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Kinetic theory of magnetized dusty plasmas with dust particles charged by collisional processes and by photoionization

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In this work, we detail the derivation of a plasma kinetic theory leading to the components of the dielectric tensor for a magnetized dusty plasma with variable charge on the dust particles, considering that the dust component of the plasma contains spherical dust particles with different sizes, which are charged both by inelastic collisions of electrons and ions and by photoionization. [http://dx.doi.org/10.1063/1.4748932]

I. INTRODUCTION

In a dusty plasma, the charge of the dust grains is acquired due to the interaction with other plasma particles and with external agents, like electromagnetic radiation. The charging process has an important role in perturbative processes and affects plasma electromagnetic and dynamical properties. A more complete study of dusty plasmas should take these charging processes into account. The electrical charge of a dust grain is determined by the current balance at its surface, which can be characterized as divided into emission and absorption processes. In a dusty plasma where for instance electron emission is not significant the equilibrium charge is predominantly negative, due to the fact that in an electrically neutral plasma the mobility of electrons toward the neutral particles is higher than that of ions. On the other hand, if electron emission is important the dust charge can be positive, what can significantly change the plasma characteristics. In nature dusty plasmas like these can be found in regions where ultraviolet radiation is significantly present, like Earth's ionosphere, interstellar medium, cometary environments, etc. The solar system, for instance, constitutes a prominent and astronomically close example of the simultaneous occurrence of these competitive effects. The dust particles of interstellar origin moving through the heliosphere collect ions and electrons from the ambient solar wind plasma. On the other hand, there is incidence of ultraviolet electromagnetic radiation from solar origin, working to make the dust particles a source of electrons. In fact, in the solar wind plasma, the amount of emitted photoelectrons is higher than the loss of electrons by absorption, with the consequence that the dust particles acquire a positive charge.[21] Moreover, taking into account that both the intensity of solar ultraviolet radiation and the solar wind plasma density decrease quadratically with increasing distance to the Sun, it can be concluded that the charge of the dust particles stays in equilibrium.[12,21]

Dusty plasmas can be classified into two different categories, depending on the relationship between the Debye length and other characteristic distance scales.[14,22] If $a \ll \lambda_D < r_d$, where a is the radius of the dust grain, λ_D the Debye length, and r_d the average distance between dust grains, the dust may be considered as a collection of isolated grains with plasma shielding. On the other hand, if $a \ll r_d < \lambda_D$, the dust itself contributes to the plasma shielding and then plays a role in its collective behaviour.

The emission of electrons by dust particles increases the density of the population of plasma electrons. In fact, in 1937 Jung[13] already considered that photoemission in interstellar plasmas was the predominant process to the accretion of plasma electrons. In 1941 and 1948, he conducted exhaustive investigations about ionization of solid particles due to photoelectric effect, by making the simplification that all particles were equally charged and neglecting the charge influence on the emission process. Many investigations have been made about dust charging by photoionization;[5,6,15] most of them assuming that the velocity distribution of the emitted photoelectrons is a Maxwellian distribution.

A critical review about the theory of photoelectric emission by metals was made by Dewdney,[1] and it was based on Fowler's hypothesis.[3] First, the probability of absorption of a photon by an electron on the material surface is independent of the electron's initial state. Second, the photon energy absorbed by the electron increases its kinetic energy in the direction normal to the surface. Third, electrons whose kinetic energy in the direction of the normal is greater than the surface potential barrier ψ will cross the surface. In the analysis, Dewdney has considered that the photoelectrons obey the Fermi-Dirac distribution. Several authors with recent works follow this approach, which also take into account the effect of the dust particle charge over the electron emission, like Sodha *et al.*[18,19] and Ignatov.[7]

In the present paper, we utilize these developments in order to obtain a kinetic formulation for dusty plasmas in which the dust particles can be charged by inelastic collisions with plasma electrons and ions, and by photoionization. We start from basic principles and proceed by presenting details of the derivation. Some of the material to be presented in the paper, connected with dust plasmas charged by collisional processes, can be found in the literature, but will be briefly reproduced here for completicity and in order to provide a framework into which to introduce the

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novelties of the formulation, mostly connected to the introduction of the photoemission process into the kinetic theory for a magnetized plasma.

The plan of the paper is the following. In Sec. II, we define the processes of charging of dust particles which will be considered in the formulation. In Sec. III, we set up general features of the formulation. In Sec. IV, we present details of the derivation of the linear theory of plasma waves in the magnetized dusty plasma, including detailed derivation of the components of the dielectric tensor. In Sec. V, we present some final remarks and commentaries about future developments and applications of the theory.

II. CHARGING PROCESSES

The starting point to every study about dusty plasmas is the definition about which dust charging processes will be considered. Dust grains immersed in a plasma can either absorb or emit ions and electrons. Therefore, the dust charge is determined by the balance of the currents of charged particles at its surface. The rate of variation of the dust electric charge can be determined by the equation[14,25]

$$\frac{dQ}{dt} = \sum_{\alpha} I_{\alpha} - \sum_{\beta} I_{\beta}^{out} = I, \tag{1}$$

where Q is the average charge of the dust grains, I is the average current at the surface, and $\sum_{\alpha} I_{\alpha}$ and $\sum_{\beta} I_{\beta}^{out}$ are, respectively, the total absorption and total emission currents. The absorption currents I_{α} are assumed to be due to inelastic collisions with plasma particles. As emission current, we consider only the current due to the emission of electrons by photoionization. The equilibrium charge of the dust particle is determined under the condition that the surface current vanishes, namely that

$$\sum_{\alpha} I_{\alpha} = \sum_{\beta} I_{\beta}^{out}. \tag{2}$$

In Eq. (1), we have used the convention that inward currents are positive, such that ions that strike the dust grain contribute with positive charge. On the other hand, electrons that strike the grain give a negative contribution.

The negative sign in I_{β}^{out} explicitly indicates that the charge carriers move in the opposite direction than those contributing to I_{α}. For example, by considering the photoemission process, one emitted electron contributes with charge e to the total charge of the dust particle. So the particle emission process gives contributions which are opposite to the sign of emitted particle charge.

The current at the dust grain surface depends on many plasma and grain conditions. For example, the absorption of plasma particles depends not only on the shape and size of the grains but also on the density and velocity distributions of the plasma particles, on the dust grain motion relative to the plasma and on the difference of potential between the plasma and the surface of the grain. The photoelectric current depends on the surface potential and also on the intensity and frequency of the incident radiation.[16]

The collisions of plasma particles with the dust grains can be described by the orbital limited motion theory (OLM). Here we make a brief description of this method, which assumes that (1) the dust grain is isolated in such a way that other dust grains do not affect the motion of ions and electrons; (2) electrons and ions in course of collision do not interact with others particles; (3) independently of the plasma electrostatic potential structure next to the dust particle, any plasma particle can hit the dust grain if permitted by conservation laws and, in that case, it will be attached to the dust grain; and (4) for spherical dust particles, the impact parameter of a plasma particle corresponds to a trajectory tangential to the dust grain. An advantage of this approach is that the collision cross section can be found by using only the laws of conservation of energy and angular *momentum*, independently of the complexity and non-linearity of the plasma potential next to the dust grain.[22]

In order to obtain an expression for the collision cross section, let us consider a particle with charge q_{α} that approaches at a long distance a spherical dust grain with charge q and radius a. Let c_{α} be the impact parameter, and v_{α} and v_{α}^{d} be, respectively, the velocity before and after collision of a particle of type α with the dust grain. For a given velocity v_{α} there is a maximum value of c_{α} for which the plasma particle will collide with the dust grain. Then by conservation laws

$$m_{\alpha} v_{\alpha} c_{\alpha} = m_{\alpha} v_{\alpha}^{d} a, \tag{3}$$

$$\left(\frac{v_{\alpha}^{d}}{v_{\alpha}}\right)^2 = 1 - \frac{2 q_{\alpha} \varphi}{m_{\alpha} v_{\alpha}^2}, \tag{4}$$

where $\varphi = \varphi_s - \bar{\varphi}$, the difference between the potential at the surface of the grain and the average plasma potential.

In this process, two cases can be distinguished: attraction between the dust and the plasma particle when $q_{\alpha}\varphi < 0$, and repulsion when $q_{\alpha}\varphi > 0$. In the last case, Eq. (4) imposes the condition

$$v_{\alpha} > \sqrt{\frac{2 q_{\alpha} \varphi}{m_{\alpha}}} = v_{\alpha}^{min}, \quad q_{\alpha}\varphi > 0. \tag{5}$$

Since the collision cross section of a spherical particle of radius a is given by $\sigma_{\alpha} = \pi c_{\alpha}^2$, it follows from Eqs. (3)–(5) that the cross section for inelastic collisions between particles of species α and a dust particle is given by

$$\sigma_{\alpha}(p, q) = \pi a^2 \left(1 - \frac{2 m_{\alpha} q_{\alpha} q}{a p_{\alpha}^2}\right) \Theta\left(1 - \frac{2 m_{\alpha} q_{\alpha} q}{a p_{\alpha}^2}\right), \tag{6}$$

where $p_{\alpha} = m_{\alpha} v_{\alpha}$, q is the charge of the grain, and Θ is the step function.

For the description of the photoelectrical charging of a dust particle, we take into account that when the surface of a given material is hit by radiation, electrons can be emitted if the energy of the radiation is greater than the work function of the material. In the case of a charged spherical dust particle, with radius a, the emitted electron must have enough energy to overcome the electrostatic attraction by the grain, in such a way that

$$\frac{p^2}{2m_e} > \frac{eq}{a}, \tag{7}$$

where $-e$ and m_e are the charge and mass of the electron, respectively. Otherwise, it will be re-absorbed by the dust grain.

The number of electrons emitted by unit area by unit time is proportional to the intensity of the radiation. For the case in which the radiation is unidirectional, incident only in one hemisphere of the grain, we can write the cross section for emission of electrons with momentum p as

$$\sigma_P(p,q) = \pi a^2 \beta(\nu)\Lambda(\nu)\Theta\left(\frac{p^2}{2m_e} - \frac{eq}{a}\right), \tag{8}$$

where $\beta(\nu)$ is the probability of an electron which arrives to the surface coming from the inside to absorb a photon of frequency ν at the surface, and $\Lambda(\nu)$ is the number of photons of frequency ν incident by unit of area by unit of time. It is assumed that all incident radiation is absorbed, in such a way that the absorption coefficient can be taken as $S_a \approx 1$. For more details on the derivation of this cross section, see Appendix.

