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On the finiteness of noncommutative supersymmetric QED$_3$ in the covariant superfield formulation

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Abstract

The three-dimensional noncommutative supersymmetric QED is investigated within the superfield approach. We prove the absence of UV/IR mixing in the theory at any loop order and demonstrate its one-loop finiteness.

During last years noncommutative gauge theories have been intensively studied. The interest in this subject has deep motivations coming mainly from string theory [1] (for a review see [2,3]). Different aspects of noncommutative gauge theories were discussed in [4–11].

One of the most remarkable properties of noncommutative theories consists of an unusual structure of divergences, the so-called UV/IR mixing, that could lead to the appearance of infrared divergences [4,12]. It should be noticed that the cancellation of quadratic and linear ultraviolet divergences in commutative theories does not guarantee the absence of harmful infrared divergences in their noncommutative counterparts [13–16]. The elimination of such divergences is crucial since they may obstruct the development of a sound renormalization scheme, leading to the breakdown of the perturbative series.

Based on experience, it is natural to expect that supersymmetry could improve this situation [4,17]. In fact, the Wess–Zumino model [14] and the three-dimensional sigma-model [18] are renormalizable at all loop orders. This is furtherly supported by the results of [19] according to which the one-loop effective action in $\mathcal{N}=1,2$ super-Yang–Mills theory contains only logarithmic divergences while for $\mathcal{N}=4$ the theory is one-loop finite [19, 20].

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In this Letter we employ the covariant superfield formalism to study noncommutative supersymmetric QED_3. We will prove that this theory is free of nonintegrable UV/IR divergences at any loop order. We shall also demonstrate that the model is one-loop finite.

The action of the three-dimensional $\mathcal{N}=1$ noncommutative supersymmetric QED is [21]

$$S=\frac{1}{2g^2}\int d^5z\, W^\alpha * W_\alpha, \tag{1}$$

where

$$W_\beta=\frac{1}{2}D^\alpha D_\beta A_\alpha-\frac{i}{2}\bigl[A^\alpha, D_\alpha A_\beta\bigr]-\frac{1}{6}\bigl[A^\alpha,\{A_\alpha,A_\beta\}\bigr] \tag{2}$$

is a superfield strength constructed from the spinor superpotential A_α. Hereafter it is implicitly assumed that all commutators and anticommutators are Moyal ones. In this work we consider only space–space noncommutativity, to evade unitarity problems [22]. This action is invariant under the gauge transformations

$$\delta A_\alpha=D_\alpha K-i[A_\alpha,K]. \tag{3}$$

Then, we must add a gauge fixing term which we choose to be

$$S_{\mathrm{GF}}=-\frac{1}{4\xi g^2}\int d^5z\,\bigl(D^\alpha A_\alpha\bigr)D^2\bigl(D^\beta A_\beta\bigr), \tag{4}$$

leading to the quadratic action

$$S_2=\frac{1}{2g^2}\int d^5z\left[\frac{1}{2}\left(1+\frac{1}{\xi}\right)A^\alpha\Box A_\alpha-\frac{1}{2}\left(1-\frac{1}{\xi}\right)A^\alpha i\partial_{\alpha\beta}D^2A^\beta\right]. \tag{5}$$

The free gauge propagator is

$$\bigl\langle A^\alpha(z_1)A^\beta(z_2)\bigr\rangle=\frac{ig^2}{2}\left[C^{\alpha\beta}\frac{1}{\Box}(\xi+1)-\frac{1}{\Box^2}(\xi-1)i\partial^{\alpha\beta}D^2\right]\delta^5(z_1-z_2), \tag{6}$$

where $C^{\alpha\beta}=-C_{\alpha\beta}$ is the second-rank antisymmetric symbol defined with the normalization $C^{12}=i$. The most convenient choice for the gauge fixing parameter is $\xi=1$, the Feynman gauge, in which the propagator collapses to

$$\bigl\langle A^\alpha(z_1)A^\beta(z_2)\bigr\rangle=ig^2C^{\alpha\beta}\frac{1}{\Box}\delta^5(z_1-z_2). \tag{7}$$

The interaction part of the classical action in the pure gauge sector is

$$\begin{aligned}S_{\mathrm{int}}=\frac{1}{g^2}\int d^5z\Bigl[&-\frac{i}{4}D^\gamma D^\alpha A_\gamma*\bigl[A^\beta,D_\beta A_\alpha\bigr]-\frac{1}{12}D^\gamma D^\alpha A_\gamma*\bigl[A^\beta,\{A_\beta,A_\alpha\}\bigr]\\&-\frac{1}{8}\bigl[A^\gamma,D_\gamma A^\alpha\bigr]*\bigl[A^\beta,D_\beta A_\alpha\bigr]+\frac{i}{12}\bigl[A^\gamma,D_\gamma A^\alpha\bigr]*\bigl[A^\beta,\{A_\beta,A_\alpha\}\bigr]\\&+\frac{1}{72}\bigl[A^\gamma,\bigl\{A_\gamma,A^\alpha\bigr\}\bigr]*\bigl[A^\beta,\{A_\beta,A_\alpha\}\bigr]\Bigr].\end{aligned} \tag{8}$$

