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FORECASTING BRAZILIAN INFLATION WITH SINGULAR SPECTRUM ANALYSIS

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Dissertação submetida ao Programa de Pós-Graduação em Economia da Faculdade de Ciências Econômicas da UFRGS, como requisito parcial para obtenção do título de Mestre em Economia, com ênfase em Economia Aplicada.

Orientador: Prof. Dr. Flávio Augusto Ziegelmann

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RESUMO

O objetivo deste artigo é avaliar previsões da inflação brasileira a partir do método não-paramétrico de Análise Espectral Singular (SSA). O exercício de previsão utiliza o esquema de janelas rolantes. Diferentes estratégias de combinação de previsões e procedimentos de seleção de variáveis para métodos multivariados foram contempladas. Para robustez, cinco horizontes de previsão foram utilizados. A avaliação das previsões considera diversos procedimentos e medidas estatísticas para oferecer conclusões confiáveis, incluindo razões de erro quadrático médio de previsão, teste de igualdade condicional de habilidade preditiva, diferenças de erro quadrático médio de previsão cumulativas e *Model Confidence Set*. Os resultados mostram que o SSA supera consistentemente os métodos competidores. Quase todas as previsões SSA superam os competidores em termos de erro quadrático médio de previsão, e em vários casos, com significância estatística. A análise da porção fora da amostra indica superioridade em performance relativa do SSA, especialmente no período de choque nos preços de energia elétrica. Adicionalmente, métodos SSA sempre foram incluídos no conjunto superior do *Model Confidence Set*. A falta de estudos relacionados com previsão da inflação brasileira e a relativa escassez de análises de previsões via métodos não-paramétricos ressaltam a relevância deste artigo. Não existem pesquisas na literatura de previsão de inflação brasileira aplicando SSA. Uma das estratégias de combinação de previsões aplicadas neste artigo não é comumente encontrada na literatura, na medida em que envolve combinações de diferentes especificações para cada método de previsão. Adicionalmente, restrições de parâmetros foram impostas nas previsões SSA, uma prática não reportada na literatura.

Palavras-chave Inflação. Análise espectral singular. Previsão. Brasil.

ABSTRACT

The purpose of this paper is to evaluate Brazilian inflation forecasts produced by the nonparametric method Singular Spectrum Analysis (SSA). This forecasting exercise employs rolling windows scheme. Different strategies of forecast combinations and variable selection procedures for multivariate methods were contemplated. For robustness, five forecast horizons were used. The forecast evaluation considers several statistical measures and procedures to offer reliable conclusions, including mean squared forecast error ratios, tests of equal conditional predictive ability, cumulative square forecast error difference and Model Confidence Set. The results show that SSA consistently outperforms the competitive methods. Almost all SSA forecasts outperforms the competitors in the mean squared forecast error sense, and several with statistical significance. Analysis of the out-of-sample portion indicates relative superior performance of SSA, especially over the period of electricity shock of prices. SSA methods were always included in the superior set of Model Confidence Set procedures. The lack of studies related to Brazilian inflation forecasting and the relative scarcity of nonparametric methods of forecasting analysis highlights the relevance of this paper. There is no research in Brazilian inflation literature applying SSA. One of the forecast combination strategies applied in this paper is not commonly found in the literature, as it involves combinations of different specifications for each forecast method. Additionally, parameter restrictions on SSA forecasts were imposed, a practice which is not reported in the literature.

Keywords Inflation. Singular Spectrum Analysis. Forecasting. Brazil.

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1 INTRODUCTION

Inflation is one of the most important macroeconomic variables, especially to Brazil, which has experienced hyperinflation between the decades of 1980's and 1990's.

Several reasons make inflation forecasting important. Once the decisions of Central Banks' monetary policy have a forward-looking nature, reliable forecasts are required. Fund managers use inflation forecasts as an auxiliary tool to define macroeconomic scenes and to anticipate interest rates that are likely to be fixed by the monetary authority. This is of interest to pension fund managers because a substantial part of federal public debt are indexed to inflation (National Treasury Notes – Series B, or NTN-B, as an example) due to strategic changes of Brazilian National Treasure on prioritizing this type of emission rather than bonds indexed to Selic interest rates from 2005 ownwards. Inflation forecasts are also important to adjust wages, to contracts that include future prices and to the investor's real returns. Clearly, producing accurate inflation forecasts has its practical relevance.

Most of the literature about inflation forecasting has focused on Phillips Curve model. Stock and Watson (1999) showed the improvements of using Phillips Curve based on measures of real aggregate activity over the traditional model based on unemployment rate. The preeminent study of Atkeson and Ohanian (2001) brought out that none of the standard Phillips Curve forecasting models outperforms a simple random walk model which motived the emergence of further studies (CHEN; TURNOVSKY; ZIVOT, 2014; GROEN; PAAP; RAVAZZOLO, 2013; STOCK; WATSON, 2007, 2008).

Studies regarding Brazilian inflation mostly focus on its determinants rather than on forecasting. This latter literature is growing, despite being relatively scarce.

Arruda, Ferreira and Castelar (2011) did a comparative study between linear and nonlinear models, and showed that the Phillips Curve model with a threshold effect had a relatively better forecasting performance. Figueiredo (2010) showed that factor models with targeted predictors performed well in context of a large data set. Medeiros, Vasconelos and Freitas ([2015]) estimated LASSO models and found relative out-of-sample superiority for short-horizons. Moreover, they found that variables selected by LASSO relate to government debt and money. Ferreira and Palma (2015) dealt with model and parameter uncertainty using Dynamic Model Averaging and found advantages for longer forecast horizons. Caldeira, Moura and Santos (2015) analyzed forecasts of macroeconomic variables using time-varying parameter vector autoregressive (TVP-VAR) and drew attention to TVP-VAR with Dynamic

Model Averaging. Particularly, they reported good results to inflation forecasting using TVP-VAR, Bayesian VAR and Factor-Augmented VAR models.

This study aims to evaluate the accuracy of Brazilian inflation forecasts using a nonparametric method called Singular Spectrum Analysis (SSA). Empirical studies applying this method in economic literature are quite recent, almost all from the mid 2000's, and have been calling attention due to appealing forecast results. The lack of studies related to Brazilian inflation forecasting and the relative scarcity of applications of nonparametric forecasting methods highlights the relevance of this paper. To the best of our knowledge, there is yet no research in Brazilian inflation literature applying this methodology.

SSA is a nonparametric method originated by the papers of Broomhead and King (1986a, 1986b) and Broomhead *et al.* (1987). Subsequently, SSA became a regular tool in some fields of natural sciences. Recently, the method started to take place in economic studies (HASSANI, 2007; HASSANI; THOMAKOS, 2010; HASSANI; ZHIGLJAVSKY, 2009). Several studies using a multivariate extension of SSA called Multichannel Singular Spectrum Analysis (MSSA) reported forecast improvements, for examples, on North American inflation (HASSANI; SOOFI; ZHIGLJAVISKY, 2013), European industrial production (HASSANI; HERAVI; ZHIGLJAVSKY, 2009), US' home sales (HASSANI *et al.*, 2014) and exchange rates (HASSANI; SOOFI; ZHIGLJAVISKY, 2009).

Many econometric methods devised for forecasting time series depend on validity of restrictive assumptions like nonstationarity and normality. The SSA do not suffer from the criticism of these assumptions and works well with time series with linear or nonlinear behavior. Furthermore, it is known that noise can reduce the accuracy of time series prediction. Instead of forecast the series ignoring the presence of the noise component, the SSA initiates with filtering the noisy series in order to forecast the new data (HASSANI; THOMAKOS, 2010).

We consider the official Brazilian inflation measure IPCA and 17 additional variables as candidates to be included in the multivariate methods, for a period from March 2003 to December 2015, at monthly frequency. Several econometric methods and survey-based measures of market inflation expectations were used as potential competitors of SSA. Under rolling windows scheme, various forecasting strategies were taken into account, including combined forecasts of different methods and forecast combinations of different specifications (or parameters) of a single method. The latter kind of forecast combinations is not commonly seen into the literature. We extend the idea behind forecast combinations, usually applied to parsimonious parametric regressions when dealing with model uncertainty, to the situation where we focus on a single method and combine different specifications of this method. Moreover, our SSA combination strategy puts restrictions on the set of possible parameters to reduce the forecast volatility. The results indicates strong evidence of superiority for SSA – MSSA over the alternative methods to forecast the Brazilian inflation rates along all the time horizons analyzed in this paper, especially to longer terms.

This paper is organized as follows: Section 2 describes the basic SSA algorithm; Section 3 describes the main SSA-forecast algorithms; Section 4 presents the data, the alternative methods, the forecasting strategies and the evaluation tools considered in this study. Section 5 discuss the empirical results. We conclude in Section 6.

2 METHODOLOGY

The main idea of SSA is to decompose the original series into a sum of other series, so that we can identify each one properly, i.e., trend, periodicity, quasi-periodicity or noise. The method does not rely on assumptions like stationarity or normality. Basically, the mechanism of SSA involves two stages: decomposition and reconstruction. Firstly, we transfer our series to a Hankel real matrices space and then apply singular value decomposition. Secondly, we select appropriate components and reconstruct them as new series.

