specially grateful to my friends Lara, Tainan, Luisa, Daiane, Clarissa and João, who have encouraged me and provided me help every time I needed. I am also grateful to all my classmates, what a such opportunity to work with all of you.

A very special gratitude goes to my forever encouraging family, my mother, my father and my brother, who have provided me through moral and emotional support in my life.

With a special mention to my husband Filipe, who encouraged me along the way and also contributed to the success of my work.

I am also grateful to the faculty of the Program for their continuous guidance, and for all university staff for their support.

A sincere gratitude for Professor Flávio Ziegelmann, Professor Edilean Silva and Professor João Caldeira, for serving as my committee members.

Finally, I would like to express my special appreciation to my advisor Professor Marcelo Portugal, for his valuable guidance and encouragement during my Master's degree. His advice on both research as well as on my career have been priceless.

Resumo

A evolução da economia brasileira durante os primeiros anos do século XXI é examinada através de um modelo microfundamentado de uma pequena economia aberta, permitindo mudanças no comportamento do Banco Central do Brasil, no parâmetro de rigidez nominal e na volatilidade dos choques estruturais. Mesmo os resultados não sendo conclusivos a respeito da presença de mudanças de regime durante o período analisado, encontramos evidências de troca de regime no âmbito da política monetária, passando em 2003 de um regime *Dove* para um regime *Hawk*, assim como evidências de choques externos mais voláteis durante períodos de incerteza. Na sequência, deixamos de lado a estimação empírica e derivamos regras de política monetária ótima para o caso brasileiro. É possível encontrar uma regra ótima capaz de estabilizar a inflação, o produto e a taxa de câmbio, mantendo uma taxa de juros estável. Por fim, o modelo trás uma discussão interessante sobre a dinâmica de determinadas variáveis macroeconômicas: uma moeda mais estável implica em uma taxa de juros mais volátil, e vice versa; um maior controle sobre a taxa de juros e/ou sobre a taxa de câmbio parece gerar uma maior instabilidade do produto e da inflação.

Palavras-chave: Markov-switching DSGE. Política monetária ótima. Estimação Bayesiana.
Classificação JEL: E31, E58, C11, C51.

Abstract

The evolution of the Brazilian economy during the first years of this century is examined through the lens of a micro-founded small open economy model that allows for changes in the behaviour of the Central Bank of Brazil, in the nominal price rigidity and in the volatility of structural shocks. Although the results are not conclusive about the presence of regime changes during the analysed sample, we find evidences in favour of shifts in the monetary policy stance, moving from a *Dove* to a *Hawk* regime in 2003, as well as evidences of more volatile external shocks during uncertainty periods. We further move away from the empirical estimation and derive optimal monetary policy rules for Brazil. It is possible to find an optimal rule that is successful in stabilizing inflation, output and exchange rates, whilst keeping interest rates stable. Finally, the model offers interesting insights about the standard deviation dynamics of macroeconomic variables: a more stable currency implies a more volatile interest rate and vice versa, and a higher control over interest rates and/or exchange rates seem to produce output and inflation instability.

Keywords: Markov-switching DSGE. Optimal monetary policy. Bayesian estimation.
JEL Codes: E31, E58, C11, C51.

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1 Introduction

The importance of considering time-varying elements in a dynamic stochastic general equilibrium (DSGE) model goes back to seminal contributions of Davig and Leeper (2007) and Farmer, Waggoner and Zha (2008), Farmer, Waggoner and Zha (2009), Farmer, Waggoner and Zha (2011). However very few studies actually sought to model the Brazilian economy in a Markov-switching (MS) DSGE framework, also generally referred to as Markov-switching rational expectations model (MSRE). With this context in mind, this paper is interested in determine possible structural changes that may have occurred in Brazil during the first years of this century by estimating a small scale two-state MS-DSGE model. Another main contribution of this study revolves around the derivation of optimal monetary policy rules for Brazil. Among the few works engaged with the subject, Andrade and Divino (2015) analysed the post-Real Plan period and proposed an optimal rule under the assumption of equal weights to inflation and output stabilization. We go further and compute a number of rules associated with different weights to interest rate, output and exchange rate stabilization. To the best of our knowledge, this paper represents the first attempt to derive optimal monetary policy rules for the Brazilian economy by means of a Markov-switching DSGE model.

The most recent vintage of DSGE models has been improved and many of the misspecified restrictions of the first generation of these models have been relaxed. They have become increasingly attractive for forecasting and quantitative policy analysis in macroeconomics. Additionally, the incorporation of Markov-switching elements in DSGE models represents a promising tool to better understand changes in the economy since it allows for stochastic and reversible regime changes of microfounded structural parameters. As an alternative, Fernandez-Villaverde, Guerron-Quintana and Rubio-Ramirez (2010) studied models with smoothly time-varying structural parameters to model parameter instability, using perturbation methods and estimating with particle filtering.

This paper is part of a research agenda that aims at representing the Brazilian economy in light of an estimated DSGE model with MS regime switches[1]. Regarding the international literature, Justiniano and Primiceri (2008) only considered regime changes in stochastic volatilities, while other authors further incorporated shifts in Taylor rule parameters or in the target inflation rate (Davig and Doh (2014), Schorfheide (2005), Liu, Waggoner and Zha (2011), Sims and Zha (2006), Bianchi (2013), Fernandez-Villaverde, Guerron-Quintana and Rubio-Ramirez (2010)). The current paper considers

[1] See also Gonçalves, Portugal and Aragón (2016) for a MS-DSGE estimation applied to Brazil.

regime changes in both structural parameters (described in what follows) and stochastic volatilities.

As a first main contribution, we incorporate MS elements in a small scale DSGE model through three different approaches, in line with Chen and MacDonald (2012): shifts in Taylor rule parameters only, shifts in the price stickiness parameter only and shifts in stochastic volatilities only. The estimation of two distinct regimes in the Taylor rule allow us to capture possible changes between a more aggressive inflation targeting (the *Hawk* regime) and a weaker response to inflation (the *Dove* regime). Changes in the nominal price rigidity may reflect firms incentive to update prices more or less frequently depending on the state of the economy. We therefore expect that prices are updated more often during uncertainty periods, and vice versa. We also introduce the possibility of changes in the volatilities of the structural disturbances to capture the "Good Luck" factor, evidenced by Sims and Zha (2006) as the key driver behind the stabilization of the U.S. economy during the "Great Moderation". As a way of including some degree of complexity, we finally consider shifts on the one hand in the Taylor rule and price stickiness parameters (as representing a single specification), and on the other hand shifts in stochastic volatilities. These changes are modelled as two independent MS processes in order to accommodate the two competing explanations.

Three important considerations emerge from the estimations. First, the periods of low and high price rigidity are not very clear, since we observe intermediate probabilities across the sample. We interpret these results as evidence of misspecification in the estimation of a two-regime MS for the price stickiness parameter, and advocate that a three-regime specification for this parameter would be more suitable for the Brazilian case. We therefore decide to estimate a last model taking the nominal price rigidity parameter as time-invariant and supposing independent shifts between the Taylor rule parameters and the stochastic volatilities. Second, the model with two independent Markov chains is found the second best performing model among those with MS elements, little behind the one with changes in stochastic volatilities only. The best performing model is the one with no regime changes, which was used as a benchmark. Third, the periods of low inflation targeting and high volatility of shocks correspond to periods generally associated with political or economic crisis. Additionally we observed a clear change in the monetary policy stance in 2003, moving from a *Dove* to a *Hawk* regime. One can easily associate this fact with the change in central bank's presidency at the beginning of 2003. In light of these evidences, we do not reject the hypothesis of regime changes during the analysed period, despite the model comparison results indicate that regime changes were not supported by the data.

The second main contribution of this paper relates to the computation of optimal monetary policy rules for Brazil, where we used a generalized Taylor rule framework.

It also allows us to compare the outcomes of the realised monetary policy of each estimated model and the associated optimal rule in terms of unconditional variances. A number of other authors were concerned with central bank preferences, and estimated the Central Bank of Brazil reaction function (please refer to Salgado, Garcia and Medeiros (2005), Silva and Portugal (2002) and Minella et al. (2003)). Nonetheless to our knowledge, the only study that actually computes an optimal rule for the Brazilian case is Andrade and Divino (2015), where the authors used a backward looking expectation model to obtain the estimates.

For each set of weights assigned to output, interest rate and exchange rate stabilization, we compute an associated optimal monetary policy rule, as well as the unconditional variances of each target variable, i.e. inflation, output, interest rates and exchange rates. Two results come up from this analysis. First, it is possible to find an optimal rule that is successful in stabilizing inflation, output and exchange rates, whilst keeping interest rates stable. This is important since it guarantees that real activity will not be affected, or at least not significantly affected. Compared to the realised, the optimal monetary policy rule is more responsive to fluctuations in inflation and exchange rates, less responsive to fluctuations in output and has a high degree of interest rate smoothing. Second, we found strong evidences that the Brazilian government was actually concerned with exchange rate stability during the analysed period. This is because the exchange rate variance of each estimated model is found to be smaller than the exchange rate variance of the associated optimal rule assigning a weight of zero to currency stability. We relate this fact with the *fear of floating* literature, concerned with the reluctance of the emerging markets in letting their currencies freely fluctuate.

Finally, this paper also explores the standard deviation dynamics of all target variables. We let vary the weights of each variable, while keeping the other weights constant, and analyse how the unconditional variances respond to these changes. Two interesting results show up. First, if the government influence currency value, it incurs in a more volatile interest rate, and the opposite is also true. We present some evidences that the Central Bank of Brazil is actually aware of this trade-off. Second, interest rate and/or exchange rate stability imply high volatilities for output and inflation, and vice versa.

The content of the paper can be summarized as follows. Section 2 presents the main equations of the small scale DSGE model. Section 3 illustrates the estimation strategy of the MS-DSGE model. Section 4 explains the empirical implementation and shows the estimation results. Section 5 explores the analysis of the optimal monetary policy rules, and section 6 concludes.

Our baseline model is the small open economy DSGE with parameter instability, derived from the work of Chen and MacDonald (2012) for the United Kingdom. The authors built up the work of Lubik and Schorfheide (2007), which is a simplified version of the small open economy with constant parameter of Galí and Monacelli (2005). Whilst the main purpose of Lubik and Schorfheide (2007) was to investigate the hypothesis whether central banks of developed countries[1] do respond to exchange rate fluctuations, Chen and MacDonald (2012) go further and analyse possible changes in the structure of the UK economy, with the inclusion of Markov-switching elements. They also derive an optimal monetary policy rule that is found to reduce the volatility of macroeconomic variables, such as output, inflation and the exchange rate.

We present here the key equilibrium conditions in log-linearized form of the model proposed by Lubik and Schorfheide (2007)[2]. It consists of an IS-equation, a Phillips curve, an exchange rate equation and an interest rate rule representing the monetary authority.

A conventional Euler equation, combined with the market clearing condition in the representative small open economy, leads to the following IS curve:

$$y_t = E_t y_{t+1} - (\tau + \lambda)(R_t - E_t \pi_{t+1} - E_t z_{t+1}) + \alpha(\tau + \lambda) E_t \Delta q_{t+1} + \frac{\lambda}{\tau} E_t \Delta y^*_{t+1} \tag{2.1}$$

where $0 < \alpha < 1$ is the import share, τ is the intertemporal substitution elasticity and $\lambda = \alpha(2-\alpha)(1-\tau)$. Endogenous variables are aggregate output, y_t, the consumer price index (CPI) inflation rate, π_t, and the nominal interest rate, R_t, while y^*_t is exogenous world output, z_t is global technology growth and q_t are the terms of trade[3], defined as the relative price of exports in terms of imports.