The action of the associated Faddeev–Popov ghosts reads

$$S_{\mathrm{FP}}=\frac{1}{2g^2}\int d^5z\,\bigl(c'D^\alpha D_\alpha c+ic'*D^\alpha[A_\alpha,c]\bigr), \tag{9}$$

implying in the propagator

$$\bigl\langle c'(z_1)c(z_2)\bigr\rangle=-ig^2\frac{D^2}{\Box}\delta^5(z_1-z_2). \tag{10}$$

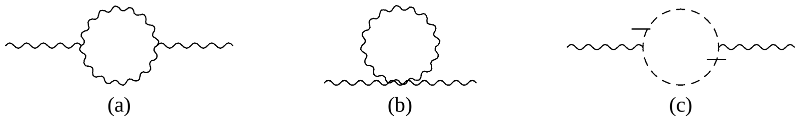

Fig. 1. Superficially linearly divergent diagrams contributing to the two-point function of the gauge field.

We assume that the ghosts are in the adjoint representation. The total action is, then, given by

$$S_{\text{total}} = S + S_{\text{GF}} + S_{\text{FP}}. \tag{11}$$

To study the divergence structure of the model we shall start by determining the superficial degree of divergence ω associated to a generic supergraph. Explicitly, ω receives contributions from the propagators and implicitly from the supercovariant derivatives. This last dependence can be unveiled by the use of the conversion rule

$$D_\alpha D_\beta = i\partial_{\alpha\beta} - C_{\alpha\beta} D^2 \tag{12}$$

and the identity $(D^2)^2 = \Box$. Each loop contributes two power of momentum. To see how this come about, notice that each integration over d^3k is decreased by one power of momentum when contracting the corresponding loop into a point. It can be seen that, if V_3, V_2, V_1, and V_0 are, respectively, the number of *pure* gauge vertices with three, two, one and none spinor derivatives, then, they altogether will contribute $\frac{3}{2}V_3 + V_2 + \frac{1}{2}V_1$ to ω. Furthermore, V_c gauge-ghost vertices will increase ω by $\frac{1}{2}V_c$. Each gauge propagator (let their number be P_A) lowers ω by two, each ghost propagator (let their number be P_c) lowers ω by one. Moving a supercovariant derivative to an external field decreases ω by $\frac{1}{2}$ (let N_D be the number of spinor derivatives moved to the external fields). Putting everything together we may conclude that ω is given by

$$\omega = 2L + \frac{3}{2}V_3 + V_2 + \frac{1}{2}(V_1 + V_c) - 2P_A - P_c - \frac{1}{2}N_D. \tag{13}$$

The number of the ghost vertices is equal to the number of the ghost propagators, $P_c = V_c$, since the ghost propagators only form closed loops. Thus, after using the topological identity $L + V - P = 1$ with $P = P_A + P_c$ and $V = V_c + V_0 + V_1 + V_2 + V_3$, we obtain

$$\omega = 2 - \frac{1}{2}V_c - 2V_0 - \frac{3}{2}V_1 - V_2 - \frac{1}{2}V_3 - \frac{1}{2}N_D. \tag{14}$$

This power counting relationship characterizes noncommutative supersymmetric QED_3 as an UV super-renormalizable theory. It is easy to realize that linear divergences may come only from the graphs with $V_3 = 2$, or $V_2 = 1$, or $V_c = 2$. These graphs are depicted in Fig. 1, they contribute to the two-point functions of A^α field. In these graphs, a crossed line corresponds to a factor D_α acting on the ghost propagator. A trigonometric factor $e^{ik\wedge l} - e^{il\wedge k} = 2i\sin(k\wedge l)$ originates from each commutator. By denoting the contributions of the graphs in Fig. 1 by $I_{1\text{a}}$, $I_{1\text{b}}$, and $I_{1\text{c}}$, respectively, we have

