



ELSEVIER

10 April 1997

PHYSICS LETTERS B

Physics Letters B 398 (1997) 12–17

$p\bar{p}$ annihilation into $\pi^+\pi^-\pi^0$ in a hadron-exchange model

M. Betz^a, E.A. Veit^{a,1}, V. Mull^b, K. Holinde^{b,2}^a Instituto de Física, Universidade Federal do Rio Grande do Sul, Cx. P. 15051, CEP 91501-970 Porto Alegre, Brazil^b Institut für Kernphysik, Forschungszentrum Jülich GmbH, 52425 Jülich, Germany

Received 30 August 1996; revised manuscript received 12 February 1997

Editor: C. Mahaux

Abstract

Proton–antiproton annihilation into three pions is studied in a hadron-exchange model. An uncorrelated three-pion channel is added to the two-meson channels considered in previous calculations. Initial-state interactions are taken into account in a distorted-wave Born approximation, using the most complete version of the Jülich $N\bar{N}$ model. The uncorrelated three-pion channel adds a 10% contribution to the $p\bar{p} \rightarrow \pi^+\pi^-\pi^0$ total cross section. The relative branching ratios for annihilation into three uncorrelated pions from states of equal orbital angular momentum, but different total angular momentum and spin, are also calculated. They compare semi-quantitatively with the corresponding quantities calculated from the phase-space components of phenomenological amplitudes fitted to experimental data for stopped antiprotons. It is concluded that hadron exchange provides a viable model for annihilation into three non-resonant pions. © 1997 Elsevier Science B.V.

PACS: 13.75.Cs; 25.43+t

Keywords: Strong interactions; Hadron-exchange models; Proton–antiproton annihilation

For more than 20 years, the $N\bar{N}$ system has attracted a great deal of interest both theoretically and experimentally [1,2], one motivation being to explore physics aspects which are less transparent in the short-range NN interaction. For instance, as the $N\bar{N}$ interaction is attractive in the inner region, it is supposedly more sensitive to effects of the quark–gluon dynamics than the NN interaction, which is repulsive in the inner region. Special attention has been given to $N\bar{N}$ annihilation, with quite different approaches ranging from statistical models to microscopic baryon exchange and explicit quark–gluon descriptions. For an overview of those models see Ref. [1].

Although considerable progress has been made in this area in the last decade, much work remains to be done in order to reach a satisfactory microscopic understanding of the $N\bar{N}$ interaction and to describe quantitatively the abundant and precise data presently existing for $N\bar{N}$ scattering as well as annihilation.

In the standard approach to $N\bar{N}$ scattering the elastic $N\bar{N}$ interaction at large and medium distances (> 1 fm) is obtained from a suitable NN model via a G -parity transformation. The short-distance part is treated in two completely different approaches. The Nijmegen [3] and the Paris [4] groups describe this region phenomenologically by using about 30 parameters. The Jülich group [5] aims to construct a dynamical model both for the large and medium distances and for the short-range region. This is undoubtedly a formidable challenge which must be pursued

¹ E-mail: eav@if.ufrgs.br.² Deceased.

in steps³. In the latest most advanced version [5], a consistent model describing $N\bar{N}$ scattering and annihilation into two mesons has been constructed by treating the $N\bar{N} \rightarrow N\bar{N}$ and $N\bar{N} \rightarrow M_1M_2$ amplitudes in a coupled-channel framework [6]. The two-meson channels explicitly considered include all possible combinations of π , η , ρ , ω , a_0 , f_0 , a_1 , f_1 , a_2 , f_2 , K and K^* . Unfortunately, these channels contribute less than 30% to the total annihilation cross section. The rest has to be attributed to channels with higher-mass mesons, three or more uncorrelated mesons, or exotic contributions. These processes are effectively described by a phenomenological optical potential in the $N\bar{N}$ interaction model. A more complete description could be achieved by including explicitly channels with more than two mesons. Hopefully, this would eventually remove the need for a purely phenomenological component.

