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2015 J. Phys.: Conf. Ser. 630 012038

(http://iopscience.iop.org/1742-6596/630/1/012038)

Journal of Physics: Conference Series **630** (2015) 012038 doi:10.1088/1742-6596/630/1/012038

Differences Between The $^3P_0$ and C$^3P_0$ model in the Charming Strange Sector

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Abstract.
The goal of this work is to establish a comparison between the very well studied $^3P_0$ model and a bound-state corrected version, the C^3P_0 model, obtained from applying the Fock-Tani transformation to the $^3P_0$ model, in the context of the charmed-strange meson sector (D_{SJ} meson). In particular, we shall calculate the decay amplitudes and decay rates of the $D_{s1}(2460)^+ \to D_s^{*+}\pi^0$ and $D_{s1}(2536)^+ \to D^*(2010)^+K^0$, showing the differences between the two models.

1. Introduction

The Fock-Tani formalism is a field theoretic method appropriated for the simultaneous treatment of composite particles and their constituents. This technique was originally used in atomic physics [1] and later in hadron physics to describe hadron-hadron scattering interactions [2, 3, 4] and meson decay [5, 6].

The $^3P_0$ model is a typical decay model which considers only OZI-allowed decay processes. The model considers a quark-antiquark pair created with the vacuum quantum numbers which interact with a meson in the initial state. It is described as the non-relativistic limit of a pair creation Hamiltonian [7].

The Fock-Tani transformation is applied to a $\bar{q}q$ pair creation Hamiltonian, producing a characteristic expansion in powers of the wave function, where the $^3P_0$ model is the lowest order term in the expansion. The corrected $^3P_0$ model (C^3P_0) is obtained from higher orders terms in this expansion, where terms contaning the bound state kernel appear [5].

Both the $^3P_0$ model and C$^3P_0$ model have been widely used in the study of meson spectroscopy. Our motivation for this work is in a comparison of these models for mesons in the charmed-strange sector. In particular, we shall calculate the decay amplitudes and decay rates of $D_{s1}(2460)^+ \to D_s^{*+}\pi^0$ and $D_{s1}(2536)^+ \to D^*(2010)^+K^0$.

2. Mesons in the Fock-Tani formalism: a brief outline

In the Fock-Tani formalism (FTf) we can write the meson creation operators in the following form: $M_\alpha^\dagger = \Phi_\alpha^{\mu\nu} q_\mu^\dagger \bar{q}_\nu^\dagger$. A single particle state in second-quantization is

$$|\alpha\rangle = M_\alpha^\dagger |0\rangle ,$$

where $\Phi_\alpha^{\mu\nu}$ is the bound-state wave-function for two-quarks. The quark and antiquark operators obey the usual anticommutation relations. The composite meson operators satisfy non-canonical commutation relations

$$[M_\alpha, M_\beta] = 0 \quad ; \quad [M_\alpha, M_\beta^\dagger] = \delta_{\alpha\beta} - \Delta_{\alpha\beta},$$

where

$$\Delta_{\alpha\beta} = \Phi_\alpha^{*\mu\gamma}\Phi_\beta^{\gamma\rho} q_\rho^\dagger q_\mu + \Phi_\alpha^{*\mu\gamma}\Phi_\beta^{\gamma\rho} \bar{q}_\rho^\dagger \bar{q}_\mu \, .$$

The idea of the FTf is to make a representation change, where the composite particle operators are described by "ideal particle" operators that satisfy canonical commutation relations, i.e.,

$$[m_\alpha, m_\beta] = 0 \quad ; \quad [m_\alpha, m_\beta^\dagger] = \delta_{\alpha\beta} \, .$$

To implement this change of representation one can define a unitary transformation U that maps the composite state $|\alpha\rangle$ into an ideal state $|\alpha)$. In the meson case, for example, we have

$$U^{-1} M_\alpha^\dagger |0\rangle = m_\alpha^\dagger |0) \equiv |\alpha),$$

where $U = \exp(tF)$ and F is the generator of the meson transformation given by

$$F = m_\alpha^\dagger \tilde{M}_\alpha - \tilde{M}_\alpha^\dagger m_\alpha \, ,$$

with $\tilde{M}_\alpha$ defined up to third order

$$\tilde{M}_\alpha = M_\alpha + \frac{1}{2}\Delta_{\alpha\beta}M_\beta + \frac{1}{2}M_\beta^\dagger[\Delta_{\beta\gamma}, M_\alpha]M_\gamma \, .$$