2.1 Decomposition: Embedding

The first step at the decomposition stage is to obtain a matrix with Hankel structure from the series at hand, known as trajectory matrix.

Consider a nonzero series $\mathbb{X} = \left(x_1, \dots, x_N\right) \in \mathbb{R}^N$, $\mathrm{N} > 2$, and a window length L, $1 < L < N$. Also, consider the lagged vectors $X_i = (x_i, \dots, x_{i+L-1})^t, 1 \le i \le K$, where $K = N - L + 1$. Denote $a_{i,j}$ as the (i,j)th element of a $L \times K$ real matrix and define the linear operator $T: \mathbb{R}^N \to \boldsymbol{\mathcal{M}}^H$, which satisfies

$$a_{i,j} = x_k,\ k = i + j - 1, 1 \le i \le L, 1 \le j \le K, \tag{1}$$

where $\mathbb{R}^N$ is the set of all ordered N-tuples of real numbers and $\boldsymbol{\mathcal{M}}^H$ is the space of the $L \times K$ real matrices with Hankel structure. The trajectory matrix $\boldsymbol{X}$ is obtained by applying $\mathbb{X} \xrightarrow{T} \boldsymbol{X}$,

$$\boldsymbol{X} = [X_1 : \dots : X_K]. \tag{2}$$

2.2 Decomposition: Singular Value Decomposition (SVD)

At the SVD step, the trajectory matrix $\boldsymbol{X}$ is decomposed into a sum of elementary matrices $\boldsymbol{X_i}$, $\boldsymbol{i} = 1, \dots, r$, where r is the rank of $\boldsymbol{X}$. These elementary matrices have properties such as being rank-one and biorthogonal.

Define $\boldsymbol{S} = \boldsymbol{X}\boldsymbol{X}^t$. Let $\lambda_1, \dots, \lambda_L$ be eigenvalues of $\boldsymbol{S}$ and $U_1, \dots, U_L$ its corresponding eigenvectors, such that $\lambda_1 \ge \cdots \ge \lambda_L \ge 0$ and $\{U_1, \dots, U_L\}$ form an orthonormal set. The SVD of the trajectory matrix $\boldsymbol{X}$ is defined as

$$\boldsymbol{X} = \sum_{i=1}^{r} \sqrt{\lambda_i} U_i V_i^t = \sum_{i=1}^{r} \boldsymbol{X_i} \tag{3}$$

where $r = rank(\boldsymbol{X}) = \max\{i: \lambda_i > 0\}$ and $V_i = \boldsymbol{X}^t U_i / \sqrt{\lambda_i}$. Since each elementary matrix is completely determined by the triple $\left(\sqrt{\lambda_i}, U_i, V_i\right)$, it is often referred as eigentriple.

Among all the rank d matrices $\boldsymbol{X}^{(\boldsymbol{d})}$, $d < r$, the matrix $\sum_{i=1}^{d} \boldsymbol{X_i}$ provides the best approximation to the trajectory matrix $\boldsymbol{X}$ in the sense that the Frobenius norm $\left\|\boldsymbol{X} - \boldsymbol{X}^{(d)}\right\|_F$ is minimum. This optimal property is desirable if few elementary matrices are selected as representative of the trajectory matrix.

2.3 Reconstruction: Grouping

The grouping step aims to form separate groups of elementary matrices so that we could identify them as meaningful components. Once SVD is applied, we partition the set of indices $\{1, \dots, r\}$ into m subsets $I_1, \dots, I_m$ such that $\boldsymbol{X_1} = \sum_{i \in I_1} \boldsymbol{X}_i$, ..., $\boldsymbol{X_m} = \sum_{i \in I_m} \boldsymbol{X}_i$. Thus, we get

$$\boldsymbol{X} = \sum_{j=1}^{m} \boldsymbol{X_j}. \tag{4}$$

It is desirable that the resulting grouped matrices, or briefly components, are separated from each other. Achieving separability is crucial to SSA. The theory of separability is well explained in literature (GOLYANDINA; NEKRUTKIN; ZHIGLJAVSKY, 2001).

Grouping eigentriples generally requires the analyst's interaction. Useful recommendations are found in Golyandina and Zhigljavsky (2013).

2.4 Reconstruction: Diagonal Averaging

In the last step, we transform the components into Hankel matrices and convert them into reconstructed series. In practice, the components are not likely to be exactly separable, implying that the inner product between each pair of components is not exactly zero (non w-orthogonal). Thus, we need an optimal procedure to bring arbitrary matrices to the Hankel matrices space of the same dimensions.

Let $Y = \left[y_{i,j}\right]_{L \times K}$, $L^* = \min(L, K)$, $K^* = \max(L, K)$ and $N = L + K - 1$. Moreover, let $y^*_{i,j} = y_{i,j}$ if $L < K$ and $y^*_{i,j} = y_{j,i}$ otherwise. Denote the spaces of arbitrary $L \times K$ matrices and $L \times K$ Hankel matrices by $\boldsymbol{\mathcal{M}}_{L \times K}$ and $\boldsymbol{\mathcal{M}}^{(H)}_{L \times K}$, respectively. The hankelization operator $\mathcal{H}: \boldsymbol{\mathcal{M}}_{L \times K} \rightarrow \boldsymbol{\mathcal{M}}^{(H)}_{L \times K}$ is defined as

$$\tilde{y}_k = \begin{cases} \frac{1}{k}\sum_{m=1}^{k} y^*_{m,k-m+1} & , 1 \le k < L^*, \\ \frac{1}{L^*}\sum_{m=1}^{L^*} y^*_{m,k-m+1} & , L^* \le k \le K^*, \\ \frac{1}{N-k+1}\sum_{m=k-K^*+1}^{N-K^*+1} y^*_{m,k-m+1} & , K^* < k \le N. \end{cases} \quad (5)$$

Applying the hankelization operator leads to

$$\boldsymbol{X} = \widetilde{\boldsymbol{X}}_{\boldsymbol{1}} + \cdots + \widetilde{\boldsymbol{X}}_{\boldsymbol{m}} \quad (6)$$

where $\mathcal{H}\boldsymbol{X}_s = \widetilde{\boldsymbol{X}}_{\boldsymbol{s}}$ and $\tilde{x}_k^{(s)}$ is the (i,j)th element of $\widetilde{\boldsymbol{X}}_{\boldsymbol{s}}$, $k = i + j - 1$, $\mathrm{s} = 1, \dots, \mathrm{m}$.

The operator $\mathcal{H}$ can be viewed as an orthogonal projection of $\boldsymbol{X}_s$ onto $\boldsymbol{\mathcal{M}}_{L\times K}^{(H)}$ space, which means that $\mathcal{H}\boldsymbol{X}_s$ is the nearest matrix to $\boldsymbol{X}_s$ in the Frobenius norm sense.

Naturally, each $\widetilde{\boldsymbol{X}}_{\boldsymbol{s}}$ is the trajectory matrix of a series $\widetilde{\mathbb{X}}_N^{(s)} = \left(\tilde{x}_1^{(s)}, \dots, \tilde{x}_N^{(s)}\right)$, $s = 1, \dots, m$. Therefore, we can write our original series as

$$\mathbb{X} = \widetilde{\mathbb{X}}_N^{(1)} + \cdots + \widetilde{\mathbb{X}}_N^{(m)}. \quad (7)$$

3 FORECASTING

There are two main algorithms to forecast time series using SSA: recurrent forecast and vector forecast. The difference between them is that the former considers a continuation of a linear recurrence relation (LRR) obtained from the reconstructed series and the latter applies a continuation of a given component into its column space before the diagonal averaging step.

Consider the series $\mathbb{X} = \left(x_1, \ldots, x_N\right)$ and the trajectory matrix $\boldsymbol{X} = [X_1 : \ldots : X_K]$. Applying the basic SSA procedure we have the components $\boldsymbol{X_1}, \ldots, \boldsymbol{X_m}$ and the corresponding reconstructed series $\widetilde{\mathbb{X}}^{(1)} = \left(\tilde{x}_1^{(1)}, \ldots, \tilde{x}_N^{(1)}\right), \ldots, \widetilde{\mathbb{X}}^{(m)} = \left(\tilde{x}_1^{(m)}, \ldots, \tilde{x}_N^{(m)}\right), 1 \leq m \leq rank(\boldsymbol{X})$. We shall call the space spanned by the L-lagged vectors, the L-trajectory space of $\mathbb{X}$, $\mathcal{L}^{(L)} = span(X_1, \ldots, X_K)$. If $rank\left(\mathcal{L}^{(L)}\right) = r$, then $rank_L(\mathbb{X}) = r$. Also, denote by $X^{\nabla} \in \mathbb{R}^{L-1}$ and $X_{\Delta} \in \mathbb{R}^{L-1}$ vectors of the first and the last $L-1$ elements of $X \in \mathbb{R}^L$, respectively.

3.1 Recurrent Forecast

The recurrent forecast rely on the following theorem[1].