The Phillips curve is obtained from optimal price setting by domestic firms, represented by the following relation:

$$\pi_t = \beta E_t \pi_{t+1} + \alpha\beta E_t \Delta q_{t+1} - \alpha \Delta q_t + \frac{\kappa}{\tau + \lambda} y_t + \frac{\kappa\lambda}{\tau(\tau + \lambda)} y^*_t \tag{2.2}$$

where β is the discount factor and $\kappa = (1-\theta)(1-\theta\beta)/\theta$ represents the degree of price stickiness in the economy. As $\kappa \to \infty$, nominal rigidities vanish. We assume Calvo price setting, where the fraction of firms $(1-\theta)$ can set prices optimally while the remaining θ firms update their prices by the steady-state inflation rate.

1 The analysis was carried out for Australia, Canada, New Zealand and the UK.
2 For a complete derivation of the model, please refer to Galí and Monacelli (2005).
3 The terms of trade enter in first difference form since it is changes in relative prices that affect inflation via the relation $\pi_t = \pi_{H,t} + \alpha \Delta q_t$, where $\pi_{H,t}$ is the domestic inflation rate.

The derivation of the exchange rate equation is obtained from the log-linearization of the CPI formula around a steady-state satisfying the purchasing power parity (PPP) condition[4]. We have

$$\Delta e_t = \pi_t - (1-\alpha)\Delta q_t - \pi_t^* \tag{2.3}$$

where e_t is the nominal exchange rate and π_t^* is an exogenous inflation shock which we treat as an unobservable.

The model is closed by incorporating the behaviour of monetary authority, where it is assumed that the central bank reacts to deviations of CPI inflation and output from their target levels. We also allow for the inclusion of nominal exchange rate depreciation Δe_t in the monetary policy rule:

$$R_t = \rho_R R_{t-1} + (1-\rho_R)[\psi_1\pi_t + \psi_2(\Delta y_t + z_t) + \psi_3\Delta e_t] + \varepsilon_t^R, \qquad \varepsilon_t^R \sim NID(0,\sigma_R^2) \tag{2.4}$$

where we suppose $\psi_1, \psi_2, \psi_3 \geq 0$ and $0 < \rho_R < 1$. Unanticipated deviations from the systematic component of the monetary policy rule are captured by ε_t^R.

Following Chen and MacDonald (2012), exogenous variables are modelled as AR(1) processes, such that

$$\Delta q_t = \rho_q \Delta q_{t-1} + \varepsilon_t^q, \qquad \varepsilon_t^q \sim NID(0,\sigma_q^2), \tag{2.5a}$$

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z, \qquad \varepsilon_t^z \sim NID(0,\sigma_z^2), \tag{2.5b}$$

$$y_t^* = \rho_{y^*} y_{t-1}^* + \varepsilon_t^{y^*}, \qquad \varepsilon_t^{y^*} \sim NID(0,\sigma_{y^*}^2), \tag{2.5c}$$

$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \varepsilon_t^{\pi^*}, \qquad \varepsilon_t^{\pi^*} \sim NID(0,\sigma_{\pi^*}^2) \tag{2.5d}$$

The assumptions of complete asset markets and perfect risk sharing from the IS equation and Phillips curve imply the relation $(\tau+\theta)\Delta q_t = \Delta y_t^* - \Delta y_t$. However the use of an endogenous equation can cause tight cross-equation restriction on the model, as indicated in Lubik and Schorfheide (2007). As a consequence the authors chose to model the terms of trade as an exogenous AR(1) process. The variables z_t, y_t^* and π_t^* are modelled as latent variables.

The model then consists of ten state variables, including two expectation terms, $X_t = [y_t, \pi_t, R_t, \Delta e_t, \Delta q_t, z_t, y_t^*, \pi_t^*, E_t y_{t+1}, E_t\pi_{t+1}]'$; five exogenous processes representing the structural shocks, $Z_t = [\varepsilon_t^R, \varepsilon_t^q, \varepsilon_t^z, \varepsilon_t^{y^*}, \varepsilon_t^{\pi^*}]'$; and two rational expectation forecast errors, $\eta_t = [\eta_t^y, \eta_t^\pi]'$, defined as the deviation of the variables from their last period expected value. We can rewrite the model as a general class of linear models as

$$\Gamma_0(\theta)X_t = \Gamma_1(\theta)X_{t-1} + \Psi(\theta)Z_t + \Pi(\theta)\eta_t \tag{2.6}$$

where Γ_0, Γ_1, Ψ and Π are conformable parameter matrices and θ collects the structural parameters of the model, $\theta = \{\psi_1, \psi_2, \psi_3, \rho_R, \alpha, \tau, \kappa, \rho_z, \rho_q, \rho_{y^*}, \rho_{\pi^*}, \sigma_R, \sigma_z, \sigma_q, \sigma_{y^*}, \sigma_{\pi^*}\}$.

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4 The CPI is defined as $P_t \equiv [(1-\alpha)(P_{H,t})^{1-eta} + \alpha(P_{F,t})^{1-eta}]^{\frac{1}{1-\eta}}$, where $P_{H,t}$ is the domestic price index and $P_{F,t}$ is the price index of imported goods. The PPP condition is satisfied if $P_{H,t} = P_{F,t}$.

Parameter instability is considered in the model by allowing different structure parameters to be subject to regime change according to a two-state Markov-Switching (MS) process. In particular, we study three MSRE versions of the model outlined above that allow for (i) regime shifts in the monetary policy parameters $(\psi_1, \psi_2, \psi_3, \rho_R)$, (ii) regime shifts only in the price stickiness parameter (κ), and (iii) regime shifts in the volatility of the exogenous shocks $(\sigma_R, \sigma_z, \sigma_q, \sigma_{y^*}, \sigma_{\pi^*})$. We also estimate a constant parameter version for benchmark purposes. The estimation of a two-state MS model for monetary policy is conceivable since we might expect a change from a more aggressive inflation targeting policy (the well known *Hawk* regime) to a weaker responsiveness to inflation (the *Dove* regime). Furthermore Liu, Waggoner and Zha (2011) show that, in the case of the United States, more than two regimes in shock variances are not favoured by the data. We then chose to follow the specification of two regimes for the other time-variant parameters, the volatilities of shocks and the price stickiness parameter.

The different versions of the model are of interest because they represent key elements of the structure of the Brazilian economy that may have changed over the analysed period. More precisely we might expect shifts in central bank's behaviour during transition periods from one central bank's president to another, as well as periods of political uncertainty, such as the election of President Lula in 2002. The change in the price stickiness parameter, as pointed out by Chen and MacDonald (2012), may reflect firm's incentive to update prices more frequently during high inflation periods (we highlight the period from 2000 until 2003) or recessions (such as the international financial crisis in 2008 and the Brazilian fiscal crisis in 2015). Finally the shifts in the volatility of exogenous shocks may capture the "good luck" factor, suggesting that a volatility decline in a particular state variable may be associated with a decline in the volatility of shocks, instead of with structural factors.

To allow for change in the structural parameters of our DSGE model, which presents forward-looking components, we need a method for solving rational expectations models that includes regime change. In this kind of model the agents must be allowed to take into account the possibility of regime changes when forming expectations. Solving such a model is not straightforward and developments into the identification of the full set of solutions and conditions guaranteeing a unique solution are still ongoing. Nevertheless a number of papers made significant improvements on that direction, including Davig and Leeper (2007), Svensson and Williams (2007), and Farmer, Waggoner and Zha (2008), Farmer, Waggoner and Zha (2009), Farmer, Waggoner and Zha (2011), to name a few. For instance, Davig and Leeper (2007) introduce regime shifts in volatilities and coefficients of the Taylor rule and conclude that some of the solutions have a linear representation, pointing out the conditions for these solutions to be unique. However, Farmer, Waggoner and Zha (2008) explains that the method exposed by Davig and Leeper (2007) is not appropriate for the class of MSRE models. Instead they propose to write an equivalent model in an expanded state space with state-invariant parameters. The authors then define a class of minimal state variable solutions (MSV) and prove that any MSV solution to the expanded model is also a solution to the MSRE model. Farmer, Waggoner and Zha (2008) argue that the MSV solution is the most interesting class to study because it is often stable under real time learning. Subsequently Farmer, Waggoner and Zha (2009) present a complete set of necessary and sufficient conditions for a large class of MSRE models to be determinate, and Farmer, Waggoner and Zha (2011) innovates into the development of an efficient method for finding MSV equilibria.

As we have lagged interest rates in the Taylor rule, the solution method of Farmer, Waggoner and Zha (2008) is better suited for the model used in this study because it allows for lagged state variables, unlike the methods exposed in Davig and Leeper (2007) or Farmer, Waggoner and Zha (2009) which only apply to purely forward-looking models, according to Chen and MacDonald (2012). Given the above restrictions, we employ the solution algorithm proposed by Farmer, Waggoner and Zha (2008). The authors propose to characterize a Markov-switching model as a generalization of equation 2.6 such that

$$\Gamma_0(\theta_{\xi_t})X_t = \Gamma_1(\theta_{\xi_t})X_{t-1} + \Psi(\theta_{\xi_t})Z_t + \Pi(\theta_{\xi_t})\eta_t \tag{3.1}$$

where the matrices $\Gamma_0(\theta_{\xi_t}), \Gamma_1(\theta_{\xi_t}), \Psi(\theta_{\xi_t})$ and $\Pi(\theta_{\xi_t})$ are functions of structural parameters and transition probabilities. The parameters are permitted to change depending on the unobserved state variable ξ_t which follows a two-state Markov chain with stationary transition matrix P, where

$$Pr[\xi_t = 1|\xi_{t-1} = 1] = p_{11}, \qquad Pr[\xi_t = 2|\xi_{t-1} = 2] = p_{22} \tag{3.2}$$

The solution method proposed by Farmer, Waggoner and Zha (2008) requires writing equation 3.1 in an expanded form with constant parameter matrices Γ_0, Γ_1, Ψ and Π:

$$\Gamma_0 X_t = \Gamma_1 X_{t-1} + \Psi Z_t + \Pi\eta_t \tag{3.3}$$

The authors define a MSV solution to the above system and find conditions under which the solution to the expanded model is also a solution to the original one, as well as how to check for uniqueness and boundedness. If a unique solution exists then this can be written as a VAR(1) process with Markov-switching elements:

$$X_t = \Phi_1(\theta_{\xi_t})X_{t-1} + \Phi_2(\theta_{\xi_t})Z_t, \qquad Z_t \sim NID(0, \Sigma_{\xi_t}) \tag{3.4}$$

Despite the existence of other solution methods, such those described in Davig and Leeper (2007) or Svensson and Williams (2007), the method proposed by Farmer, Waggoner and Zha (2008) appears to work well in our small-scale DSGE framework with the iterative procedure converging rapidly in most cases.

