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The controlling role of envelope mismatches in intense inhomogeneous charged beams

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Inhomogeneous cold beams undergo wave breaking as they move along the axis of a magnetic focusing system. All the remaining control parameters fixed, the earliest wave breaking is a sensitive function of the inhomogeneity parameter: the larger the inhomogeneity, the sooner the breaking. The present work analyzes the role of envelope size mismatches in the wave breaking process. The analysis reveals that the wave breaking time is also very susceptible to the mismatch; judiciously chosen mismatches can largely extend beam lifetimes. The work is extended to include recently discussed issues on the presences of fast and slow regimes of wave breaking, and the theory is shown to be accurate against simulations. © *2010 American Institute of Physics*.
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A fundamental issue in the dynamics of magnetically focused beams designed to meet requirements in vacuum electronics, concerns relaxation when the beam profile is not homogeneous.[1] On general grounds of energy conservation one concludes that relaxation takes place as the coherent fluctuations of beam inhomogeneities are converted into particle kinetic energy and field energy.[2] Recent works actually show that in the case of cold beams relaxation proceeds in two basic steps.[3] Wave breaking initially pushes particles off the beam, and then ejected particles form a relaxing hot halo as they absorb energy from macroscopic oscillations of the remaining beam core. Wave breaking is therefore a key feature in the process.

Attention has been mostly drawn to the effects the degree of inhomogeneity has on wave breaking. Two instances were then identified. Originally, a threshold was obtained in terms of gradients in the amplitude of waves propagating across the beam.[4] As particles largely displaced from their equilibrium positions are released, they overtake each other in less than one plasma wave cycle creating density singularities and wave breaking; for small displacements, breaking is absent. A more thorough analysis however shows that not only amplitude gradients, but also formerly neglected gradients of the spatially varying frequency of the density waves is a key factor determining wave breaking.[3,5] The physical process is different from the previous, as one shows that no threshold exists in this latter case. Particles slowly move out of phase due to small differences in their oscillatory frequencies, until a time when one eventually catch up with another creating again infinite densities and breaking.

In addition to the inhomogeneity effect, one should also note that since wave breaking is essentially dictated by compressions and rarefactions of beam densities, it may be quite possible that expansions or contractions of the beam transversal size have a noticeable effect on the process. In particular we will show that, contrarily to the homogeneous beam case where envelope mismatch is an undesirable feature, for inhomogeneous beams it may largely delay wave breaking, extending beam lifetime.

We focus on crystalline cold beams which have been attracting a growing amount of interest lately,[6] also serving as a step towards warmer beams. Crystalline beams are not only cold, but also spatially ordered. Consider thus an axially symmetric, collisionless, unbunched beam moving with constant velocity along z. In the para-axial approximation the equation for the radial motion of any cylindrical layer takes the form[3]

$$r'' = -\kappa r + Q(r)/r. \tag{1}$$

Primes indicate derivatives with respect to z for stationary beams. $Q(r)$ is a measure of the total charge up to radial layer r. It reads $Q(r)=KN(r)/N_t$, where $K=N_t q^2/\gamma^3 m\beta^2 c^2$ is the beam perveance, with $N(r)$ denoting the number of particles up to radial coordinate r, and N_t their total number. q and m denote the beam particle charge and mass, respectively. $\gamma=(1-\beta^2)^{-1/2}$ is the relativistic factor where $\beta=v_z/c$, v_z is the constant axial beam velocity, and c is the speed of light. $\kappa\equiv(qB/2\gamma m\beta c^2)^2$ where B is the constant axial focusing magnetic field. Beams with perfectly matched envelopes are the ones for which the initially farthest radial layer r_{b0} is in equilibrium: $r_{b0}^2=K/\kappa$ from Eq. (1).

We suppose that the beam starts off from rest as a cold fluid. Then, while particles do not overtake each other, $Q(r,z)$ may be evaluated for any layer at r as the initial value $Q(r_0)$, where $r(z=0)\equiv r_0$. In a likewise fashion, one can compute the amount of charge contained between two neighbor layers located at r and $r+dr$ in the form $dQ=2\pi r\rho(r,z)dr=2\pi r_0\rho(r_0,0)dr_0$, where ρ denotes the charge (Q) density of the system. The expression for dQ tells us that the density evolves as

$$\rho(r,z)=\rho(r_0,0)(\partial r/\partial r_0)^{-1}(r_0/r). \tag{2}$$

Equation (2) reveals that the density function develops a singularity when the orbital equation $r=r(r_0,z)$ becomes multi-

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valued with $\partial r/\partial r_0=0$. This point corresponds to a potential barrier not all particles can move across. Some particles are reflected relaxing the beam via kinetic effects associated with emittance growth.