III. GENERAL FEATURES OF THE MODEL

We consider a dusty plasma in the presence of an external magnetic field of strength B_0, whose direction is taken to be the $\hat{z}$ direction, $\vec{B}_0 = B_0\hat{z}$, in an environment with incidence of anisotropic radiation, whose direction of propagation is taken by considering two limiting cases: perpendicularly and parallelly to the magnetic field direction. In our model, we consider that there are n dust populations characterized by different radius a_j and electric charge q_j.[4]

The dielectric tensor will be derived in the scope of kinetic theory, approach which has already been used to demonstrate that effects due to the dust charging can lead to significant modification in the damping of low frequency waves.[11] Our approach is based on previous works of Vladimirov[24] and of de Juli and Schneider,[8] from which we follow the collisional model and make the necessary modifications to include photoionization effects. More specifically, we include in the kinetic formulation a source term related to photoelectrons, modeled in such a way to satisfy the Vlasov equilibrium and requirements of photoelectric effect. Such a term can be written in the form

$$J_e^P = \beta(\nu)\Lambda(\nu)\int dq \sum_j \sigma_P^j(p,q)(f_d^j - f_{d0}^j)$$
$$\times \left[\frac{p_x}{m_e}\Theta(p_x)\delta_{1,-n_x} + \frac{p_z}{m_e}\Theta(p_z)\delta_{1,-n_z}\right]F(p), \tag{9}$$

where $\hat{n}_\Lambda \cdot \hat{x} = n_x$ and $\hat{n}_\Lambda \cdot \hat{z} = n_z$, with $\hat{n}_\Lambda$ being an unitary vector pointing in the direction of propagation of the radiation, and where $\beta(\nu)$, $\Lambda(\nu)$ and σ_P^j are quantities mentioned in Sec. II, which have a more detailed specification in Appendix. f_d^j and f_{d0}^j are, respectively, the distribution function for dust particles of species j and its equilibrium form. It is assumed that the emitted photoelectrons obey Fermi-Dirac statistics through the distribution

$$F(p) = \frac{2}{h^3}\left[1 + \exp\left(\frac{p^2}{2m_e k_B T_d} - \zeta\right)\right]^{-1}, \tag{10}$$

where $\zeta = (k_B T_d)^{-1}(h\nu - \phi)$.

In Eq. (9), we have assumed that radiation is incident from a given direction, striking only one side of the dust particle, and considered two limiting cases. One is such that the direction of radiation incidence is perpendicular to the direction of the external magnetic field. The other case assumes the parallel direction. The term $(p_x/m_e)\Theta(p_x)$ serves to assure that the emitted electrons have $p_x > 0$, in the case of incidence along the x-axis, and the term $(p_z/m_e)\Theta(p_z)$ plays the same role in the case of parallel incidence.

The development of a linear analysis will show that the perturbation of the distribution function of plasma particles of kind α splits into three terms. One is due to the self-consistent fields and the other two are due to the presence of dust. More specifically, one is due to the absorption currents of plasma particles and the other is due to the photoelectric emission current. The three parts of the distributions will be indexed, respectively, by the labels C, A, and P, that also will be used to identify the three terms of the dielectric tensor that correspond to the respective perturbative process.

IV. LINEAR ANALYSIS OF THE KINETIC EQUATIONS

The system defined by the Vlasov-Maxwell equations furnishes a closed system describing a plasma. For a dusty plasma, one must supply the kinetic equation for the dust component. In the case of different populations of dust particles with radius a_j, considering the non-relativistic limit, collisional charging of the dust particles and electron emission by photoelectric effect, and also considering motionless dust particles, the Vlasov-Maxwell system can be written as follows:

$$\frac{\partial f_d^j}{\partial t} + \frac{\partial}{\partial q}[I^j f_d^j] = 0,$$
$$\left\{\frac{\partial}{\partial t} + \frac{\vec{p}}{m_\alpha}\cdot\vec{\nabla} + q_\alpha\left[\vec{E} + \frac{\vec{p}}{m_\alpha c}\times\vec{B}\right]\cdot\vec{\nabla}_{\vec{p}}\right\}f_\alpha$$
$$= -\int dq\ \frac{p}{m_\alpha}\sum_j \sigma_\alpha^j(p,q)(f_d^j f_\alpha - f_{d0}^j f_{\alpha 0})$$
$$+ \delta_{\alpha e}\beta(\nu)\Lambda(\nu)\int dq\ \sum_j \sigma_P^j(p,q)(f_d^j - f_{d0}^j)$$
$$\times\left[\frac{p_x}{m_\alpha}\Theta(p_x)\delta_{1,-n_x} + \frac{p_z}{m_\alpha}\Theta(p_z)\delta_{1,-n_z}\right]F(p),$$
$$\nabla\cdot\vec{E} = 4\pi\sum_\alpha q_\alpha\int d\vec{p}\ f_\alpha + 4\pi\sum_j\int dq\ q f_d^j(\vec{r},q,t),$$
$$\nabla\times\vec{B} = \frac{1}{c}\frac{\partial}{\partial t}\vec{E} + \frac{4\pi}{c}\sum_\alpha\frac{q_\alpha}{m_\alpha}\int d\vec{p}\ \vec{p}f_\alpha,$$
$$\nabla\times\vec{E} = -\frac{1}{c}\frac{\partial}{\partial t}\vec{B},$$
$$\nabla\cdot\vec{B} = 0, \tag{11}$$

where

$$I^j(\vec{r},q,t) = \sum_\alpha I^j_\alpha(\vec{r},q,t) - I^j_P(q) \tag{12}$$

is the average current attaining the surface of a dust particle of radius a_j, and

$$I^j_P(q) = -e\beta(\nu)\Lambda(\nu)\int d\vec{p}\ \sigma^j_P(p,q) \times \left[\frac{p_x}{m_\alpha}\Theta(p_x)\delta_{1,-n_x} + \frac{p_z}{m_\alpha}\Theta(p_z)\delta_{1,-n_z}\right]F(p) \tag{13}$$

and

$$I^j_\alpha(\vec{r},q,t) = q_\alpha\int d\vec{p}\ \sigma^j_\alpha(q,p)\frac{p}{m_\alpha}f_\alpha$$

are the photoemission and absorption currents.

Assuming that the distribution functions can be written as a summation of an equilibrium part and a small perturbation, $f_\alpha = f_{\alpha 0} + f_{\alpha 1}$, and assuming the same for the fields, the system of Eq. (11) can be separated into a set of zeroth order equations and a set of first order equations. The equations for the $f_{\alpha 1}$ distributions can be solved by integrating over the unperturbed orbits. We introduce the *ansatz*[8]

$$f_{\alpha 1}(\vec{r},\vec{p},t) = e^{-\sum_j \nu^{j0}_{\alpha d}(p)t} f^{aux}_\alpha(\vec{r},\vec{p},t)$$

and after integration obtain the following form for the Fourier transform of the perturbation

$$f_{\alpha\vec{k}} = -\int_{-\infty}^0 d\tau\ e^{i\{\vec{k}\cdot\vec{R}-[\omega+i\sum_j \nu^{j0}_{\alpha d}(p)]\tau\}} \times \left\{q_\alpha\left[\bar{E}(\vec{k},\omega) + \frac{\vec{p}'}{m_\alpha c}\times\bar{B}(\vec{k},\omega)\right]\cdot\vec{\nabla}_{\vec{p}'}f_{\alpha 0}(\vec{p}) + \sum_j \hat{\nu}^j_{\alpha d}(\vec{k},\vec{p},\omega)f_{\alpha 0}(\vec{p}) - \sum_j \hat{\nu}^j_P(\vec{k},p,\omega)F(p)\right\}, \tag{14}$$

where we have defined

$$\hat{\nu}^j_{\alpha d}(\vec{k},p,\omega) \doteq \int_{-\infty}^{\infty} dq\ \sigma^j_\alpha(p,q)\frac{p}{m_\alpha}\hat{f}^j_d(\vec{k},q,\omega), \tag{15}$$

$$\hat{\nu}^j_P(\vec{k},p,\omega) \doteq \delta_{\alpha e}\beta(\nu)\Lambda(\nu)\int_{-\infty}^{\infty} dq\ \sigma^j_P(p,q) \times\left[\frac{p_x}{m_\alpha}\Theta(p_x)\delta_{1,-n_x} + \frac{p_z}{m_\alpha}\Theta(p_z)\delta_{1,-n_z}\right]\hat{f}^j_d(\vec{k},q,\omega). \tag{16}$$

The other first order equations can be written, in terms of Fourier transforms, as follows:

$$\begin{aligned} &-i\omega\hat{f}^j_d(\vec{k},\omega,q) + \frac{\partial}{\partial q}[\hat{I}^j(\vec{k},\omega,q)f^j_{d0} + I^j_0\hat{f}^j_d(\vec{k},\omega,q)] = 0,\\ &i\vec{k}\cdot\bar{E}(\vec{k},\omega) = 4\pi\sum_\alpha q_\alpha\int d\vec{p}\ f_{\alpha\vec{k}} + 4\pi\sum_j\int_{-\infty}^{\infty} dq\ q\hat{f}^j_d,\\ &i\vec{k}\times\bar{B}(\vec{k},\omega) = -i\frac{\omega}{c}\bar{E} + \frac{4\pi}{c}\sum_\alpha\frac{q_\alpha}{m_\alpha}\int d\vec{p}\ \vec{p}f_{\alpha\vec{k}},\\ &\vec{k}\times\bar{E}(\vec{k},\omega) = \frac{\omega}{c}\bar{B}(\vec{k},\omega), \quad \nabla\cdot\vec{B}_1 = 0, \end{aligned} \tag{17}$$

where

$$\bar{j}(\vec{k},\omega) = \sum_\alpha\frac{q_\alpha}{m_\alpha}\int d\vec{p}\ \vec{p}f_{\alpha\vec{k}}(\vec{k},\vec{p},\omega), \tag{18}$$

$$\hat{I}^j(\vec{k},q,\omega) = \sum_\alpha q_\alpha\int d\vec{p}\ \sigma^j_\alpha(q,p)\frac{p}{m_\alpha}f_{\alpha\vec{k}}, \tag{19}$$

$$I^j_0(q) = -I^j_P + \sum_\alpha q_\alpha\int d\vec{p}\ \sigma^j_\alpha(q,p)\frac{p}{m_\alpha}f_{\alpha 0}. \tag{20}$$