The solution in equation 3.4 is combined with a system of observation equations, $Y_t = ZX_t$, specified as[1]:

$$\begin{bmatrix} \Delta GDP_t \\ INF_t \\ INT_t \\ \Delta EX_t \\ \Delta TOT_t \end{bmatrix} = \begin{bmatrix} \gamma^{(A)} + \Delta y_t + z_t \\ \pi^{(A)} + 4\pi_t \\ r^{(A)} + \pi^{(A)} + 4\gamma^{(A)} + 4R_t \\ \Delta e_t \\ \Delta q_t \end{bmatrix} \tag{3.5}$$

where Y_t represents a data matrix[2]. Our observed variables, which are fully described in the next section, are output growth (ΔGDP_t), inflation (INF_t), nominal interest rates (INT_t), nominal exchange rate depreciation (ΔEX_t) and changes in the terms of trade (ΔTOT_t). The parameters $\gamma^{(A)}, \pi^{(A)}$ and $r^{(A)}$ represent respectively the steady state values of output growth, inflation and interest rates, and they are estimated in conjunction with the other structural parameters of the model.

We adopt the Bayesian approach to model estimation. The likelihood is computed with the modified Kalman filter introduced by Kim and Nelson (2000)[3] and then combined with a prior distribution of the parameters to obtain the posterior.

To characterize the posterior distribution of the structural parameters, a Markov Chain Monte Carlo simulation is used. We first use a numerical optimization to find a mode of the posterior density, denoted $\tilde{\theta}$, by maximizing $lnL(Y^T|\theta)+lnp(\theta)$, where $L(\bullet)$ represents the likelihood function. One of the practical problems is that the solution for the DSGE model may not be unique or may not exist at some parameter values. To circumvent these issues we use the `csminwel` optimization routine written by Christopher Sims, which randomly perturbs the search direction if it reaches a cliff caused by indeterminacy or nonexistence. We then use a random-walk Metropolis-Hastings algorithm to generate $200,000$ draws from the posterior distribution with 20% of those being discarded. The proposal density is a Normal distribution whose mean is the previous

1 More details about observation equations can be found in An and Schorfheide (2007).

2 We considered no measurement errors.

3 The presence of the unobserved states X_t and the unobserved Markov states implies that the standard Kalman filter can no longer be used to make inferences on the state vector X_t and to calculate the likelihood. The unobserved Markov states implies that the inference on X_t is conditioned on the value of ξ_t in the current and past periods. Each iteration of the Kalman filter then implies an M fold increase in the number of cases to consider (where M denotes the number of regimes) making the filter intractable very quickly, as noted by Kim and Nelson (2000). The approximation limits the number of states that are carried forward in the Kalman filter, and these states are then collapsed at the end of each iteration.

successful draw and whose variance is the inverse of the (negative) Hessian at the posterior mode found before the simulation. The inverse Hessian is scaled to yield a target acceptance rate of 30%.

4 Empirical Implementation and Estimation Results

This section first presents the observable data series, and then moves to the exposition and analysis of the main estimation results of the MS models.

4.1 Data Description

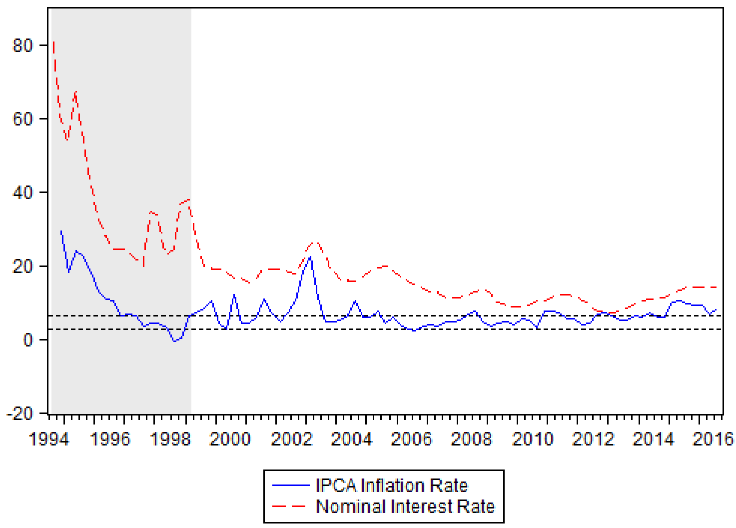
As observables, we use five series of Brazilian quarterly data: output growth, inflation rate, nominal interest rate (the *Selic* rate), nominal exchange rate and changes in the terms of trade. All data are seasonally adjusted, except for the nominal interest rate, and the sample spans 2000:Q1 to 2016:Q3. The series are obtained from the Central Bank of Brazil database and the Brazilian Institute of Geography and Statistics (*IBGE*). More precisely the data treatment was the following: (i) output growth is the log difference of seasonally adjusted real GDP scaled by 100; (ii) inflation rate is obtained from the log difference of the consumer price index (*IPCA*, mean)[1] scaled by 400; (iii) nominal depreciation rate is computed from log differences of the Real/US Dollar exchange rate index (mean) scaled by 100; and (iv) the terms of trade are the relative price of exports in terms of imports (price index), converted to log differences and scaled by 100.

Our sample is restricted by the most recent monetary policy regime. Specifically the Central Bank of Brazil is committed to an inflation targeting regime as well as a managed (or dirty) float exchange rate regime since 1999. Implemented in 1994, the *Real Plan* was a successful attempt at controlling hyperinflation in the country, which presented in 1990 an annual inflation rate of about 4000%. But it was only in June 1999 that Brazilian government formally implemented the inflation targeting regime. The inflation target oscillated between 8% and 3.5% in the first six years of the new policy, and since 2005 the inflation target has been stable at 4.5% ($\pm 2\%$)[2]. Figure 1 shows the path of inflation and nominal interest rates since the implementation of the *Real Plan*. One can observe the sustainable decline of the inflation rate, in particular after the implementation of the inflation targeting regime, in 1999. It is also worth noting that the government has been found difficulties in reaching the inflation target, as indicated by the average inflation rate of 6.5% over our sample. In particular we highlight the last four years of the sample in which the inflation rate moved up after a period of convergence to the target. In 1999 Brazilian government also modified its exchange rate policy, switching from a target zone arrangement regime to a managed float regime,

[1] We pass the IPCA series through the X13 filter in EViews.
[2] For the years of 2017 and 2018, only the bands of the inflation target were modified from $\pm 2\%$ to $\pm 1.5\%$.

in which the exchange rate is market determined but where the central bank may occasionally intervene. In practice the Central Bank of Brazil intervene quite often in the currency value through a number of mechanisms, including currency swap and buying or selling currency directly.

Figure 1 – Post-*Real Plan* Inflation and Interest Rates in Brazil



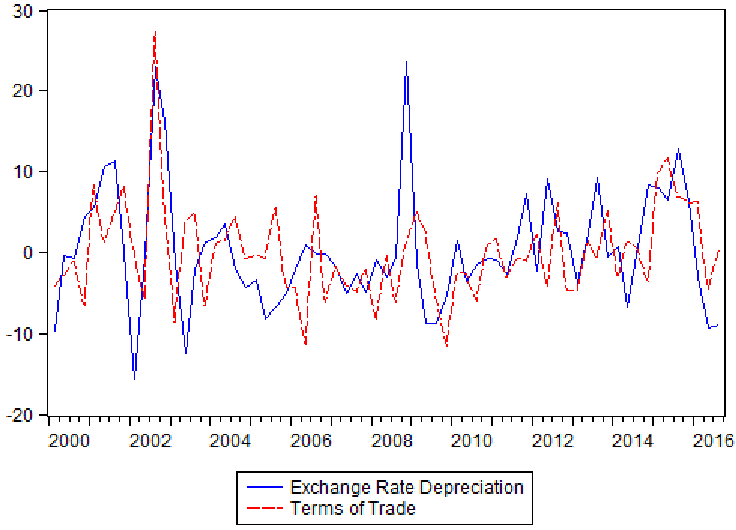
Source: Own construction (2017) based on *IBGE* and Central Bank of Brazil databases (2016).

Similar to the analysis proposed by Negro and Schorfheide (2008), we plot in figure 2 the Real/US$ exchange rate depreciation together with changes in the terms of trade. As one can see, the series do not present a clear correlation and are very volatile. More precisely we compute a Pearson correlation coefficient of 0.41. If we recall the specification of our DSGE model, the exchange rate movements are a function of inflation differentials and the terms of trade:

$$\Delta\hat{e}_t = \hat{\pi}_t - \hat{\pi}_t^* - (1-\alpha)\Delta\hat{q}_t$$

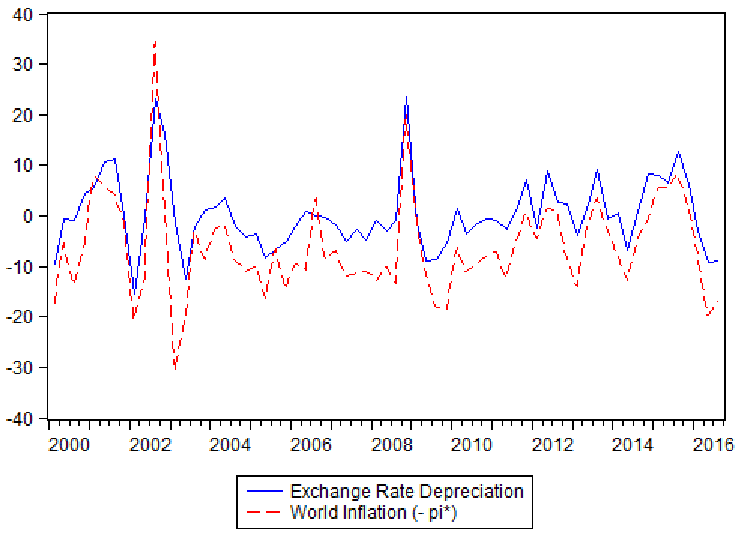
As mentioned earlier the world inflation $\hat{\pi}_t^*$ is treated as a latent variable. From the relation above, we can isolate $\hat{\pi}_t^*$ to obtain $-\hat{\pi}_t^* = \Delta\hat{e}_t - \hat{\pi}_t + (1-\alpha)\Delta\hat{q}_t$. Figure 3 exposes exchange rate movements together with the implicit world inflation, where we supposed $\alpha = 0.2$ (in accordance with our prior distribution for α). As indicated by the graph, most of the fluctuations of the exchange rate can be explained by the exogenous process $\hat{\pi}_t^*$, illustrating the well-known exchange rate disconnect (see Obstfeld and Rogoff (2001)). In this case, we compute a Pearson correlation of 0.86.

Figure 2 – Exchange Rate and Terms of Trade Movements



Source: Own construction (2017) based on the Central Bank of Brazil database (2016).

Figure 3 – Exchange Rate and World Inflation



Source: Own construction (2017) based on the Central Bank of Brazil database (2016).

4.2 Parameters estimates and regime probabilities

The procedure of setting the prior distribution of the estimated parameters was primarily based on evidences from national and international literature. In general our priors are quite loose, as prior information is surrounded by a large degree of uncertainty. The prior distributions are presented in table 1 and are the same for all the estimated models.

More specifically, priors of the Taylor rule parameters were based on working papers of the Central Bank of Brazil, including Castro et al. (2015), Linardi (2016) and Costa (2016). We adapted the mean of the prior distribution of ψ_3 to be 0.2, in line with the posterior estimate for this parameter by Linardi (2016). The prior for τ is chosen to be consistent with the estimation of the inverse of the intertemporal substitution elasticity by Castro et al. (2015), such that $\tau \equiv 1/\sigma = 1/1.3 = 0.77$. Regarding the price stickiness parameter, κ, we learned from our sample that this parameter fluctuates around 1.5, so we set this value as the mean of a gamma distribution. The prior mean of the steady state interest rate is selected according to the relation $\beta = (1 + r^{(A)}/400)^{-1}$, where β is the discount factor and takes the value of 0.989[3]. The priors of the parameters $\pi^{(A)}$ and $\gamma^{(A)}$ were set to be consistent with the sample average of the inflation and the growth rate of output series respectively. We use diffuse priors for the four parameters related to variables modelled as AR(1) processes and for the standard deviations of the five innovations. We follow Chen and MacDonald (2012) for the prior specification of the diagonal elements of the transition matrix, where we used beta distributions with mean 0.9 and standard deviation 0.05.