Calculations of $N\bar{N}$ annihilation into more than two uncorrelated pions have already been performed [7], although in a simplified model and without taking into account interactions in the initial state, which are known to have a rather drastic effect on the magnitude of the cross sections.

The present work represents a first step in the program of extending the Jülich model of annihilation to include channels with more than two mesons. We restrict ourselves to the three-pion final state, more specifically to the $p\bar{p} \rightarrow \pi^+\pi^-\pi^0$ process. In addition to the contributions from channels composed of a pion and a heavier meson, considered in previous works, we include the contribution from the channel of three uncorrelated pions. Since, as in previous works, the heavy mesons are treated as stable particles, the contribution of this new channel adds incoherently to those of all relevant two-meson channels. In this first calculation, the transition amplitude for $N\bar{N} \rightarrow 3\pi$ is built from nucleon exchange only, while the $N\bar{N} \rightarrow M_1M_2$ amplitudes include Δ exchange also. This simplification requires some readjustment of the form-factor cutoff, to be explained. Interactions in the initial state are considered in a distorted-wave Born approximation (DWBA), using the most advanced version of the Jülich model.

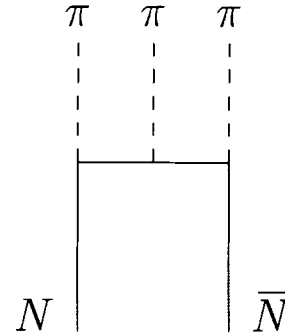


Fig. 1. Born transition amplitude for $N\bar{N} \rightarrow 3\pi$.

The basis of the present model is given in Ref. [5]. Starting from the standard pseudovector-coupling Lagrangian, we calculate the Born transition amplitude $V^{N\bar{N} \rightarrow 3\pi}$ from the Feynman tree diagram involving nucleon exchanges shown in Fig. 1. The complete amplitude is obtained by summing such diagrams over all permutations of the final-state pions. The value of the coupling constant is taken to be $f_{NN\pi}^2/4\pi = 0.0778$, as in the Bonn NN model. The vertices include form factors parametrized by the function

$$F(p) = \frac{\Lambda_{NN\pi}^2 - M^2}{\Lambda_{NN\pi}^2 - p^2}, \quad (1)$$

where M is the nucleon mass and p the four-momentum of the off-shell nucleon. For the inner vertex, there appears the product of two such form factors since there are two off-shell nucleons. The procedure for fixing the value of the cutoff parameter $\Lambda_{NN\pi}$ will be discussed shortly.

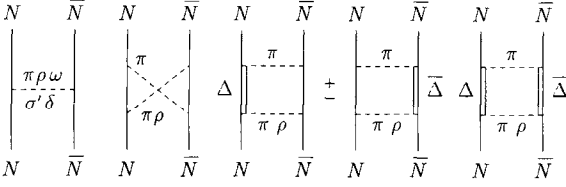
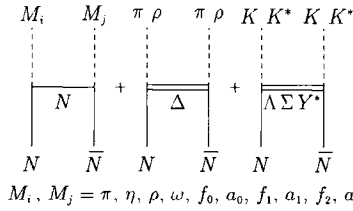
Interactions in the initial state are taken into account in DWBA, in which the annihilation amplitude into three pions is given by

$$T^{N\bar{N} \rightarrow 3\pi} = V^{N\bar{N} \rightarrow 3\pi} + V^{N\bar{N} \rightarrow 3\pi} G^{N\bar{N} \rightarrow N\bar{N}} T^{N\bar{N} \rightarrow N\bar{N}}, \quad (2)$$

where $G^{N\bar{N} \rightarrow N\bar{N}}$ is the propagator for the $N\bar{N}$ pair and the $N\bar{N}$ scattering amplitude $T^{N\bar{N} \rightarrow N\bar{N}}$ is obtained by the solution of a Lippmann–Schwinger equation [5]:

$$T^{N\bar{N} \rightarrow N\bar{N}} = V^{N\bar{N} \rightarrow N\bar{N}} + V^{N\bar{N} \rightarrow N\bar{N}} G^{N\bar{N} \rightarrow N\bar{N}} T^{N\bar{N} \rightarrow N\bar{N}}. \quad (3)$$

³ A review of the steps accomplished so far and a complete list of references may be found in Ref. [5].