The distribution (14) can be splitted into three different terms, as a consequence of the model adopted for the charging of the dust particles. The first term is due to the field perturbations, the second is due to the current associated to collisional charging, and the third is due to the current associated to photoelectric emission:

$$f_{\alpha\vec{k}} = f^C_{\alpha\vec{k}} + f^A_{\alpha\vec{k}} + f^P_{\alpha\vec{k}}, \tag{21}$$

where

$$\begin{aligned} f^C_{\alpha\vec{k}} &= -q_\alpha\int_{-\infty}^0 d\tau\ e^{i\{\vec{k}\cdot\vec{R}-[\omega+i\sum_j \nu^{j0}_{\alpha d}(p)]\tau\}} \times\left[\bar{E}(\vec{k},\omega) + \frac{\vec{p}'}{m_\alpha c}\times\bar{B}(\vec{k},\omega)\right]\cdot\vec{\nabla}_{\vec{p}'}f_{\alpha 0}(\vec{p}),\\ f^A_{\alpha\vec{k}} &= -\int_{-\infty}^0 d\tau\ e^{i\{\vec{k}\cdot\vec{R}-[\omega+i\sum_j \nu^{j0}_{\alpha d}(p)]\tau\}} \times\left[\sum_j \hat{\nu}^j_{\alpha d}(\vec{k},p,\omega)\right]f_{\alpha 0}(p),\\ f^P_{\alpha\vec{k}} &= \int_{-\infty}^0 d\tau\ e^{i\{\vec{k}\cdot\vec{R}-[\omega+i\sum_j \nu^{j0}_{\alpha d}(p)]\tau\}} \times\left[\sum_j \hat{\nu}^j_P(\vec{k},p,\omega)\right]F(p). \end{aligned} \tag{22}$$

From Ampère's law, and using the Fourier transform of the current density, after a fair amount of algebra, the following identity can be obtained

$$\sum_j \sigma_{ij}(\vec{k},\omega)\bar{E}_j(\vec{k},\omega) = \sum_\alpha\frac{q_\alpha}{m_\alpha}\int d\vec{p}\ p_i(f^C_{\alpha\vec{k}} + f^A_{\alpha\vec{k}} + f^P_{\alpha\vec{k}}), \tag{23}$$

where the σ_{ij} are the components of the conductivity tensor, which should not be confused with the cross-sections σ_α and σ_P.

The dependence of the term associated with $f^C_{\alpha\vec{k}}$ is easily demonstrated. Using the identity

$$\left[\bar{E}(\vec{k},\omega) + \frac{\vec{p}'}{m_\alpha c}\times\bar{B}(\vec{k},\omega)\right]\cdot\vec{\nabla}_{\vec{p}'}f_{\alpha 0} = \left[\left(1 - \frac{\vec{p}'\cdot\vec{k}}{m_\alpha\omega}\right)\vec{\nabla}_{\vec{p}'}f_{\alpha 0} + \frac{\vec{k}\cdot\vec{\nabla}_{\vec{p}'}f_{\alpha 0}}{m_\alpha\omega}\vec{p}'\right]\cdot\bar{E},$$

we obtain

$$f^C_{\alpha\vec{k}} = -q_\alpha \vec{A}^\alpha \cdot \bar{E}(\vec{k},\omega), \tag{24}$$

where

$$\vec{A}^\alpha \doteq \int_{-\infty}^0 d\tau \; e^{i\{\vec{k}\cdot\vec{R}-[\omega+i\Sigma_j \nu^{j0}_{\alpha d}(p)]\tau\}} \times \left[\left(1-\frac{\vec{p}'\cdot\vec{k}}{m_\alpha\omega}\right)\vec{\nabla}_{\vec{p}'} f_{\alpha 0} + \frac{\vec{k}\cdot\vec{\nabla}_{\vec{p}'} f_{\alpha 0}}{m_\alpha \omega}\vec{p}'\right]. \tag{25}$$

The field dependency of the terms associated with $f^A_{\alpha\vec{k}}$ and $f^P_{\alpha\vec{k}}$ is not so explicit, appearing through the average charge of the dust particle. In what follows, we present further details about the derivation of the relationship between the perturbed current and the electric field components.

A. The contribution associated with $f^C_{\alpha\vec{k}}$

The derivation of the contribution associated with $f^C_{\alpha\vec{k}}$ is fairly conventional, and will be reproduced here for completicity. From Eqs. (23) and (24), we obtain

$$\sigma^C_{ij}(\vec{k},\omega) = -\sum_\alpha \frac{q^2_\alpha}{m_\alpha}\int d\vec{p}\; p_i A^\alpha_j. \tag{26}$$

Since the external magnetic field introduces anisotropy in the system, it is convenient to introduce cylindrical coordinates in *momentum* space. Without loss of generality, the wave vector $\vec{k}$ is assumed to be in xz plane,

$$\vec{k} = k_\perp \hat{x} + k_\parallel \hat{z}.$$

Therefore, the x, y, and z components of Eq. (25), which appear in the integrand of Eq. (26), contain the following:

$$\left[\left(1-\frac{\vec{p}'\cdot\vec{k}}{m_\alpha\omega}\right)\vec{\nabla}_{\vec{p}'} f_{\alpha 0} + \frac{\vec{k}\cdot\vec{\nabla}_{\vec{p}'} f_{\alpha 0}}{m_\alpha\omega}\vec{p}'\right]_{x,y,z} = \left\{\begin{array}{c} \cos(\phi-\omega_\alpha\tau)\mathcal{L}(f_{\alpha 0}) \\ \sin(\phi-\omega_\alpha\tau)\mathcal{L}(f_{\alpha 0}) \\ \dfrac{\partial f_{\alpha 0}}{\partial p_\parallel} + \cos(\phi-\omega_\alpha\tau)\dfrac{k_\perp}{m_\alpha\omega}L_\alpha(f_{\alpha 0}) \end{array}\right\}, \tag{27}$$

where the operators $\mathcal{L}$ and L_α are defined by

$$\mathcal{L} = \left[\frac{\partial}{\partial p_\perp} - \frac{k_\parallel}{m_\alpha\omega}L_\alpha\right], \qquad L_\alpha = p_\parallel\frac{\partial}{\partial p_\perp} - p_\perp\frac{\partial}{\partial p_\parallel}. \tag{28}$$

By substituting Eq. (27) in (26), the integrations over τ variable can be easily performed

$$\int_{-\infty}^0 d\tau e^{i\{\vec{k}\cdot\vec{R}-[\omega+i\Sigma_j\nu^{j0}_{\alpha d}(p)]\tau\}}\left\{\begin{array}{c} 1 \\ \cos(\phi-\omega_\alpha\tau) \\ \sin(\phi-\omega_\alpha\tau)\end{array}\right\} = i\sum_{n,l=-\infty}^{\infty}\frac{e^{i(n-l)\phi}}{\omega - l\omega_\alpha - \dfrac{k_\parallel p_\parallel}{m_\alpha} + i\Sigma_j\nu^{j0}_{\alpha d}(p)} \times \left\{\begin{array}{c} J_n(b_\alpha)J_l(b_\alpha) \\ \dfrac{l}{b_\alpha}J_n(b_\alpha)J_l(b_\alpha) \\ iJ_n(b_\alpha)J'_l(b_\alpha)\end{array}\right\},$$

where $b_\alpha = k_\perp p_\perp/m_\alpha\omega_\alpha$. Then, after integration over ϕ we obtain the components of the dielectric tensor that correspond to $f^C_{\alpha\vec{k}}$

$$\epsilon^C_{ij} = \delta_{ij} + \sum_{l=-\infty}^{\infty}\sum_\alpha \frac{\omega^2_{p\alpha}}{\omega n_\alpha}\int d\vec{p}\;\frac{p_\parallel^{\delta_{iz}}p_\perp^{\delta_{ix}+\delta_{iy}}\Pi^{l\alpha}_{ij}}{\omega - l\omega_\alpha - \dfrac{k_\parallel p_\parallel}{m_\alpha} + i\Sigma_\ell \nu^{\ell 0}_{\alpha d}(p)} \times \left(\frac{p_\parallel}{p_\perp}\right)^{\delta_{jz}}\left[\mathcal{L}(f_{\alpha 0}) + \delta_{jz} i\left(\sum_\ell \frac{\nu^{\ell 0}_{\alpha d}(p)}{\omega}\right)\frac{L_\alpha(f_{\alpha 0})}{p_\parallel}\right] - \delta_{iz}\sum_\alpha \frac{\omega^2_{p\alpha}}{\omega^2 n_\alpha}\int d\vec{p}\;\frac{p_\parallel}{p_\perp}L_\alpha(f_{\alpha 0}), \tag{29}$$

where the expression

$$\omega^2_{p\alpha} = \frac{4\pi n_\alpha q^2_\alpha}{m_\alpha}$$

defines the plasma angular frequency of particles of kind α and the $\Pi^{l\alpha}_{ij}$ define the components of the tensor

$$\tilde{\Pi}^{l\alpha}(p_\perp) \doteq \left(\begin{array}{ccc} \dfrac{l^2 J_l^2}{b_\alpha^2} & i\dfrac{lJ_l' J_l}{b_\alpha} & \dfrac{lJ_l^2}{b_\alpha} \\ -i\dfrac{lJ_l' J_l}{b_\alpha} & J_l'^2 & -iJ_l' J_l \\ \dfrac{lJ_l^2}{b_\alpha} & iJ_l J_l' & J_l^2 \end{array}\right). \tag{30}$$

It is seen that the ϵ^C_{ij} differ from the components of the tensor of a conventional plasma by an additional term due to the collisional charging frequency, and a new term in the iz components ϵ^C_{iz}.[9]