Table 1 also indicates the means and 90% error bands for the DSGE model with no regime changes, called Model 1. As suggested by Chen and MacDonald (2012) habit formation in consumption (h) is introduced in our estimations since it is found a significant autocorrelation pattern in the residuals of the IS equation in the original model proposed by Lubik and Schorfheide (2007).

In general our estimates are broadly in line with commonly used values in literature. We observe that according to these estimates the Central Bank of Brazil pursue a high anti-inflationary monetary policy, and it appears to have concerns about output volatility, with a response coefficient of 0.39 (lower than what was set in the prior). In the same time the monetary authority does not seem to pay too much attention to exchange rate movements. We highlight the relative high inflation targeting coefficient of 2.91 compared to other author's findings, such as 2.43 in Castro et al. (2015) or 1.29 in Costa (2016). The estimates also indicate a relatively high degree of interest-rate smoothing, of about 0.85.

The price stickiness estimate of 1.43 reveals that domestic firms optimise their prices approximately every two and a half months. It is worth noting however the relatively large confidence interval for this variable, indicating a certain amount of uncertainty about its estimated mean. We can relate this feature to other author's findings about price stickiness in Brazil. According to Barros et al. (2009) price-setting decisions are strongly influenced by the macroeconomic environment[4] which could explain the

[3] Estimated value for β according to Castro et al. (2015).

[4] During Barros et al. (2009)'s sample period, from March 1996 to December 2008, a number of important events in Brazil produced substantial macroeconomic variability, such as two emerging

Table 1 – Priors, means and 90% error bands for the time-invariant parameter model

	Posterior			Prior			
Parameter	Mean	5%	95%	Density	Domain	Mean	Std Dev
ψ_1	2.9062	2.1582	3.6203	Gamma	$\mathbb{R}^+$	3.00	0.50
ψ_2	0.3909	0.2176	0.5532	Gamma	$\mathbb{R}^+$	0.50	0.15
ψ_3	0.0345	0.0023	0.0648	Gamma	$\mathbb{R}^+$	0.20	0.15
ρ_R	0.8470	0.8040	0.8879	Beta	$[0,1)$	0.60	0.15
τ	0.5424	0.4143	0.6627	Beta	$[0,1)$	0.77	0.20
κ	1.4266	0.8139	2.0053	Gamma	$\mathbb{R}^+$	1.50	0.50
ρ_z	0.8653	0.7852	0.9453	Beta	$[0,1)$	0.50	0.25
ρ_q	0.4620	0.3460	0.5734	Beta	$[0,1)$	0.80	0.10
ρ_{π^*}	0.3654	0.1831	0.5440	Beta	$[0,1)$	0.50	0.25
ρ_{y^*}	0.7266	0.5768	0.8769	Beta	$[0,1)$	0.50	0.25
α	0.1574	0.0970	0.2197	Beta	$[0,1)$	0.19	0.05
h	0.7103	0.5589	0.8786	Normal	$\mathbb{R}^+$	0.70	0.10
$r^{(A)}$	4.5260	3.1843	5.9010	Gamma	$\mathbb{R}^+$	4.50	1.00
$\pi^{(A)}$	6.5992	5.8941	7.3157	Gamma	$\mathbb{R}^+$	6.00	2.00
$\gamma^{(A)}$	0.5737	0.2706	0.8698	Normal	$\mathbb{R}^+$	0.57	0.20
σ_R	0.3037	0.2509	0.3571	Inv Gamma	$\mathbb{R}^+$	0.50	5.00
σ_z	0.5372	0.3913	0.6759	Inv Gamma	$\mathbb{R}^+$	0.85	5.00
σ_{π^*}	9.5656	8.1913	10.9668	Inv Gamma	$\mathbb{R}^+$	0.55	5.00
σ_{y^*}	1.0786	0.8583	1.2946	Inv Gamma	$\mathbb{R}^+$	1.50	5.00
σ_q	6.5426	5.5630	7.4943	Inv Gamma	$\mathbb{R}^+$	1.20	5.00
p_{11}	-	-	-	Beta	$[0,1)$	0.90	0.05
p_{22}	-	-	-	Beta	$[0,1)$	0.90	0.05
q_{11}	-	-	-	Beta	$[0,1)$	0.90	0.05
q_{22}	-	-	-	Beta	$[0,1)$	0.90	0.05

Source: Own construction (2017).

large price-setting volatility observed between 1996 and 2008. This means that price-setting frequency in Brazil, linked to the variable κ, can vary substantially between one period to another, which may undermine the accuracy of the estimation of κ in a time-invariant parameter model. For instance Gouvea (2007) explains that substantial disturbances to average inflation impose high costs of not adjusting prices and trigger more frequent price reviews, which was the case during the confidence shock before 2002 presidential elections. The author also points out that on average prices remain unchanged for 2.7 to 3.8 months[5], a result quite above our estimate.

4.2.1 Markov-switching models

Table 2 presents the posterior estimates of the estimated Markov-switching models, where we allow for shifts between two regimes. The estimated model that permits changes only in the monetary policy rule parameters (ψ_1, ψ_2, ψ_3 and ρ_R) is called Model 2. According to Gonçalves, Portugal and Aragón (2016), this specification was found superior than the constant parameter model for the Brazilian economy, where the data used spans from 1996 to 2012.

As indicated by the posterior estimates, regime 1 is characterized by strong inflation targeting with an inflation response coefficient of 3.81, compared to 1.35 for regime 2. There is no overlap in the confidence intervals across the two regimes. The other parameters of the Taylor rule do not present a significant change across regimes, except for a slight decrease in the interest rate smoothing coefficient, from 0.81 to 0.74.

The filtered and smoothed probabilities of regime 2, which we called the low inflation targeting regime, are illustrated on the top panel of figure 4. According to the model, monetary policy in Brazil switched from a low inflation targeting regime to a more aggressive one approximately in the end of 2003, remaining in regime 1 until the end of the sample. One possible explanation for this behaviour is the change in the presidency of the central bank at the beginning of 2003. The first action of the new central bank's president, Mr Henrique Meirelles, was rising interest rates in response to a jump in inflation after the election of President Lula at the end of 2002. Throughout his presidency[6], both inflation and interest rates sustainably declined, which might suggest an anti-inflationary behaviour of the central bank at that time.

In what is called Model 3, only the price stickiness parameter, κ, is permitted to change over time. According to Galí and Monacelli (2005), this parameter also represents the slope coefficient of the Phillips curve, where the domestic inflation is a function of the expected domestic inflation and the real marginal cost. As mentioned

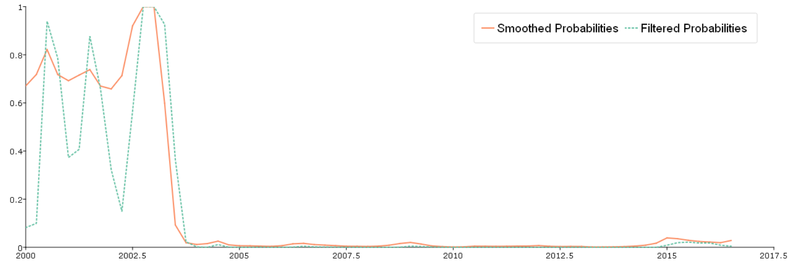
markets crisis, changes in monetary policy, blackouts and energy rationing and an election crisis.

5 The author's sample period ranges from March 1996 to to April 2006.

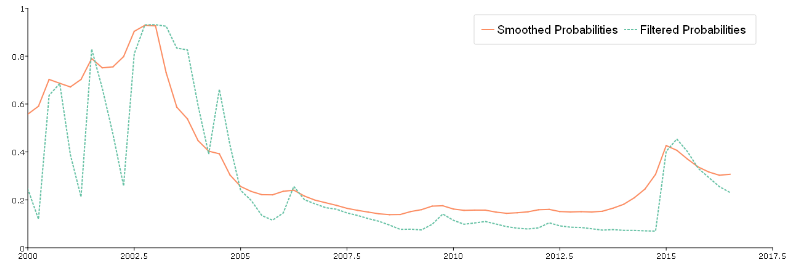
6 Mr Meirelles remained in office from 2003 to 2011.

Figure 4 – Smoothed and Filtered Probabilities for Models 2, 3 and 4

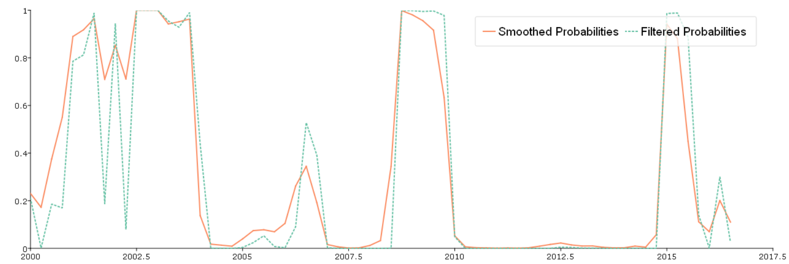
(a) Probability of low inflation targeting



(b) Probability of low price rigidity



(c) Probability of high volatility of shocks



Source: Own construction (2017).
Notes: Top panel: probability of regime 2 for the monetary policy parameters (Model 2). Second panel: probability of regime 2 for the parameter κ (Model 3). Lower panel: probability of regime 2 for the stochastic volatilities (Model 4).

earlier, this model investigates whether firms update their prices more (less) frequently in periods of economic instability (stability). As shown in table 2, the posterior estimates of κ change significantly over the two regimes. Regime 1 presents a lower κ value, with 0.84, rising to 2.12 in regime 2. This means that in regime 1 firms optimize their prices every four and a half months, while in regime 2, called the low price rigidity regime, the frequency of price adjustment changes to approximately 15 days.

The second panel of figure 4 shows that the low price rigidity regime was prevailing in Brazil at the end of 2002 and in 2003. This is a very plausible result since at that time the inflation rate passed the mark of 20%, the highest rate registered in our sample, and confirms our presumption that firms have a stronger incentive to change prices more often in periods of economic uncertainty. It is interesting to note however that the probabilities do not always permit to be absolutely conclusive about the state of the economy, specifically on the first two years of the sample and in 2015, where we observe intermediate probabilities. We also note that again the confidence intervals of both regime estimates are quite large in comparison to the other variables, and we point out a little overlap across regimes. All these evidences might indicate that this model could be misspecified for the Brazilian economy, in which case we would suggest that a markov-switching model with three different regimes for κ could better explain the economy dynamics, given the information provided earlier in favour of a variable value for the price stickiness in Brazil.

The next markov-switching model allows changes only in the standard deviations of exogenous shocks, that is Model 4. As one can see in table 2 greater volatilities are associated with regime 2, and only σ_R presents a little overlap in the confidence intervals. The probabilities presented in the bottom panel of figure 4 show three main periods where the economy was affected by exogenous shocks: (i) between 2001 and 2003, where we highlight the unstable inflation and political uncertainty linked to the election of President Lula; (ii) in 2008 and 2009, during the world financial crisis; and (iii) in 2015 where the Brazilian economy faced an unprecedented fiscal crisis. The standard deviations of the technological growth shock and terms of trade shock quite double in regime 2, while the standard deviations of the foreign inflation shock and foreign output shock are three times larger in regime 2 than in the first regime.