Theorem 1. Let $P_1, \ldots, P_d$ be an orthonormal base of a linear space $\mathcal{L}_d$ and $span(P_1^{\nabla}, \ldots, P_d^{\nabla}) = \mathcal{L}_d^{\nabla}$. Furthermore, denote π_i as the last element of the vector P_i, $i = 1, \ldots, d$, and $v^2 = \pi_1^2 + \cdots + \pi_d^2$. Suppose that $1 \leq rank_L\left(\mathbb{X}^{(j)}\right) = d < L, 1 \leq j \leq m$, and $e_L \notin \mathcal{L}_d$. Then the following LRR is valid

$$x_{i+L-1} = \sum_{k=1}^{L-1} a_k x_{i+L-1-k}, \; 1 \leq i \leq K \tag{8}$$

where the coefficients are defined by

$$\mathcal{R} = (a_{L-1}, \ldots, a_1)^t = \frac{1}{1-v^2} \sum_{i=1}^{d} \pi_i P_i^{\nabla}. \tag{9}$$

If we focus on forecast the (j)th component of the time series $\mathbb{X}$, the recurrent forecast defines the time series $\widetilde{\mathbb{F}}_{N+M} = \left(\tilde{f}_1, \ldots, \tilde{f}_N, \tilde{f}_{N+1}, \ldots, \tilde{f}_{N+M}\right)$ as

$$\tilde{f}_i = \begin{cases} \tilde{x}_i^{(j)}, & i = 1, \ldots, N, \\ \sum_{j=1}^{L-1} a_j y_{i-j}, & i = N+1, \ldots, N+M, \end{cases} \tag{10}$$

[1] Proof can be found in Golyandina, Nekrutkin and Zhigljavsky (2001, p. 247).

where the coefficients $a_{L-1}, \dots, a_1$ is given by $\mathcal{R}$.

The basic recurrent forecast uses the left eigenvectors provided by SSA as the orthonormal basis for the linear space $\mathcal{L}_d$. For a given L, suppose $\mathbb{X} = S + N$ where S is the signal term and N is related to the noise. Denote by $\mathcal{L}^{L,S}$ the L-trajectory space of the series S. If (strong) separability is achieved, then the continuation of the LRR that governs the reconstructed series is the same of those results produced by the basic recurrent forecast, ie., $\mathcal{L}_d = \mathcal{L}^{L,S}$ and the recurrent algorithm does not produce error by using the wrong LRR. In practice, approximate separability is required to carry out problems related to the differences between $\mathcal{L}_d$ and $\mathcal{L}^{L,S}$, and errors contained in the reconstructed series which affects the initial data of the algorithm.

3.2 Vector Forecast

The series $\mathbb{X}$ admits L-continuation in $\mathcal{L}^{(L)}$ if there exists a unique number $\tilde{x}_{N+1}$ such that all the L-lagged vectors of $\tilde{\mathbb{X}}_{N+1} = (x_1, \dots, x_N, \tilde{x}_{N+1})$ belong to $\mathcal{L}^{(L)}$. The vector forecast, firstly, do L-continuation and then reconstruct the series.

Consider the same notation of Theorem and that $e_L \notin \mathcal{L}_d$. Define the linear operator $P^{(v)}: \mathcal{L}_d \to \mathbb{R}^L$ as

$$P^{(v)}Y = \begin{pmatrix} \Pi Y_\Delta \\ \mathcal{R}^t Y_\Delta \end{pmatrix},\ Y \in \mathcal{L}_d \tag{11}$$

where $\Pi Y_\Delta = (V^\nabla)(V^\nabla)^t Y_\Delta + y(1 - v^2)\mathcal{R}$ and $V^\nabla = [P_1^\nabla : \dots : P_d^\nabla]$. The operator Π can be viewed as an orthogonal projection $\Pi: \mathbb{R}^{L-1} \mapsto \mathcal{L}_d^\nabla$. But since $\Pi Y_\Delta \in \mathcal{L}_d^\nabla$ and it's assumed that $e_L \notin \mathcal{L}_d$, then there is a unique vector $P^{(v)}Y$ which the last element is given by $\mathcal{R}^t Y_\Delta$.

Focusing on the (j)th component $\boldsymbol{X_j} = \sum_{i \in I_j} \boldsymbol{X}_i = [\hat{X}_{1,j} : \dots : \hat{X}_{K,j}]$, define

$$W_i = \begin{cases} \hat{X}_{i,\boldsymbol{j}}, & 1 \le i \le K \\ P^{(v)} W_{i-1}, & K + 1 \le i \le K + M + L - 1 \end{cases} \tag{12}$$

Making the diagonal averaging of the matrix $\boldsymbol{W} = [W_1 : \dots : W_{K+M+L-1}]$ we obtain the series $\tilde{\mathbb{G}}_{N+M} = (\tilde{g}_1, \dots, \tilde{g}_N, \tilde{g}_{N+1}, \dots, \tilde{g}_{N+M})$ where $\tilde{g}_{N+1}, \dots, \tilde{g}_{N+M}$ form the M terms of the vector forecast. Again, as in the basic recurrent forecast case we can use the left eigenvectors provided by SSA. If $\boldsymbol{X_j}$ belongs to $\mathcal{L}_d = span(U_i : i \in I_j)$ and strong separability is achieved, then recurrent and vector forecasts coincide with the exact continuation of the series $\mathbb{X}^{(j)}$ because $\mathcal{L}_d$ equals the trajectory space of $\mathbb{X}^{(j)}$, consequently, the matrix Π is the identity matrix and W has Hankel structure.

3.3 Multichannel Singular Spectrum Analysis

The extension of the model to the multivariate case is straightforward. The main difference relies on the trajectory matrix where we use a stacked version. For a given window length L, assume a multivariate series $f_i = \left(f_i^{(1)}, \dots, f_i^{(M)}\right)$, where $i = 1, \dots, N$. Denote the trajectory matrices of the individual series $\left\{f_i^{j}\right\}_{i=1}^{N}$ as $\boldsymbol{X}^{(j)}$, $j = 1, .., M$. The trajectory matrix of the multivariate series can be defined as

$$X = \begin{pmatrix} \boldsymbol{X}^{(1)} \\ \vdots \\ \boldsymbol{X}^{(M)} \end{pmatrix}. \tag{13}$$

Note that the eigentriples reflect information of the entire set of variables, including the cross products

$$\boldsymbol{X}_v \boldsymbol{X}_v^t = \begin{bmatrix} \boldsymbol{X}^{(1)}\boldsymbol{X}^{(1)t} & \boldsymbol{X}^{(1)}\boldsymbol{X}^{(2)t} & \cdots & \boldsymbol{X}^{(1)}\boldsymbol{X}^{(m)t} \\ \boldsymbol{X}^{(2)}\boldsymbol{X}^{(1)t} & \boldsymbol{X}^{(2)}\boldsymbol{X}^{(2)t} & \cdots & \boldsymbol{X}^{(2)}\boldsymbol{X}^{(m)t} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{X}^{(m)}\boldsymbol{X}^{(1)t} & \boldsymbol{X}^{(m)}\boldsymbol{X}^{(2)t} & \cdots & \boldsymbol{X}^{(m)}\boldsymbol{X}^{(m)t} \end{bmatrix}_{(ML)\times(ML)} . \tag{14}$$

4 FORECASTING EXERCISE AND PERFORMANCE EVALUATION

Our forecasting analysis is based on 18 variables (see APPENDIX D) between March 2003 and December 2015, a total of $T = 154$ monthly observations. The accuracy of SSA forecasts will be analyzed considering alternative models, including: Autoregressive model (AR); Random Walk (RW); Seasonal Autoregressive Integrated Moving Average model (SARIMA); Exponential Smoothing in State Space model (ETS); Neural Networks Autoregressive model (NNAR); Unobservable Components model (UCM); Generalized Phillips Curve model[2] (GPC); and Vector Autoregressive model (VAR).

The evaluation of the forecast methods mentioned above will be conducted by measures of Mean Squared Forecast Error (MSFE) ratios, Cumulative Square Forecast Error Difference (CSFED) as those used by Welch and Goyal (2008), Giacomini and White (2006) tests[3] and the Model Confidence Set (MCS) procedure proposed by Hansen, Lunde and Nason (2011).