3. The Microscopic Model

The Hamiltonian used in this model is inspirated in the $^3P_0$ model, deduced in [7]:

$$H_I = g\int d^3x \Psi^\dagger(\vec{x})\gamma^0\Psi(\vec{x}) \tag{1}$$

where $\Psi(\vec{x})$ is the Dirac quark field, one should note that the bilinear $\Psi^\dagger\gamma^0\Psi$ leads to the decay $(q\bar{q})_A \to (q\bar{q})_B + (q\bar{q})_C$ through the $b^\dagger d^\dagger$ term. Introducing the following notation $b \to q$; $d \to \bar{q}$; $\mu = (\vec{p}\,', s')$ e $\nu = (\vec{p}, s)$, after the expansion in the momentum representation, one obtains a compact notation for H_I:

$$H_I = V_{\mu\nu}\ q^\dagger_\mu\, \bar{q}^\dagger_\nu$$

where the sum (integration) is applied over repeated indeces and

$$V_{\mu\nu} \equiv -\gamma\ \delta_{f_\mu f_\nu}\delta_{c_\mu c_\nu}\delta\left(\vec{p}_\mu + \vec{p}_\nu\right)\chi^*_{s_\mu}\left[\vec{\sigma}\cdot\left(\vec{p}_\mu - \vec{p}_\nu\right)\right]\chi^c_{s_\nu}\,. \tag{2}$$

In Eq. (2) γ is the free parameter pair production strength with $\gamma = g/2m_q$, where m_q is the quark mass of the pair creation. Applying the Fock-Tani transformation to H_I one obtains the effective Hamiltonian

$$H^{C3P0}_{FT} = U^{-1}\, H_I\, U = H_0 + \delta H_1$$

The decay amplitude h_{fi} for $m_\gamma \to m_\alpha + m_\beta$, is given by

$$\langle f \mid H^{C3P0}_{FT} \mid i\rangle = \delta(P_\gamma - P_\alpha - P_\beta)\, h_{fi} \tag{3}$$

where $|i\rangle = m^\dagger_\gamma|0\rangle$, $|f\rangle = m^\dagger_\alpha m^\dagger_\beta|0\rangle$ and

$$\begin{aligned} h_{fi} = & -V_{\mu\nu}\left\{\Phi^{*\rho\nu}_\beta\Phi^{*\mu\eta}_\alpha + \Phi^{*\mu\eta}_\beta\Phi^{*\rho\nu}_\alpha\right\}\Phi^{\rho\eta}_\gamma \\ & -\frac{1}{4}V_{\mu\nu}\left\{\Phi^{*\rho\tau}_\alpha\Phi^{*\mu\eta}_\beta + \Phi^{*\rho\tau}_\beta\Phi^{*\mu\eta}_\alpha\right\}\Delta\left(\rho\eta;\lambda\nu\right)\Phi^{\lambda\tau}_\gamma \\ & -\frac{1}{4}V_{\mu\nu}\left\{\Phi^{*\rho\nu}_\alpha\Phi^{*\sigma\eta}_\beta + \Phi^{*\rho\nu}_\beta\Phi^{*\sigma\eta}_\alpha\right\}\Delta\left(\rho\eta;\mu\xi\right)\Phi^{\sigma\xi}_\gamma \\ & +\frac{1}{2}V_{\mu\nu}\left\{\Phi^{*\rho\tau}_\alpha\Phi^{*\sigma\eta}_\beta + \Phi^{*\sigma\eta}_\alpha\Phi^{*\rho\tau}_\beta\right\}\Delta\left(\rho\eta;\mu\nu\right)\Phi^{\sigma\tau}_\gamma \end{aligned} \tag{4}$$