Table 2 – Parameter estimates of the Markov-switching models

Parameter	Model 2		Model 3		Model 4		Model 5	
	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2
ψ_1	3.8117	1.3463	3.2154	-	3.3356	-	3.6288	1.6859
	(3.0475,4.5956)	(0.9678,1.7112)	(2.5071,3.8883)		(2.6319,4.0130)		(2.8333,4.4907)	(1.0108,2.3396)
ψ_2	0.4429	0.4453	0.4259	-	0.4663	-	0.4559	0.4751
	(0.2546,0.6271)	(0.2262,0.6458)	(0.2267,0.6128)		(0.2620,0.6715)		(0.2517,0.6457)	(0.2462,0.6930)
ψ_3	0.0473	0.0466	0.0532	-	0.0467	-	0.0506	0.0844
	(0.0034,0.0876)	(0.0019,0.0905)	(0.0049,0.0967)		(0.0038,0.0852)		(0.0038,0.0959)	(0.0006,0.1860)
ρ_R	0.8090	0.7408	0.8642	-	0.8407	-	0.8053	0.7670
	(0.7560,0.8631)	(0.6520,0.8349)	(0.8234,0.9045)		(0.7991,0.8829)		(0.7495,0.8659)	(0.6551,0.8838)
τ	0.7247	-	0.5342	-	0.6508	-	0.6713	-
	(0.5769,0.8656)		(0.3285,0.7330)		(0.5344,0.7606)		(0.5586,0.7780)	
κ	2.0842	-	0.8393	2.1236	2.1092	-	2.1497	-
	(1.3183,2.8580)		(0.0793,1.7925)	(1.3405,2.9235)	(1.3409,2.8926)		(1.3607,2.8824)	
ρ_z	0.0960	-	0.2013	-	0.1488	-	0.1367	-
	(0.0056,0.1754)		(0.0225,0.3646)		(0.0276,0.2604)		(0.0187,0.2444)	
ρ_q	0.4252	-	0.5304	-	0.4221	-	0.4203	-
	(0.3256,0.5196)		(0.3387,0.7221)		(0.3278,0.5218)		(0.3260,0.5204)	
ρ_{π^*}	0.3609	-	0.3456	-	0.4288	-	0.4164	-
	(0.1828,0.5336)		(0.1757,0.5246)		(0.2595,0.5961)		(0.2512,0.5815)	
ρ_{y^*}	0.9437	-	0.8606	-	0.8996	-	0.9089	-
	(0.8988,0.9964)		(0.7728,0.9495)		(0.8323,0.9675)		(0.8467,0.9727)	
α	0.1360	-	0.0937	-	0.1485	-	0.1425	-
	(0.0805,0.1896)		(0.0323,0.1519)		(0.0889,0.2033)		(0.0879,0.1956)	
h	0.6655	-	0.6900	-	0.6713	-	0.6583	-
	(0.4969,0.8280)		(0.5277,0.8586)		(0.5099,0.8342)		(0.4986,0.8191)	

Source: Own construction (2017).

Parameter	Model 2		Model 3		Model 4		Model 5	
	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2
$r^{(A)}$	4.8572	-	4.2863	-	4.1789	-	4.2475	-
	(3.4106,6.3226)		(3.0080,5.5632)		(2.9458,5.4161)		(2.9712,5.4634)	
$\pi^{(A)}$	6.2580	-	6.3095	-	6.2883	-	6.0805	-
	(5.5006,7.0146)		(5.3714,7.1897)		(5.4299,7.1551)		(5.3346,6.8485)	
$\gamma^{(A)}$	0.5880	-	0.5673	-	0.6287	-	0.6348	-
	(0.3870,0.7990)		(0.3515,0.7888)		(0.4315,0.8299)		(0.4418,0.8356)	
σ_R	0.4051	-	0.3237	-	0.3353	0.4546	0.3704	0.4472
	(0.3191,0.4896)		(0.2660,0.3822)		(0.2600,0.4097)	(0.3156,0.5859)	(0.2757,0.4545)	(0.3127,0.5733)
σ_z	1.2299	-	1.1667	-	0.9371	1.5902	0.9220	1.5975
	(1.0487,1.4133)		(0.9720,1.3516)		(0.7442,1.1219)	(1.1992,1.9803)	(0.7551,1.0975)	(1.2034,1.9771)
σ_{π^*}	9.4796	-	9.6296	-	5.2450	13.9458	5.0410	14.0283
	(8.1775,10.7717)		(8.2393,10.9607)		(4.0677,6.3500)	(10.6040,17.1544)	(3.9056,6.1057)	(10.5987,17.5308)
σ_{y^*}	2.0348	-	2.1101	-	0.9205	2.7153	0.9860	2.4995
	(0.7873,3.3238)		(0.8280,3.0278)		(0.5598,1.2651)	(1.5669,3.8520)	(0.5832,1.3564)	(1.4277,3.4691)
σ_q	6.3431	-	6.5865	-	4.3635	8.7042	4.2392	8.6216
	(5.4661,7.2358)		(5.5134,7.6309)		(3.4127,5.3071)	(6.5593,10.7479)	(3.2309,5.1815)	(6.6007,10.5630)
p_{11}	0.9651	-	0.9291	-	0.9074	-	0.9079	-
	(0.9390,0.9911)		(0.8735,0.9853)		(0.8500,0.9646)		(0.8516,0.9619)	
p_{22}	0.9177	-	0.8905	-	0.8603	-	0.8662	-
	(0.8643,0.9751)		(0.8164,0.9699)		(0.7791,0.9408)		(0.7919,0.9423)	
q_{11}	-	-	-	-	-	-	0.9421	-
							(0.8975,0.9864)	
q_{22}	-	-	-	-	-	-	0.8792	-
							(0.7939,0.9661)	

Source: Own construction (2017).

4.2.2 Model 5: A two-Markov chain model

Whilst each model specified above may explain in some extent the economy dynamics, it would be preferable to aggregate in one single model three independent Markov chains in order to accommodate the three explanations. However this would increase significantly the complexity of the estimation. To circumvent this issue, we follow Chen and MacDonald (2012) and combine changes in the monetary policy parameters with changes in the parameter κ as being part of the same specification. This is because the probability of being in the low inflation targeting regime (regime 2) occurred at approximately the same time of being in the low price rigidity regime (regime 2). In the same time changes in the volatility of exogenous shocks occurred separately. Therefore we estimate another model, called Model 5, combining two independent Markov chains.

The first estimation results suggested however that shifts between two regimes in the price stickiness parameter are not supported by the data. Indeed we observe a κ value of 2.17 for regime 1 and of 2.10 for regime 2, indicating that we are not in presence of two distinct regimes for this variable. We then decided to estimate model 5 supposing that κ is time-invariant, allowing for shifts in the Taylor rule and in the stochastic volatilities as two independent Markov chains.

The estimation method differs somewhat from those with one single Markov chain, due to the construction of the transition probability matrix. Instead of having two separate transition matrices, we combine them into one single transition matrix with four states, P^*, such that

$$P^* = Q \otimes P \tag{4.1}$$

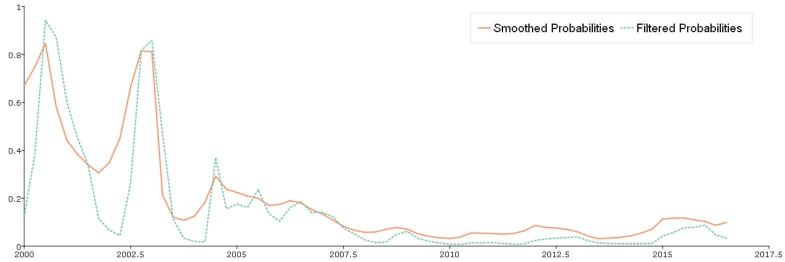
where Q and P represent respectively the transition probability matrix of the unobservable state variable governing changes in the standard deviations of shocks, and changes in the monetary policy parameters.

The estimated parameters for model 5 are generally in line with those found in models 2 and 4. Here again we observe a more aggressive inflation targeting behaviour in regime 1, with a coefficient of 3.63, compared to regime 2 with 1.68. There is no overlap in the confidence intervals.

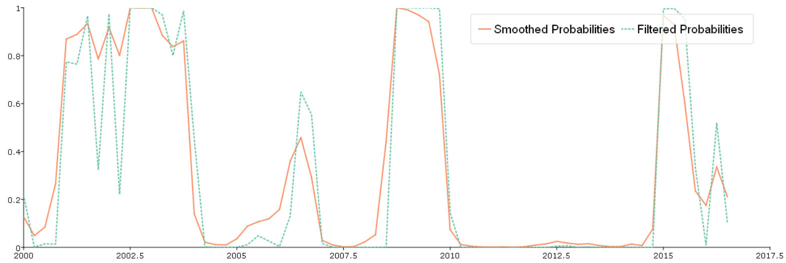
Figure 5 shows the probabilities of regime 2 for both specifications. With a probability of approximately 80%, monetary policy followed a low inflation targeting regime in 2000 and by the end of 2002 and 2003, very similar to what was inferred from model 2. In the same way of what was indicated by model 4, the bottom panel of figure 5 suggests that high volatilities of external shocks have hit the economy from 2001 to 2003, during the financial crisis in 2009, and also during the more recent fiscal crisis in 2015.

Figure 5 – Smoothed and Filtered Probabilities of Model 5

(a) Probability of low inflation targeting



(b) Probability of high volatility of shocks



Source: Own construction (2017).
Notes: Probabilities of Model 5: Top panel, probability of regime 2 for the monetary policy parameters. Lower panel, probability of regime 2 for the stochastic volatilities.

Regarding the probabilities of low inflation targeting, some considerations are in order. Interestingly the models do not point out a weaker monetary policy reaction against inflation during the presidency of Alexandre Tombini (January 2011 to June 2016), specially in 2012 where the Selic rate reached its record low at 7.25%.

First of all, it is worth noting that, differently from his recent predecessors, Tombini faced an arduous macroeconomic situation, being part of a presumed irresponsible government regarding fiscal policy and not so attentive to inflation. On the other hand, we may wonder why interest rates were cut down to its lowest historical level in 2012[7] (7.25%) in a context of considerable economic uncertainty. Tombini was therefore very criticised because of the presumption of political intervention in central bank decisions and because of his failure in maintaining inflation within the target bands. During his presidency, the inflation target (4.5%) was never reached. The minimum level of inflation between 2011 and 2015 registered 5.8%, very close to the upper band of 6.5%, and in 2015 we observed the highest level of inflation since 2003, 10.67%. The average inflation during the presidency of the former central bank president, Henrique Meirelles, registered 5.78%, climbing to 7.06% under Tombini's administration.

[7] In 2012 the government also cut down the interest rates of two important public financial institutions, *Banco do Brasil* and *Caixa Econômica Federal*. The primarily goal behind this public policy was to reduce the price of money to households and small and medium-sized enterprises. As an indirect effect, the government also aimed to influence the average loan interest rate of the private sector.

Table 3 – Model comparison through log marginal likelihood

Model 1	Constant parameter model	-833
Model 2	Markov-switching in the monetary policy parameters only	-862
Model 3	Markov-switching in the price stickiness parameter only	-867
Model 4	Markov-switching in volatility of shocks only	-843
Model 5.1	Two Markov chains model with Markov-switching in κ	-846
Model 5.2	Two Markov chains model without Markov-switching in κ	-844

Source: Own construction (2017).