So, it all depends on the behavior of the compressibility factor $\partial r/\partial r_0$ as a function of "time" z. An approximate solution for small oscillations can be obtained from Eq. (1) in the fluid state where $Q(r)$ can be replaced with $Q(r_0)$ as explained earlier

$$r(z) \approx r_{\mathrm{eq}} + A\cos(\omega z). \tag{3}$$

The solution describes an oscillatory motion of amplitude $A\equiv r_0-r_{\mathrm{eq}}$ around an equilibrium point r_{eq} promptly recognized as $r_{\mathrm{eq}}=\sqrt{Q(r_0)/\kappa}$ from Eq. (1). The amplitude depends on r_0, and the nonlinearly corrected frequency also does: canonical perturbative theories show that[3,7] $\omega(r_0)=\sqrt{2\kappa}+\sqrt{\kappa}A^2/(6\sqrt{2}r_{\mathrm{eq}}^2)$. Therefore, if from Eq. (3) one writes down the compressibility factor one arrives at

$$\frac{\partial r}{\partial r_0} = \frac{\partial r_{\mathrm{eq}}}{\partial r_0} + \frac{\partial A}{\partial r_0}\cos(\omega z) - z\frac{\partial \omega}{\partial r_0}A\sin(\omega z). \tag{4}$$

If the inhomogeneity in A is sufficiently large that $\partial A/\partial r_0 > \partial r_{\mathrm{eq}}/\partial r_0$, wave breaking takes place within a cycle of oscillation as the cosine's phase slips from zero towards π. In this case the last term on the right hand side of Eq. (4) can be safely neglected as a small $\mathcal{O}(A^3/r_{\mathrm{eq}}^3)$ quantity. The threshold condition for fast wave breaking dominated by the amplitude gradient can also be written in the convenient form $Q^{-1/2}(\partial Q/\partial r_0)<\sqrt{\kappa}$. In typical configurations of beams with humped cores and dilute populations near the border, $\partial Q/\partial r_0|_{r_0\to r_{b0}}\to 0$ and the condition for wave breaking is easily satisfied there. This is the fast regime analyzed in Ref. 4, as mentioned earlier. In addition to this fast regime, another clear fast regime is found as one considers hollow beams, where densities are extremely high near the beam border, but small at the center. Looking at the beam center where $Q\sim\rho r_0^2$ for a local density ρ, one sees that the wave breaking condition is fulfilled there as $\rho\to 0$.

When the threshold for the fast wave breaking is not attained across the beam, a simple oscillatory process cannot bring the compressibility factor to zero. This is where the last term of Eq. (4) begins to play its crucial role. Corrections to the frequency are small as mentioned, but the respective term present in Eq. (4) grows linearly with the time z. Thus, no matter how small is the inhomogeneity, for sufficiently long periods of time the term involving the frequency gradient becomes large enough that $\partial r_{\mathrm{eq}}/\partial r_0 \sim z^* A\,\partial\omega/\partial r_0$ for a given $z^*=z^*(r_0)$,

$$z^* \sim (\partial r_{\mathrm{eq}}/\partial r_0)/(A\,\partial\omega/\partial r_0). \tag{5}$$

Neglecting the nonsecular term, at this point the wave breaking singularity $\partial r/\partial r_0=0$ is reached again. The earliest breaking time is the one of physical relevance. It is obtained here as the minimum of $z^*(r_0)$ over all r_0's in the form $z_{\mathrm{wb}}\equiv\min_{r_0}[z^*(r_0)]$, from which convenient approximations shall be discussed later.

For now, let us summarize our findings based on typically varying beam profiles. (i) Starting from humped core beams with very low densities at the borders, wave breaking is fast and occurs at the beam border.[4] (ii) Next, as one diminishes the density contrast between beam core and beam border, one enters a slow regime where the rapidly oscillating compressibility factor modulates linearly with z, reaching the wave breaking state $\partial r/\partial r_0=0$ after long time periods.[3] Importantly, the slow regime of wave breaking does not involve any threshold. As long as beam inhomogeneities are present, the beam is bound to undergo wave breaking. (iii) Finally, with further increase in the density contrast, now with higher densities near the border, a different zone of fast wave breaking is reached where breaking occurs near the beam center.