In the Eq. (4), the terms dependent on Δ are the bound state corrections, where the kernel represents an intermediate state of transition of the particle from the initial state to the final state. In this kernel a sum is performed over mesons with the quantum numbers of the final state. The meson wave function is defined as

$$\Phi^{\mu\nu}_\alpha = \chi^{s_1 s_2}_{S_\alpha} f^{f_1 f_2}_{f_\alpha} C^{c_1 c_2}\Phi^{\vec{P}_\alpha - \vec{p}_1 - \vec{p}_2}_{nl}\ ,$$

where χ is spin; f is flavor and C are color coefficients. The spatial part is given by the SHO wave-functions [8].

4. Applications and Results

Now we shall consider some specific processes for a comparative study between the $^3P_0$ and C$^3P_0$ models. In particular, the decay processes studied are: $D_{s1}(2460)^+ \to D_s^{*+}\pi^0$ and $D_{s1}(2536)^+ \to D^*(2010)^+K^0$. The full expressions for the decay amplitudes h_{fi} has the following form

$$h_{fi} = \left[\frac{\gamma}{\pi^{1/4}(\rho+1)^2}\right]\sum_{LS}^{N}\mathcal{C}_{LS}Y_{LM}\left(\Omega\right)\ ,$$

where the coefficients $\mathcal{C}_{LS}$ are polynomials which have a dependence on the momentum P and in β gaussian width of the mesons involved in the processes. The decay amplitude h_{fi} can be combined with relativistic phase space to give the decay rate [5, 7]:

$$\Gamma_{A\rightarrow BC} = 2\pi\, P\, \frac{E_B E_C}{M_A} \left(\frac{\gamma}{\pi^{1/4}(\rho+1)^2}\right)^2 \sum_{LS}^{N} (\mathcal{C}_{LS})^2 \,.$$

The experimental values are extracted from "Particle Data Group 2010" (PDG) [9] and the theoretical values obtained with $^3P_0$ and C$^3P_0$ model for these processes are shown in tables 1 and 2. In Tab. 1 we can see that the $^3P_0$ model is zero for the decay processes with final state $D_{SJ}\,\pi$. In Tab. 2 the two models obtain the equal results. For the theoretical results presented in tables the γ and β (in GeV) are $\gamma = 0.420$, $\beta_{\pi^0} = 0.410$, $\beta_{K^0} = 0.399$, $\beta_{D^*(2010)^+} = 0.280$, $\beta_{D_s^{*+}} = 0.200$ and for the intermediate state $\beta_6 = 0.100$ and $\beta_7 = 0.630$ is the state 1^1S_0 and the intermediate state $\beta_8 = 0.300$ and $\beta_9 = 0.200$ is the state 1^3S_1.

Table 1. Experimental values of the total decay rates and branching ratios for the meson $D_{s1}(2460)^+$.

$D_{s1}(2460)^+$ Branching ratios			
Process	Exp. (PDG)	$^3P_0$	C^3P_0
$\Gamma_{D_s^{*+}\pi^0}/\Gamma_{tot}$	0.48 ± 0.11	0	0.014

Table 2. Experimental values of the total decay rates and branching ratios for the meson $D_{s1}(2536)^+$.

$D_{s1}(2536)^+$ Branching ratios			
Process	Exp. (PDG)	$^3P_0$	C^3P_0
$\Gamma^{S-wave}_{D^*(2010)^+K^0}/\Gamma_{D^*(2010)^+K^0}$	0.72 ± 0.05 ± 0.01	0.995	0.995

5. Conclusions

Briefly we presented a comparsion of the $^3P_0$ model and the Corrected $^3P_0$ model applied for two meson decay processes of the charmed-strange sector. In this sector the decay processes are of two forms: $D^*_{SJ} \rightarrow D_{SJ}\pi$ and $D^*_{SJ} \rightarrow D^*K$. The first can not be obtained in the $^3P_0$ model, but the C$^3P_0$ model can be applied for both decay processes. The next step will be to consider the other D decay channels in the Corrected $^3P_0$ model.

6. Acknowledgements

This work is partially financed by CNPq, Brazil.

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