Therefore it is a very conceivable assumption that a *Dove* regime was actually in place at some point during the presidency of Tombini. The models may have concealed this information because of the excessive data volatility until 2003, specially of the inflation series. In other words, the presence of a remarkably identified *Dove* regime until 2003 may have underestimated the possible weaker inflation targeting regime after 2011. In our case specifically, the misinterpretation can also be explained by the relative small sample: while studies interested in the U.S. or the Euro zone economy use samples with at least four decades of data, we worked here with only 16 years of data series.

4.2.3 Model comparison

In order to compare the different specifications, the log marginal likelihood of each model is approximated with Geweke (1999) modified harmonic-mean estimator, which provides a coherent framework to compare non-nested models.

As one can see in table 3, the log likelihood comparison suggests that the constant parameter model would more likely represent the Brazilian economy than the Markov-switching models. This result is at odds with a recent work for the Brazilian economy, in which parameter instability models are found superior to the constant version. More specifically, through the estimation of different MS models, Gonçalves, Portugal and Aragón (2016) found that the version allowing for changes in the coefficients of the Taylor rule and shock volatilities provides the best fit. The authors used a very similar model to the present work, except for the inclusion of incomplete asset markets and indexation of prices to past inflation, which are absent in the model presented here.

Considering only the MS models, we observe that the one with shifts in both monetary policy parameters and shock variances is the second best fitted model, little behind the one with changes only in shock variances. The model with shifts only in the price stickiness parameter presented the worse fit to the data, followed by the model with only changes in the monetary policy rule. It is interesting to note that if we do not allow for κ to vary between two regimes, which we called model 5.2, we have a better fit compared to the one allowing this parameter to change (model 5.1).

Given the empirical evidences risen by the regime probability analysis, it would be at least unwise to reject the hypothesis of a time-varying parameter model for the Brazilian economy. Indeed the periods of weak inflation targeting and high volatility of shocks pointed out by the probability graphs correspond to the most unstable periods of our sample. Moreover we have strong reasons to believe that a change in monetary policy stance indeed occurred at the time the new president of the central bank took office, in 2003. Therefore we decided to undertake the optimal monetary policy analysis for all estimated models, and then compare the outcomes of each one of them.

In this section, we focus on the derivation of optimal monetary policy rules for the Brazilian economy, moving away from the empirical estimation. We use all posterior estimates, except those related to the monetary policy rule, to compute the optimal policies within the following generalised Taylor rule framework:

$$R_t = \rho_R R_{t-1} + \psi_\pi \pi_t + \psi_y(\Delta y_t + z_t) + \psi_e \Delta e_t \tag{5.1}$$

The variables presented in this optimal rule are the same as those in equation 2.4. The parameters of the optimal monetary policy rule (ρ_R, ψ_π, ψ_y and ψ_e) are chosen to minimize the intertemporal loss function at period t such as

$$W = E_t \sum_{\tau=0}^{\infty} \beta^\tau L_{t+\tau} \tag{5.2}$$

where $0 < \beta \leq 1$ is the household's discount factor and the period loss function L_t is given by

$$L_t = Y_t^{'} \Lambda Y_t \tag{5.3}$$

where $Y_t \equiv \{\pi_t, \Delta y_t, R_t, \Delta e_t\}^{'}$ is a vector of targeted variables. The matrix Λ is a diagonal weighting matrix and determine the relative priority given to each of the targeted variables, with diagonal elements $(1, \Lambda_{\Delta y}, \Lambda_R, \Lambda_{\Delta e})$. We consider an unconditional welfare loss function where β goes to unity. In our analysis, we look at a range of different weights given to output, interest rates and exchange rates, since the weight assigned to inflation is normalized to unity. For each set of weights ($\Lambda_{\Delta y}, \Lambda_R, \Lambda_{\Delta e}$), we compute the associated optimal monetary rule, as well as the unconditional variances of each targeted variable, as illustrated in the following subsections.

Regarding the methodology, we are not allowed to use traditional frameworks to derive policy maker's optimal rules, such the one exposed in Dennis (2004). Since we included Markov-switching in some of the parameters, our solution must take into account the possibility of parameter uncertainty. We then apply the so called Markov jump-linear-quadratic (MJLQ) framework, developed by Svensson and Williams (2007), which is considered a very flexible tool for the analysis of optimal policy under uncertainty. It builds on the control-theory literature and uses recursive methods, permitting its application in relatively general models.

5.1 Unconditional variances analysis

We were first interested in understanding the unconditional variances dynamics. It is important to know, for instance, how the economy responds to a government intervention towards the exchange rate in terms of the volatility of macroeconomic variables. Similarly we want to anticipate the effect of a greater government concern with inflation or output. In order to address this issues, we control the weights given to each variable and analyse the unconditional variances of the target variables.

We started by assigning zero weight to output and a weight of 0.4 to the interest rate[1], while varying the weight on the exchange rate. Here in particular, we varied over a quite restricted grid, from 0 to 0.15, since our estimations indicated that the Brazilian government pay little attention to exchange rates compared to the weight given to inflation, for instance. As indicated in the top section of table 4, as government devotes more attention to exchange rate movements, we observe a rise in the volatility of the other three targeted variables. When controlling for the interest rate, we assigned again zero weight to output and a weight of 0.05 to the exchange rate. The middle section of table 4 indicates that as government puts more weight on the control of the interest rate, we can lose stability of the other three variables. Finally, we fixed at 0.9 and 0.1 the weights on the interest rate and the exchange rate respectively in order to control for the output. The bottom section of table 4 shows that as we rise the efforts at controlling output, we give up interest rate and exchange rate stability, while inflation volatility is reduced.

To summarize this short analysis, two interesting insights show up. First, the volatility dynamics of interest rates and exchange rates clearly oppose each other. If the government tries to influence the exchange rate, it must give up interest rate stability. Similarly when interest rate stability is the main concern, we become exposed to a more unstable currency (we explore this first point in what follows). Second, both interest rates and exchange rate dynamics present an opposite behaviour when compared to output and inflation dynamics. When government tries to stabilize interest rates or exchange rates, we have strong reasons to believe that the volatility of output and inflation will move up, and vice versa. This conclusions illustrate a number of macroeconomic trade-offs and indicate the magnitude at which they are applied to the Brazilian economy.

Concerning the first idea, some facts suggest that the Central Bank of Brazil seems to be aware of the trade-off between exchange rate and interest rate volatilities. To elucidate this point, we present here some considerations. The volatility of Brazilian exchange rate emerges among the highest in the world. Figure 6 compares currency

[1] As a counterfactual, we also varied the weights on the output and interest rates over a grid on the unit square to validate our conclusions. We did so for each of our analysis.

Table 4 – Unconditional variances dynamics

Weights		Output	Inflation	Interest rate	Exchange rate
Exchange rate	0.00	0.1568	0.0045	0.0091	18.3807
	0.05	0.1597	0.0123	0.0133	16.8030
	0.10	0.1645	0.0351	0.0188	15.3485
	0.15	0.1705	0.0690	0.0251	14.0759
Interest rate	0.2	0.1584	0.0110	0.0156	16.7892
	0.4	0.1597	0.0123	0.0133	16.8030
	0.7	0.1608	0.0130	0.0112	16.8094
	0.9	0.1613	0.0145	0.0101	16.8175
Output	0.1	0.1658	0.0355	0.0145	15.4028
	0.4	0.1648	0.0337	0.0148	15.5010
	0.6	0.1642	0.0326	0.0150	15.5629
	0.9	0.1634	0.0312	0.0152	15.6511

Source: Own construction (2017).

volatility between a number of developed and emerging countries, presenting two distinct measures of exchange rate uncertainty: the standard deviation of the effective exchange rate (EER) index series in log differences[2], and the percentage difference between the maximum and the minimum of the EER index[3]. As one can see, developed countries present a lower currency volatility compared to developing economies. The standard deviation of Brazilian exchange rate figures as the second largest among the studied economies and the percentage difference between the extreme points of the series is found to be the highest in Brazil. The Brazilian statistics are also the largest among those of the other BRICS members (Russia, India, China and South Africa).

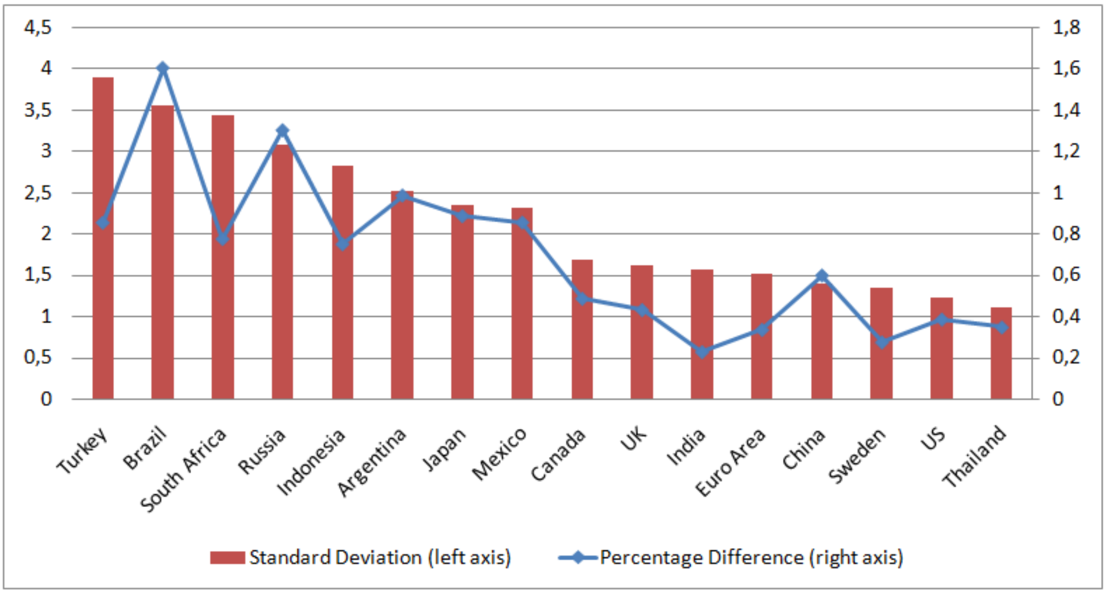
As discussed earlier, a higher exchange rate volatility spurs government intervention in controlling currency. This interference can occur through a number of tools, such as currency swaps, direct buying or selling currency or simply interest rate movements, which indirectly affects currency value. In Brazil, we observe an increasing

[2] The EER index is computed by the Bank of International Settlements (https://www.bis.org/statistics/eer.htm, May 16, 2017) and represents the geometric weighted averages of bilateral exchange rates adjusted by relative consumer prices. The weighting patter is time-varying.

[3] We used monthly data from February 2000 to March 2017, except for the Argentine economy, where we decided to sample between October 2002 and March 2017. This is because Argentina experienced the end of its fixed exchange rate regime in January 2002, which produced a large reduction of the value of the peso during that year. After a few months, the exchange rate was mostly a floating exchange rate.

interest on reverse and traditional currency swaps in order to control exchange rate volatilities, being used to weaken and strengthen real respectively. By March 2016, the Central Bank of Brazil registered a passive currency account of more than US$ 100 billion (since then however the central bank has reduced the passive amount to roughly US$ 22 billion by March 2017). In other words policy makers appear to prefer currency swaps instead of interest rates movements to control exchange rates. This is not at odds with the previous findings, suggesting that the use of interest rates to stabilize currency can be successful only at the expense of a more volatile interest rate. As a way to avoid interferences in real activity, the Central Bank of Brazil mostly adopts currency swaps to control exchange rate movements.