B. Dust particle electrical charge

The distributions $f^A_{\alpha\vec{k}}$ e $f^P_{\alpha\vec{k}}$ determine two additional terms to the dielectric tensor that correspond, respectively, to absorption and emission currents. In order to obtain these components, it is necessary to express the distributions explicitly in terms of the electric field. This can be accomplished by considering moments of the dust distribution in q variable as described in this section. The approach follows that developed in the Ph.D. thesis by de Juli, except that here we consider the case of several populations of dust particles with different radii.[10]

We start by taking zeroth and first moments of distribution f_d^j in q variable. By linearizing the average charge, $Q^j(\vec{r},t) = Q_0^j + Q_1^j(\vec{r},t)$, where Q_0^j is the equilibrium average charge of dust particle of radius a_j, we obtain in zero order

$$\int_{-\infty}^{\infty} dq\ f_{d0}^j(q) = n_{d0}^j, \qquad \int_{-\infty}^{\infty} dq\ q f_{d0}^j(q) = n_{d0}^j Q_0^j, \tag{31}$$

and in first order

$$\int_{-\infty}^{\infty} dq\ f_{d1}^j(\vec{r},q,t) = n_{d1}^j(\vec{r},t), \qquad \int_{-\infty}^{\infty} dq\ q f_{d1}^j(\vec{r},q,t) = n_{d0}^j Q_1^j(\vec{r},t). \tag{32}$$

We consider dust particles with radius a_j sufficiently large, such that their charge is much larger than the elementary charge. In such a case, the charge of a dust particle can be approximated as a continuous quantity, with small fluctuations relative to the equilibrium value.[8,23] All dust particles of population j can be considered to have the same equilibrium charge. Moreover, we have assumed that the dust distribution is independent of momentum. With these considerations, Eq. (31) implies that the equilibrium dust distribution can be written as

$$f_{d0}^j(q) = n_{d0}^j \delta(q - Q_0^j). \tag{33}$$

It is also reasonable to suppose that for waves whose frequencies are much higher than the dust plasma frequency we may assume that the form of distribution f_d^j does not differ much from the form of equilibrium f_{d0}^j, in such a way that the Fourier transform of the perturbation f_{d1}^j can be written as

$$\hat{f}_d^j(\vec{k},q,\omega) = n_{d0}^j\{\delta[q - \hat{Q}_T^j(\vec{k},t)] - \delta[q - Q_0^j]\} \tag{34}$$

with $\hat{Q}_T^j = Q_0^j + \hat{Q}^j(\vec{k},\omega)$.

We can determine the charge $\hat{Q}^j(\vec{k},\omega)$ by taking the first moment of kinetic equation for the dust component (17) and by considering the Fourier transform of Eq. (32), obtaining for $\hat{Q}^j$

$$\hat{Q}^j(\vec{k},\omega) = \frac{i}{\omega}[I_0^j(\hat{Q}_T^j) + \hat{I}^j(\vec{k},Q_0^j,\omega)], \tag{35}$$

where we have used the condition $I_0^j(Q_0) = 0$. Expanding $I_0^j(\hat{Q}_T^j)$ in a Taylor series around Q_0^j one obtains

$$I_0^j(\hat{Q}_T^j) \simeq -\nu_{ch}^j \hat{Q}^j(\vec{k},\omega), \tag{36}$$

where

$$\nu_{ch}^j \equiv -\left.\frac{\partial I_0^j}{\partial \hat{Q}_T^j}\right|_{\hat{Q}_T^j = Q_0^j} \tag{37}$$

is the charging frequency and we have assumed that $|\hat{Q}^j| \ll |Q_0^j|$.

In terms of $f_{\alpha\vec{k}} = f_{\alpha\vec{k}}^C + f_{\alpha\vec{k}}^A + f_{\alpha\vec{k}}^P$, the perturbation current $\hat{I}^j$ reads

$$\hat{I}^j(\vec{k},Q_0^j,\omega) = \hat{I}^{jC}(\vec{k},Q_0^j,\omega) + \hat{I}^{jA}(\vec{k},Q_0^j,\omega) + \hat{I}^{jP}(\vec{k},Q_0^j,\omega), \tag{38}$$

where

$$\hat{I}^{j\ C,A,P}(\vec{k},q,\omega) = \frac{1}{n_{d0}^j}\sum_\alpha q_\alpha \int d\vec{p}\ \nu_{\alpha d}^{j0}(p) f_{\alpha\vec{k}}^{C,A,P}, \tag{39}$$

$$\nu_{\alpha d}^{j0}(p) = \frac{n_{d0}^j p}{m_\alpha}\sigma_\alpha^j(p,Q_0^j). \tag{40}$$

It is possible to express the currents (39) in terms of the equilibrium charge. We start by substituting the perturbation (34) in (16) and then integrating over q

$$\hat{\nu}_P^j(\vec{k},p,\omega) = \delta_{\alpha e}\frac{2}{3}\pi a_j^2 \beta(\nu)\Lambda(\nu) n_{d0}^j \times \left[\Theta\left(1 - \frac{2m_e e\hat{Q}_T^j}{a_j p^2}\Theta(\hat{Q}_T^j)\right) - \Theta\left(1 - \frac{2m_e e Q_0^j}{a_j p^2}\Theta(Q_0^j)\right)\right] \times \left[\frac{p_x}{m_e}\Theta(p_x)\delta_{1,-n_x} + \frac{p_z}{m_e}\Theta(p_z)\delta_{1,-n_z}\right]. \tag{41}$$

By expanding the first term in a Taylor series around Q_0^j

$$\Theta\left(1 - \frac{2m_e e\hat{Q}_T^j}{a_j p^2}\Theta(\hat{Q}_T^j)\right) \simeq \Theta\left(1 - \frac{2m_e e Q_0^j}{a_j p^2}\Theta(Q_0^j)\right) - \frac{2m_e\Theta(Q_0^j)}{a_j p^2}\delta\left(1 - \frac{2m_e e Q_0^j}{a_j p^2}\Theta(Q_0^j)\right)\hat{Q}^j(\vec{k},\omega)$$

and then substituting into (41), we obtain

$$\hat{\nu}_P^j(\vec{k},p,\omega) \simeq \delta_{\alpha e} n_{d0}^j \sigma_P^{j\prime}(p) \times \left[\frac{p_x}{m_e}\Theta(p_x)\delta_{1,-n_x} + \frac{p_z}{m_e}\Theta(p_z)\delta_{1,-n_z}\right]\hat{Q}^j(\vec{k},\omega), \tag{42}$$

where we have defined

$$\sigma_P^{j\prime}(p) \doteq -\frac{2\pi a_j m_e e}{p^2}\beta(\nu)\Lambda(\nu)\frac{2}{3} \times \delta\left(1 - \frac{2m_e e Q_0^j}{a_j p^2}\Theta(Q_0^j)\right)\Theta(Q_0^j). \tag{43}$$

By a similar procedure we obtain for $\hat{\nu}_{\alpha d}^j$, Eq. (15),

$$\hat{\nu}_{\alpha d}^j(\vec{k},p,\omega) \simeq n_{d0}^j \sigma_\alpha^{j\prime}(p)\frac{p}{m_\alpha}\hat{Q}^j(\vec{k},\omega), \tag{44}$$

where

$$\sigma_\alpha^{j\prime}(p) \doteq -\frac{2\pi a_j q_\beta m_\beta}{p^2}\left[\Theta\left(1 - \frac{2q_\alpha m_\alpha Q_0^j}{a_j p^2}\right) + \left(1 - \frac{2q_\alpha m_\alpha Q_0^j}{a_j p^2}\right)\delta\left(1 - \frac{2q_\alpha m_\alpha Q_0^j}{a_j p^2}\right)\right]. \tag{45}$$

The frequencies (42) and (44), written in terms of the average dust charge, allow the following form for the distributions $f^A_{\alpha\vec{k}}$ and $f^P_{\alpha\vec{k}}$:

$$f^A_{\alpha\vec{k}} = -\int_{-\infty}^{0} d\tau\ e^{i\{\vec{k}\cdot\vec{R}-[\omega+i\sum_j \nu^{j0}_{\alpha d}(p)]\tau\}} \times \sum_j n^j_{d0}\sigma^{j\prime}_{\alpha}(p)\frac{p}{m_\alpha}f_{\alpha 0}(p)\hat{Q}^j(\vec{k},\omega),$$

$$f^P_{\alpha\vec{k}} = \delta_{\alpha e}\int_{-\infty}^{0} d\tau\ e^{i\{\vec{k}\cdot\vec{R}-[\omega+i\sum_j \nu^{j0}_{\alpha d}(p)]\tau\}} \times \sum_j n^j_{d0}\sigma^{j\prime}_{P}(p)F(p)\hat{Q}^j(\vec{k},\omega) \times \left[\frac{p_x}{m_e}\Theta(p_x)\delta_{1,-n_x}+\frac{p_z}{m_e}\Theta(p_z)\delta_{1,-n_z}\right]. \quad (46)$$

By substituting (24) and (46) in (39),

$$\hat{I}^{jC}(\vec{k},q,\omega) = -\vec{A}^{j0}\cdot\bar{E}(\vec{k},\omega), \quad (47)$$

$$\hat{I}^{jA}(\vec{k},q,\omega) = -\sum_\ell \nu^{j\ell}_1\hat{Q}^\ell(\vec{k},\omega), \quad (48)$$

$$\hat{I}^{jP}(\vec{k},q,\omega) = \sum_\ell \nu^{j\ell}_{P1}\hat{Q}^\ell(\vec{k},\omega), \quad (49)$$

where

$$\nu^{j\ell}_1 = \frac{n^\ell_{d0}}{n^j_{d0}}\sum_\alpha q_\alpha\int d\vec{p}\ \nu^{j0}_{\alpha d}(p)\sigma^{\ell\prime}_{\alpha}(p)\frac{p}{m_\alpha}f_{\alpha 0}(p) \times \int_{-\infty}^{0} d\tau\ e^{i\{\vec{k}\cdot\vec{R}-[\omega+i\sum_m \nu^{m0}_{\alpha d}(p)]\tau\}}, \quad (50)$$

$$\nu^{j\ell}_{P1} = \frac{n^\ell_{d0}}{n^j_{d0}}\sum_\alpha \delta_{\alpha e}q_\alpha\int d\vec{p}\ \nu^{j0}_{\alpha d}(p)\sigma^{\ell\prime}_{P}(p) \times \left[\frac{p_x}{m_e}\Theta(p_x)\delta_{1,-n_x}+\frac{p_z}{m_e}\Theta(p_z)\delta_{1,-n_z}\right]F(p) \times \int_{-\infty}^{0} d\tau\ e^{i\{\vec{k}\cdot\vec{R}-[\omega+i\sum_m \nu^{m0}_{\alpha d}(p)]\tau\}}, \quad (51)$$

and

$$\vec{A}^{j0} \doteq \frac{1}{n^j_{d0}}\sum_\alpha q^2_\alpha\int d\vec{p}\ \nu^{j0}_{\alpha d}(p)\vec{A}^{\alpha}. \quad (52)$$