$$\begin{aligned}
I_{1\text{a}} = {} & \frac{1}{32}\int \frac{d^3p}{(2\pi)^3}\, d^2\theta_1\, d^2\theta_2 \int \frac{d^3k}{(2\pi)^3} \frac{\sin^2(k\wedge p)}{k^2(p-k)^2} A^\beta(-p,\theta_1) A^{\beta'}(p,\theta_2) \\
& \times \biggl[-D^\gamma D^\alpha \left(C_{\gamma\gamma'} \frac{\xi+1}{k^2} + k_{\gamma\gamma'} \frac{\xi-1}{k^4} D^2 \right) D^{\alpha'} D^{\gamma'} \delta_{12} \\
& \quad \times D_\beta \left(C_{\alpha\alpha'} \frac{\xi+1}{k^2} + (p-k)_{\alpha\alpha'} \frac{\xi-1}{(p-k)^4} D^2 \right) D_{\beta'} \delta_{12} \\
& \quad + D^\gamma D^\alpha \left(C_{\gamma\alpha'} \frac{\xi+1}{k^2} + k_{\gamma\alpha'} \frac{\xi-1}{k^4} D^2 \right) D_{\beta'} \delta_{12} \\
& \quad \times D_\beta \left(C_{\alpha\gamma'} \frac{\xi+1}{k^2} + (p-k)_{\alpha\gamma'} \frac{\xi-1}{(p-k)^4} D^2 \right) D^{\alpha'} D^{\gamma'} \delta_{12} \biggr] + \cdots,
\end{aligned} \tag{15a}$$

$$
\begin{aligned}
I_{1\mathrm{b}} = {} & \frac{1}{3}(\xi+1)\int \frac{d^3p}{(2\pi)^3}\, d^2\theta_1 \int \frac{d^3k}{(2\pi)^3}\frac{\sin^2(k\wedge p)}{k^2} \\
& \times \left[A^\beta(-p,\theta_1)A_\beta(p,\theta_1)C_{\gamma\alpha}D^\gamma D^\alpha\delta_{12}\big|_{\theta_1=\theta_2} - A^\beta(-p,\theta_1)A_\alpha(p,\theta_1)C_{\gamma\beta}D^\gamma D^\alpha\delta_{12}\big|_{\theta_1=\theta_2}\right] \\
& + \frac{1}{3}(\xi-1)\int \frac{d^3p}{(2\pi)^3}\, d^2\theta_1 \int \frac{d^3k}{(2\pi)^3}\frac{\sin^2(k\wedge p)}{k^4} \\
& \times \left[A^\beta(-p,\theta_1)A_\beta(p,\theta_1)k_{\gamma\alpha}D^\gamma D^\alpha D^2\delta_{12}^2\big|_{\theta_1=\theta_2} - A^\beta(-p,\theta_1)A_\alpha(p,\theta_1)k_{\gamma\beta}D^\gamma D^\alpha D^2\delta_{12}\big|_{\theta_1=\theta_2}\right] \\
& - \frac{1}{4}(\xi+1)\int \frac{d^3p}{(2\pi)^3}\, d^2\theta_1 \int \frac{d^3k}{(2\pi)^3}\frac{\sin^2(k\wedge p)}{k^2}A^\gamma(-p,\theta_1)A^\beta(p,\theta_1)\delta^\alpha_\alpha D_{\gamma 1}D_{\beta 2}\delta_{12}\bigg|_{\theta_1=\theta_2} \\
& - \frac{1}{4}(\xi-1)\int \frac{d^3p}{(2\pi)^3}\, d^2\theta_1 \int \frac{d^3k}{(2\pi)^3}\frac{\sin^2(k\wedge p)}{k^2}A^\gamma(-p,\theta_1)A^\beta(p,\theta_1)k^\alpha_\alpha D_{\gamma 1}D^2 D_{\beta 2}\delta_{12}\bigg|_{\theta_1=\theta_2} + \cdots,
\end{aligned}
\tag{15b}
$$

$$
I_{1\mathrm{c}} = \frac{1}{2}\int \frac{d^3p}{(2\pi)^3}\, d^2\theta_1\, d^2\theta_2 \int \frac{d^3k}{(2\pi)^3}\frac{\sin^2(k\wedge p)}{k^2(k+p)^2}A_\alpha(-p,\theta_1)A_\beta(p,\theta_2)D_1^\alpha D^2\delta_{12}D^2 D_2^\beta\delta_{12}. \tag{15c}
$$

Where not otherwise indicated it must be understood that the supercovariant derivatives act on the Grassmann variable θ_1, also $\delta_{12} = \delta^2(\theta_1-\theta_2)$. In the expressions for the I_1's the terms where covariant derivatives act on external fields were omitted because they do not produce linear divergences and UV/IR mixing (as we shall shortly verify, such terms give only finite contributions). In the formulae above they are indicated by the ellipsis. After some D-algebra transformations we arrive at

$$
I_{1\mathrm{a}} = -\frac{1}{2}\xi\int \frac{d^3p}{(2\pi)^3}\, d^2\theta_1 \int \frac{d^3k}{(2\pi)^3}\frac{\sin^2(k\wedge p)}{k^2}A^\beta(-p,\theta_1)A_\beta(p,\theta_1) + \cdots, \tag{16a}
$$

$$
I_{1\mathrm{b}} = \frac{1}{2}(1+\xi)\int \frac{d^3p}{(2\pi)^3}\, d^2\theta_1 \int \frac{d^3k}{(2\pi)^3}\frac{\sin^2(k\wedge p)}{k^2}A^\beta(-p,\theta_1)A_\beta(p,\theta_1) + \cdots, \tag{16b}
$$

$$
I_{1\mathrm{c}} = -\frac{1}{2}\int \frac{d^3p}{(2\pi)^3}\, d^2\theta_1 \int \frac{d^3k}{(2\pi)^3}\frac{\sin^2(k\wedge p)}{k^2}A^\beta(-p,\theta_1)A_\beta(p,\theta_1) + \cdots. \tag{16c}
$$