Fig. 2. Elastic part of the $N\bar{N}$ interaction. $M_i, M_j = \pi, \eta, \rho, \omega, f_0, a_0, f_1, a_1, f_2, a_2$ Fig. 3. Transition potential for $N\bar{N} \rightarrow 2$ mesons.

The $N\bar{N}$ interaction $V^{N\bar{N} \rightarrow N\bar{N}}$ contains an elastic and an annihilation part:

$$V^{N\bar{N} \rightarrow N\bar{N}} = V_{\text{el}} + V_{\text{ann}}. \quad (4)$$

The elastic interaction is obtained through a G -parity transformation of the full Bonn NN potential [8], corresponding to the diagrams shown in Fig. 2.

The annihilation interaction consists of a microscopic and a phenomenological piece:

$$V_{\text{ann}} = \text{Im} \left[\sum_{ij} V^{M_i M_j \rightarrow N\bar{N}} G^{M_i M_j} V^{N\bar{N} \rightarrow M_i M_j} \right] + V_{\text{opt}}. \quad (5)$$

The microscopic component is the imaginary part of a sum of box diagrams with two-meson intermediate states resulting from all possible combinations of π , η , ρ , ω , a_0 , f_0 , a_1 , f_1 , a_2 , f_2 , K and K^* mesons. The $V^{N\bar{N} \rightarrow M_i M_j}$ transition potentials are given by the baryon-exchange diagrams represented in Fig. 3. The same transition potentials are used in the calculation of the amplitudes for annihilation into two mesons (M_1, M_2), which is given by an equation analogous to (2), with $V^{N\bar{N} \rightarrow M_1 M_2}$ as the Born term. The coupling constants and cutoff parameters at the vertices of these transition potentials are quoted in Table 1 of Ref. [5].

The phenomenological optical potential simulates the effect of other contributions, such as annihilation

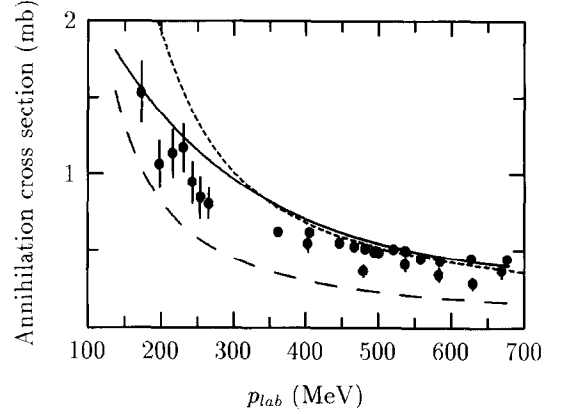


Fig. 4. $p\bar{p} \rightarrow \pi^+\pi^-$ cross section as a function of the incident momentum. The solid line corresponds to a calculation that includes N exchange as well as Δ exchange, with a cutoff $\Lambda_{NN\pi} = 1.5$ GeV [5]. Results of calculations including only t -channel N exchange for $\Lambda_{NN\pi} = 1.7$ GeV are shown by the short dashed line and for $\Lambda_{NN\pi} = 1.5$ GeV by the long dashed line. The data are from Refs. [9–11].

into more than two mesons, and is parametrized in coordinate space as:

$$V_{\text{opt}} = -iW \exp\left(-\frac{r^2}{2r_0^2}\right), \quad (6)$$

with $W = 1$ GeV and $r_0 = 0.4$ fm. These values have been obtained previously [5] through an overall fit to $N\bar{N}$ integrated cross-section data.