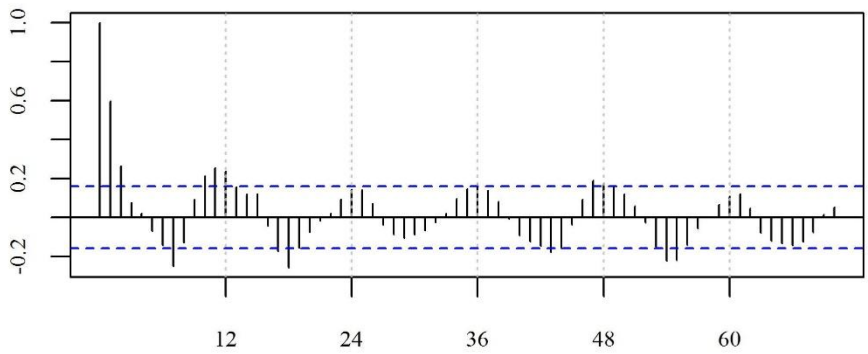
Our focus relies on rolling windows forecasts. We split the sample according to the theoretical viewpoint of SSA. For relatively short time series, if we know the period p of the periodic component of a time series of length N, then $N - 1$ must be proportional to p to satisfy conditions of separability (GOLYANDINA; ZHIGLJAVSKY, 2013, p. 50). For monthly data, generally period 12 is natural (see Figure 1). Therefore, avoiding small window lengths to properly capture data structures, we choose an estimation window of length[4] $E = 97$. In our exercise, the model parameters are selected by cross validation with a validation period of length $V = 15$. Finally, the in-sample portion has length $R = E + V = 112$ and the out-of-sample portion has length $P = T - R = 42$. Regardless the forecast horizon τ, the estimation window and the number of out-of-sample forecasts produced will always be the same, but the number of forecasts produced in the validation period will be $\vartheta = V - \tau + 1$. Thus, the longer the forecast horizon, the less will be the information used for parameter or model selection. We choose this way because of the equal predictive ability test asymptotics and the fact that our time series is short.

[2] We consider a bivariate version of that used by Koop and Korobilis (2012).

[3] Under rolling windows framework, Giacomini and White (2006) test of equal conditional predictive ability is adequate to treat nested and nonnested models, and general estimation procedures including Bayesian, semi- and nonparametric methods.

[4] The associated results (not reported) of this window length are robust to $E = 85$ and $E = 109$.

Figure 1 - IPCA Series Autocorrelation Function



Source: elaborated by the author (2016).

To deal with model uncertainty, we consider combinations of different forecast methods and combinations of different specifications of the same forecast method. The former uses the simple arithmetic mean combination and the latter uses a weighted average based on the mean squared forecast error obtained from the validation period. In respect to SSA forecasts, we restrict the set of possible eigentriples to be chosen such that only those relevant remains (explaining 85% of the data variation). By doing this, we expect less volatile forecasts. In practice, for noisy series, considering just the main eigentriples prevent us from noise-signal mixing as well as turn the algorithm computationally faster.

In respect to the forecast combination of specifications of the same method, the number of possible combinations of parameters in SSA and MSSA is huge. Then, we only choose the 5% best models to produce the final combined forecasts. For SARIMA, AR, SETAR and NNAR models, we set the maximum parameter values equal to 6. For all the univariate techniques, except SSA, we consider the 50% best models to calculate the final forecast combinations. Even so, the number of SSA models remains greater than each of these univariate cases.

There are four different forecast combination strategies in this work. The first strategy uses forecast combinations of different specifications of a given univariate method. Denote B_{ij} as the j-th n_i-tuple of parameters of the method i, which defines the model used to produce forecasts. In addition, let

$$MSE\left(e_{i|B_{ij}}\right) = \sum_{t=t_0}^{m} \left(\hat{f}_{i,t|B_{ij}} - f_{i,t}\right)^2 / (m - t_0 + 1) \tag{15}$$

where $\hat{f}_{i,t|B_{ij}} - f_{i,t}$ is the error between the forecasted value produced by the method i using parameters B_{ij}, at time t, in the validation period that stands from t_0 to m. The final forecast of the first strategy can be written as

$$\hat{f}_{i,m+1} = \sum_{j=1}^{K_i} \omega_j \hat{f}_{i,m+1|B_{ij}} \tag{16}$$

$$\omega_j = \left(\frac{1}{MSE\left(e_{i|B_{ij}}\right)}\right) \Bigg/ \sum_{j=1}^{K_i} \left(\frac{1}{MSE\left(e_{i|B_{ij}}\right)}\right). \tag{17}$$

where K_i is the number of possible n_i-tuple combinations. One can see that those models associated with lower MSEs in the validation period receive higher weights.

The second strategy applies the same scheme of forecast combination previously mentioned, but in a bivariate framework. The bivariate model associated with the lowest MSE, in the first validation window, defines the secondary variable that is going to be used in the remaining forecasting exercise. The third strategy is similar to the second, with the difference that possibly new secondary variables can be selected as the estimation window rolls, depending on the performance over the validation periods. That is, the third strategy is a variable-updating strategy whereas the second is a variable-fixed strategy. The fourth strategy corresponds to the simple arithmetic mean of a set of different final forecast methods.

As explained above, there are two algorithms for the SSA or MSSA to produce forecasts: vector forecast (V) and recurrent forecast (R). The model RSSA and VSSA corresponds to the univariate SSA forecasts; RMSSA and VMSSA are related to MSSA forecasts under the variable-fixed strategy; RMSSAu and VMSSAu also corresponds to MSSA forecasts, but under variable-updating selection scheme. Additionally, GPC and VAR forecasts consider the variable-updating strategy.

For horizon τ, the FOCUS forecast is the median of expectations from the last day of the previous τ-month.

The following forecasts belongs to the fourth forecast strategy. COMB1 is the mean of all forecasts produced in our exercise, except those related to SSA or MSSA. COMB2 is similar to COMB1, but includes SSA and MSSA. SSAb averages basic univariate recurrent and vector forecasts, without combinations of specifications and without eigentriple's restrictions. SSAm is the mean of all forecasts associated with SSA or MSSA, excluding SSAb. The last two forecasts can be thought as a representative forecast of a very basic SSA setup and as representative of the general SSA – MSSA methodology, respectively.

5 RESULTS

In general, out-of-sample performance evaluation shows appealing results for SSA and MSSA methods. Under sliding forecasts scheme, the method is able to incorporate the new data structure more efficiently.

Tables 1-5 (APPENDIX A) present MSFE ratios of relevant pairs of models and indicates superiority of SSA-MSSA against the alternative methods, which can be seen in the rectangular blocks with dashed border in each table. Inside these blocks, almost all MSFE ratios are less than unity in magnitude, and statistically less in several cases, indicating that SSA – MSSA forecasts outperform the competitor's methods. Particularly, FOCUS showed a relatively good out-of-sample ability when aiming one-step ahead forecasting despite not being statistically significant. For other forecast horizons, SSA and MSSA methods consistently indicates performance gains over all alternative methods, especially in 12-steps ahead case where statistical significances were found against FOCUS. In addition, note that the first line of each table indicates relative gains of using the forecast combination strategies presented in this paper over the basic SSAb.

For every horizon, COMB2 outperformed COMB1 in MSE sense. With the exception of the one-step ahead horizon, the performance gains were statistically significant, which means that the inclusion of SSA-MSSA forecasts in the global average improves the final forecasts.

Figures 2-6 (APPENDIX B) plots the differences between the cumulative square forecast error (CSFE) of the SSAm forecasts and the CSFE of the alternative methods. This kind of plot offers special advantages because it allows us to evaluate the forecasting performance over the whole out-of-sample period and to identify periods of relative gains. When the curves in each plot increases, the SSAm outperforms the alternative methods. Viewing SSAm as representative of SSA and MSSA methods, the first impression is that the plots have in general positively sloped curves.

The CSFED plots bring out a distinct behavior over the period between January 2015 and March 2015, characterized by shocks of administered prices. During this post-election period, government decided to raise fuel taxes and primarily unfreeze electricity prices, resulting in a great elevation of inflation rates. SSAm forecasts exhibit strong relative gains over the period, especially for longer horizons. We refer to this period as the Electricity Shock.

The CSFED patterns are rather similar, with the exception of the FOCUS behavior for one-step ahead case. The key aspect of the CSFEDs is that they are generally increasing, sometimes slowly and sometimes rapidly. This increase means that SSAm performs well not just in a unique time interval, but also in the entire out-of-sample period. We could also mention the reasonable degree of smoothness of the CSFEDs, with the exclusion of the electricity shock period. It indicates that the performance behavior is reliable and is not so erratic over time. To illustrate the robustness of SSAm, we displayed the CSFEs for other (estimation) window lengths. We see that the behavior of these CSFEs remains quite homogeneous. Despite being visual features, these aspects offer robustness to SSA-MSSA methods.

Table 6 (APPENDIX C) shows the results of the Hansen, Lunde and Nason (2011) procedure to summarize the relative performance of an entire set of methods. The advantage of the MCS is that we do not need a benchmark to generate the set of best models with a given confidence level. Practically, every method considered statistically superior was related on SSA or MSSA.

Figures 7-8 (APPENDIX C) offer insights about what kind of additional variables the MSSA methodology have selected, under the mentioned variable-updating forecasting. Along all forecast horizons, Recurrent and Vector MSSA have selected variables related to other existing consumer price indexes such as IPC_FIPE, IPC_FGV and IPC_M.

6 CONCLUSIONS

We investigate the relative performance of SSA methodology to forecast Brazilian inflation rates.

We find that SSA consistently outperforms the competitor's forecasts over the entire out-of-sample period, especially in the Electricity Shock, from January 2015 to March 2015, using measures of MSFE, Giacomini-White tests and MCS procedures. Performance superiority were more evident for longer horizons. Particularly, for 12-step ahead horizon, some SSA and MSSA strategies showed to be statistically more accurate than the expectations of Brazilian market analysts.