Hence, the total one-loop two-point function of the gauge superfield, given by $I_1 = I_{1\mathrm{a}} + I_{1\mathrm{b}} + I_{1\mathrm{c}}$, is free from both UV and UV/IR infrared singularities. The same situation takes place in the four-dimensional noncommutative supersymmetric QED [15,16]. It is also easy to show that the logarithmically divergent parts of $I_{1\mathrm{a}}$, $I_{1\mathrm{b}}$ and $I_{1\mathrm{c}}$, which involve derivatives of the gauge fields, turn out to be proportional to the integral

$$
\int \frac{d^3k}{(2\pi)^3}\frac{k_{\alpha\beta}\sin^2(k\wedge p)}{k^2(k+p)^2} \tag{17}
$$

and are therefore finite by symmetric integration. Thus, the logarithmic divergences in $I_{1\mathrm{a}}$, $I_{1\mathrm{b}}$, and $I_{1\mathrm{c}}$ are also absent, i.e., the two-point function of A^α field is *finite* in the one-loop approximation. We already mentioned that linear divergences are possible only for $V_2 = 1$, or $V_3 = 2$, or $V_c = 2$. Nevertheless, it is easy to see that two- and higher-loop graphs satisfying these conditions are just vacuum ones. Then, there are no linear UV and UV/IR infrared divergences beyond one-loop and, as consequence, *the Green functions are free of nonintegrable infrared divergences at any loop order*.

We examine next the structure of potentially logarithmic divergent diagrams. They correspond to $0 \leqslant \omega < 1$, which is possible if $V_0 = 1$, or $V_1 = 1$, or $V_2 = 2$, or $V_c = 3, 4$, or $V_c = 2$ with $V_2 = 1$, or $V_3 = 2$ with $V_2 = 1$, or $V_3 = 2$ with $V_c = 2$, or $V_3 = 3, 4$, or $V_2 = V_3 = 1$. Notwithstanding, the contributions of these graphs turn out to be very similar among themselves so that the same mechanism of cancellation of divergences applies. As a prototype

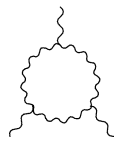

Fig. 2. A typical logarithmically divergent diagram.

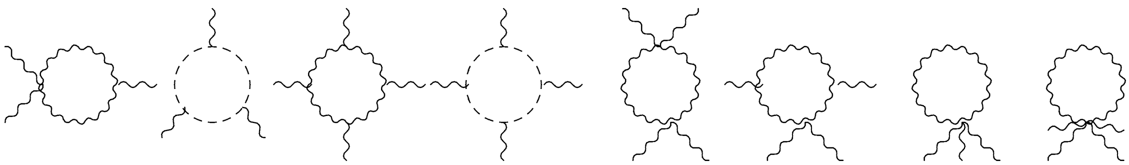

Fig. 3. Other superficially divergent contributions.

of this mechanism let us consider the supergraph with $V_3 = 3$ in Fig. 2. Its amplitude in the Feynman gauge reads

$$\begin{aligned} I_2 = {} & -\frac{1}{3}\left(\frac{i}{2}\right)^3 \int \frac{d^3p_1\, d^3p_2}{(2\pi)^6} \int d^2\theta_1\, d^2\theta_2\, d^2\theta_3 \int \frac{d^3k}{(2\pi)^3} \frac{\sin(k\wedge p_1)\sin[k\wedge(p_1+p_2)]\sin[(k+p_1)\wedge p_2]}{k^2(k+p_1)^2(k+p_1+p_2)^3} \\ & \times A_\beta(p_1,\theta_1)A_{\beta'}(p_2,\theta_2)A_{\beta''}(-p_1-p_2,\theta_3) \\ & \times D^\gamma D^\alpha D^{\beta'}\delta_{12} D^{\gamma'} D^{\alpha'} D^{\beta''}\delta_{23} D^{\gamma''} D^{\alpha''} D^{\beta}\delta_{12} C_{\gamma\alpha'} C_{\gamma'\alpha''} C_{\gamma''\alpha}. \end{aligned} \tag{18}$$

