Linear and non-linear flow mode in Pb–Pb collisions at √s_{NN} = 2.76 TeV

ALICE Collaboration

**Abstract**

The second and the third order anisotropic flow, V₂ and V₃, are mostly determined by the corresponding initial spatial anisotropy coefficients, ε₂ and ε₃, in the initial density distribution. In addition to their dependence on the same order initial anisotropy coefficient, higher order anisotropic flow, Vₙ (n > 3), can also have a significant contribution from lower order initial anisotropy coefficients, which leads to mode-coupling effects. In this Letter we investigate the linear and non-linear modes in higher order anisotropic flow Vₙ for n = 4, 5, 6 with the ALICE detector at the Large Hadron Collider. The measurements are done for particles in the pseudorapidity range |η| < 0.8 and the transverse momentum range 0.2 < p_T < 5.0 GeV/c as a function of collision centrality. The results are compared with theoretical calculations and provide important constraints on the initial conditions, including initial spatial geometry and its fluctuations, as well as the ratio of the shear viscosity to entropy density of the produced system.

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1. Introduction

The primary goal of the ultra-relativistic heavy-ion collision programme at the Large Hadron Collider (LHC) is to study the properties of the Quark–Gluon Plasma (QGP), a novel state of strongly interacting matter that is proposed to exist at high temperatures and energy densities [1,2]. Studies of azimuthal correlations of produced particles have contributed significantly to the characterisation of the matter created in heavy-ion collisions [3,4]. Anisotropic flow, which quantifies the anisotropy of the momentum distribution of final state particles, is sensitive to the event-by-event fluctuating initial geometry of the overlap region, together with the transport properties and equation of state of the system [4–7]. The successful description of anisotropic flow results by hydrodynamic calculations suggests that the created medium behaves as a nearly perfect fluid [4,5] with a shear viscosity to entropy density ratio, η/s, close to a conjectured lower bound 1/4π [8]. Anisotropic flow is characterised using a Fourier decomposition of the particle azimuthal distribution in the plane transverse to the beam direction [9,10]:

\[
\frac{dN}{d\phi} \propto 1 + 2 \sum_{n=1}^{\infty} v_n \cos(n(\phi - \Psi_n)),
\]

where N is the number of produced particles, \(\phi\) is the azimuthal angle of the particle and \(\Psi_n\) is the nth order flow symmetry plane.

The nth order (complex) anisotropic flow \(V_n\) is defined as: \(V_n \equiv \frac{v_n}{\langle v_n \rangle} e^{i\phi_n}\), where \(v_n = |V_n|\) is the flow coefficient, and \(\Psi_n\) represents the azimuth of \(V_n\) in momentum space. For non-central heavy-ion collisions, the dominant flow coefficient is \(V_2\), referred to as elliptic flow. Non-vanishing values of higher flow coefficients \(V_3\)–\(V_6\) at the LHC are ascribed primarily to the response of the produced QGP to fluctuations of the initial energy density profile of the colliding nucleons [11–15].

The standard (moment-defined) initial anisotropy coefficients \(\varepsilon_n\) together with their corresponding initial symmetry planes (also called participant planes) \(\Phi_n\) can be calculated from the transverse positions \((r, \phi)\) of the participating nucleons

\[
\varepsilon_n e^{i\phi_n} = \frac{\langle r^{2+2n} e^{i\phi} \rangle}{\langle r^{2n} \rangle} \quad (\text{for } n > 1),
\]

where \(\langle \rangle\) denotes the average over the transverse position of all participating nucleons, \(\phi\) is azimuthal angle, and \(n\) is the order of the coefficient [11,16]. It has been shown in [17,18] that \(V_2\) and \(V_3\) are mostly determined with the same order initial spatial anisotropy coefficients \(\varepsilon_2\) and \(\varepsilon_3\), respectively. Considering that \(\eta/s\) reduces the hydrodynamic response of \(v_n\) to \(\varepsilon_n\), it was proposed in [18–21] that \(\varepsilon_n/\varepsilon_2\) (for \(n = 2, 3\)) could be a direct probe to quantitatively constrain the \(\eta/s\) of the QGP in hydrodynamic calculations. However, \(\varepsilon_n\) cannot be determined experimentally. Instead, they are obtained from various theoretical models, resulting in large uncertainties in the estimated \(\eta/s\) derived indirectly from \(V_2\) and \(V_3\) measurements [17,19]. On the other hand, higher order anisotropic flow \(V_n\) with \(n > 3\) probe smaller spatial scales and...
thus are more sensitive to $\eta/s$ than $V_2$ and $V_3$ due to more pronounced viscous corrections [16,22]. Thus, the study of the full set of flow coefficients is expected to constrain both $\epsilon_n$ and $\eta/s$ simultaneously. However, it was realised later that $V_n$ with $n > 3$ is not linearly correlated with the corresponding $\epsilon_n$ [16,22,23], which makes the extraction of $\eta/s$ from measurements of higher order flow coefficients less straightforward. In addition to the study of flow coefficients, the results of correlations between different order anisotropic flow angles and amplitudes shed light on both the early stage dynamics and the transport properties of the created QGP [24–32]. In particular, the characteristic pattern of flow symmetry plane correlations (also known as angular correlations of flow-vectors) observed in experiments is reproduced quantitatively by theoretical calculations [29–33]. However, the correlations between flow coefficients (also known as amplitude correlations of flow-vectors), investigated using symmetric cumulants, provide stricter constraints on initial conditions and $\eta/s$ than the individual $\epsilon_n$ measurements [24–28,31,32]. It is a challenge for current theoretical models to provide quantitative descriptions of the correlations between different order flow coefficients.