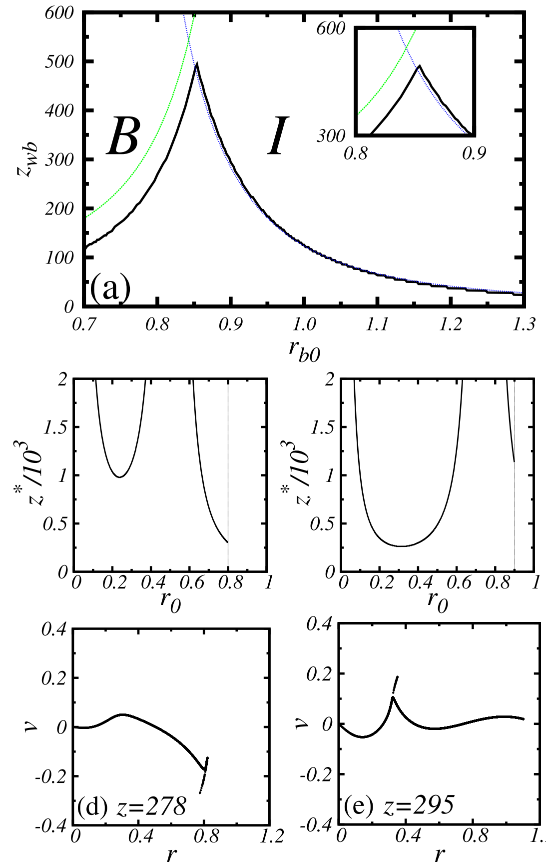

FIG. 1. (Color online) Wave breaking time vs initial beam radius in panel (a). $z^*=z^*(r_0)$ for $r_{b0}=0.8$ in (b) and for $r_{b0}=0.9$ in (c). Beam phase-spaces just after the breaking: $r_{b0}=0.8$ in panel (d) and $r_{b0}=0.9$ in (e). In all cases, $\chi=0.6$. Simulations are based on Gauss's law using 50000 cylindrical shells.

We now add the effect of a mismatched beam border and investigate the effect in slow regimes where it is prominent. The density profile is presently needed and is specified in the general parabolic form $\rho(r_0\le r_{b0})=2K/\pi r_{b0}^2[1+\chi(2r_0^2/r_{b0}^2-1)]$, $\rho(r_0>r_{b0})=0$, where r_{b0} is the initial beam size and $-1\le\chi\le+1$ measures the degree of inhomogeneity; $\chi\to-1$ for humped, $\chi=0$ for homogeneous, and $\chi\to+1$ for hollow beams. With $Q(r_0)=\int_0^{r_0}2\pi r\rho(r)dr$ we see that the slow region lies within the limits $\chi_{\min}=-0.5$ and $\chi_{\max}=0.75$ for the matched beam $r_{b0}=\sqrt{K/\kappa}$. With that information we construct Fig. 1(a) using $\chi=0.6$, where z_{wb} is displayed as a function of beam size; in all numerics, radial coordinates are given in units of $\sqrt{K/\kappa}$ and z in units of $\kappa^{-1/2}$. Note that because of our choice of the inhomogeneity $\chi=0.6$, we do fall in a slow region, at least in the vicinity of the matched beam. The thick line is obtained exactly as one integrates Eq. (1) and its derivative with respect to r_0, looking for the earliest time where $\partial r/\partial r_0\to 0$, all in the fluid state where we can replace $Q(r)\to Q(r_0)$. The thin lines are based on the perturbative solution Eq. (5) and approximate the exact curve on the right and left hand sides of the peak. In addition to the peak the plot reveals strong sensitivity to the choice of r_{b0}. We note that the matched beam is not the one with the largest lifetime before breaking. The longest living beam is the one at the peak where $r_{b0} \approx 0.85$, and its breaking time is around five times larger than the matched beam's time.

The reason for the sharp peak can be understood in panels (b) and (c) where we plot the local wave breaking time $z^*(r_0)$ as a function of the initial position of the corresponding fluid element; we recall that the smallest $z^*(r_0)$ is of major physical significance. Panel (b) represents point $r_{b0} = 0.8$, on the left side of the peak. For this point and all others on the left side the earliest breaking occurs at the beam border (B). Panel (c) represents point $r_{b0}=0.9$ on the right of the peak and reveals that the earliest breaking time for this point (and all others on the right side) occurs in the inner (I) body of the beam. The curves for $r_{b0}<1$ always reveal two local minima separated by a divergent z^* corresponding to a fixed equilibrium point r_{fp} located inside the beam; that portion of the beam extending up to the fixed point behaves like a matched beam of radius r_{fp} and, as one can show, a renormalized $\chi \rightarrow 1+(\chi-1)K/\kappa r_{b0}^2$. This allows to obtain the wave breaking time z_{wb}^I in the inner region (right-hand side approximation) as

