A Soft Bag Quark-Meson Coupling Model for High Density Nuclear Matter

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Abstract. The structure of high density nuclear matter is studied in the framework of a new approach, a mean-field description of non-overlapping neutron and proton bags with soft surfaces bound by the self-consistent exchange of the σ, ω and ρ meson fields. We consider in this formulation each bag with an internal region, where quarks are free, a finite surface region, where the quark wave functions are gradually suppressed, and an exterior region, where the quark wave functions vanish. The finite surface region with a typical scale of about half a Fermi models the smooth transition region between the perturbative and non-perturbative vacua, as expected from a gradual outside-inside restoration of chiral symmetry. As an application we determine the structure of the equation of state for neutron stars. We then consistently include in the formulation lepton degrees of freedom, the chemical equilibrium conditions for neutron stars and baryon number and electric charge conservation. We investigate properties of both nuclear and neutron matter. Using the TOV equations, the role of the softness of quark bags degrees of freedom on bulk static properties of neutron stars are determined.

NEUTRON STARS WITH A BAG

The starting point of our analysis is the lagrangian density model formulated in Ref.[1] combined with scalar-isoscalar, vector-isoscalar and vector-isovector meson-quark couplings[2]:

$$
\begin{aligned}
\mathscr{L}_{FBM} &= \frac{i}{2}[\bar{\psi}_q\gamma^\mu\partial_\mu\psi_q-\partial_\mu\bar{\psi}_q\gamma^\mu\psi_q]-B(r)F(r)-\bar{\psi}_q[m_q-g^q_\sigma\sigma+(1+\gamma^0)V(r)]\psi_q \\
&+ \frac{i}{2}[\bar{\psi}_q\gamma^\mu(ig^q_\omega\omega_\mu+ig^q_\rho\frac{1}{2}\tau\cdot\rho_\mu)\psi_q]+\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma-\frac{1}{2}m^2_\sigma\sigma^2 \\
&- \frac{1}{2}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}m^2_\omega\omega_\mu\omega^\mu-\frac{1}{4}\mathbf{L}_{\mu\nu}\cdot\mathbf{L}^{\mu\nu}+\frac{1}{2}m^2_\rho\rho_\mu\cdot\rho^\mu+\sum_\lambda\bar{\psi}_\lambda[i\gamma_\mu\partial^\mu-m_\lambda]\psi_\lambda
\end{aligned}
$$

with $F_{\mu\nu}=\partial_\mu\omega_\nu-\partial_\nu\omega_\mu$, $\mathbf{L}_{\mu\nu}=\partial_\mu\rho_\nu-\partial_\nu\rho_\mu$. In these expressions: ψ_q, σ, ω and ρ represent the quark, scalar-isoscalar, vector-isoscalar and vector-isovector meson fields and g^q_σ, g^q_ω, g^q_ρ are the coupling constants; moreover, as leptons are an important ingredient for maintaining chemical equilibrium and charge neutrality in neutron stars, these particles are included in the formalism with ψ_λ representing lepton fields. The main steps in our formulation involve: a) the determination of the Euler-Lagrange equations

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and their solutions for a given confining potential or, alternatively, for chosen suppression functions $F(r)$ and $G(r)$ (with appropriate distributional behaviors); b) the calculation of the components of the energy-momentum tensor in the mean-field approximation (in the following discussion in this section, the mean fields σ, ω and ρ fields are denoted by $\bar{\sigma}$, $\bar{\omega}$ and $\bar{\rho}$; for the details see[2, 3]); c) the determination of the EoS for nuclear matter.

The Pauli principle induces Fermi motion of the nucleons. Thus the energy of a moving bag with momentum $\mathbf{k}$ is $\varepsilon(\mathbf{k}) = \sqrt{M_N^{*2} + \mathbf{k}^2} + 3g_\omega^q \bar{\omega} \pm 1/2 g_\rho^q \bar{\rho}$, where $(+)$ refers to protons (p) and $(-)$ to neutrons (n) and M_N^* to the nucleon effective mass. The time-like mean-field component of the vector field is determined by imposing baryon number conservation from which we get $\bar{\omega} = g_\omega \rho_B / m_\omega^2$, with $g_\omega = 3g_\omega^q$, while the $\bar{\sigma}$ and $\bar{\rho}$ mean-fields are determined from the thermodynamic condition:

$$\left(\frac{\partial E_{tot}}{\partial \bar{\sigma}}\right)_{R,\rho_B} = \left(\frac{\partial E_{tot}}{\partial \bar{\rho}}\right)_{R,\rho_B} = 0 \; ; \; \bar{\rho} = \frac{g_\rho}{2m_\rho^2}\rho_3$$

applied to the total energy per nucleon E_{tot} of nuclear matter[3]. We obtain

$$\bar{\sigma} = \frac{-\gamma}{(2\pi)^3 m_\sigma^2}\left(\int^{k_{F_p}} d\mathbf{k}_p \frac{M_p^*}{\sqrt{M_p^{*2} + \mathbf{k}_p^2}}\left(\frac{\partial M_p^*}{\partial \bar{\sigma}}\right)_R + \int^{k_{F_n}} d\mathbf{k}_n \frac{M_n^*}{\sqrt{M_n^{*2} + \mathbf{k}_n^2}}\left(\frac{\partial M_n^*}{\partial \bar{\sigma}}\right)_R\right)$$

and

$$E_{tot} = \frac{\gamma}{\rho_B (2\pi)^3}\left(\int^{k_{F_p}} d\mathbf{k}_p \sqrt{M_p^{*2} + \mathbf{k}_p^2} + \int^{k_{F_n}} d\mathbf{k}_n \sqrt{M_n^{*2} + \mathbf{k}_n^2}\right) + \frac{m_\sigma^2 \bar{\sigma}^2}{2\rho_B} + \frac{g_\omega^2 \rho_B}{2m_\omega^2} + \frac{g_\rho^2 \rho_3^2}{8m_\rho^2 \rho_B}.$$