Figure 6 – Exchange Rate Comparison by Country



Source: Own construction (2017).
Notes: Left axis: standard deviation of the effective exchange rate index in log differences (monthly data). Right axis: Percentage difference between the maximum and the minimum of the effective exchange rate index.

5.2 Realised and optimal policy rules

As a way to compare the different models and associated rules, a key question is how to choose a specific set of weights, or putting it differently, how to propose the weight assigned to each variable. With the same set of weights for each model, the comparison is straightforward. In what follows, we explain our choice in adopting the rule that gives a weight of 0.9 to interest rate stability and 0.1 to output stability. All results presented below were computed under this specification. Concerning the weight assigned to the exchange rate, we consider two different values, 0.00 and 0.05, and analyse both outcomes.

A high weight on the interest rate control ($\Lambda_R = 0.9$) may limit the central bank reaction to economy conditions. In other words, it avoids an optimal rule assigning excessive values for the interest rate, and consequently it can prevent an unsustainable government debt, for instance. At the same time, the previous analysis showed that a high control over interest rate stability leads to a more unstable exchange rate. In order to offset this undesirable effect, we chose a low weight for output (a 0.1 weight), which restricts currency swings according to our results.

Please refer to table 5 for the unconditional variances of the estimated models and the associated optimal models[4]. Figure 7 gives a more visual insight over the different values of output and inflation variances. It plots the variances of the estimated models as well as of the optimal rules assigning 0.05 weight to currency variance. Regarding the results, some considerations are in order. First, we may distinguish between the time-invariant model (model 1) and the Markov-switching models. Indeed model 1 presented a much higher inflation volatility than the other models, which are concentrated on the right bottom zone in the graph. Even though it may be reasonable to believe that Brazilian inflation presents a volatility at the same order of output volatility, what is suggested by model 1, we chose to focus our study on the Markov-switching models. This is because of the strong evidences provided earlier indicating that parameter instability appears to explain the economy dynamics.

Second, as we might expect, output and inflation volatility of the optimal rules are all smaller than the estimated rules. Third, the exchange rate volatility of the rule assuming zero weight to this variable is higher than the associated estimated rule for all models. Nonetheless if a little attention is given to exchange rate movements, represented by a weight of 0.05, we obtain smaller currency volatilities than the estimated rule. And finally, if we suppose $\Lambda_{\Delta e} = 0.05$ among the Markov-switching models, the only model that delivers a similar interest rate volatility than its estimated rule is model 5.2, all other models presenting higher interest rate movements. We highlighted this model in bold type in table 5.

As a counterfactual, we compute optimal rules supposing that the high inflation targeting regime is applied to the whole sample (models 2 and 5.2). As one could expect, the unconditional variances of output, inflation and interest rates are reduced, compared to the estimated rule. The higher exchange rate volatility is explained by the greater weight given to inflation stability in this case.

An interesting remark is in order. As noted earlier, the exchange rate variance of all estimated models is smaller than the exchange rate variance of the associated optimal rule assigning zero weight on exchange rate movements. The evidence sug-

4 We chose model 5.2 for this analysis since it presented the best fit compared to model 5.1, as indicated in table 3.

Table 5 – Unconditional variances of the estimated and optimal models

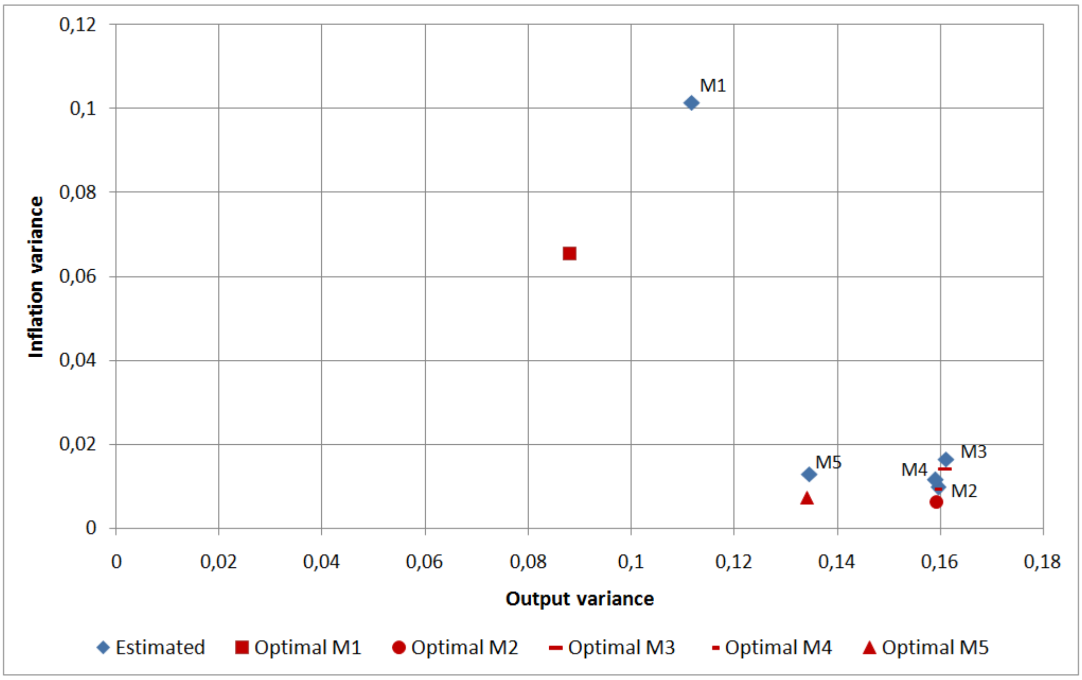
		Output	Inflation	Interest rate	Exchange rate
Model 1	Estimated	0.1117	0.1015	1.0526	16.9414
	$\Lambda_{\Delta e} = 0$	0.0852	0.0589	0.8016	16.9536
	$\Lambda_{\Delta e} = 0.05$	0.0881	0.0655	0.8070	15.5031
Model 2	Estimated	0.1596	0.0098	0.0048	15.7106
	$\Lambda_{\Delta e} = 0$	0.1578	0.0016	0.0037	16.5733
	$\Lambda_{\Delta e} = 0.05$	0.1592	0.0064	0.0057	15.1861
	regime 1	0.1585	0.0052	0.0042	16.0199
Model 3	Estimated	0.1611	0.0165	0.0086	17.5121
	$\Lambda_{\Delta e} = 0$	0.1581	0.0070	0.0073	18.3352
	$\Lambda_{\Delta e} = 0.05$	0.1610	0.0142	0.0102	16.8412
Model 4	Estimated	0.1591	0.0115	0.0054	15.2901
	$\Lambda_{\Delta e} = 0$	0.1579	0.0042	0.0049	15.9294
	$\Lambda_{\Delta e} = 0.05$	0.1590	0.0094	0.0065	14.6426
Model 5.2	Estimated	0.1346	0.0130	0.0058	14.7563
	$\Lambda_{\Delta e} = 0$	0.1332	0.0031	0.0041	15.7970
	$\Lambda_{\Delta e} = 0.05$	**0.1342**	**0.0074**	**0.0059**	**14.5113**
	regime 1	0.1337	0.0078	0.0052	15.2096

Source: Own construction (2017).

Notes: (i) All optimal models were computed supposing a 0.9 weight for interest rate stability and a 0.1 weight for output stability. (ii) $\Lambda_{\Delta e} = 0$ and $\Lambda_{\Delta e} = 0.05$ refer to the optimal rules assigning zero and 0.05 weights respectively on the exchange rate. (iii) regime 1 refers to the rule that applies regime 1 policy (the high inflation targeting regime) to the whole sample.

gests therefore that Brazilian government was indeed concerned with currency stability. This concern is not at odds with a branch of literature focused on the so called *fear of floating*, introduced by Calvo and Reinhart (2000). The authors claim that the apparent reluctance of the emerging markets in letting their currencies fluctuate is related to a lack of credibility in the first place, even at the expense of engaging in procyclical policies, which seems to be the case in Brazil[5]. Calvo and Reinhart (2000) also point out that exchange rate volatility appears to be more damaging to trade in emerging markets, since hedging opportunities are more limited as trade is predominantly invoiced in US$. Additionally, the passthrough from exchange rate swings to inflation is far higher in emerging economies than in developed ones, suggesting that governments concerned with inflation will have a tendency to cap exchange rate movements, a very credible assumption for the Brazilian case.

Figure 7 – Unconditional Variances: Estimated Models and Optimal Rules



Source: Own construction (2017).

Under the assumption that $\Lambda_{\Delta e} = 0.05$, $\Lambda_{\Delta y} = 0.1$ and $\Lambda_R = 0.9$, we compute the coefficients of the optimal rule for the Brazilian economy, and compare it to the coefficients obtained from the estimation of model 5.2 (please refer to table 6). As mentioned earlier, this optimal rule delivers a similar interest rate volatility than the estimated one, and successfully reduces output, inflation and exchange rate volatility, as shown in table 5. Compared to the estimated monetary policy rule, the optimal policy has a higher degree of interest rate smoothing, it is more responsive to fluctuations in

5 If we establish a comparative analysis between the state of the economy (recession or expansion) and the level of government expenditure, it is possible to conclude about a procyclical general pattern of the Brazilian fiscal policy during the last two decades.

Table 6 – Realised and optimal monetary policy rule parameters

Realised policy rule				
Regime	R_{t-1}	$\Delta y_t + z_t$	π_t	Δe_t
High inflation targeting	0.8053	0.0887	0.7065	0.0098
Low inflation targeting	0.7670	0.1107	0.3928	0.0196
Optimal monetary policy rule				
Regime	R_{t-1}	$\Delta y_t + z_t$	π_t	Δe_t
Low volatility of shocks	0.9990	0.0000	1.0687	0.0420
High volatility of shocks	0.9990	0.0000	1.2035	0.0508

Source: Own construction (2017).

Notes: (i) The realised policy rule in the upper panel is obtained from the estimation of model 5.2. (ii) The optimal rule in the lower panel is derived under the assumption that $\Lambda_{\Delta e} = 0.05$, $\Lambda_{\Delta y} = 0.1$ and $\Lambda_R = 0.9$ for model 5.2.

inflation and exchange rates, and less responsive to fluctuations in output across both regimes.

As shown before, it is very likely that Brazilian government is actually concerned with currency movements, at least a little. Taking this aspect into consideration, a rule giving a 0.05 weight to exchange rate volatility was proposed. But then a key question arises: What does this weight truly mean in terms of policy maker's efforts, or more technically, in terms of policy rule coefficients? From table 6, we know that in average the realised exchange rate coefficient was 0.0147, while the optimal rule proposes an exchange rate coefficient of 0.0464, as an average across regimes. At the same time, table 5 indicates that the estimated exchange rate volatility is 14.7563, while for the chosen optimal rule it is only 14.5113. This aspects point to an important conclusion: If policy makers follow the optimal rule, they are permitted to even increase their attention to exchange rates, compared to what was realised according to model 5.2 (more precisely, they can move from a coefficient of 0.0147 to a coefficient of 0.0464) with the guarantee of being at most at the weight $\Lambda_{\Delta e} = 0.05$.