Performance gains on using forecast combinations and restrictions on eigentriples rather than using forecasts without such strategies were found. Advantages on including SSA – MSSA in the average of all forecast methods were found. Additionally, the most selected type of variable, in the SSA bivariate methods, relate to alternative consumer price indexes.

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APPENDIX A – MEAN SQUARED FORECAST ERROR RATIOS

Table 1 - MSFE ratios (1-step ahead forecasts)

h=1	SSAm	RSSA	VSSA	RMSSA	VMSSA	RMSSAu	VMSSAu	COMB2	COMB1	SARIMA	ETS	NNAR	UCM	AR	PHILLIPS	VAR	RW	FOCUS
SSAb	1,19	1,11	1,14	1,01	1,3	1,1	1,19	1,15	1,01	0,96*	0,82*	0,62	0,9	0,87*	0,84**	0,84**	0,84	1,34
SSAm		0,93	0,96	0,85	1,09	0,93*	1	0,97	0,85	0,81	0,69	0,52*	0,76	0,73*	0,7	0,7	0,71	1,13
RSSA			1,03	0,91	1,18	1	1,07	1,04	0,91	0,87	0,74	0,56	0,81	0,79	0,76	0,76	0,76	1,21
VSSA				0,89	1,14	0,97	1,04	1,01	0,89	0,85	0,72	0,55	0,79	0,77	0,74	0,74	0,74	1,18
RMSSA					1,29	1,09	1,17	1,14	1	0,95	0,81	0,61	0,89	0,86	0,83	0,83	0,83	1,33
VMSSA						0,85	0,91	0,89	0,77	0,74	0,63	0,48*	0,69	0,67**	0,64**	0,64**	0,64	1,03
RMSSAu							1,08	1,05	0,91	0,87	0,75	0,56*	0,82	0,79	0,76	0,76	0,76	1,21
VMSSAu								0,97	0,85	0,81	0,69	0,52*	0,76	0,74	0,71	0,71	0,71	1,13
COMB2									**0,87**	0,83*	0,71	0,54**	0,78	0,76***	0,73**	0,73**	0,73	1,16

Note: This table presents statistics on out-of-sample errors for inflation forecasts. The statistics are the mean squared forecast errors (MSFE) ratios between methods in the columns and in the rows. A MSFE ratio less than one (cross-hatching cells) means that the method in the row is more accurate in magnitude than the method in the column. Asterisks "***", "**" and "*" indicates rejection of Giacomini-White's null hypothesis of equal conditional predictive ability of forecasts at the 1%, 5% and 10% level of significance, respectively. The rectangle with dashed border brings a clearer view of the relative performance between the set of SSA and MSSA methods and the alternative methods.
Source: elaborated by the author (2016).

Table 2 - MSFE ratios (3-steps ahead forecasts)

h=3	SSAm	RSSA	VSSA	RMSSA	VMSSA	RMSSAu	VMSSAu	COMB2	COMB1	SARIMA	ETS	NNAR	UCM	AR	PHILLIPS	VAR	RW	FOCUS
SSAb	1,1	1,08	0,99	1,02**	1,15	1,08	1,08	0,87	0,61	0,54**	0,34***	0,32***	0,69	0,52**	0,59	0,66	0,34***	0,79
SSAm		0,98	0,9*	0,93	1,05	0,99	0,98	0,79	0,56*	0,49**	0,3***	0,29**	0,63	0,47**	0,54	0,6	0,31***	0,72
RSSA			0,92	0,95	1,07	1	1	0,8	0,57	0,5**	0,31***	0,3**	0,64	0,48**	0,55	0,61	0,31***	0,73
VSSA				1,03	1,16	1,09*	1,09	0,88	0,62	0,55	0,34***	0,33**	0,69	0,52*	0,6	0,67	0,34***	0,8
RMSSA					1,13	1,06	1,05	0,85	0,6*	0,53**	0,33***	0,32**	0,67	0,51**	0,58	0,65	0,33***	0,77
VMSSA						0,94**	0,93	0,75*	0,53*	0,47**	0,29***	0,28**	0,6	0,45**	0,51*	0,57	0,29***	0,68
RMSSAu							0,99	0,8	0,57*	0,5**	0,31***	0,3**	0,64	0,48**	0,55	0,61	0,31***	0,73
VMSSAu								0,81	0,57	0,5**	0,31***	0,3**	0,64	0,48**	0,55	0,61	0,31***	0,73
COMB2									**0,71*** **	0,62***	0,39***	0,37**	0,79	0,6***	0,68	0,76	0,39***	0,91

Source: elaborated by the author (2016).

Table 3 - MSFE ratios (6-steps ahead forecasts)

h=6	SSAm	RSSA	VSSA	RMSSA	VMSSA	RMSSAu	VMSSAu	COMB2	COMB1	SARIMA	ETS	NNAR	UCM	AR	PHILLIPS	VAR	RW	FOCUS
SSAb	1,29*	1,28**	1,28**	1,31**	1,16	1,29*	1,35**	0,95*	0,68*	0,65	0,31***	0,63**	0,65*	0,68**	0,62**	0,88	0,31***	1,02
SSAm		0,99	0,99*	1,02	0,9***	1	1,04	0,73*	0,53	0,5	0,24***	0,49*	0,5**	0,53**	0,48***	0,68**	0,24***	0,79
RSSA			1	1,03	0,91**	1,01	1,05	0,74*	0,53	0,5*	0,24***	0,49	0,51**	0,53*	0,49***	0,69**	0,24***	0,79
VSSA				1,03	0,91***	1,01	1,05	0,74**	0,54	0,51	0,24***	0,5*	0,51*	0,53***	0,49***	0,69**	0,24***	0,8
RMSSA					0,88***	0,98	1,02	0,72*	0,52	0,49*	0,24***	0,48	0,5**	0,52*	0,47***	0,67**	0,23***	0,77
VMSSA						1,11**	1,16***	0,82	0,59	0,56	0,27***	0,54*	0,56*	0,59***	0,54***	0,76	0,26***	0,88
RMSSAu							1,04	0,74*	0,53	0,5*	0,24***	0,49	0,51**	0,53*	0,48***	0,68**	0,24***	0,79
VMSSAu								0,71**	0,51	0,48*	0,23***	0,47*	0,49**	0,51**	0,46***	0,65**	0,23***	0,76
COMB2									**0,72*****	0,68***	0,33***	0,67**	0,69*	0,72**	0,66**	0,93	0,32***	1,07

Source: elaborated by the author (2016).

Table 4 - MSFE ratios (9-steps ahead forecasts)

h=9	SSAm	RSSA	VSSA	RMSSA	VMSSA	RMSSAu	VMSSAu	COMB2	COMB1	SARIMA	ETS	NNAR	UCM	AR	PHILLIPS	VAR	RW	FOCUS
SSAb	1,18	1,23	1,2	1,07	1,02	1,21	1,24	0,93	0,74	0,75	0,47***	0,41*	0,86*	0,62**	0,36***	0,72*	0,43***	0,91
SSAm		1,04**	1,02	0,91***	0,87***	1,03	1,05	0,79***	0,63***	0,64***	0,4***	0,35*	0,73***	0,53**	0,31***	0,61*	0,37***	0,77
RSSA			0,98	0,87***	0,83***	0,98	1,01	0,76***	0,61***	0,61***	0,39***	0,33*	0,7**	0,51**	0,3***	0,58**	0,35***	0,74
VSSA				0,89***	0,85*	1	1,03	0,78**	0,62**	0,63**	0,4***	0,34*	0,72**	0,52**	0,3***	0,6*	0,36***	0,76
RMSSA					0,95	1,13*	1,16**	0,87**	0,7***	0,7***	0,44***	0,38*	0,81**	0,58**	0,34***	0,67*	0,4***	0,85
VMSSA						1,18**	1,22***	0,92*	0,73**	0,74**	0,47***	0,4*	0,85*	0,61*	0,36***	0,7	0,43***	0,89
RMSSAu							1,03	0,77***	0,62***	0,62***	0,39***	0,34*	0,72**	0,52**	0,3***	0,59**	0,36***	0,75
VMSSAu								0,75**	0,6**	0,61***	0,38***	0,33*	0,7***	0,5**	0,29***	0,58*	0,35***	0,73
COMB2									**0,8*****	0,8***	0,51***	0,44	0,93	0,67**	0,39***	0,77	0,46***	0,98*

Source: elaborated by the author (2016).