It is still necessary to obtain the perturbed charge $\hat{Q}^j$ in terms of the electric field. By considering (36), (38), (47), (48), and (49), one obtains from Eq. (35) the identity

$$(\omega + i\nu^j_{ch})\hat{Q}^j(\vec{k},\omega) + i\sum_\ell(\nu^{j\ell}_1-\nu^{j\ell}_{P1})\hat{Q}^\ell(\vec{k},\omega) = -i\vec{A}^{j0}\cdot\bar{E}. \quad (53)$$

Equation (53) has a coupling in the $\hat{Q}^j$ variable. We look for a solution using a recursive method. By neglecting the "interaction" between different dust populations we can write

$$\hat{Q}^j(\vec{k},\omega) \simeq \frac{-i\vec{A}^{j0}\cdot\bar{E}}{[\omega+i\nu^j_{ch}+i\nu^{jj}_1-i\nu^{jj}_{P1}]}.$$

Using this approximation in Eq. (53), one obtains

$$\hat{Q}^j(\vec{k},\omega) = \frac{-i}{\omega+i\nu^j_{ch}+i\nu^{jj}_1-i\nu^{jj}_{P1}} \times \sum_\ell\left[\delta_{j\ell} - i\frac{(1-\delta_{j\ell})(\nu^{j\ell}_1-\nu^{j\ell}_{P1})}{\omega+i\nu^\ell_{ch}+i\nu^{\ell\ell}_1-i\nu^{\ell\ell}_{P1}}\right](\vec{A}^{\ell 0}\cdot\bar{E}).$$

Using this new result in Eq. (53),

$$\hat{Q}^j(\vec{k},\omega) = \frac{-i}{(\omega+i\nu^j_{ch}+i\nu^{jj}_1-i\nu^{jj}_{P1})} \times \sum_{\ell_1}\sum_{\ell_2}\left[\delta_{j\ell_1}\delta_{j\ell_2} - i\frac{(1-\delta_{j\ell_2})(\nu^{j\ell_2}_1-\nu^{j\ell_2}_{P1})}{\omega+i\nu^{\ell_2}_{ch}+i\nu^{\ell_2\ell_2}_1-i\nu^{\ell_2\ell_2}_{P1}}\left[\delta_{\ell_2\ell_1} - i\frac{(1-\delta_{\ell_2\ell_1})(\nu^{\ell_2\ell_1}_1-\nu^{\ell_2\ell_1}_{P1})}{\omega+i\nu^{\ell_1}_{ch}+i\nu^{\ell_1\ell_1}_1-i\nu^{\ell_1\ell_1}_{P1}}\right]\right](\vec{A}^{\ell_1 0}\cdot\bar{E})$$

and again

$$\hat{Q}^j(\vec{k},\omega) = \frac{-i}{(\omega+i\nu^j_{ch}+i\nu^{jj}_1-i\nu^{jj}_{P1})}\sum_{\ell_1}\sum_{\ell_2}\sum_{\ell_3}\Bigg[\delta_{j\ell_1}\delta_{j\ell_2}\delta_{j\ell_3} - i\frac{(1-\delta_{j\ell_3})(\nu^{j\ell_3}_1-\nu^{j\ell_3}_{P1})}{(\omega+i\nu^{\ell_3}_{ch}+i\nu^{\ell_3\ell_3}_1-i\nu^{\ell_3\ell_3}_{P1})} \times \left[\delta_{\ell_3\ell_1}\delta_{\ell_3\ell_2} - i\frac{(1-\delta_{\ell_3\ell_2})(\nu^{\ell_3\ell_2}_1-\nu^{\ell_3\ell_2}_{P1})}{\omega+i\nu^{\ell_2}_{ch}+i\nu^{\ell_2\ell_2}_1-i\nu^{\ell_2\ell_2}_{P1}}\left[\delta_{\ell_2\ell_1} - i\frac{(1-\delta_{\ell_2\ell_1})(\nu^{\ell_2\ell_1}_1-\nu^{\ell_2\ell_1}_{P1})}{\omega+i\nu^{\ell_1}_{ch}+i\nu^{\ell_1\ell_1}_1-i\nu^{\ell_1\ell_1}_{P1}}\right]\right]\Bigg](\vec{A}^{\ell_1 0}\cdot\bar{E}),$$

and so on. The iteration can be taken until an arbitrary number of times.

By defining the quantity

$$C_{\ell_1\ell_2} = -i\frac{(1-\delta_{\ell_1\ell_2})(\nu_1^{\ell_1\ell_2}-\nu_{P1}^{\ell_1\ell_2})}{\omega+i\nu_{ch}^{\ell_2}+i\nu_1^{\ell_2\ell_2}-i\nu_{P1}^{\ell_2\ell_2}},$$

we write

$$\hat{Q}^j(\vec{k},\omega)=\frac{-i}{(\omega+i\nu_{ch}^j+i\nu_1^{jj}-i\nu_{P1}^{jj})}\sum_{\ell_1}\sum_{\ell_2}\sum_{\ell_3}(\vec{A}^{\ell_1 0}\cdot\bar{E})\times[\delta_{j\ell_1}\delta_{j\ell_2}\delta_{j\ell_3}+C_{j\ell_3}[\delta_{\ell_3\ell_1}\delta_{\ell_3\ell_2}+C_{\ell_3\ell_2}[\delta_{\ell_2\ell_1}+C_{\ell_2\ell_1}]]].$$

The different terms can be simplified by running some of the indicated summations, and one finally obtains the expression

$$\hat{Q}^j(\vec{k},\omega)=\frac{-i}{(\omega+i\nu_{ch}^j+i\nu_1^{jj}-i\nu_{P1}^{jj})}\sum_{\ell}B_{j\ell}(\vec{A}^{\ell 0}\cdot\bar{E}), \quad (54)$$

where

$$B_{j\ell}=\left[\delta_{j\ell}+C_{j\ell}+\sum_{\ell_2}C_{j\ell_2}C_{\ell_2\ell}+\sum_{\ell_2}\sum_{\ell_3}C_{j\ell_3}C_{\ell_3\ell_2}C_{\ell_2\ell}+\cdots\right].$$

Equation (54) can be used in Eq. (46), leading to the following expressions for the distributions:

$$f_{\alpha\vec{k}}^A=\sum_j n_{d0}^j\sigma_\alpha^{j\prime}(p)\frac{p}{m_\alpha}f_{\alpha 0}(p)\frac{i}{\omega+i\nu_{ch}^j+i\nu_1^{jj}-i\nu_{P1}^{jj}}\times\sum_\ell B_{j\ell}(\vec{A}^{\ell 0}\cdot\bar{E})\int_{-\infty}^0 d\tau\; e^{i\{\vec{k}\cdot\vec{R}-[\omega+i\sum_m\nu_{\alpha d}^{m0}(p)]\tau\}},$$

$$f_{\alpha\vec{k}}^P=\delta_{\alpha e}\sum_j n_{d0}^j\sigma_P^{j\prime}(p)\frac{-i}{\omega+i\nu_{ch}^j+i\nu_1^{jj}-i\nu_{P1}^{jj}}\times\left[\frac{p_x}{m_e}\Theta(p_x)\delta_{1,-n_x}+\frac{p_z}{m_e}\Theta(p_z)\delta_{1,-n_z}\right]F(p)\times\sum_\ell B_{j\ell}(\vec{A}^{\ell 0}\cdot\bar{E})\int_{-\infty}^0 d\tau\; e^{i\{\vec{k}\cdot\vec{R}-[\omega+i\sum_m\nu_{\alpha d}^{m0}(p)]\tau\}}. \quad (55)$$

These expressions show the distributions $f_{\alpha\vec{k}}^A$ and $f_{\alpha\vec{k}}^P$ explicitly expressed in terms of the electric field components. They can be used to derive the contributions ε_{ij}^A and ε_{ij}^P to the components of the dielectric tensor.

C. The contribution associated with $f_{\alpha\vec{k}}^A$

From Eq. (23), it follows that

$$\sum_j\sigma_{ij}^A(\vec{k},\omega)\bar{E}_j(\vec{k},\omega)=\sum_\alpha\frac{q_\alpha}{m_\alpha}\int d\vec{p}\; p_i f_{\alpha\vec{k}}^A,$$

which, by considering Eq. (55), leads to

$$\sigma_{ij}^A(\vec{k},\omega)=i\sum_k\frac{n_{d0}^k}{\omega+i\nu_{ch}^k+i\nu_1^{kk}-i\nu_{P1}^{kk}}\times\sum_\alpha\frac{q_\alpha}{m_\alpha}\int d\vec{p}\; p_i\sigma_\alpha^{k\prime}(p)\frac{p}{m_\alpha}f_{\alpha 0}(p)\sum_\ell B_{k\ell}A_j^{\ell 0}\times\int_{-\infty}^0 d\tau\; e^{i\{\vec{k}\cdot\vec{R}-[\omega+i\sum_m\nu_{\alpha d}^{m0}(p)]\tau\}}. \quad (56)$$

The integration over the τ variable must be made for x, y, and z components, separately. Considering that this integral appears multiplied by the components p_i, we obtain

$$p_\parallel\left(\frac{p_\perp}{p_\parallel}\right)^{\delta_{ix}+\delta_{iy}}\sum_{n,l=-\infty}^{\infty}\left(\frac{n}{b_\alpha}\right)^{\delta_{ix}}J_nJ_l\left(-i\frac{J_l'}{J_l}\right)^{\delta_{iy}}\times\frac{ie^{i(n-l)\phi}}{\omega-l\omega_\alpha-\frac{k_\parallel p_\parallel}{m_\alpha}+i\sum_m\nu_{\alpha d}^{m0}(p)}.$$