As discussed above, it is known that the lower order anisotropic flow $V_n$ ($n = 2, 3$) is largely determined by a linear response of the system to the corresponding $\epsilon_n$ (except in peripheral collisions). Higher order anisotropic flow $V_n$ with $n > 3$ have contributions not only from the linear response of the system to $\epsilon_n$, but also contributions proportional to the product of $\epsilon_2$ and/or $\epsilon_3$. These contributions are usually called non-linear response [25,34] in higher order anisotropic flow. For a single event, $V_n$ with $n = 4, 5$ and 6 can be decomposed into the so-called linear and the non-linear contributions, according to

$$V_4 = V_4^{NL} + V_4 = \chi_{4,22}(V_2^2)^2 + V_4^L,$$

$$V_5 = V_5^{NL} + V_5 = \chi_{5,32}V_2V_3 + V_5^L,$$

$$V_6 = V_6^{NL} + V_6 = \chi_{6,22}(V_2^3) + \chi_{6,33}(V_3^2) + \chi_{6,42}V_2V_4^L + V_6^L. \quad (5)$$

Here $\chi_{n,m:k}$ is a new observable called the non-linear mode coefficient [34] and $V_n^{NL}$ represents the non-linear mode which has contributions from modes with lower order anisotropic coefficients. The $V_2^L$ term represents the linear mode, which was naively expected from the linear response of the system to the same order $\epsilon_n$. However, a recent hydrodynamic calculation showed that $V_2^L$ is not driven by the linear response to the standardsly moment-defined $\epsilon_2$ introduced in Eq. (2), but the corresponding cumulant-defined anisotropy coefficient $\epsilon_2^{c} [30,35]$. For example, $V_2^L$ is expected to be driven by the 4th-order cumulant-defined anisotropy coefficient and its corresponding initial anisotropy plane which can be calculated as

$$\epsilon_2^{c} = -\frac{(z^4) - 3(z^2)^2}{\langle z^4 \rangle} = \epsilon_2^{a4} + \frac{3\langle r^2 \rangle^2}{\langle r^4 \rangle} \epsilon_2^{2a4},$$

where $z = re^{i\phi}$. The calculations for other order anisotropy coefficients and their corresponding initial symmetry planes can be found in [30,35]. If the non-linear and linear modes of higher order anisotropic flow, $V_4^{NL}$ and $V_4^L$, are uncorrelated (e.g. $V_4^L$ is perpendicular to $V_4^{NL}$), they can be isolated. One of the proposed approaches to validate the assumption that $V_n^{NL}$ and $V_n^L$ are uncorrelated is testing the following relations [25]:

$$\frac{\langle V_4 (V_2^2) v_2 \rangle}{\langle V_4^2 \rangle} = \frac{\langle v_6 \rangle}{\langle v_2^4 \rangle \langle v_2 \rangle};$$

$$\frac{\langle V_5 (V_2^2) v_2 \rangle}{\langle V_5 \rangle} = \frac{\langle v_4^4 \rangle}{\langle v_2^2 \rangle \langle v_2^2 \rangle};$$

$$\frac{\langle V_5 (V_2^2) v_2 \rangle}{\langle V_5 \rangle} = \frac{\langle v_4^4 \rangle}{\langle v_2^2 \rangle \langle v_2^2 \rangle}. \quad (8)$$

If the above relations are valid, one could combine the analyses of higher order anisotropic flow with respect to their corresponding symmetry planes and to the planes of lower order anisotropic flow $V_2$ or $V_3$ to eliminate the uncertainty from initial state assumptions and extract $\eta/s$ with better precision [34].

In this Letter, the linear and non-linear modes in higher order anisotropic flow generation are studied in Pb–Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV with the ALICE detector. The main observables are introduced in Section 2 and the experimental setup is described in Section 3. Section 4 presents the study of the systematic uncertainties of the above mentioned observables. The results and their discussion are provided in Section 5. Section 6 contains the summary and conclusions.