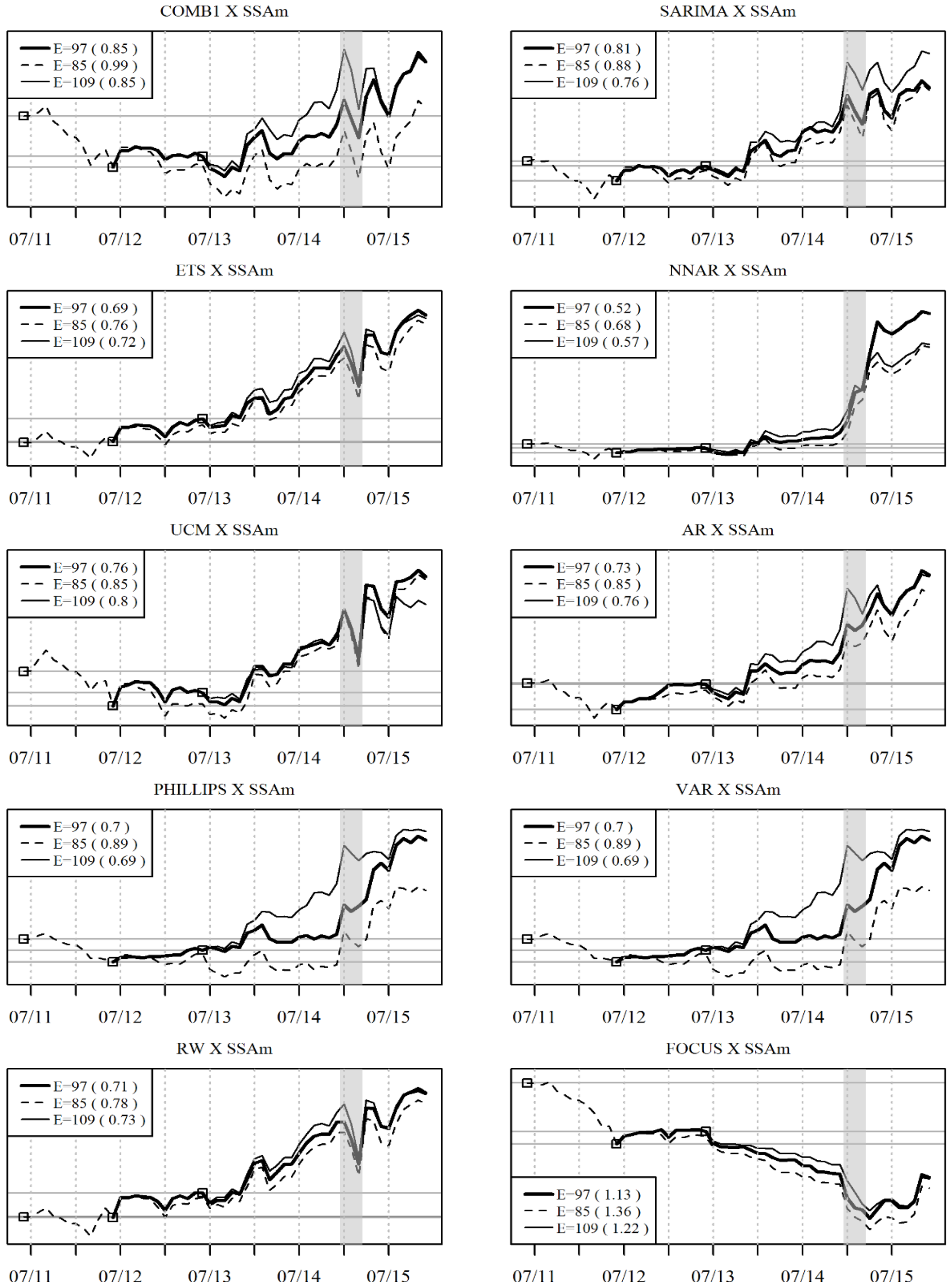
Table 5 - MSFE ratios (12-step ahead forecasts)

h=12	SSAm	RSSA	VSSA	RMSSA	VMSSA	RMSSAu	VMSSAu	COMB2	COMB1	SARIMA	ETS	NNAR	UCM	AR	PHILLIPS	VAR	RW	FOCUS
SSAb	4,29**	4,17**	3,92**	3,93	4,3***	4,22**	4,23**	3,47	2,84	2,83	3,42*	1,9	3,42	2,1**	1,4	2,38**	3,08*	3,04
SSAm		0,97	0,91**	0,92*	1	0,98**	0,98***	0,81***	0,66***	0,66***	0,8	0,44**	0,8	0,49***	0,33***	0,55***	0,72	0,71
RSSA			0,94***	0,94***	1,03	1,01	1,01***	0,83***	0,68***	0,68***	0,82	0,46**	0,82	0,5***	0,34***	0,57***	0,74	0,73*
VSSA				1**	1,1	1,08***	1,08	0,88**	0,72***	0,72**	0,87	0,48**	0,87	0,54***	0,36***	0,61***	0,79	0,78*
RMSSA					1,09*	1,07***	1,07**	0,88**	0,72**	0,72***	0,87	0,48**	0,87	0,53***	0,36***	0,6***	0,78	0,77*
VMSSA						0,98	0,98*	0,81***	0,66***	0,66***	0,79	0,44**	0,79	0,49***	0,32***	0,55***	0,72	0,71
RMSSAu							1***	0,82**	0,67***	0,67***	0,81	0,45**	0,81	0,5***	0,33***	0,56***	0,73	0,72
VMSSAu								0,82***	0,67***	0,67***	0,81	0,45**	0,81	0,5***	0,33***	0,56***	0,73	0,72
COMB2									**0,82*****	0,81***	0,98	0,55***	0,99	0,61***	0,4***	0,69***	0,89	0,88

Source: elaborated by the author (2016).

APPENDIX B – CUMULATIVE SQUARED FORECAST ERROR DIFFERENCES

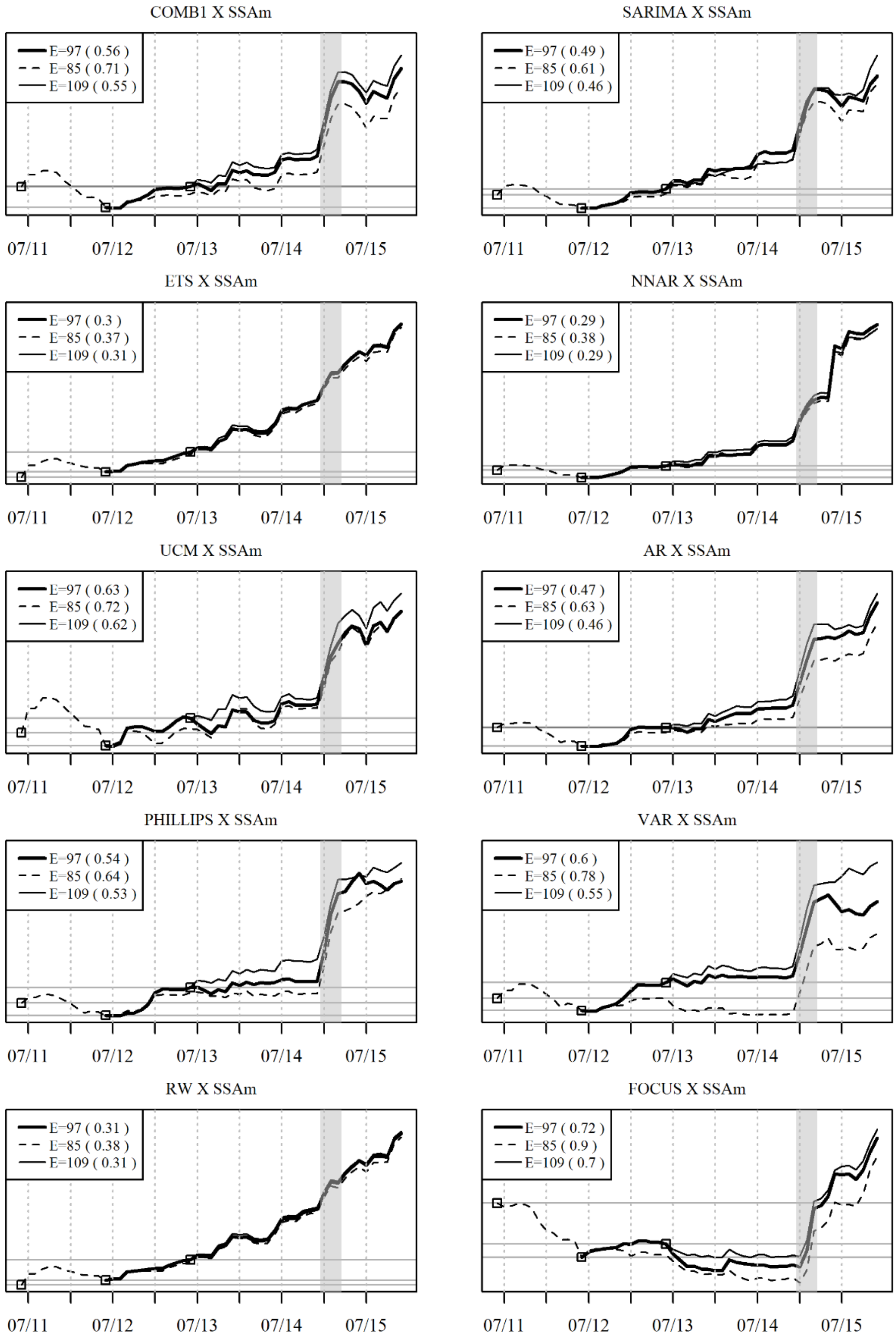
Figure 2 - CSFED (1-step ahead forecast)



Note: This figure presents the CSFEs, similar to those used by Welch and Goyal (2008). An increase in line indicates superior performance to SSAm over the alternative method. In addition to the case where the estimation window has length 97, the CSFEs for estimation windows of lengths 85 and 109 are displayed to give notions of robustness. For each plot, the values between parentheses inside the legend are MSE ratios of the corresponding pairwise methods. The gray period in the plots indicates shocks of unfreezing administered prices (basically electricity).