The only ϕ dependence is in the exponential, and the integration in ϕ results in

$$\int_0^{2\pi}d\phi\; e^{i(n-l)\phi}=2\pi\delta_{n,l} \quad (57)$$

and leads to

$$\sigma_{ij}^A(\vec{k},\omega)=-\sum_k\frac{n_{d0}^k}{\omega+i\nu_{ch}^k+i\nu_1^{kk}-i\nu_{P1}^{kk}}\sum_{l=-\infty}^{\infty}\sum_\alpha\frac{q_\alpha}{m_\alpha^2}\times\int d\vec{p}\;\frac{p_\parallel^{\delta_{iz}}p_\perp^{\delta_{ix}+\delta_{iy}}p\sigma_\alpha^{k\prime}(p)f_{\alpha 0}(p)\Pi_{iz}^{l\alpha}}{\omega-l\omega_\alpha-\frac{k_\parallel p_\parallel}{m_\alpha}+i\Sigma_m\nu_{\alpha d}^{m0}(p)}\sum_\ell B_{k\ell}A_j^{\ell 0}. \quad (58)$$

Using Eqs. (52) and (58), the contribution to the plasma dielectric tensor which is associated with $f_{\alpha\vec{k}}^A$ can be written more conveniently as a product of two dimensionless vector quantities[4]

$$\epsilon_{ij}^A=\frac{4\pi i}{\omega}\sigma_{ij}^A=\sum_k\mathcal{U}_i^k\mathcal{S}_j^k, \quad (59)$$

with

$$\mathcal{U}_i^k=-\frac{1}{a_k}\frac{1}{\omega+i\nu_{ch}^k+i\nu_1^{kk}-i\nu_{P1}^{kk}}\sum_\alpha\frac{q_\alpha}{m_\alpha^2}\sum_{l=-\infty}^{+\infty}\times\int d^3p\;\frac{p_\perp p\sigma_\alpha^{k\prime}(p)f_{\alpha 0}}{\omega-\frac{k_\parallel p_\parallel}{m_\alpha}-l\Omega_\alpha+i\Sigma_m\nu_{\alpha d}^{m0}(p)}\left(\frac{p_\parallel}{p_\perp}\right)^{\delta_{iz}}\Pi_{iz}^{l\alpha}, \quad (60)$$

$$\mathcal{S}_j^k=-a_k\sum_\ell B_{k\ell}\frac{n_{d0}^k}{n_{d0}^\ell}\left[\sum_\alpha\frac{\omega_\alpha^2 m_\alpha}{n_{\alpha 0}}\sum_{l=-\infty}^{+\infty}\times\int d\vec{p}\;\frac{\nu_{\alpha d}^{\ell 0}(p)}{\omega}\frac{1}{\omega-\frac{k_\parallel p_\parallel}{m_\alpha}-l\Omega_\alpha+i\Sigma_m\nu_{\alpha d}^{m0}(p)}\times\mathcal{L}(f_{\alpha 0})\left(\frac{p_\parallel}{p_\perp}\right)^{\delta_{jz}}\Pi_{zj}^{l\alpha}+i\,\delta_{jz}\sum_\alpha q_\alpha^2\sum_{l=-\infty}^{+\infty}\int d\vec{p}\;\frac{[\nu_{\alpha d}^{\ell 0}(p)/\omega]}{\omega-\frac{k_\parallel p_\parallel}{m_\alpha}-l\Omega_\alpha+i\Sigma_m\nu_{\alpha d}^{m0}(p)}\times\left(\sum_m\frac{\nu_{\alpha d}^{m0}(p)}{\omega}\right)\frac{L(f_{\alpha 0})}{p_\perp}\Pi_{zj}^{l\alpha}-\delta_{jz}\sum_\alpha q_\alpha^2\int d\vec{p}\;\frac{\nu_{\alpha d}^{\ell 0}(p)}{\omega}\frac{L(f_{\alpha 0})}{p_\perp}\right]. \quad (61)$$

D. The contribution associated with $f^P_{\alpha\vec{k}}$

From Eq. (23), it follows that

$$\sum_j \sigma^P_{ij}(\vec{k},\omega)\bar{E}_j(\vec{k},\omega) = \sum_\alpha \frac{q_\alpha}{m_\alpha}\int d\vec{p}\; p_i f^P_{\alpha\vec{k}},$$

which, by considering Eq. (55), leads to

$$\sigma^P_{ij}(\vec{k},\omega) = -i\sum_k \frac{n^k_{d0}}{\omega + i\nu^k_{ch} + i\nu^{kk}_1 - i\nu^{kk}_{P1}}\sum_\alpha \frac{\delta_{\alpha e}q_\alpha}{m_\alpha m_e} \times \int d\vec{p}\;\; \sigma^{k\prime}_P(p)p_i\left[\frac{p_x}{m_e}\Theta(p_x)\delta_{1,-n_x} + \frac{p_z}{m_e}\Theta(p_z)\delta_{1,-n_z}\right] \times F(p)\sum_\ell B_{k\ell}A^{\ell 0}_j\int_{-\infty}^0 d\tau\;\; e^{i\{\vec{k}\cdot\vec{R} - [\omega + i\sum_m \nu^{m0}_{\alpha d}(p)]\tau\}}. \tag{62}$$

In this component, it is immediate the identification of the two possibilities of incidence of radiation, perpendicular and parallel to the external magnetic field direction, as defined in Eq. (9).

Due to the presence of the functions $\Theta(p_x)$ and $\Theta(p_z)$, there are different limits of integration in $\vec{p}$ space. For the term with $\Theta(p_x)$, the integrand does not vanish for $-\pi/2 < \phi < \pi/2$ and $0 < \theta < \pi$. For the term with $\Theta(p_z)$ function, the integration intervals are $0 < \phi < 2\pi$ and $0 < \theta < \pi/2$. The integration on the variable θ must be considered for future calculations by utilizing the expressions that will be obtained in this section. In both cases, the ϕ integration can be readily performed.

For incidence along the x axis, the integrand of Eq. (62) contains the product $p_i p_x$. The integral over the τ variable can be performed, so that the integrand of Eq. (62) will contain, for the components with $i = x, y, z$,

$$p_\perp p_\parallel \left(\frac{p_\perp}{p_\parallel}\right)^{\delta_{ix}+\delta_{iy}} \sum_{n,l=-\infty}^{\infty}\left(\frac{n}{b_\alpha}\right)^{\delta_{ix}}\left(-i\frac{J_l'}{J_l}\right)^{\delta_{iy}} J_n J_l \times \frac{ie^{i(n-l)\phi}\cos\phi}{\omega - l\omega_\alpha - \frac{k_\parallel p_\parallel}{m_\alpha} + i\Sigma_m \nu^{m0}_{\alpha d}(p)}. \tag{63}$$

The use of Eq. (63) with (62) shows that the integration over ϕ is the same for all components, and after performed leads to the following:

$$\int_{-\pi/2}^{\pi/2} d\phi\;\cos\phi e^{i(n-l)\phi} = \frac{\pi}{2}[\delta_{n,l-1} + \delta_{n,l+1}] + \frac{1}{n-l+1}\sin\left[\frac{\pi}{2}(n-l+1)\right]_{n\neq l-1} + \frac{1}{n-l-1}\sin\left[\frac{\pi}{2}(n-l-1)\right]_{n\neq l+1}.$$

It is seem that there is an infinite number of values for the argument for which the sin function is non-vanishing. For the second term, with $n \neq l-1$,

$$\frac{1}{n-l+1}\sin\left[\frac{\pi}{2}(n-l+1)\right]_{n\neq l-1} = \begin{cases} \frac{1}{n-l+1}, & n = l+4m \\ -\frac{1}{n-l+1}, & n = 2+l+4m \end{cases}.$$

For the third with, with $n \neq l+1$,

$$\frac{1}{n-l-1}\sin\left[\frac{\pi}{2}(n-l-1)\right]_{n\neq l+1} = \begin{cases} \frac{1}{n-l-1}, & n = 2+l+4m \\ -\frac{1}{n-l-1}, & n = 4+l+4m \end{cases},$$

where $m = 0, \pm 1, \pm 2, \ldots$.

Therefore, the integration over the ϕ variable may be written in the following form:

$$\int_{-\pi/2}^{\pi/2} d\phi\;\cos\phi e^{i(n-l)\phi} = \frac{\pi}{2}[\delta_{n,l-1} + \delta_{n,l+1}] + \frac{\delta_{n,l+4m} + \delta_{n,2+l+4m}}{1+4m} - \frac{\delta_{n,2+l+4m} + \delta_{n,4+l+4m}}{3+4m}. \tag{64}$$

For incidence along the z axis, the integrand of Eq. (62) contains the product $p_i p_z$. After the τ integration the integrand of Eq. (62) will therefore contain, for the components with $i = x, y, z$,

$$p_\parallel^2 \left(\frac{p_\perp}{p_\parallel}\right)^{\delta_{ix}+\delta_{iy}} \sum_{n,l=-\infty}^{\infty}\left(\frac{n}{b_\alpha}\right)^{\delta_{ix}}\left(-i\frac{J_l'}{J_l}\right)^{\delta_{iy}} J_n J_l \times \frac{ie^{i(n-l)\phi}}{\omega - l\omega_\alpha - \frac{k_\parallel p_\parallel}{m_\alpha} + i\Sigma_m \nu^{m0}_{\alpha d}(p)}. \tag{65}$$

The integration in ϕ is from 0 to 2π and is the same as in Eq. (57).