By using the relationship (12) and the identity $\{D_\alpha, D^2\} = 0$ we find that I_2 vanishes. The fact that this graph is finite is actually a gauge independent statement. Indeed, in an arbitrary gauge and after D-algebra transformations, $I_2^{(\xi)}$ is given by

$$\begin{aligned} I_2^{(\xi)} = {} & I_2 - i\frac{1}{6}\int d^2\theta \int \frac{d^3p_1\, d^3p_2}{(2\pi)^6} \int \frac{d^3k}{(2\pi)^3}\left[\sin(p_1\wedge p_2) - \sum_{i=1}^{3}\sin(2k\wedge p_i + p_1\wedge p_2)\right] \\ & \times \frac{1}{k^2(k+p_1)^2(k+p_1+p_2)^3} k^2\xi\left(\xi^2-1\right) A_\beta(p_1,\theta)A_{\beta'}(p_2,\theta) \\ & \times \left[k^{\beta'\beta''} D^\beta A_{\beta''}(p_3,\theta) + k^{\beta\beta''} D^{\beta'} A_{\beta''}(p_3,\theta) + k^{\beta'\beta} D^{\beta''} A_{\beta''}(p_3,\theta)\right], \end{aligned} \tag{19}$$

whose planar part is proportional to that of the integral in Eq. (17), which is finite. The nonplanar part of $I_2^{(\xi)}$ is composed of two terms, one proportional to

$$\int \frac{d^3k}{(2\pi)^3} \frac{k_{\alpha\beta}\cos(2k\wedge p)}{k^2(k+p)^2}, \tag{20}$$

which is evidently finite, and the other proportional to a linear combination of integrals of the form

$$\int \frac{d^3k}{(2\pi)^3} \frac{k_{\alpha\beta}\sin(2k\wedge p)}{k^4} = -\frac{i}{4\pi}\frac{\tilde{p}_{\alpha\beta}}{\sqrt{\tilde{p}^2}}. \tag{21}$$

Here, $\tilde{p}_{\alpha\beta} = \Theta_{mn} p^n (\sigma^m)_{\alpha\beta}$, and Θ_{mn} is the constant antisymmetric matrix characterizing the noncommutativity of the underlying space–time. As $\Theta^{0i} = 0$, this last expression does not produce logarithmic divergences, which confirms the finiteness of the contribution $I_2^{(\xi)}$.

The above mechanism also enforces the vanishing of UV logarithmic divergences and of UV/IR infrared logarithmic singularities from the graphs in Fig. 3. The UV finiteness of all these one-loop graphs may be proved in an analogous way. For example, in the Feynman gauge the one-loop graph with $V_2 = 2$ contains four spinor derivatives and its UV leading contribution is proportional to the finite integral in Eq. (17). A similar situation arises for the one-loop graph with $V_2 = V_3 = 1$. The one-loop graph with $V_3 = 2$ and $V_2 = 1$ contains 8 D-factors and, after using the identity $(D^2)^2 = \square$, either a finite contribution proportional to that in Eq. (17) or a finite term in which some derivatives are moved to the external fields could emerge. The others potentially divergent one-loop graphs correspond to $V_c = 4$ or $V_3 = 4$ and for them the same mechanism applies and, hence, they are finite. As it can be checked, the same happens in an arbitrary covariant gauge. The vanishing of UV/IR infrared singularities for all these graphs has the same origin as that for the graph in Fig. 2.

Up to this point, the net result of our study is that *the theory without matter turns out to be one-loop UV and IR finite*. It is interesting to note that, in the framework of the background field method [21,23], all contributions to the effective action are superficially finite. From a formal viewpoint this is caused by the presence of two spinor derivatives in the expression for the strength W_α in Eq. (2), which makes $N_D \geqslant 4$ in Eq. (14), since loop corrections must be at least of second order in the background strengths (compare with [19]). We also remark that Eq. (14) implies in the absence of divergences at three- and higher-loop orders, in agreement with the super-renormalizability of the theory. This concludes our analysis of the $\mathcal{N} = 1$ supersymmetry.

We next study the interaction of the spinor gauge field with matter. To this end we add to (36) the matter action

$$S_m = -\int d^5z \left[\frac{1}{2}\big(D^\alpha\bar{\phi}_a + i[\bar{\phi}_a, A^\alpha]\big) * \big(D_\alpha\phi_a - i[A_\alpha, \phi_a]\big) + m\bar{\phi}_a\phi_a\right]. \tag{22}$$

Here, ϕ_a, $a = 1, \ldots, N$, are scalar superfields and $\bar{\phi}_a$ their corresponding conjugate ones. We may also write

$$S_m = \int d^5z \left[\bar{\phi}_a\big(D^2 - m\big)\phi_a - i\frac{1}{2}\big([\bar{\phi}_a, A^\alpha] * D_\alpha\phi_a - D_\alpha\bar{\phi}_a * [A^\alpha, \phi_a]\big) - \frac{1}{2}[\bar{\phi}_a, A^\alpha] * [A_\alpha, \phi_a]\right]. \tag{23}$$