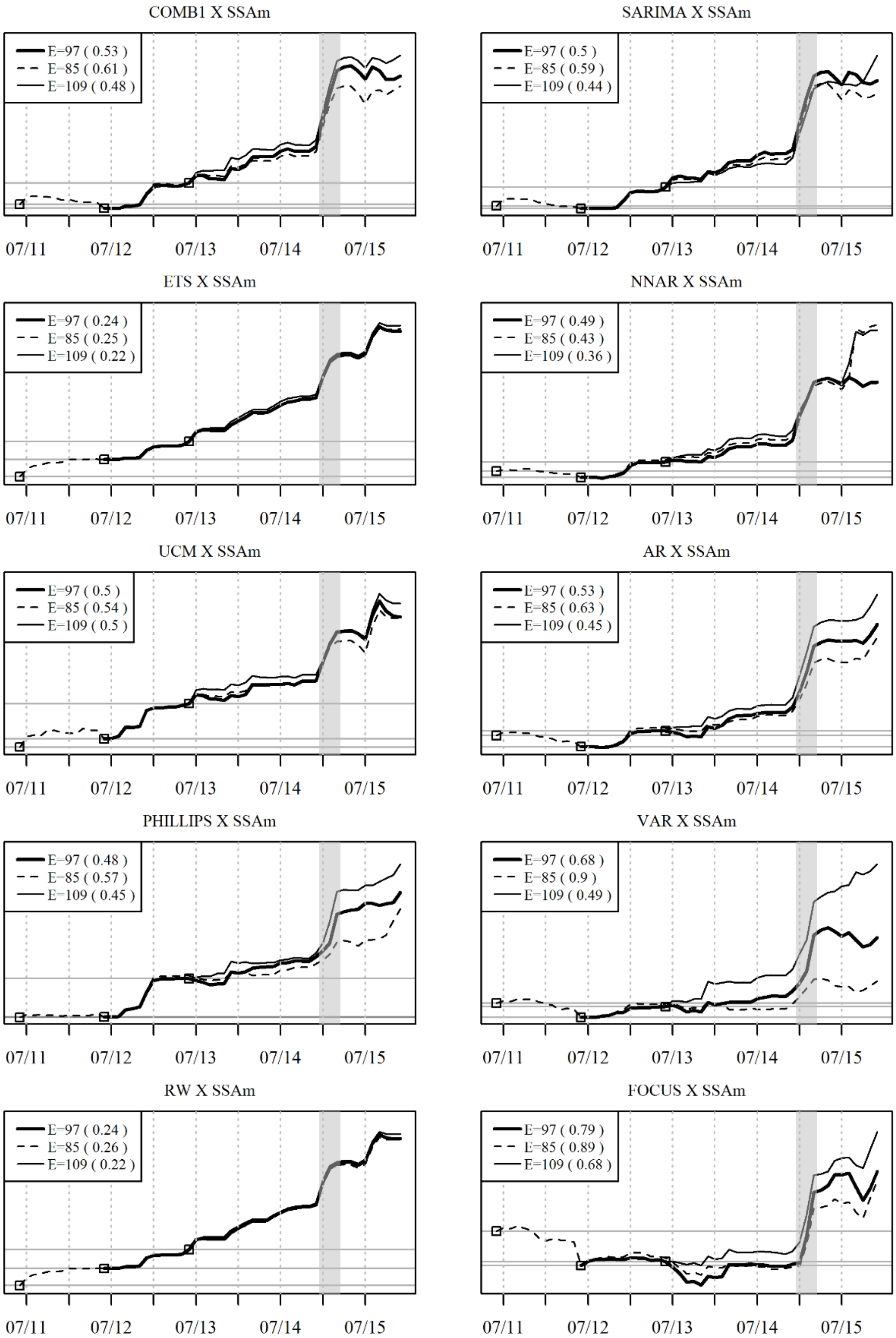
Source: elaborated by the author (2016).

Figure 3 - CSFED (3-step ahead forecast)



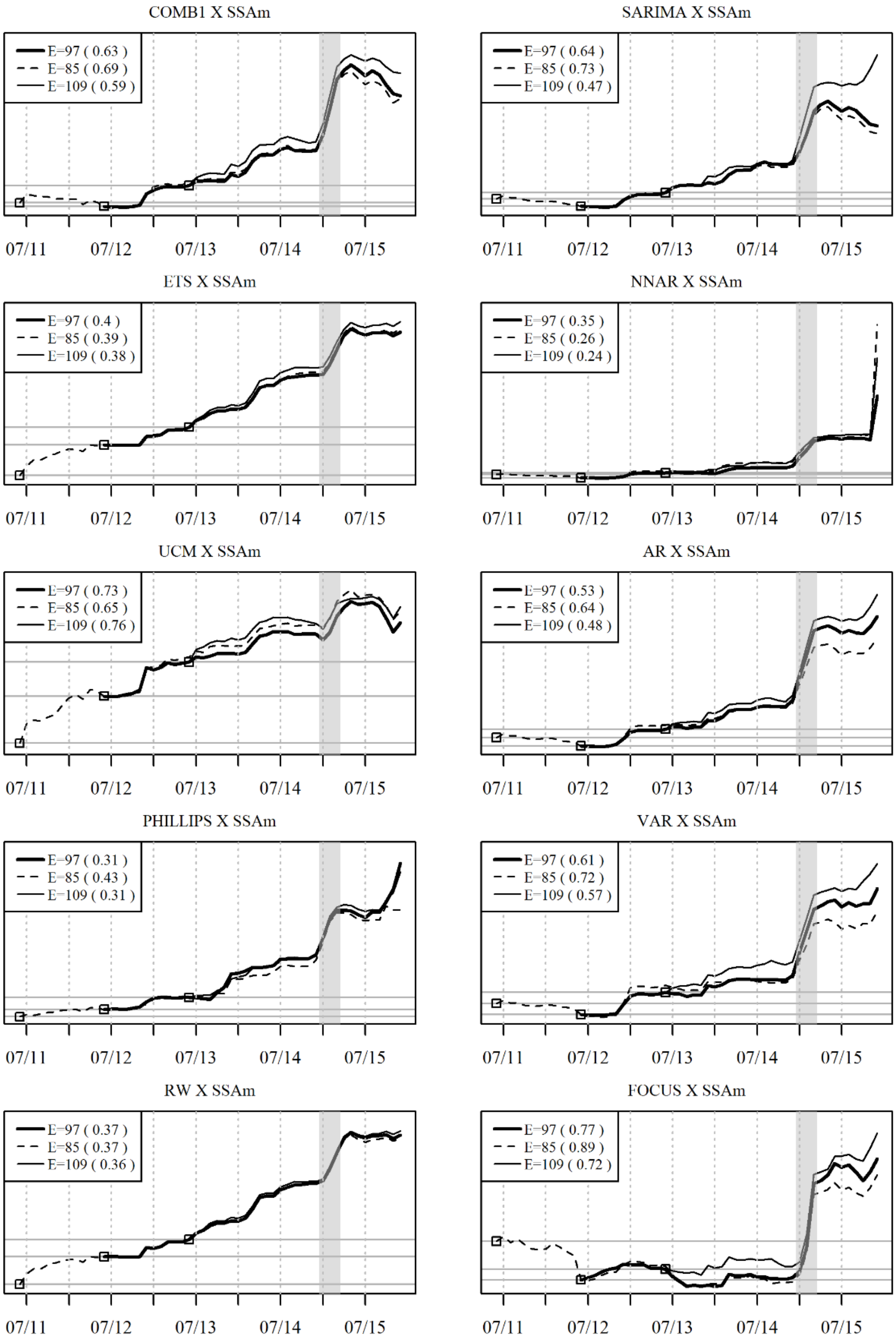
Source: elaborated by the author (2016).