2. Observables and analysis methods

Ideally, the flow coefficient $v_n$ can be measured via the azimuthal correlations of emitted particles with respect to the symmetry plane $\Psi_n$ as $v_n = \langle \cos(n\phi - \Psi_n) \rangle$. Since $\Psi_n$ is unknown experimentally, the simplest approach to obtain $v_n$ is using 2-particle correlations:

$$v_n[2] = \langle \langle \cos(n\phi_1 - \Psi_n) \rangle \rangle^{1/2} = \left( \frac{v_n^2}{v_2} \right)^{1/2}. \quad (9)$$

Here $\langle \langle \rangle \rangle$ denotes the average over all particles in a single event and then an average of over all events, $\langle \rangle$ indicates the event average of over all events, and $\phi_i$ represents the azimuthal angle of the i-th particle. The analysed events are divided into sub-events A and B, separated by a pseudorapidity gap, to suppress non-flow effects. The latter are the azimuthal correlations not associated to the common symmetry plane $\Psi_4$, such as jets and resonance decays. Thus, we modify Eq. (9) to

$$v_n[2] = \langle \langle \cos(n\phi_1^A - n\phi_2^B) \rangle \rangle^{1/2} = \left( \frac{v_n^2}{v_2^A} \right)^{1/2}. \quad (10)$$

Here $\phi_1^A$ and $\phi_2^B$ are selected from subevent A and B, respectively.

Before introducing observables related to the linear and non-linear modes in higher order anisotropic flow, it is crucial to verify whether Eqs. (7)–(8) are applicable. The left and right hand sides of Eq. (7) are obtained by constructing suitable multi-particle correlations [34]:

$$\langle V_4 (V_2^2) v_2^A \rangle = \frac{\langle V_4 \rangle^2 \langle V_2^2 \rangle}{\langle V_2 \rangle} \langle v_2^A \rangle$$

$$= \frac{\langle \cos(4\phi_1^A + 2\phi_2^A - 2\phi_3^A - 2\phi_4^B - 2\phi_5^B) \rangle}{\langle \cos(4\phi_1^A - 2\phi_2^B - 2\phi_3^B) \rangle} \langle \cos(2\phi_1^A - 2\phi_2^B) \rangle \langle \cos(2\phi_1^A - 2\phi_2^B) \rangle, \quad (11)$$

Similarly, we can validate Eq. (8) by calculating both sides with [34]:

$$\langle V_5 (V_2^2) v_2^A \rangle = \frac{\langle V_5 \rangle^2 \langle V_2^2 \rangle}{\langle V_2 \rangle} \langle v_2^A \rangle$$

$$= \frac{\langle \cos(5\phi_1^A + 2\phi_2^A - 3\phi_3^A - 2\phi_4^B - 2\phi_5^B) \rangle}{\langle \cos(5\phi_1^A - 3\phi_2^B - 2\phi_3^B) \rangle} \langle \cos(2\phi_1^A - 2\phi_2^B) \rangle. \quad (13)$$
\[
\frac{v^A_{4,22}}{v^B_{4,22}} = \frac{\langle \cos(4\psi^A_4 + 2\psi^A_2 - 2\psi^B_4) \rangle}{\langle \cos(2\psi^A_4 + 2\psi^A_2 - 2\psi^B_4) \rangle}.
\]

The magnitude of \(V_{mk}^A\) was denoted as \(v_n[\psi_m]\) (here \(\psi_m\) is the lower order flow symmetry plane and \(m = 2, 3\) in [34]). The notation \(v_{n,m,k}\) where \(n\) specifies the order of the flow term while \(m\) and \(k\) etc. denote the contributing lower order flow symmetry planes, is used in this Letter. If the linear and non-linear modes are independent, then the non-linear mode in higher order anisotropic flow can be analysed by correlating \(V_n\) with \(\psi_2\) or/and \(\psi_3\) [34]. For sub-event A we can define:

\[
v^A_{4,22} = \sqrt{\langle \cos(4\psi^A_4 + 2\psi^A_2 - 2\psi^B_4) \rangle},
\]

\[
v^A_{5,32} = \sqrt{\langle \cos(5\psi^A_5 + 3\psi^A_3 - 2\psi^B_5) \rangle},
\]

\[
v^A_{6,222} = \sqrt{\langle \cos(6\psi^A_6 + 4\psi^A_4 - 3\psi^B_6) \rangle},
\]

\[
v^A_{6,33} = \sqrt{\langle \cos(6\psi^A_6 + 3\psi^A_3 - 3\psi^B_6) \rangle}.
\]