We may compare our results for the estimated monetary policy with those found by Palma and Portugal (2014). The authors estimate the preferences of the Central Bank of Brazil within a DSGE framework (with data from January 2000 to December 2013) and conclude that policy makers were most concerned with inflation stabilization, followed by interest rate smoothing, exchange rate stabilization and output stabilization, in that order. A possible intuition behind our results, that point to a greater weight for interest rate smoothing rather than for inflation stabilization, and in the same time a higher concern with output stabilization compared to exchange rate movements, grounds in the choice of weights assigned to each variable. Interest rate volatility indeed received a relative high weight ($\Lambda_R = 0.9$) compared to the other variables, while output stabilization received a greater weight compared to exchange rates ($\Lambda_{\Delta y} = 0.1$ and $\Lambda_{\Delta e} = 0.05$).

It is worth noting however the interest rate cost of applying the specified optimal rule. If Brazilian government would have followed this rule from 2000:Q2 to 2016:Q3, interest rates would have been 7.13 percentage points higher than it actually was in average. This difference turns out to be considerable, especially for a country with an already large interest rate for international standards. It is very much possible that interest rates of this magnitude would have damaged the sustainable path of national debt, not representing a real option for the government to follow. This aspect reveals an important flaw in our model: It neglects fiscal authority. If a fiscal rule is introduced in the model, such the one proposed by Castro et al. (2015), the government's ultimate goal would be to stabilise (or even reduce) the public sector debt-to-GDP ratio, where the implied fiscal instrument is government spending, also not included in the present model. As a consequence, it is very likely that fiscal restrictions would limit the rise of interest rates by the monetary authority.

6 Concluding Remarks

This paper is first interested in determine possible structural changes that may have occurred in the Brazilian economy in the first years of the century. We do so through the estimation of a small scale open economy DSGE model that incorporates Markov-switching elements. In particular, we find that a model allowing for shifts in the monetary policy rule, as well as shifts in the volatilities of exogenous shocks, can explain quite well the economy dynamics. Monetary policy is therefore identified by a switch from a weaker inflation targeting stance, that prevailed mostly before 2003, to a more aggressive inflation targeting regime from 2003 onwards.

In the second main part of the paper, we move away from the empirical estimation and derive optimal monetary policy rules for Brazil under distinct model specifications. To the best of our knowledge, this is the first attempt to compute optimal rules in the Brazilian case under a MS-DSGE structure. By assigning different weights to output, interest rate and exchange rate stabilization, we show that it is possible to find an optimal rule that is successful in stabilizing output, inflation and exchange rates, whilst keeping interest rates stable. We highlight however that the adoption of this rule could have damaging consequences for the path of national debt, since it implies interest rates being 7.13 percentage points higher in average than the realised. This points out to an important flaw in our model, that is the absence of a fiscal authority. Further research could, for instance, incorporate a fiscal rule in the model, such the one proposed by Castro et al. (2015), forcing the government to also control for the public sector debt-to-GDP ratio. In this case, it is very much likely that the influence of monetary authority in terms of interest rate movements would be quite limited.

Our study also offers interesting insights about the standard deviation dynamics of macroeconomic variables. Two important conclusions show up. First, the volatility dynamics of interest rates and exchange rates oppose each other, in other words, the government must give up interest rate stability to control the volatility of exchange rates, and vice versa. We explore some facts suggesting that the Central Bank of Brazil is actually aware of this trade-off, and mostly adopts currency swaps to control exchange rate movements, instead of using the interest rate instrument. Second, a higher control over interest rates and/or exchange rates seem to create output and inflation instability.

It is worth reminding that the role of fiscal policy has often been neglected when studying the evolution of inflation and output, as firmly pointed out by Bianchi and Ilut (2016). The authors innovate by constructing a model that incorporates monetary/fiscal policy mix changes, in other words they allow for shifts of power between the monetary and fiscal authorities (an active/leading authority versus a passive authority)[1]. They advocate that these models can better explain the path of inflation than a model that only considers the monetary authority as the leading one (when the fiscal side is the leading authority, fiscal imbalances produce long lasting and persistence increases in inflation and the monetary authority loses its ability to control inflation). As a consequence, Bianchi and Ilut (2016) state that a model only considering monetary as the leading authority may induce a misinterpretation concerning inflation stability. If they are correct, this could indicate that our model may be misspecified, in which case our conclusions about inflation stabilization could be flawed. Nevertheless it would be interesting to estimate the model with Brazilian data and verify its suitability before drawing any conclusions, since the model was first created to shed light on U.S. economy dynamics.

Another possible cause of debate relates to equilibrium indeterminacy. As in Lubik and Schorfheide (2007), we rule out the possibility of indeterminacy by restricting our estimations to the region in the parameter space for which a unique equilibrium exists. However if the central bank (inadvertently) implement indeterminate rules, our empirical model may be misspecified.

[1] Leeper (1991) define an active policy as an authority that pays no attention to the state of government debt and is free to set its control variable as it sees fit. A passive authority, on the other hand, responds to government debt shocks. It behaviour is constrained by private optimization and the active authority's actions.

Bibliography

AN, S.; SCHORFHEIDE, F. Bayesian analysis of dsge models. *Econometric Reviews*, v. 26, n. 2-4, p. 113–172, 2007.

ANDRADE, J. P.; DIVINO, J. A. Optimal rules for monetary policy in brazil. *Institute for Applied Economic Research Discussion Paper*, v. 101, Jan 2015.

BARROS, R. et al. Price setting in a variable macroeconomic evironment: Evidence from brazilian cpi. *Working Paper*, 2009.

BIANCHI, F. Regime switches, agents' beliefs, and post-world war ii us macroeconomic dynamics. *Review of Economic Studies*, v. 80, n. 2, p. 463–490, 2013.

BIANCHI, F.; ILUT, C. *Monetary/Fiscal Policy Mix and Agents' Beliefs*. [S.l.]: Duke University, 2016. Manuscript.

CALVO, G. A.; REINHART, C. M. Fear of floating. *NBER Working Paper Series*, v. 7993, Nov 2000.

CASTRO, M. R. D. et al. Samba: Stochastic analytical model with a bayesian approach. *Brazilian Review of Econometrics*, v. 35, n. 2, p. 103–170, 2015.

CHEN, X.; MACDONALD, R. Realized and optimal monetary policy rules in an estimated markov-switching dsge model of the united kingdom. *Journal of Money, Credit and Banking*, v. 44, n. 6, p. 1091–1116, 2012.

COSTA, S. M. A. Structural trends and cycles in a dsge model for brazil. *Central Bank of Brazil Working Paper Series*, v. 434, May 2016.

DAVIG, T.; DOH, T. Monetary policy regime shifts and inflation persistence. *The Review of Economics and Statistics*, v. 96, n. 5, p. 862–875, 2014.

DAVIG, T.; LEEPER, E. M. Generalising the taylor rule principle. *American Economic Review*, v. 97, n. 3, p. 607–635, 2007.

DENNIS, R. Inferring policy objectives from economic outcomes. *Oxford Bulletin of Economics and Statistics*, v. 66, n. S1, p. 735–764, 2004.

FARMER, R. E. A.; WAGGONER, D. F.; ZHA, T. Minimal state variable solutions to markov-switching rational expectations models. *Federal Reserve Bank of Atlanta Working Paper*, v. 23, 2008.

FARMER, R. E. A.; WAGGONER, D. F.; ZHA, T. Understanding markov-switching rational expectations models. *Journal of Economic Theory*, v. 144, p. 1849–1867, 2009.

FARMER, R. E. A.; WAGGONER, D. F.; ZHA, T. Minimal state variable solutions to markov-switching rational expectations models. *Journal of Economic Dynamics and Control*, v. 35, n. 12, 2011.

FERNANDEZ-VILLAVERDE, J.; GUERRON-QUINTANA, P.; RUBIO-RAMIREZ, J. F. Fortune or virtue: Time-variant volatilities versus parameter drifting in us data. *NBER Working Papers*, n. 15928, 2010. National Bureau of Economic Research, Inc.

GALÍ, J.; MONACELLI, T. Monetary policy and exchange rate volatility in a small open economy. *Review of Economic Studies*, v. 72, p. 707–734, 2005.

GEWEKE, J. Using simulation methods for bayesian econometric models: Inference, development, and communication. *Econometric Reviews*, v. 18, n. 1, p. 1–73, 1999.

GONÇALVES, C. C. S.; PORTUGAL, M. S.; ARAGÓN, E. K. Assessing brazilian macroeconomic dynamics using a markov-switching dsge model. *EconomiA*, v. 17, p. 23–42, 2016.

GOUVEA, S. Price rigidity in brazil: Evidence from cpi micro data. *Central Bank of Brazil Working Paper Series*, v. 143, Sep 2007.

JUSTINIANO, A.; PRIMICERI, G. The time varying volatility os macroeconomic fluctuations. *American Economic Review*, v. 98, n. 3, p. 604–641, 2008.

KIM, C.; NELSON, C. *State-Space Models with Regime Switching. Classical and Gibbs-Sampling Approaches with Applications*. Cambridge, Massachusetts: The MIT Press, 2000.

LEEPER, E. M. Equilibria under active and passive monetary and fiscal policies. *Journal of Monetary Economics*, v. 27, p. 129–147, 1991.

LINARDI, F. M. Assessing the fit of a small open-economy dsge model for the brazilian economy. *Central Bank of Brazil Working Paper Series*, v. 424, Apr 2016.

LIU, Z.; WAGGONER, D.; ZHA, T. Sources of the great moderation: A regime-switching dsge approach. *Quantitative Economics*, v. 2, n. 2, p. 251–301, 2011.

LUBIK, T. A.; SCHORFHEIDE, F. Do central banks respond to exchange rate movements? a structural investigation. *Journal of Monetary Economics*, v. 54, n. 4, p. 1069–1087, 2007.

MINELLA, A. et al. Inflation targeting in brazil: Contructing credibility under exchange rate volatility. *Central Bank of Brazil Working Paper Series*, v. 77, Jul 2003.

NEGRO, M. D.; SCHORFHEIDE, F. Inflation dynamics in a small open economy model under inflation targeting: some evidence from chile. *Federal Reserve Bank of New York Staff Reports*, n. 329, p. 1–49, 2008.

OBSTFELD, M.; ROGOFF, K. The six major puzzles in international macroeconomics: Is there a common cause? *NBER Macroeconomics Annual 2000*, v. 15, p. 339–390, 2001.

PALMA, A. A.; PORTUGAL, M. S. Preferences of the central bank of brazil under the inflation targeting regime: Estimation using a dsge model for a small open economy. *Journal of Policy Modeling*, v. 36, n. 5, p. 824–839, 2014.

SALGADO, M. J.; GARCIA, M. G.; MEDEIROS, M. C. Monetary policy during brazil's real plan: Estimating the central bank's reaction function. *Revista Brasileira de Economia*, v. 59, n. 1, p. 61–79, 2005.

SCHORFHEIDE, F. Learning and monetary policy shifts. *Review of Economic Dynamics*, v. 8, n. 2, p. 392–419, 2005.

SILVA, M. E. D.; PORTUGAL, M. S. Inflation targeting in brazil: an empirical evaluation. *Revista de Economía Banco Central del Uruguay*, v. 9, n. 2, p. 85–122, 2002.

SIMS, C. A.; ZHA, T. Were there regime switches in us monetary policy? *American Economic Review*, v. 91, n. 1, p. 54–81, 2006.

SVENSSON, L. E. O.; WILLIAMS, N. Monetary policy with model uncertainty: distribution forecast targeting. *CEPR Discussion Papers*, v. 6331, 2007.