The pressure can be found from $p = \rho_B^2 (\partial E_{tot}/\partial \rho_B)$, from which we get the EoS of nuclear matter. In this expression, $\rho_3 = \rho_p - \rho_n$, $m_\sigma = 550 MeV$, $m_\omega = 783 MeV$ and $m_\rho = 770 MeV$. Moreover,

$$\left(\frac{\partial M_N^*}{\partial \bar{\sigma}}\right)_R = -g_\sigma \frac{E_{bag}}{M_N^*}\left[\left(1 - \frac{E_q}{E_{bag} R}\right) S(\bar{\sigma}) + \frac{m_q^*}{E_{bag}}\right] = -g_\sigma C_N(\sigma)$$

with $g_\sigma \equiv 3g_\sigma^q S_N(0)$, $C_N(\sigma) \equiv S_N(\sigma)/S_N(0)$ and

$$S(\bar{\sigma}) = \frac{E_q/2 + R m_q^*(E_q - 1)}{E_q(E_q - 1) + R m_q^*/2}.$$

E_{bag} is defined as

$$E_{bag} = M_N = \sum_q E_q + \frac{2}{3}\sum_q \int_{R_0}^{R_1} dr\, r\, u(r)^2 \frac{d\mathscr{V}_c(r)}{dr} - \frac{Z}{R}$$

where the parameter Z accounts for the energy of the zero-point motion and m_q^* is the quark effective mass $m_q^* = m_q - g_\sigma^q \bar{\sigma}$. Details on the quark substructure of the nucleons are thus entirely contained in the effective coupling $(\partial M_N^*/\partial\bar{\sigma})_R$. The coupling constants, $g_\sigma = 3g_\sigma^q$ and g_ω, are determined to fit the binding energy of nuclear matter (-16 MeV) at the saturation density, ρ_0=0.17 fm^{-3}. If the nucleon were simply made of three massive constituent quarks, the nucleon mass in vacuum, M_N, and that in the meson condensate, M_N^*, would have to satisfy $M_N \approx 3m_q$, $M_N^* \approx 3m_q^*$ and hence the effective nucleon mass would be[4] $M_N^* = M_N - g_\sigma\bar{\sigma}$ where we have defined $g_\sigma = 3g_\sigma^q$. Since one finds, from the expression of the nucleon effective mass, $(\partial M_N^*/\partial\bar{\sigma}) = -g_\sigma C_N(\sigma)$, the following self-consistent condition (SCC) then holds

$$g_\sigma\bar{\sigma} = -\frac{\gamma g_\sigma}{(2\pi)^3 m_\sigma^2}\left(\int^{k_{F_p}} d\mathbf{k}_p \frac{M_p^*}{\sqrt{M_p^{*2}+\mathbf{k}_p^2}}\left(-3g_\sigma^q S_p(\sigma) = -g_\sigma C_p(\sigma)\right)_R + \int^{k_{F_n}} d\mathbf{k}_n \frac{M_n^*}{\sqrt{M_n^{*2}+\mathbf{k}_n^2}}\left(-3g_\sigma^q S_n(\sigma) = -g_\sigma C_n(\sigma)\right)_R\right) \quad (1)$$

with the multiplicity $\gamma = 2$ for neutron matter and $\gamma = 4$ for nuclear matter. The effective nucleon mass is, in this case, given by $M_N^* = M_N - 3g_\sigma^q S_N(0)\,\bar{\sigma}$ or equivalently by $M_N^* = M_N - (C_N(\sigma)/S_N(\sigma))\,\bar{\sigma}$. Neutron star configurations may then be obtained by combining the resulting EoS for nuclear matter with the TOV equations, chemical equilibrium, baryon number and electric charge conservation.

RESULTS AND CONCLUSIONS

Our results[2] indicate that the softness of the surface of the bag has an influence in the bulk static properties of nuclear matter under extreme conditions, as in neutron stars. Our quark-meson coupling model, with protons and neutrons confined in soft bags and interacting with scalar, vector and isovector mesons, predicts a maximum mass for neutron stars around $1.85M_\odot$. The corresponding results of the MIT bag model and the non-linear QHD model of Glendenning (see Refs. in [2]) are respectively $1.4M_\odot$ and $1.9M_\odot$. Improvement in our results may be achieved by the inclusion of the fundamental baryon octect, gluon degrees of freedom and trapped neutrinos in chemical equilibrium. The present version of the model provides already a reasonable description of the role of quark bags with soft surfaces in high-density nuclear matter.

REFERENCES

1. F.G. Pilotto, *Eur. Phys.J.* **C28**:535-543,2003.
2. D.T. da Silva, D. Hadjimichef, M. Razeira, M. Dillig and C.A.Z. Vasconcellos, *Int. J. Mod. Phys.* **D** (2004) (accepted for publication).
3. P.A.M. Guichon, K. Saito, E. Rodionov, A.W. Thomas, *Nucl. Phys.* **A601**, 349 (1996).
4. J.D. Walecka, *Ann. Phys.* **83**, 491 (1974).