In the construction of the transition amplitude for $N\bar{N} \rightarrow 3\pi$, we do not consider Δ exchange as this would bring about considerable complications which seem uncalled for in an exploratory calculation. Of course, complete consistency is thus given up with the treatment of the $N\bar{N} \rightarrow 2$ mesons amplitudes, in which Δ exchange is taken into account. Although Δ exchange is known to be necessary to obtain a good description of the charge dependence of $p\bar{p} \rightarrow 2\pi$ data, its effect in a given charge channel can be parametrized by using an effective value for the cutoff $\Lambda_{NN\pi}$. This can be seen in Fig. 4, which shows results for the $p\bar{p} \rightarrow \pi^+\pi^-$ total cross section. The results of Ref. [5], obtained including N and Δ exchanges with $\Lambda_{NN\pi} = 1.5$ GeV, are well reproduced in the range $300 \text{ MeV} \leq p_{\text{lab}} \leq 700 \text{ MeV}$ by a calculation with N exchange only and $\Lambda_{NN\pi} = 1.7$ GeV. We shall assume that the effect of Δ exchange on the uncorre-

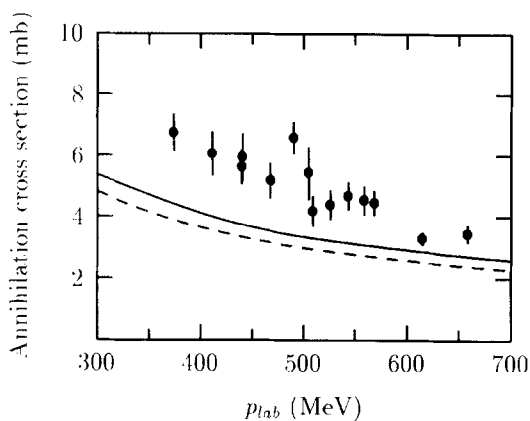


Fig. 5. $p\bar{p} \rightarrow \pi^+\pi^-\pi^0$ cross section as a function of the incident momentum. The dashed line corresponds to the sum of two-meson channels; the solid line, the sum of two-meson plus three-pion channels. The data are from Ref. [9].

lated $\pi^+\pi^-\pi^0$ channel can be similarly parametrized by using the same effective cutoff at the $NN\pi$ vertex as in the $\pi^+\pi^-$ channel. Thus we present calculations of the $N\bar{N} \rightarrow \pi^+\pi^-\pi^0$ cross section for $\Lambda_{NN\pi} = 1.7$ GeV, since the data are in the range $300 \text{ MeV} \leq p_{\text{lab}} \leq 700 \text{ MeV}$ where this effective cutoff seems reasonable.

Annihilation cross sections are calculated using the Monte Carlo method for the phase-space integration. The $p\bar{p}$ annihilation cross section into three pions has contributions from three uncorrelated pions in addition to some two-meson channels. The two-meson channels which contribute are $\rho\pi$, $f_0\pi$ and $f_2\pi$, since the ρ , f_0 and f_2 mesons decay into two pions, with branching ratios 100%, 78% and 85%, respectively.

Our results for the $p\bar{p} \rightarrow \pi^+\pi^-\pi^0$ cross section are compared with the experimental data [9] in Fig. 5. The theoretical results take into account the percentages of decay of the heavy mesons into 2 pions, quoted above, as well as the isospin weights. Even though most of this cross section is provided by the $\rho\pi$ channel, the uncorrelated three-pion channel is not negligible, amounting to about 10%.