Similarly, one can obtain \(v^B_{n,m,k}\) for sub-event B. The average of \(v^A_{n,m,k}\) and \(v^B_{n,m,k}\), denoted as \(v_{n,m,k}\), quantifies the magnitude of the non-linear mode in higher order anisotropic flow, which can be written as [36]:

\[
v^A_{4,22} = \frac{\langle v^A_4 v^A_2 \cos(4\psi^A_4 - 4\psi^B_2) \rangle}{\langle v^A_2 \rangle} \approx \langle v^A_4 \cos(4\psi^A_4 - 4\psi^B_2) \rangle,
\]

\[
v^A_{5,32} = \frac{\langle v^A_5 v^A_3 \cos(5\psi^A_5 - 3\psi^A_3 - 2\psi^B_5) \rangle}{\langle v^A_3 \rangle} \approx \langle v^A_5 \cos(5\psi^A_5 - 3\psi^A_3 - 2\psi^B_5) \rangle,
\]

\[
v^A_{6,222} = \frac{\langle v^A_6 v^A_2 \cos(6\psi^A_6 - 6\psi^B_2) \rangle}{\langle v^A_2 \rangle} \approx \langle v^A_6 \cos(6\psi^A_6 - 6\psi^B_2) \rangle,
\]

\[
v^A_{6,33} = \frac{\langle v^A_6 v^A_3 \cos(6\psi^A_6 - 6\psi^B_3) \rangle}{\langle v^A_3 \rangle} \approx \langle v^A_6 \cos(6\psi^A_6 - 6\psi^B_3) \rangle.
\]

The approximation is valid if the correlation between lower \((n = 2, 3)\) and higher \((n > 3)\) flow coefficients is weak.

3. Experimental setup and data analysis

The data samples analysed in this Letter were recorded by ALICE during the Pb–Pb runs of the LHC at a centre-of-mass energy of \(\sqrt{s_{NN}} = 2.76\) TeV in 2010. Minimum bias Pb–Pb collision events were triggered by the coincidence of signals in the V0 detector [37,38], with an efficiency of 98.4% of the hadronic cross section [39]. The V0 detector is composed of two arrays of scintillator counters, V0-A and V0-C, which cover the pseudorapidity ranges \(2.8 < \eta < 5.1\) and \(-3.7 < \eta < -1.7\), respectively. Beam background events were rejected using the timing information from the V0 and the Zero Degree Calorimeter (ZDC) [37] detectors and by correlating reconstructed clusters and tracklets with the Silicon Pixel Detectors (SPD). The fraction of pile-up events in the data sample is found to be negligible after applying dedicated pile-up removal criteria [40]. Only events with a reconstructed primary vertex within \(\pm 10\) cm from the nominal interaction point along the beam direction were used in this analysis. The primary vertex was estimated using tracks reconstructed by the Inner Tracking System (ITS) [37,41] and Time Projection Chamber (TPC) [37,42]. The collision centrality was determined from the measured V0 amplitude and centrality intervals were defined following the procedure described in [39]. About 13 million Pb–Pb events passed all of the event selection criteria.
Tracks reconstructed using the combined information from the TPC and ITS are used in this analysis. This combination ensures a high detection efficiency, optimum momentum resolution, and a minimum contribution from photon conversions and secondary charged particles produced either in the detector material or from weak decays. To reduce the contributions from secondaries, charged tracks were required to have a distance of closest approach to the primary vertex in the longitudinal (z) direction and transverse (xy) plane smaller than 3.2 cm and 2.4 cm, respectively. Additionally, tracks were required to have at least 70 TPC space points out of the maximum 159. The average chi^{2} per degree of freedom of the track fit to the TPC space points was required to be below 2. In this study, tracks were selected in the pseudorapidity range |\eta| < 0.8 and the transverse momentum range 0.2 < p_{T} < 5.0 GeV/c.

4. Systematic uncertainties

Numerous sources of systematic uncertainty were investigated by varying the event and track selection as well as the uncertainty associated with the possible remaining non-flow effects in the analysis. The variation of the results with the collision centrality is calculated by alternatively using the TPC or SPD to estimate the event multiplicity and is found to be less than 3\% for all observables. Results with opposite polarities of the magnetic field within the ALICE detector and with narrowing the nominal ±10 cm range of the reconstructed vertex along the beam direction from the centre of the ALICE detector to 9, 8 and 7 cm do not show a difference of more than 2\% compared to the default selection criteria for various measurements. The contributions from pile-up events to the final systematic uncertainty are found to be negligible. The sensitivity to the track selection criteria was explored by varying the number of TPC space points and by using tracks reconstructed in the TPC alone. Varying the number of TPC space points from 70 to 80, 90 and 100 out of a possible 158, results in a 1–3\% variation of the results for \vn, within 1.5\% for \rho_{n}mk and \eta_{m}mk. Using TPC-only tracks leads to a difference of less than 14\%, 17\% and 8\% for \vn, \rho_{n}mk and \eta_{m}mk respectively. Both effects were included in the evaluation of the systematic uncertainty. Several different approaches have been applied to estimate the effects of non-flow. These include the investigation of multi-particle correlations with various |\Delta\eta| gaps, the application of the like-sign technique which correlates two particles with either all positive or negative charges and suppress such non-flow as due to resonance decays, as well as the calculations using HIJING Monte Carlo simulations [43], which do not include anisotropic flow. It was found that the possible remaining non-flow effects are less than 10.5\%, 11\% and 7\% for \vn, \rho_{n}mk and \eta_{m}mk respectively. They are taken into account in the final systematic uncertainty. The systematic uncertainties evaluated for each source mentioned above were added in quadrature to obtain the total systematic uncertainty of the measurements.