The free propagator of the scalar fields is

$$\big\langle\bar{\phi}_a(z_1)\phi_b(z_2)\big\rangle = i\delta_{ab}\frac{D^2 + m}{\square - m^2}\delta^5(z_1 - z_2), \tag{24}$$

which, in momentum space, reads

$$\big\langle\bar{\phi}_a(-k, \theta_1)\phi_b(k, \theta_2)\big\rangle = -i\delta_{ab}\frac{D^2 + m}{k^2 + m^2}\delta_{12}. \tag{25}$$

The superficial degree of divergence when matter is present is given by

$$\omega = 2 - \frac{1}{2}V_c - 2V_0 - \frac{3}{2}V_1 - V_2 - \frac{1}{2}V_3 - \frac{1}{2}E_\phi - \frac{1}{2}V_\phi^D - \frac{1}{2}N_D - V_\phi^0, \tag{26}$$

where, as before, V_i is the number of pure gauge vertices with i spinor derivatives, E_ϕ is the number of external scalar lines, N_D is the number of spinor derivatives associated to external lines, V_ϕ^D is the number of triple vertices $A^\alpha * \bar{\phi}_a * \overleftrightarrow{D}_\alpha\phi_a$, and V_ϕ^0 is the number of quartic vertices $\phi_a * \bar{\phi}_a * A^\alpha * A_\alpha$.

Graphs can now be split into those with $E_\phi = 0$ and those with $E_\phi \neq 0$. The leading UV divergence for those with $E_\phi = 0$ is $\omega = 3/2$, corresponding to a tadpole graph which vanishes identically. What comes next are graphs with two external A_α legs which are UV linearly divergent. They are depicted in Fig. 4. Graphs with three and four external A_α legs are UV logarithmically divergent. The remaining ones are finite. As for the graphs with $E_\phi \neq 0$, only those with $E_\phi = 2$ are potentially UV logarithmically divergent, those with $E_\phi > 2$ are finite.

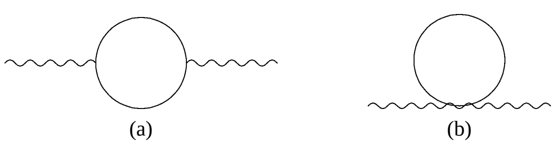

Fig. 4. One-loop corrections to the self-energy of the spinor gauge field.

Graphs with $E_\phi = 0$ verify the conditions $V_\phi^0 > 0$ or $V_\phi^D > 0$ which, unless for the tadpole graph already mentioned, imply that $\frac{1}{2}V_\phi^D + V_\phi^0 \geqslant 1$. On the other hand, if $\frac{1}{2}V_\phi^D + V_\phi^0 > 2$, the corresponding supergraphs are superficially finite, according to (26). Since there are no external matter legs, each vertex of the one-loop graph must involve matter. Hence, we arrive at the following condition for ω being nonnegative

$$1 \leqslant \frac{1}{2}V_\phi^D + V_\phi^0 \leqslant 2. \tag{27}$$

The lower limit of the inequality corresponds to $\omega = 1$, whereas the upper limit corresponds to $\omega = 0$.

The UV linearly divergent case is only realized by the one-loop matter correction to the two-point function of the gauge field A_α (Fig. 4). The graph (a) in Fig. 4 furnishes

$$\begin{aligned} I_{4\mathrm{a}} = -\int \frac{d^3p}{(2\pi)^3}\, d^2\theta_1\, d^2\theta_2 \int \frac{d^3k}{(2\pi)^3} A^\alpha(-p,\theta_1) A^\beta(p,\theta_2)\sin^2(k\wedge p) \\ \times \big[D_{\alpha 1}\langle\phi_a(1)\bar\phi_b(2)\rangle\big(D_{\beta 2}\langle\bar\phi_a(1)\phi_b(2)\rangle\big) - \big(D_{\alpha 1}D_{\beta 2}\langle\phi_a(1)\bar\phi_b(2)\rangle\big)\langle\bar\phi_a(1)\phi_b(2)\rangle\big], \end{aligned} \tag{28}$$

where the indices 1 and 2 in the supercovariant derivatives designate the field to which the D operator is applied. Taking into account the explicit form of the propagators, we found