$$z_{\text{wb}}^I = \min_{r_0<r_{\text{fp}}} [z^*(r_0)], \tag{6}$$

a form that can benefit from minimizing procedures previously applied to fully matched beams.[3] As for the breaking at the beam border z_{wb}^B (left-hand side approximation) one simply evaluates

$$z_{\text{wb}}^B = z^*(r_0 = r_{b0}). \tag{7}$$

There is thus an abrupt transition at the peak, where the beam simultaneously breaks at the center and at the border: r_{b0} at the peak is obtained from $z_{\text{wb}}^I = z_{\text{wb}}^B$. Full simulations in panels (d) (r_{b0}=0.8) and (e) (r_{b0}=0.9), further clarify the case, again indicating the abrupt change in location of the initial breaking (where the jets are seen) across the peak. When $\chi<0$ the behavior is reversed, but otherwise equivalent, with the internal fixed point appearing when the beam is stretched with $r_{b0}>1$.

We can now investigate beam inhomogeneity and size in a unified way. To do so we construct Fig. 2, where the earliest breaking time is coded in colors, as a function of the control parameters r_{b0} and χ. A wide parametric extension is covered enabling the see the fast wave breaking regions and all details of the slow region. The analytical bent dotted line represents the loci of the largest wave breaking time. Expectations are confirmed: wave breaking strongly depends not only on the beam profile χ, but also on the beam size r_{b0}. And more: given χ Fig. 2 teaches how a judicious mismatching applied to r_{b0} may help to control detrimental effects of nonuniformities across the beam section. We also note that near the borders, a small shift in r_{b0} can bring the system from a fast to the slower region and that previous estimates for the matched beam are accurate.

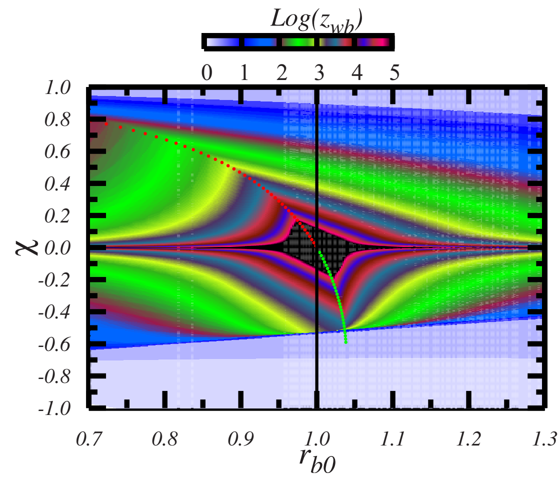

FIG. 2. (Color online) Color coded wave breaking time map in the $\chi \times r_{b0}$-space. Doted line comes from the analytical approach, and indicate the loci of maximum wave breaking time. Black means $z_{\text{wb}} > 10^5$.

To summarize, we investigated two types of wave breaking cases in space charge inhomogeneous beams: a fast breaking commanded by amplitude gradients and a slow breaking commanded by frequency gradients. The latter has no threshold and is bound to happen no matter how small is the beam nonuniformity. Then, in all instances we showed how a judiciously chosen envelope size mismatch can significantly extend the beam life time as compared with the matched case.

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[1]S. M. Lund, D. P. Grote, and R. C. Davidson, Nuc. Instrum. Methods Phys. Res. A **544**, 472 (2005); M. Hess and C. Chen, Phys. Plasmas **13**, 053108 (2006); J. Zhou, K. R. Samokhvalova, and C. Chen, *ibid.* **15**, 023102 (2008).
[2]S. Bernal, R. A. Kishek, M. Reiser, and I. Haber, Phys. Rev. Lett. **82**, 4002 (1999).
[3]F. B. Rizzato, R. Pakter, and Y. Levin, Phys. Plasmas **14**, 110701 (2007); R. P. Nunes R. Pakter, F. B. Rizzato, A. Endler, and E. G. Souza, *ibid.* **16**, 033107 (2009).
[4]O. A. Anderson, Part. Accel. **21**, 197 (1987); S. G. Anderson and J. B. Rosenzweig, Phys. Rev. ST Accel. Beams **3**, 094201 (2000).
[5]J. M. Dawson, Phys. Rev. **113**, 383 (1959).
[6]H. Okamoto, Phys. Plasmas **9**, 322 (2002).
[7]F. B. Rizzato and R. Pakter, Phys. Rev. Lett. **89**, 184102 (2002).