5. Results and discussion

As discussed in Sec. 2, one can validate the assumption that linear and non-linear modes in higher order anisotropic flow are uncorrelated via Eqs. (7) and (8). These have been tested in a Multi-Phase Transport (AMPT) model [25] as well as in the hydrodynamic calculations [44]. Good agreement between left- and right-hand sides of Eqs. (7) and (8) is found for all centrality classes, independent of the initial conditions and the ideal or viscous fluid dynamics used in the calculations. Thus, it is crucial to check these equalities in data, to further confirm the assumption that the two components are uncorrelated and can be isolated independently. Fig. 1 confirms that the agreement observed in theoretical calculations is also present in the data despite small deviations found in central collisions when testing Eqs. (8). Their centrality dependency are similar as the previous theoretical predictions [25,44]. The measurements support the assumption that higher order anisotropic flow \vn_{n} (n > 3) can be modeled as the sum of independent linear and non-linear modes.

The magnitudes of linear and non-linear modes in higher order anisotropic flow are reported as a function of collision centrality in Fig. 2. In this Letter, sub-events A and B are built in the pseudorapidity ranges −0.8 < \eta < −0.4 and 0.4 < \eta < 0.8, respectively, which results in a pseudorapidity gap of |\Delta\eta| > 0.8 for all presented measurements. It can be seen that the linear mode \vn_{4} depends weakly on centrality and is the larger contribution to \vn_{4}(2) for the centrality range 0−30\%. The non-linear mode, \vn_{4,22}, increases monotonically as the centrality decreases and saturates around centrality percentile 50\%, becoming the dominant source for centrality intervals above 40\%. Similar trends of centrality dependence have been observed for \vn_{5}, although \vn_{5,32} becomes the dominant contribution in centrality percentile above 30\%. Only two non-linear components \vn_{6,22} and \vn_{6,33} are discussed for \vn_{6}. It is shown in Fig. 2 (right) that \vn_{6,22} increases monotonically as the centrality decreases to centrality 50\%, while \vn_{6,33} has a weaker centrality dependence compared to \vn_{6,22}.

The linear and non-linear modes in higher order anisotropic flow were investigated by the ATLAS Collaboration [26] using a different approach based on “Event Shape Engineering” [45]. With this method one can utilise large fluctuations in the initial geometry of the system to select events corresponding to a specific initial shape. The conclusion is qualitatively consistent with what is reported here, although a direct comparison is not possible due to the different kinematic cuts (especially the integrated \pt range) used in the two measurements. The higher order anisotropic flow induced by lower order anisotropic flow were also measured using the event-plane method at the LHC [14,46]. However, the measurements of the non-linear mode presented in this Letter are based on the multi-particle correlations method with a |\Delta\eta| gap. This method makes it easier to measure an observable like \vn_{3,2}, which is less straightforward to define using the event plane method [14,46]. In addition, as pointed out in [25,34,47], this new multi-particle correlations method should strongly suppress short-range (in pseudorapidity) non-flow effects and provides a robust measurement without any dependence on the experimental acceptance. The measurements are compared to recent hadronic flow measurements from a hybrid IP-Glasma + MUSIC + UrQMD model [48], in which realistic event-by-event initial conditions are used and the hydrodynamic evolution takes into account both shear and bulk viscosity. It is shown that this hydrodynamic calculation could describe quantitatively the total magnitudes of \vn_{4} and \vn_{6} as well as the magnitudes of their linear and non-linear modes, while it slightly overestimates the results for \vn_{5}.