The calculation of Eq. (62) may proceed by elimination of $\delta_{\alpha\neq e}$ in the α summation and by the use of Eqs. (63) and (64) for the case of perpendicular incidence and Eqs. (57) and (65) for the case of parallel incidence. The summation over n from $-\infty$ to ∞ can give place to another summation, over index m, for the terms using Eq. (63). After these operations, and using the components of the tensor defined by Eq. (30), and rebuilding the ϕ integration, Eq. (62) becomes as follows:

$$\sigma^P_{xj}(\vec{k},\omega) = -\frac{e}{2m_e^2}\sum_k \frac{n^k_{d0}}{\omega + i\nu^k_{ch} + i\nu^{kk}_1 - i\nu^{kk}_{P1}} \sum_{l=-\infty}^{\infty} \int d\vec{p}\ \frac{p_\perp \sigma^{k\prime}_P(p) F(p)}{\omega - l\omega_e - \frac{k_\parallel p_\parallel}{m_e} + i\Sigma_n \nu^{n0}_{ed}(p)} \sum_\ell B_{k\ell} A^{\ell 0}_j$$
$$\times \left\{ 2p_\parallel \Pi^{le}_{xz} \Theta(\vec{p}\cdot\hat{z})\delta_{1,-n_z} + p_\perp \delta_{1,-n_x}\left[\Pi^{le}_{xx} - i\Pi^{le}_{yz} + \frac{1}{\pi b_e}\sum_m \left[\frac{(l+4m)J_{l+4m} + (2+l+4m)J_{2+l+4m}}{1+4m} - \frac{(2+l+4m)J_{2+l+4m} + (4+l+4m)J_{4+l+4m}}{3+4m}\right] J_l\right]\right\},$$

$$\sigma^P_{yj}(\vec{k},\omega) = -\frac{e}{2m_e^2}\sum_k \frac{n^k_{d0}}{\omega + i\nu^k_{ch} + i\nu^{kk}_1 - i\nu^{kk}_{P1}} \sum_{l=-\infty}^{\infty} \int d\vec{p}\ \frac{p_\perp \sigma^{k\prime}_P(p) F(p)}{\omega - l\omega_e - \frac{k_\parallel p_\parallel}{m_e} + i\Sigma_n \nu^{n0}_{ed}(p)} \sum_\ell B_{k\ell} A^{\ell 0}_j$$
$$\times \left\{ 2p_\parallel \Pi^{le}_{yz} \Theta(\vec{p}\cdot\hat{z})\delta_{1,-n_z} + p_\perp \delta_{1,-n_x}\left[\Pi^{le}_{yx} + \frac{2}{\pi b_e}\sum_m \left[\frac{(1+l+4m)J_{1+l+4m}}{1+4m} - \frac{(3+l+4m)J_{3+l+4m}}{3+4m}\right] J'_l\right]\right\},$$

$$\sigma^P_{zj}(\vec{k},\omega) = -\frac{e}{2m_e^2}\sum_k \frac{n^k_{d0}}{\omega + i\nu^k_{ch} + i\nu^{kk}_1 - i\nu^{kk}_{P1}} \sum_{l=-\infty}^{\infty} \int d\vec{p}\ \frac{p_\parallel \sigma^{k\prime}_P(p) F(p)}{\omega - l\omega_\alpha - \frac{k_\parallel p_\parallel}{m_\alpha} + i\Sigma_n \nu^{n0}_{ed}(p)} \sum_\ell B_{k\ell} A^{\ell 0}_j$$
$$\times \left\{ 2p_\parallel \Pi^{le}_{zz} \Theta(\vec{p}\cdot\hat{z})\delta_{1,-n_z} + p_\perp \delta_{1,-n_x}\left[\Pi^{le}_{zx} + \frac{2}{\pi b_e}\sum_m \left[\frac{(1+l+4m)J_{1+l+4m}}{1+4m} - \frac{(3+l+4m)J_{3+l+4m}}{3+4m}\right] J_l\right]\right\}.$$

The components of the contribution to the dielectric tensor which is associated with $f^P_{\alpha k}$ can be more conveniently written as a product of two dimensionless vector quantities,

$$\epsilon^P_{ij} = \sum_k U^{P,k}_i S^{P,k}_j, \tag{66}$$

where

$$U^{P,k}_i = -\frac{1}{\omega + i\nu^k_{ch} + i\nu^{kk}_1 - i\nu^{kk}_{P1}} \frac{e}{2a_k m_e^2} \times \sum_{l=-\infty}^{\infty}\int d\vec{p}\ \frac{p_\parallel^{\delta_{iz}} p_\perp^{\delta_{ix}+\delta_{iy}} \sigma^{k\prime}_P(p) F(p)}{\omega - l\omega_e - \frac{k_\parallel p_\parallel}{m_e} + i\Sigma_n \nu^{n0}_{ed}(p)} \times [p_\perp(\Pi^{le}_{ix} - i\delta_{ix}\Pi^{le}_{yz} + \Pi^{le,P}_{ix})\delta_{1,-n_x} + 2p_\parallel \Pi^{le}_{iz}\Theta(\vec{p}\cdot\hat{z})\delta_{1,-n_z}], \tag{67}$$

and

$$S^{P,k}_j = \frac{4\pi i}{\omega} a_k \sum_\ell B_{k\ell}\frac{n^k_{d0}}{n^\ell_{d0}} \sum_\alpha q_\alpha^2 \int d\vec{p}\ \nu^{\ell 0}_{\alpha d}(p) A^\alpha_j, \tag{68}$$

where we have made use of Eq. (52) and where we have defined the tensorial components

$$\Pi^{le,P}_{xx} \doteq \frac{1}{\pi b_e}\sum_m \left[\frac{(l+4m)J_{l+4m} + (2+l+4m)J_{2+l+4m}}{1+4m} - \frac{(2+l+4m)J_{2+l+4m} + (4+l+4m)J_{4+l+4m}}{3+4m}\right] J_l, \tag{69}$$

$$\Pi^{le,P}_{yx} \doteq \frac{2}{\pi b_e}\sum_m \left[\frac{(1+l+4m)J_{1+l+4m}}{1+4m} - \frac{(3+l+4m)J_{3+l+4m}}{3+4m}\right] J'_l, \tag{70}$$

$$\Pi^{le,P}_{zx} \doteq \frac{2}{\pi b_e}\sum_m \left[\frac{(1+l+4m)J_{1+l+4m}}{1+4m} - \frac{(3+l+4m)J_{3+l+4m}}{3+4m}\right] J_l. \tag{71}$$

The expression for the components A^α_j can be further developed by following along procedures analogous to those of Sec. IV A. As a result, the $S^{P,k}_j$ components can be written in the following compact form:

$$S^{P,k}_j = -a_k \sum_\ell B_{k\ell}\frac{n^k_{d0}}{n^\ell_{d0}} \sum_{l=-\infty}^{\infty}\sum_\alpha \frac{\omega^2_{p\alpha} m_\alpha}{n_{\alpha 0}} \times \int d\vec{p}\ \frac{[\nu^{\ell 0}_{\alpha d}(p)/\omega]\Pi^{l\alpha}_{zj}}{\omega - l\omega_\alpha - \frac{k_\parallel p_\parallel}{m_\alpha} + i\Sigma_m \nu^{m0}_{\alpha d}(p)} \left(\frac{p_\parallel}{p_\perp}\right)^{\delta_{jz}} \times \left[\mathcal{L}(f_{\alpha 0}) + \delta_{jz} i \left(\sum_j \frac{\nu^{j0}_{\alpha d}(p)}{\omega}\right)\frac{L_\alpha(f_{\alpha 0})}{p_\parallel}\right] + \delta_{jz}\frac{a_k}{\omega}\sum_\ell B_{k\ell}\frac{n^k_{d0}}{n^\ell_{d0}}\sum_\alpha \frac{\omega^2_{p\alpha} m_\alpha}{n_{\alpha 0}} \times \int d\vec{p}\ \frac{\nu^{\ell 0}_{\alpha d}(p)}{\omega}\frac{L_\alpha(f_{\alpha 0})}{p_\perp}, \tag{72}$$

in which we have taken into account that $\sum_l J_l^2 = 1$.

V. FINAL REMARKS

We have developed a kinetic theory for magnetized dusty plasmas in which the dust particles can be charged by inelastic collisions with plasma electrons and ions and by photoemission. The development started from basic principles, with emphasis on the similarities between the contributions due to the collisional charging and photoemission, as well as on the peculiarities associated with both contributions. The development associated with the collisional charging has already been presented in the literature, but the development of the terms associated with the emission of photoelectrons represents a novel contribution.

The outcome of the formulation is a dielectric tensor whose components are divided into three parts, denoted as ε_{ij}^{C}, ε_{ij}^{A}, and ε_{ij}^{P}. The ε_{ij}^{C} are closely related to the dielectric tensor of a dustless plasma, except for the addition of a imaginary contribution related to the collisional charging of the dust particles to the resonant denominator, and except for the addition of a term proportional to the inelastic collision frequencies between electrons and ions and the dust particles, to the component with $j=z$. The ε_{ij}^{A} only exists due to the presence of dust particles collisionally charged. In our formulation, the ε_{ij}^{A} were written as a product of two quantities which contain integrals over the distribution function of the plasma particles, electrons, and ions. The ε_{ij}^{P} are associated with the occurrence of photoelectric effect on the surface of the dust particles and have also been written as a product of two integral terms, one of them depending on a integral over the distribution functions of plasma electrons and ions and the other depending on a integration over the distribution function of the photoelectrons.

The dielectric tensor which has been obtained shall be useful to be used in the dispersion relation for the study of waves in dusty environments, allowing analysis of different sets of conditions, ranging from those where collisional charging is dominant to those where the incidence of radiation is such that it makes the photoemission dominant. These two limiting situations can be very different, since the charge in the dust charge is negative in the case of dominant collisional charging and becomes positive in the case of dominance of photoemission. To the best of our knowledge, studies on waves and instabilities in dusty plasmas, taking into account the combined effect of collisional charging and photoelectric emission, have not yet been made. We intend to utilize the formulation developed in this paper on the study of wave propagation in a forthcoming publication.

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APPENDIX: PHOTOELECTRIC EMISSION

The Sommerfeld model proposes that the energy states for free electrons in a metal are uniformly distributed in the *momenta* space, and that the probability of occupation of a state is given by the Fermi-Dirac distribution[17]

$$F(p') = \frac{2}{h^3}\left[1+\exp\left(\frac{p'^2}{2m_e k_B T} - \frac{\varepsilon_F}{k_B T}\right)\right]^{-1}, \tag{A1}$$

where m_e is the electron mass, h the Planck constant, k_B the Boltzmann constant, T the temperature, and ε_F the Fermi energy. Therefore, the number of electrons by unit volume with *momentum* components in the intervals p'_x to $p'_x + dp'_x$, p'_y to $p'_y + dp'_y$, and p'_z to $p'_z + dp'_z$ is given by

$$dN' = F(p')d\vec{p}\,'. \tag{A2}$$

By defining the z direction normal to the surface of the material, in a semi-classical approximation we can say that

$$d\Sigma_s = \frac{p'_z}{m_e} dN' \tag{A3}$$

is the density of electrons that strike the surface by unit area by unit time. Electrons with energy such that

$$\frac{p'^2_z}{2m_e} > \psi,$$

where ψ is the required energy for an electron overcome the surface potential barrier, may scape from the material.