Although the additional contribution due to the uncorrelated final state is welcome, the calculation still underestimates the data for the total cross section. A possible origin of the remaining discrepancy could be the absence, in the present model, of interferences between the different channels. Such interferences would appear if the heavy mesons were explicitly allowed to

decay into pions, rather than being treated as stable particles. Another effect which could enhance the theoretical cross section might be the final-state interactions which tend to increase the $\rho\pi$ contribution [12]. It is fairly clear from Fig. 5 that the discrepancy between the calculation and the data could be reduced or even eliminated by choosing a larger value of the effective cutoff at the $NN\pi$ vertex in the three-pion annihilation amplitude. However, we have refrained from adjusting any parameter to fit directly the cross section for annihilation into three pions. As explained above, the effective cutoff $\Lambda_{NN\pi}$ has been constrained by fitting the experimental $p\bar{p} \rightarrow \pi^+\pi^-$ total cross section. All other cutoff masses and all coupling constants, as well as the optical potential parameters, have been kept identical to the values previously adjusted.

In the case of stopped antiprotons, a more detailed comparison can be made by considering the experimental information from a specific $N\bar{N}$ initial state. The G -parity of the initial antiprotonium state is $G = (-1)^{L+S+I}$. For a three-pion final state $G = -1$. Therefore, in S -waves, three pions can be produced only from the $(^3S_1, I = 0)$ and $(^1S_0, I = 1)$ $N\bar{N}$ states. In P -waves, they can be produced from the $(^1P_1, I = 0)$, $(^3P_1, I = 1)$ and $(^3P_2, I = 1)$ states. Data for annihilation from these various initial states has been presented by the ASTERIX Collaboration [13]. It has been analysed in terms of resonant and non-resonant (phase-space) contributions. The observed contributions from the $\rho\pi$ channel have already been compared with the results of the Jülich model [12]. Here we concentrate on the phase-space component, which we compare to the uncorrelated three-pion contribution calculated in the present work. Following the procedure used previously [12], we assume that all the spin states of given orbital angular momentum in protonium are populated with about the same probability and identify the relative branching ratios for decay from given atomic states to the ratios between the contributions from the corresponding partial waves to the annihilation cross section at low energy ($E_{\text{lab}} = 5 \text{ MeV}$). At such small energies the employed parametrization of the Δ piece by an effective N exchange (with $\Lambda_{NN\pi} = 1.7 \text{ GeV}$) does not work anymore, as can be seen from Fig. 4. It leads to an overestimation of the empirical cross section. On the other hand, the use of the original cutoff value of $\Lambda_{NN\pi} = 1.5 \text{ GeV}$ underestimates the data. Therefore,

Table 1

Ratios of branching ratios for $p\bar{p} \rightarrow \pi^+\pi^-\pi^0$ at rest. The experimental values are taken from the phenomenological analysis of Ref. [13]. The theoretical results are obtained as relative cross sections at $E_{\text{lab}} = 5$ MeV for two values of the effective cutoff at the $NN\pi$ vertex. The ratios between numbers of states $(2J+1)$ are also listed

	$\Lambda_{NN\pi}$ (GeV)	${}^3S_1/{}^1S_0$	${}^3P_1/{}^1P_1$	${}^3P_2/{}^1P_1$
Dynamical model	1.50	3.2	1.6	1.4
	1.70	3.0	1.7	1.4
Experiment		1.9	2.9	3.8
State counting		3.0	1.0	1.67

we present theoretical results for both values of the cutoff, cf. Table 1. Clearly, the dependence of such ratios upon the value of the effective cutoff turns out to be quite weak.

As can be seen in Table 1, the ratios of spin triplet to singlet contributions for S -waves show little dependence of the dynamics upon the spin-isospin quantum numbers. The theoretical value is quite close to the value of 3 expected on the basis of simple state counting, and also not too far from the phenomenological value. In P -waves with total angular momentum $J = 1$, the spin-isospin dependence is significant, and qualitatively similar for the theoretical model and the phenomenological analysis. The situation is less satisfactory as far as the 3P_2 state is concerned. The theoretical ${}^3P_2/{}^1P_1$ ratio is close to the statistical value of $5/3$, whereas the phenomenological value is more than twice as large. One should note, however, that the separation of the $\rho\pi$ production into 3P_1 and 3P_2 contributions is ambiguous in the phenomenological analysis [13]. As a consequence, the phase-space contribution to annihilation from the 3P_2 state may have been overestimated. It should also be kept in mind that the assumption of purely statistical population of the initial states may be questioned [14].