The centrality dependence of \rho_{n}mk, which quantifies the angular correlations between different order flow symmetry planes, is
presented in Fig. 3. It is observed that $\rho_{4,22}$ increases from central to peripheral collisions, which suggests that the correlations between $\Psi_2$ and $\Psi_4$ are stronger in peripheral than in central collisions. It implies that $V_4^{NL}$ tends to align with $V_4$ in more peripheral collisions. The results of $\rho_{5,33}$, which measures the correlation of $\Psi_3$ and $\Psi_5$, do not exhibit a strong centrality dependence within the statistical uncertainties. As mentioned above, $\rho_{4,22}$ and $\rho_{6,33}$ are similar to the previous “event-plane correlation” measurements $\langle \cos(4\Phi_4 - 4\Phi_2) \rangle_w$ and $\langle \cos(6\Phi_6 - 6\Phi_3) \rangle_w$ in [33]. The comparisons between measurements of these observables are also presented in Fig. 3. The results are compatible with each other, despite the different kinematic ranges used by ATLAS and this analysis. It should be also noted that the measurements of $\rho_{n,mk}$ presented in this Letter show the symmetry plane correlations at mid-pseudorapidity $|\eta| < 0.8$ while ATLAS measured the symmetry plane correlations using $-0.5 < \eta < 0.5$ and $0.5 < \eta < 4.8$ for two-plane correlations, and using $-2.7 < \eta < -0.5$, $0.5 < \eta < 2.7$ and $3.3 < |\eta| < 4.8$ for 3-plane correlations [33]. Previous investigations suggest that there might be $\eta$-dependent fluctuations of the flow symmetry plane and the flow magnitude [49,50]. As a consequence, one might expect a difference when measuring the correlations of flow symmetry planes from different pseudorapidity regions. However, Fig. 3 shows good agreement between the ALICE and ATLAS measurements. Therefore, no obvious indication that the flow symmetry plane varies with $\eta$ can be deduced from this comparison. It is noticeable in Fig. 3 that the $\rho_{5,32}$ measurement seems slightly higher than the $\langle \cos(5\Phi_5 - 3\Phi_3 - 2\Phi_2) \rangle_w$ measurement. This is mainly due to a small difference between the definitions of the observable as introduced in Sec. 2: the term $\langle v_2^3 v_3^{1/2} \rangle$ is used in $\rho_{5,32}$, whereas $\langle v_2^{1/2} v_3^{3/2} \rangle$ is used in the “event-plane correlations” [33]. Considering the known anti-correlations between $v_2$ and $v_3$ [26,27], $\langle v_2^{1/2} v_3^{3/2} \rangle$ could be up to 10% lower than $\langle v_2^{1/2} v_3^{1/2} \rangle$ depending on the centrality class [27], leading to a slightly larger $\rho_{5,32}$ than $\langle \cos(5\Phi_5 - 3\Phi_3 - 2\Phi_2) \rangle_w$ from ATLAS.

It has been observed in hydrodynamic and transport model calculations that the symmetry plane correlations, e.g. correlations of second and fourth order symmetry planes, change sign during the system evolution [29,30,32]. The measured flow symmetry plane correlations could be nicely explained by the combination of contributions from linear and non-linear modes in higher order anisotropic flow [30]. This indicates that the flow symmetry plane correlation $\rho_{n,mk}$ carries important information about the dynamic evolution of the created system. In addition, the model calculations suggest that stronger initial symmetry plane correlations are reflected in stronger correlations between the flow symmetry planes in the final state [29,32]. And a larger value of $\eta/s$ of the QGP leads to weaker flow symmetry plane correlations in the final state. As pointed out in [29], the hydrodynamic calculations from VISH2 + 1 using Monte Carlo Glauber (MC-Gib) or Monte Carlo Kharzeev–Levin–Nardi (MC-KLN) initial conditions can only describe qualitatively the trends of the centrality dependence of the event-plane correlation measurements by ATLAS. It is therefore expected that these hydrodynamic calculations cannot describe the presented ALICE measurements, which are compatible with the ATLAS event-plane correlation measurements. Fig. 3 shows that the hydrodynamic calculations from IP-Glasma + MUSIC + UrQMD [48] reproduce nicely the measurements of symmetry plane correlations $\rho_{n,mk}$. The measurements of $\rho_{n,mk}$ presented in this Letter, together with the comparison to hydrodynamic calculations, should place constraints on the initial conditions and $\eta/s$ of the QGP in hydrodynamic calculations.

Fig. 4 presents the measurements of the non-linear mode coefficients as a function of collision centrality. It is observed that $\chi_{4,22}$ and $X_{6,222}$ decrease modestly from central to mid-central collisions, and stay almost constant from mid-central to more peripheral collisions. For $X_{5,32}$ and $X_{6,33}$ strong centrality dependence is not observed either. Thus, the dramatic increase of $v_{n,mk}$ shown in Fig. 2 appears to be mainly due to the increase of $v_2$ and/or $v_3$ from central to peripheral collisions and not the increase of the non-linear mode coefficient. It is also noteworthy that the relationship of $\chi_{4,22} \sim X_{6,32} \approx X_{6,33}$ is approximately valid, as predicted by hydrodynamic calculations [34]. The comparisons to event-by-event viscous hydrodynamic calculations from VISH2 + 1 [44] and from IP-Glasma + MUSIC + UrQMD [48] are also presented in Fig. 4. VISH2 + 1 shows that $\chi_{4,22}$ calculations with MC-Gib initial conditions are larger than those with MC-KLN initial conditions, i.e. $\chi_{4,22}$ depends on the initial conditions. At the same time,
the curves with different $\eta/s$ values for VISH2 + 1 are very similar, indicating that $\chi_{4,22}$ is insensitive to $\eta/s$. The measurements favour IP-Glasma and MC-KLN over MC-Gib initial conditions regardless of $\eta/s$. This suggests that the $\chi_{4,22}$ measurement can be used to constrain the initial conditions, with less concern of the setting of $\eta/s(T)$ in hydrodynamic calculations than previous flow observables.