The incidence of radiation on the metal surface may cause electrons to be emitted, provided that the frequency ν of the radiation is greater than the characteristic cut-off frequency ν_0 of the material. This is the so-called photoelectric effect, and the emitted electrons are called photoelectrons. The number of photoelectrons is proportional to the number of photons incident by unit of area by unit of time, which we denote as $\Lambda(\nu)$. Defining $\beta(\nu)$ as the ratio between the number of electrons absorbing photon energy by unit of area by unit of time and the product of the number of photons by unit of area by unit of time with the number of electrons which strike the surface from inside by unit of area by unit of time, the electron density at the surface that can lead to photoelectric emission is

$$d\Sigma_{ph} = \beta(\nu)\Lambda(\nu)d\Sigma_s. \tag{A4}$$

The product $\beta(\nu)\Lambda(\nu)$ is a dimensionless quantity, with $\Lambda(\nu)$ and $\beta(\nu)$ depending the frequency of the radiation. The quantity $\beta(\nu)$ can be considered as the probability of photon absorption at the surface.

The kinetic energy of the emitted photoelectrons will be

$$\frac{p^2}{2m_e} = \frac{p'^2}{2m_e} - \psi + h\nu, \tag{A5}$$

where $h\nu$ is the photon energy. If the emitting material has an electrostatic potential $\varphi_s > 0$ at the surface, photoelectrons that do not have energy greater then $e\varphi_s$ will return and will be captured. Therefore, the number of photoelectrons by unit of area by unit of time is given by

$$\begin{aligned}\Sigma_{ph} = \beta(\nu)\Lambda(\nu)\frac{2}{h^3}\int d\vec{p}\;\frac{p_z}{m_e}\Theta(p_z)\\ \times\Theta\left\{\frac{p^2}{2m_e} - e\varphi_s\Theta[\varphi_s]\right\}\\ \times\left[1+\exp\left(\frac{p^2}{2m_e k_B T_d} - \zeta\right)\right]^{-1},\end{aligned} \tag{A6}$$

where

$$\zeta = \frac{1}{k_B T_d}(h\nu - \phi)$$

and where $\phi = \psi - \varepsilon_F$ is the work function of the material.

In the case of a spherical particle of radius a with charge q uniformly distributed over the surface, the electrostatic potential at the surface is given by $\varphi_s = q/a$. In the case of anisotropic incidence of radiation, only one hemisphere of the particle will emit photoelectrons. By defining the z direction as the direction of propagation of radiation, and by considering that a fraction S_a of the incident radiation is absorbed, the intensity of the radiation over the surface varies, by considering azimuthal symmetry, according to

$$\Lambda(\nu) S_a \cos\theta, \tag{A7}$$

where θ is the angular variable in relation to z axis. To obtain the number of photoelectrons emitted by the particle, it is necessary to integrate over the spherical surface by taking into account the limit of integration $0 < \theta < \frac{\pi}{2}$. By considering that the component of *momentum* normal to the surface is given by $\vec{p}\cdot\hat{r}$, where $\hat{r}$ is a unit vector normal to the surface, the number of photoelectrons emitted by a spherical particle by unit of time is given by

$$\Omega_{ph,q} = \int_0^{2\pi} d\phi \int_0^{\frac{\pi}{2}} d\theta \left| \frac{\partial \vec{r}}{\partial \theta} \times \frac{\partial \vec{r}}{\partial \phi} \right| \beta(\nu)\Lambda(\nu) S_a \cos\theta \times \int d\vec{p}\, \frac{(\vec{p}\cdot\hat{r})}{m_e} \Theta(\vec{p}\cdot\vec{r}) \Theta\left[\frac{p^2}{2m_e} - \frac{eq}{a}\Theta(q)\right] F(p). \tag{A8}$$

A fair approximation is obtained by assuming a disk of radius a. After integration, one obtains

$$\Omega_{ph,q} = \pi a^2 \beta(\nu)\Lambda(\nu) S_a \times \int d\vec{p}\, \frac{p_z}{m_e} \Theta\left[\frac{p^2}{2m_e} - \frac{eq}{a}\Theta(q)\right] F(p).$$

We may define the quantity

$$\sigma_P(p,q) \doteq \pi a^2 \beta(\nu)\Lambda(\nu) S_a \Theta\left[1 - \frac{2m_e eq}{ap^2}\Theta(q)\right] \tag{A9}$$

as the photoemission cross section, and write

$$\Omega_{ph,q} = \frac{2}{h^3}\int d\vec{p}\; \sigma_P(p,q)\frac{p_z}{m_e} \times \left[1 + \exp\left(\frac{p^2}{2m_e k_B T_d} - \zeta\right)\right]^{-1}. \tag{A10}$$

Expression (A7) has been written accordingly with Mie theory,[20] in such a way that $S_a = S_e - S_s$, where S_e and S_s are, respectively, the extinction and scattering coefficients. The parameter λ is the wave length of the radiation and μ is the complex refractive index of the material. For the case where $2\pi a/\lambda \geq 10$, one may consider $S_a \approx 1$. Tables with values of S_e and S_s with dependence to $2\pi a/\lambda$ may be found in Refs. 2 and 26.

[1] J. W. Dewdney, "Energy distributions of photoelectrons from metals due to surface effect," Phys. Rev. **125**(2), 399 (1962).
[2] J. Dorschner, "Theoretische Untersuchungen über den interstellaren Staub II. Optische Eigenschaften kugelförmiger Staubteilchen aus meteoritischen Silikaten und aus schmutzigem Eis," Astron. Nachr. **292**, 71 (1970).
[3] R. H. Fowler, "The analysis of photoelectric sensitivity curves for clean metals at various temperatures," Phys. Rev. **38**, 456 (1931).
[4] R. A. Galvão, L. F. Ziebell, R. Gaelzer, and M. C. de Juli, "The dielectric tensor for magnetized dusty plasmas with superthermal plasma populations and dust particles of different sizes," Braz. J. Phys. **41**(4–6), 258–274 (2011).
[5] C. K. Goertz, "Dusty plasmas in the solar system," Rev. Geophys. **27**, 271, doi:10.1029/RG027i002p00271 (1989).
[6] J. Goree, "Charging of particles in a plasma," Plasma Sources Sci. Technol. **3**, 400 (1994).
[7] A. M. Ignatov, "Photoelectric charging of dust grains," Plasma Phys. Rep. **35**(8), 647–650 (2009).
[8] M. C. de Juli and R. S. Schneider, "The dielectric tensor for dusty magnetized plasmas with variable charge on dust particles," J. Plasma Phys. **60**(2), 243–263 (1998).
[9] M. C. de Juli and R. S. Schneider, "The spatial absorption of the magnetosonic waves in dusty magnetized plasmas," J. Plasma Phys. **64**, 57–56 (2000).
[10] M. C. de Juli, "Ondas em plasmas empoeirados magnetizados, com carga variável das partículas de poeira," *Tese de Doutorado* (Instituto de Física, Universidade Federal do Rio Grande do Sul, 2000).
[11] M. C. de Juli, R. S. Schneider, L. F. Ziebell, and V. Jatenco-Pereira, "Effects of dust-charge fluctuation on the damping of Alfvén waves in dusty plasmas," Phys. Plasmas **12**, 052109 (2005).
[12] M. Horányi, "Charged dust dynamics in the solar system," Annu. Rev. Astron. Astrophys. **34**, 383–418 (1996).
[13] B. Jung, "The origin of the solid particles in interstellar space," Astron. Nachr. **263**, 426 (1937).
[14] D. A. Mendis and M. Rosenberg, "Cosmic dusty plasma," Annu. Rev. Astron. Astrophys. **32**, 419–463 (1994).
[15] M. Rosenberg, D. A. Mendis, and D. P. Sheehan, "Positively charged dust crystals induced by radiative heating," IEEE Trans. Plasma Sci. **27**(1), 239–242 (1999).
[16] P. K. Shukla and A. A. Mamun, *Introduction to Dusty Plasma Physics* (IOP, 2005).
[17] M. S. Sodha, "Thermal and photoelectric ionization of solid particles," Br. J. Appl. Phys. **14**, 172–176 (1963).
[18] M. S. Sodha, S. Misra, and S. K. Mishra, "Charging of dust particles in an illuminated open complex plasma system," Phys. Plasmas **16**, 123705 (2009).
[19] M. S. Sodha, S. K. Mishra, S. Misra, and S. Srivastava, "Fluctuation of charge of dust particles in a complex plasma," Phys. Plasmas **17**, 073705 (2010).
[20] M. S. Sodha, S. K. Mishra, and S. Misra, "Kinetics of illuminated complex plasmas considering Mie scattering by spherical dust particles with a size distribution," J. Appl. Phys. **109**, 013303 (2011).
[21] V. J. Sterken, N. Altobelli, S. Kempf, G. Schwehm, R. Srama, and E. Grün, "The flow of interstellar dust into the solar system," Astron. Astrophys. **538**, A102 (2012).
[22] V. N. Tsytovich, "Dust plasma crystals, drops, and clouds," Phys. Usp. **40**(1), 53–94 (1997).
[23] V. N. Tsytovich and U. de Angelis, "Kinetic theory of dusty plasmas. I. General approach," Phys. Plasmas **6**(4), 1093–1106 (1999).
[24] S. V. Vladimirov, "Propagation of waves in dusty plasmas with variable charges on dust particles," Phys. Plasmas **1**(8), 2762–2767 (1994).
[25] S. V. Vladimirov, K. Ostrikov, and A. A. Samarian, *Physics and Applications of Complex Plasmas* (Imperial College Press, 2005).
[26] N. C. Wikramsinghe, *Light Scattering Functions for Small Particles* (Wiley, New York, 1973).