$$\begin{aligned} I_{4\mathrm{a}} = N\int \frac{d^3p}{(2\pi)^3}\, d^2\theta_1\, d^2\theta_2 \int \frac{d^3k}{(2\pi)^3} A^\alpha(-p,\theta_1) A^\beta(p,\theta_2)\sin^2(k\wedge p) \\ \times \left[\frac{D_{\alpha 1}(D_1^2+m)}{k^2+m^2}\delta_{12}\frac{(D_1^2+m)D_{\beta 2}}{(k+p)^2+m^2}\delta_{12} - \frac{D_{\alpha 1}(D_1^2+m)D_{\beta 2}}{k^2+m^2}\delta_{12}\frac{D_1^2+m}{(k+p)^2+m^2}\delta_{12}\right], \end{aligned} \tag{29}$$

which, after using $D_{\beta 2}\delta_{12} = -D_{\beta 1}\delta_{12}$, can be cast as

$$\begin{aligned} I_{4\mathrm{a}} = N\int \frac{d^3p}{(2\pi)^3}\, d^2\theta_1\, d^2\theta_2 \int \frac{d^3k}{(2\pi)^3} J(k,p) \\ \times \big[2\big(D_1^2+m\big)\delta_{12}D_{\alpha 1}\big(D_1^2+m\big)D_{\beta 1}\delta_{12}A^\alpha(-p,\theta_1)A^\beta(p,\theta_2) \\ + \big(D_1^2+m\big)\delta_{12}\big(D_1^2+m\big)D_{\beta 1}\delta_{12}\big(D^\alpha A_\alpha\big)(-p,\theta_1)A^\beta(p,\theta_2)\big], \end{aligned} \tag{30}$$

where we have introduced the notation

$$J(k,p) = \frac{\sin^2(k\wedge p)}{(k^2+m^2)[(k+p)^2+m^2]}. \tag{31}$$

It is convenient to split $I_{4\mathrm{a}}$ into two parts, $I_{4\mathrm{a}} = I_{4\mathrm{a}}^{(1)} + I_{4\mathrm{a}}^{(2)}$, where $I_{4\mathrm{a}}^{(1)}$ and $I_{4\mathrm{a}}^{(2)}$ are, respectively, associated to the first and second terms in the large brackets in the right-hand side of Eq. (30). It is straightforward to verify that

$$\begin{aligned} I_{4\mathrm{a}}^{(1)} = 2N\int \frac{d^3p}{(2\pi)^3}\, d^2\theta \int \frac{d^3k}{(2\pi)^3} J(k,p) \\ \times\big[-\big(k^2+m^2\big)C_{\alpha\beta}A^\alpha(-p,\theta)A^\beta(p,\theta) + (k_{\alpha\beta} - mC_{\alpha\beta})\big(D^2A^\alpha(-p,\theta)\big)A^\beta(p,\theta)\big]. \end{aligned} \tag{32}$$

For the second term in the right-hand side of Eq. (30) one analogously finds

$$I_{4\mathrm{a}}^{(2)} = N\int \frac{d^3p}{(2\pi)^3}\, d^2\theta \int \frac{d^3k}{(2\pi)^3} J(k,p)\big[D^\gamma D^\alpha A_\alpha(-p,\theta)(k_{\gamma\beta} - mC_{\gamma\beta})A^\beta(p,\theta)\big]. \tag{33}$$

By adding Eqs. (32) and (33) we can cast the contribution from the graph (a) in Fig. 4 as

$$I_{4\mathrm{a}} = 2N\int \frac{d^3p}{(2\pi)^3}\, d^2\theta \int \frac{d^3k}{(2\pi)^3} \frac{\sin^2(k\wedge p)}{(k^2+m^2)[(k+p)^2+m^2]}$$

$$\times\left[-\left(k^2+m^2\right)C_{\alpha\beta}A^\alpha(-p,\theta)A^\beta(p,\theta)+(k_{\alpha\beta}-mC_{\alpha\beta})\left[D^2A^\alpha(-p,\theta)\right]A^\beta(p,\theta)\right.$$
$$\left.+\frac{1}{2}D^\gamma D^\alpha A_\alpha(k_{\gamma\beta}-mC_{\gamma\beta})A^\beta(p,\theta)\right]. \tag{34}$$

The algebraic manipulations for the graph (b) in Fig. 4 are simpler and yield

$$I_{4\mathrm{b}}=2N\int\frac{d^3p}{(2\pi)^3}\,d^2\theta\int\frac{d^3k}{(2\pi)^3}\frac{\sin^2(k\wedge p)}{(k+p)^2+m^2}C_{\alpha\beta}A^\alpha(-p,\theta)A^\beta(p,\theta). \tag{35}$$

The complete correction to the two-point function is, therefore,

$$I_4=2N\int\frac{d^3p}{(2\pi)^3}\,d^2\theta\int\frac{d^3k}{(2\pi)^3}\frac{\sin^2(k\wedge p)}{(k^2+m^2)[(k+p)^2+m^2]}$$
$$\times(k_{\gamma\beta}-mC_{\gamma\beta})\left[\left(D^2A^\gamma(-p,\theta)\right)A^\beta(p,\theta)+\frac{1}{2}D^\gamma D^\alpha A_\alpha(-p,\theta)A^\beta(p,\theta)\right]. \tag{36}$$

We stress that the dangerous linear divergences have disappeared, i.e., the two-point function of A^α field turns out to be free of UV/IR infrared singularities and, moreover, finite. This two-point function can be used for deriving the effective propagators in the $\frac{1}{N}$ expansion [24].