It was predicted that $\chi_{6,222} < \chi_{6,33}$ based on the ideal hydrodynamic calculation using smooth initial Gaussian density profiles [34], whereas an opposite prediction was obtained in the ideal hydrodynamic calculation evolving genuinely bumpy initial conditions obtained from a Monte Carlo sampling of the initial nucleon positions in the colliding nuclei [44]. It is seen in Fig. 4 that $\chi_{6,222} \sim \chi_{6,33}$ within the current uncertainties. The data are not able to discriminate the different predictions in [34] and [44]. Hydrodynamic calculations using MC-KLN and IP-Glasma initial conditions give better descriptions of $\chi_{6,222}$, compared to the ones using MC-Gib initial conditions. For $\chi_{5,32}$ none of the combinations of initial conditions and $\eta/s$ in the hydrodynamic calculations agree quantitatively with data. This might be due to the current difficulty of describing the anti-correlations between $v_2$ and $v_3$ in hydrodynamic calculations [27,51], which are involved in the calculation of $\chi_{5,32}$. Furthermore, VISH2 + 1 calculations show that $\chi_{5,32}$ and $\chi_{6,33}$ are very weakly sensitive to the initial conditions, but decrease as $\eta/s$ increases. The investigation with the VISH2 + 1 hydrodynamic framework shows that the sensitivity of $\chi_{5,32}$ and $\chi_{6,33}$ to $\eta/s$ is not due to sensitivity to shear viscous effects during the build-up of hydrodynamic flow. Instead, as found in [44], it is due to the $\eta/s$ at freeze-out. The measurements of $\chi_{5,32}$ and $\chi_{6,33}$ do not further constrain the $\eta/s$ during system evolution, however, they provide unique information on $\eta/s$ at freeze-out which was poorly known and cannot be obtained from other anisotropic flow related observables. Further improvement of model calculations on correlations between different order flow coefficients are necessary to better understand the comparison of $\chi_{5,32}$ obtained from data and hydrodynamic calculations. The $\chi_{6,33}$ results are consistent with hydrodynamic calculations from VISH2 + 1 with MC-KLN initial conditions using $\eta/s = 0.08$ and IP-Glasma + MUSIC + UrQMD with a $\eta/s = 0.095$. It is shown that $\chi_{5,32}$ and $\chi_{6,33}$ have a weak centrality dependence if a smaller $\eta/s$ is used in the hydrodynamic calculations. Such a weak centrality dependence of $\chi_{5,32}$ and $\chi_{6,33}$ is observed in data as well. The measurements presented here suggest a small $\eta/s$ value at freeze-out, which can be useful to constrain the temperature dependence of the shear viscosity over entropy density ratio, $\eta/s(T)$, in the development of hydrodynamic frameworks. These results suggest that future tuning of the parameterisations of $\eta/s(T)$ in hydrodynamic frameworks using the presented measurements is necessary.

6. Summary

The linear and non-linear modes in higher order anisotropic flow generation were studied with 2- and multi-particle correlations in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. The results presented in this Letter show that higher order anisotropic flow can be isolated into two independent contributions: the component that arises from a non-linear response of the system to the lower order initial anisotropy coefficients $\varepsilon_2$ and/or $\varepsilon_3$, and a linear mode which is driven by linear response of the system to the same order cumulant-defined anisotropy coefficient. A weak centrality dependence is observed for the contributions from linear mode whereas the contributions from non-linear mode increase dramatically as the collision centrality decreases, and it becomes the dominant source in higher order anisotropic flow in mid-central to peripheral collisions. It is shown that this is mainly due to the increase of lower order flow coefficients $v_2$ and $v_3$. The correlations between different flow symmetry planes are measured. The results are compatible with the previous “event-plane correlation” measurements, and can be quantitatively described by calculations using the IP-Glasma + MUSIC + UrQMD framework. Furthermore, non-linear mode coefficients, which have different sensitivities to the shear viscosity over entropy density ratio $\eta/s$ and the initial conditions, are presented in this Letter. Comparisons to hydrodynamic calculations suggest that the data is described better by hydrodynamic calculations with smaller $\eta/s$. In addition, the MC-Gib initial condition is disfavoured by the presented results.

Measurements of linear and non-linear modes in higher order anisotropic flow and their comparison to hydrodynamic calculations provide more precise constraints on the initial conditions and temperature dependence of $\eta/s$. These results could also offer new insights into the geometry of the fluctuating initial state and into the dynamical evolution of the strongly interacting medium produced in relativistic heavy-ion collisions at the LHC.

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[31] H. Niemi, K.J. Eskola, R. Paatelainen, Event-by-event fluctuations in a perturba-
tive QCD + saturation + hydrodynamics model: determining QCD matter shear 
viscosity in ultrarelativistic heavy-ion collisions, Phys. Rev. C 93 (2) (2016) 

[32] Y. Zhou, K. Xiao, Z. Fang, F. Liu, R. Steinelis, Anisotropic distributions in a mul-
[nucl-ex].

in $\sqrt{s_{NN}} = 2.76$ TeV lead-lead collisions with the ATLAS detector, Phys. Rev. C 

[34] L. Yan, J.-Y. Ollitrault, v$_{2}$, v$_{3}$, v$_{4}$, v$_{5}$, v$_{6}$, nonlinear hydrodynamic response versus 

[35] J. Qian, U. Heinz, R. He, L. Shu, Differential flow correlations in relativis-
[nucl-th].