On the theoretical side, previous experience indicates that these ratios may be sensitive to the kind of initial state interaction included [5], as well as to the final state interactions [12]. The latter are not taken into account in this first calculation with three uncorrelated pions. Interferences between resonant and non-resonant amplitudes could also affect the results significantly. With due concession for these uncertainties, the present model seems generally compatible with the experimental data.

In summary, the inclusion of an uncorrelated three-pion channel in the Jülich model improves the description of the total cross section for the $p\bar{p} \rightarrow \pi^+\pi^-\pi^0$ annihilation process. We stress that this is by no means a trivial result since, in view of the cumulative effect of form factors at the three vertices, the contribution in question could a priori easily have come out as insignificant or unacceptably large. The hadron-exchange model also describes semi-quantitatively the experimentally determined relative branching ratios for annihilation into three uncorrelated pions from states of equal orbital angular momentum, but different total angular momentum and spin. Although it may be argued that this does not constitute a stringent test, we note that it would not be passed by a model implying strong dynamical selection rules in this channel, since the experimental ratios do not differ very much from those deduced from state counting.

These results indicate that the hadron-exchange model, which has so far been successfully used to describe $N\bar{N}$ annihilation into two mesons, is also capable of accounting for annihilation into more than two uncorrelated mesons. Besides motivating a more thorough study of the three-pion channels – taking into account Δ exchange and interference effects – this conclusion suggests also that the inclusion of three-meson channels containing all relevant combinations of light and heavy mesons in the model might provide a unified description of a large fraction of all annihilations. With ever increasing computational capacity, the extension of this type of calculation to states with four or more uncorrelated mesons might also be soon within reach.

Acknowledgements

We thank “Centro Nacional de Supercomputação da Universidade Federal do Rio Grande do Sul (UFRGS)” where much of the computational work has been done. Financial support for this work was provided by the international agreement KFA (Germany)–CNPq (Brazil).

References

- [1] C.B. Dover, T. Gutsche, M. Maruyama and A. Fäbller, Prog. Part. Nucl. Phys. 29 (1992) 87.

- [2] A. Amsler and F. Myhrer, *Ann. Rev. Nucl. Part. Sci.* 41 (1991) 219.
- [3] P.H. Timmers, W.A. van der Sanden and J.J. de Swart, *Phys. Rev. D* 29 (1984) 1928;
R. Timmermans, Th.A. Rijken and J.J. de Swart, *Phys. Rev. C* 50 (1994) 48.
- [4] M. Pignone, M. Lacombe, B. Loiseau and R. Vinh Mau, *Phys. Rev. C* 50 (1994) 2710.
- [5] V. Mull and K. Holinde, *Phys. Rev. C* 51 (1995) 2360.
- [6] G.Q. Liu and F. Tabakin, *Phys. Rev. C* 41 (1990) 665.
- [7] E.A. Veit, M.H. Steffani, J. Moraes and T. Kodama, PANIC93, Italy, Book of Abstract, Vol. II, p. 462;
J. Moraes, Ms. Thesis, Instituto de Física, UFRGS, unpublished.
- [8] R. Machleidt, K. Holinde and Ch. Elster, *Phys. Rep.* 149 (1987) 1; version given in Table 9 therein.
- [9] F. Sai, S. Sakamoto and S.S. Yamamoto, *Nucl. Phys. B* 213 (1983) 371.
- [10] Y. Sugimoto et al., *Phys. Rev. D* 37 (1988) 583.
- [11] G. Bardin et al., *Phys Lett. B* 192 (1987) 471.
- [12] V. Mull, G. Jansen, J. Speth and K. Holinde, *Phys. Lett. B* 347 (1995) 193.
- [13] B. May et al., *Z. Phys. C* 46 (1990) 191, 203.
- [14] A. Rotondi, *Nucl. Phys. A* 558 (1993) 235c.