It remains to consider the graphs with $\omega=0$. It follows from (27), that the only remaining one-loop logarithmically divergent graphs involving matter are those ones depicted in Fig. 5. Nevertheless, a direct calculation shows that the planar contributions of the first two of these supergraphs is proportional to the integral in Eq. (17) whose divergent part is known to vanish. The divergent parts of their nonplanar contributions vanish in a way similar to that of the graphs in Figs. 2 and 3. As for the third graph, it is evidently finite.

We shall next deal with the graphs with $E_\phi>0$. Such graphs do not contain linear divergences, according to Eq. (26). Furthermore, the number of external scalar legs must be even since any vertex carries an even number of scalar fields, and only an even number of them can be contracted into propagators. As stated before, the logarithmic divergences in this case are possible only for $E_\phi=2$, $V_\phi^D=2$ and for $E_\phi=2$, $V_\phi^0=1$. These graphs are shown in Fig. 6. The graph (a) in Fig. 6 gives the contribution

$$I_{6\mathrm{a}}=2g^2\int\frac{d^3p}{(2\pi)^3}\,d^2\theta_1\,d^2\theta_2\int\frac{d^3k}{(2\pi)^3}\bar\phi_a(-p,\theta_1)\phi_a(p,\theta_2)\frac{\sin^2(k\wedge p)}{k^2[(k+p)^2+m^2]}D^\alpha\left(D^2-m\right)D^\beta\delta_{12}$$
$$\times\left[\frac{1}{2}(\xi+1)C_{\alpha\beta}+\frac{1}{2}(\xi-1)\frac{k_{\alpha\beta}}{k^2}D^2\right]\delta_{12}+\cdots. \tag{37}$$

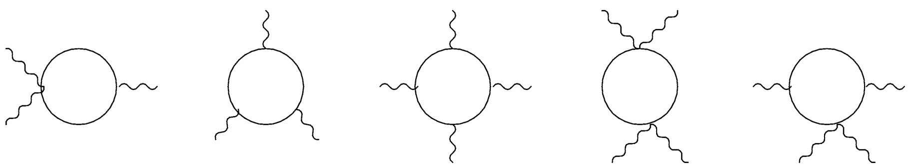

Fig. 5. Contributions to the three and four point functions of the spinor gauge field.

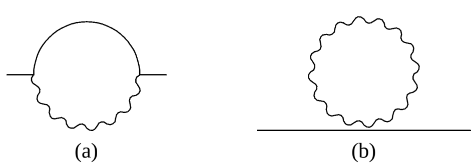

Fig. 6. One-loop corrections to the self-energy of the ϕ field.

As before, the ellipsis stands for manifestly finite terms. After some simplifications, one obtains

$$I_{6\mathrm{a}} = -2\xi g^2 m \int \frac{d^3p}{(2\pi)^3}\, d^2\theta \int \frac{d^3k}{(2\pi)^3} \bar{\phi}_a(-p,\theta)\phi_a(p,\theta) \frac{\sin^2(k\wedge p)}{k^2[(k+p)^2+m^2]}, \tag{38}$$

which is finite. The second graph in Fig. 6 yields the amplitude

$$I_{6\mathrm{b}} = (\xi - 1)\int \frac{d^3p}{(2\pi)^3}\, d^2\theta_1 \int \frac{d^3k}{(2\pi)^3} \bar{\phi}_a(-p,\theta_1)\phi_a(p,\theta_2) \frac{k^\alpha{}_\alpha}{k^4} \sin^2(k\wedge p) D^2\delta_{12}\bigg|_{\theta_1=\theta_2}, \tag{39}$$

which vanishes identically because of $k^\alpha{}_\alpha = 0$.

Therefore the two-point function of the scalar field is free from UV/IR mixing and, moreover, finite in any covariant gauge. It follows from Eq. (26) that the supergraphs with two or more external scalar legs and one or more gauge legs are also superficially finite.

To sum up we conclude that *the three-dimensional noncommutative supersymmetric QED is one-loop UV and UV/IR infrared finite both without and with matter*. A natural development of this work consists in the investigation of the possibility of appearance of divergences at two-loop order. Other possible developments are a detailed study of the $1/N$ expansion for the model involving many scalar fields and the analysis of spontaneous symmetry breaking and the Higgs mechanism.

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