[37] ALICE Collaboration, K. Aamodt, et al., The ALICE experiment at the CERN LHC, 

[38] ALICE Collaboration, E. Abbas, et al., Performance of the ALICE VZERO system, 

[39] ALICE Collaboration, B. Abelev, et al., Centrality determination of Pb–Pb col-
lisions at $\sqrt{s_{NN}} = 2.76$ TeV with ALICE, Phys. Rev. C 88 (4) (2013) 044909, 

[40] ALICE Collaboration, B.B. Abelev, et al., Performance of the ALICE experiment at 
ex].

[41] ALICE Collaboration, K. Aamodt, et al., Alignment of the ALICE inner tracking 
[physics.ins-det].

[42] J. Aline, et al., The ALICE TPC, a large 3-dimensional tracking device with 

[43] M. Gyulassy, X.-N. Wang, HIJING 1.0: a Monte Carlo program for parton and 
particle production in high-energy hadronic and nuclear collisions, Comput. 

[44] J. Qian, U.W. Heinz, J. Liu, Mode-coupling effects in anisotropic flow in heavy-

[45] J. Schukraft, A. Timmins, S.A. Voloshin, Ultra-relativistic nuclear collisions: 
[nucl-th].

[46] ALICE Collaboration, B. Abelev, et al., Anisotropic flow of charged hadrons, 
pi–(anti-)protons measured at high transverse momentum in Pb–Pb col-
[nucl-th].

044907, arXiv:1209.2232 [nucl-exp].

predictions for Pb + Pb collisions at 5.02 A TeV, arXiv:1609.02958 [hep-ph].

[49] CMS Collaboration, V. Khachatryan, et al., Evidence for transverse momentum 
and pseudorapidity dependent event plane fluctuations in PbPb and pPb colli-

[50] L.-G. Pang, G.-Y. Qin, V. Roy, X.-N. Wang, G.-L. Ma, Longitudinal decorrelation 
of anisotropic flows in heavy-ion collisions at the CERN Large Hadron Collider, 

[51] X. Zhu, Y. Zou, H. Xu, H. Song, Correlations of flow harmonics in 2.76 A TeV 
119 Technical University of Košice, Košice, Slovakia
120 Technical University of Split FESB, Split, Croatia
121 The Henryk Niewodniczanski Institute of Nuclear Physics, Polish Academy of Sciences, Cracow, Poland
122 The University of Texas at Austin, Physics Department, Austin, TX, United States
123 Technical University of Košice, Košice, Slovakia
124 Universidad Autónoma de Sinaloa, Culiacán, Mexico
125 Universidad de São Paulo (USP), São Paulo, Brazil
126 Universidad de São Paulo (USP), São Paulo, Brazil
127 University of the Witwatersrand, Johannesburg, South Africa
128 University of Tokyo, Tokyo, Japan
129 University of Tsukuba, Tsukuba, Japan
130 University of the Witwatersrand, Johannesburg, South Africa
131 Université de Lyon, Université Lyon 1, CNRS/IN2P3, IPN-Lyon, Villeurbanne, Lyon, France
132 University of the Witwatersrand, Johannesburg, South Africa
133 University of the Witwatersrand, Johannesburg, South Africa
134 University of the Witwatersrand, Johannesburg, South Africa
135 University of the Witwatersrand, Johannesburg, South Africa
136 Università degli Studi di Pavia, Pavia, Italy
137 Università di Brescia, Brescia, Italy
138 Università di Brescia, Brescia, Italy
139 V. Fock Institute for Physics, St. Petersburg State University, St. Petersburg, Russia
140 Warsaw University of Technology, Warsaw, Poland
141 Wayne State University, Detroit, MI, United States
142 Wigner Research Centre for Physics, Hungarian Academy of Sciences, Budapest, Hungary
143 Yale University, New Haven, CT, United States
144 Yonsei University, Seoul, South Korea
145 Zentrum für Technologietransfer und Telekommunikation (ZTT), Fachhochschule Worms, Worms, Germany

i Deceased.
ii Also at: Dipartimento DET del Politecnico di Torino, Turin, Italy.
iii Also at: Georgia State University, Atlanta, Georgia, United States.
iv Also at: M.V. Lomonosov Moscow State University, D.V. Skobeltsyn Institute of Nuclear Physics, Moscow, Russia.
v Also at: Department of Applied Physics, Aligarh Muslim University, Aligarh, India.
vi Also at: Institute of Theoretical Physics, University of Wroclaw, Poland.