Figure 4 - CSFED (6-step ahead forecast)



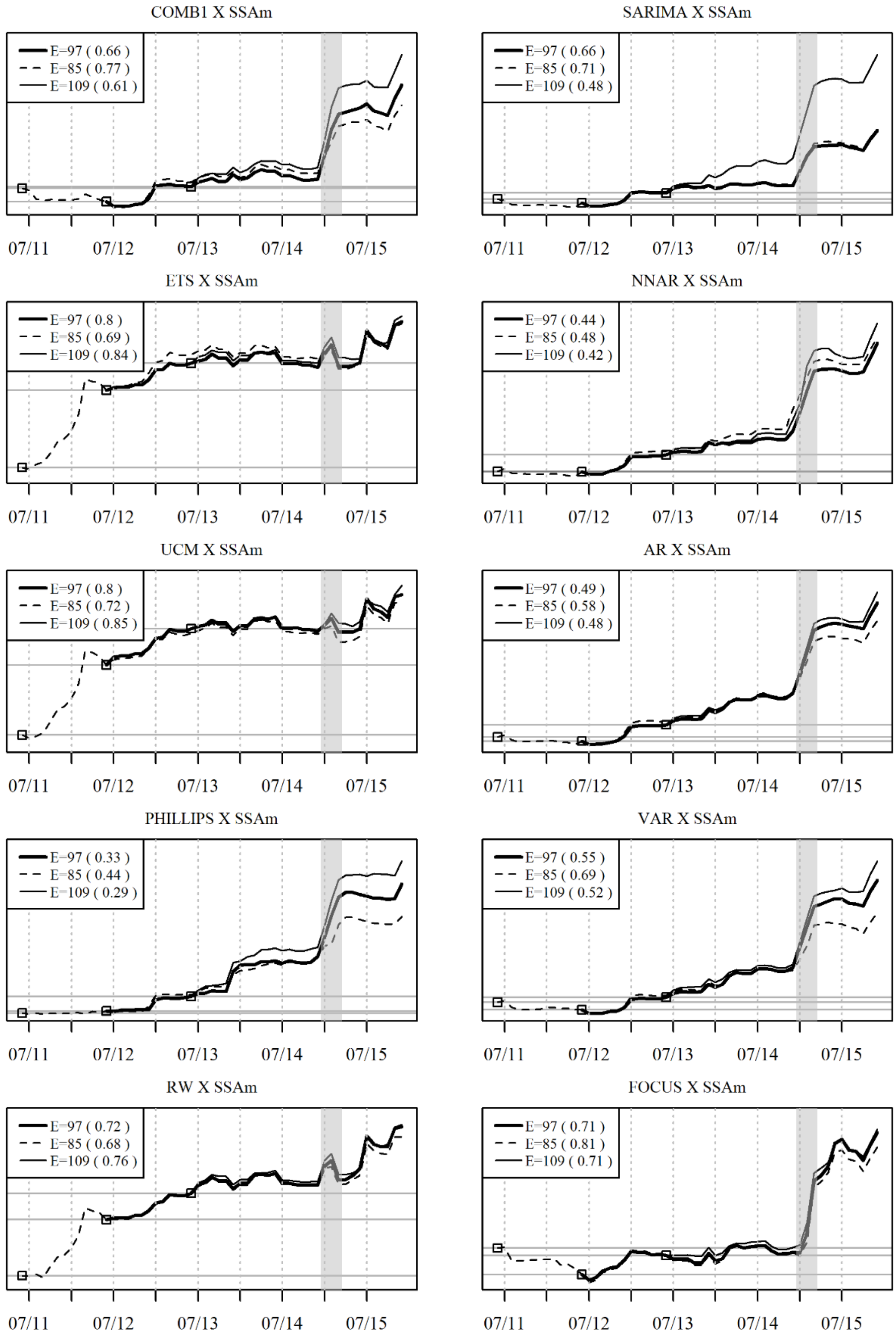
Source: elaborated by the author (2016).

Figure 5 - CSFED (9-step ahead forecast)



Source: elaborated by the author (2016).

Figure 6 - CSFED (12-step ahead forecast)



Source: elaborated by the author (2016).

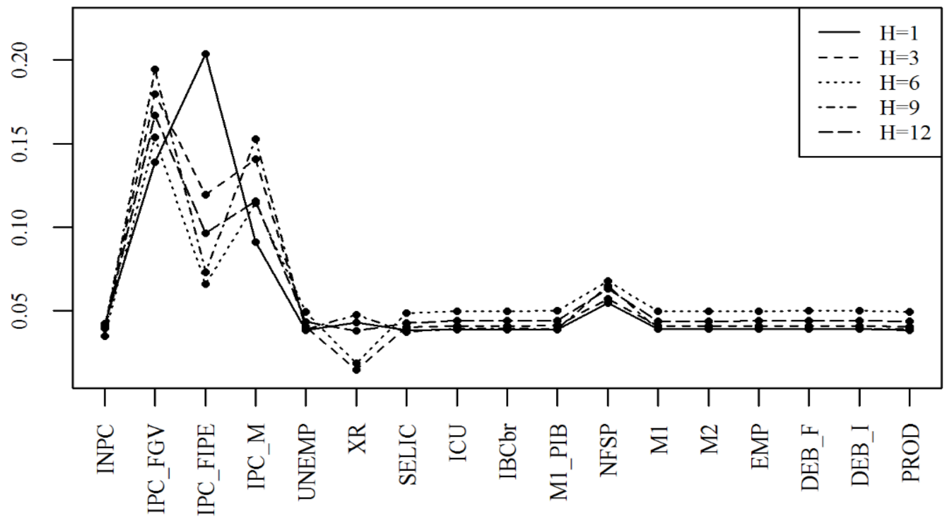
APPENDIX C – ADDITIONAL RESULTS

Table 6 - Model Confidence Sets

Horizons	1	3	6	9	12
Models	VMSSA	SSAb	SSAm	RSSA	SSAm
	FOCUS	SSAm	RSSA	VSSA	RSSA
		RSSA	VSSA	RMSSAu	VSSA
		VSSA	RMSSA	VMSSAu	RMSSA
		RMSSA	RMSSAu		RMSSAu
		VMSSA	VMSSAu		VMSSAu
		RMSSAu			
		VMSSAu			

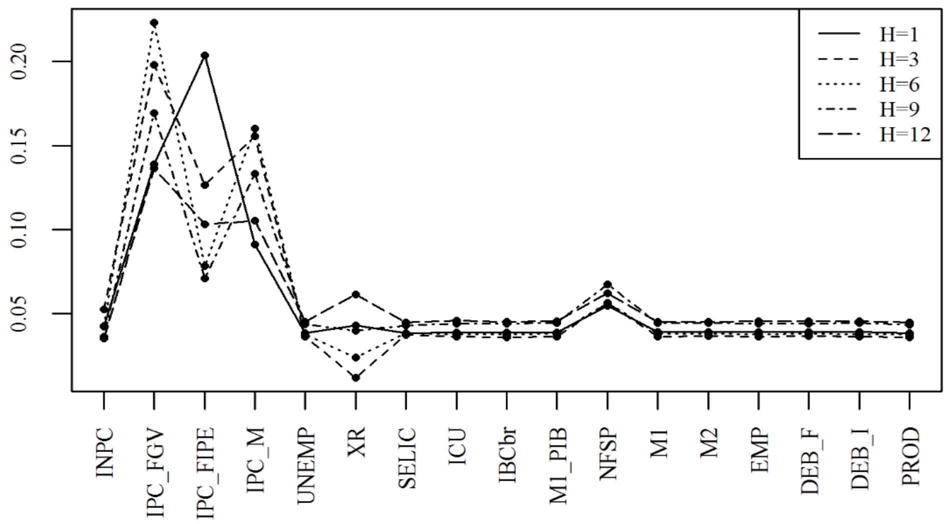
Note: This table presents the model confidence sets obtained by Hansen, Lunde and Nason (2011) procedure, for a confidence level of 20%.
Source: elaborated by the author (2016).

Figure 7- RMSSAu selected variables



Source: elaborated by the author (2016).

Figure 8 - VMSSAu selected variables



Source: elaborated by the author (2016).

APPENDIX D – DATA

Variables	Data	Source	Transformations
IPCA	Extended National Consumer Price Index	IBGE	Δ%
INPC	National Consumer Price Index	IBGE	Δ%
IPC_FGV	Consumer Price Index	FGV	Δ%
IPC_FIPE	Consumer Price Index	Fipe	Δ%
IPC_M	Consumer Price Index - Market	FGV	Δ%
UNEMP	Unemployment rate - six largest metropolitan areas	IBGE	X-12 ARIMA+diff_log
SELIC	Selic accumulated in the month in annual terms	BCB	diff_log
ICU	Installed capacity utilization, seasonaly adjusted	CNI	diff_log
IBCbr	Brazilian Economic Activity Index, seasonally adjusted	BCB	diff_log
M1_PIB	M1 money supply, end-of-period balance (% GDP)	BCB	X-12 ARIMA+diff_log
NFSP	Borrowing requirements of central government, total debt	STN	IGP-DI deflation+X-12 ARIMA+diff_log
M1	M1 money supply, working day balance average	BCB	IGP-DI deflation+X-12 ARIMA+diff_log
M2	M2 broad money supply, end-of-period balance	BCB	IGP-DI deflation+X-12 ARIMA+diff_log
EMP	Industrial employment index, seasonaly adjusted	CNI	diff_log
DEB_F	Fiscal net debt (% GDP)	BCB	diff_log
DEB_I	Net public debt (% GDP) - Internal - public sector	BCB	diff_log
PROD	Quantum index of industrial physical production, seasonaly adjusted	IBGE	diff_log

Constructed variables	Comments
XR	Exchange rate pass-through constructed from exchange (sell) rate (BCB) and seasonally adjusted finished goods PPI (U.S. Bureau of Labor Statistics) data. The variable is calculated as the sum of the log-differenced exchange rate and the north american PPI.