Chaotic interaction of Langmuir solitons and long wavelength radiation

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In this work we analyze the interaction of isolated solitary structures and ion-acoustic radiation. If the radiation amplitude is small solitary structures persist, but when the amplitude grows energy transfer towards small spatial scales occurs. We show that transfer is particularly fast when a fixed point of a low dimensional model is destroyed. [S1063-651X(98)11811-1]

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I. INTRODUCTION

Langmuir turbulence has been one of the most studied problems in modern nonlinear plasma physics. Over the last years a great deal of effort has been directed to its analysis, as well as to the analysis of related subjects such as soliton dynamics, collapse, nucleation of cavitons, electromagnetic emission, and others [1]. More recently, attempts have been made to understand the turbulence in terms of concepts of nonlinear dynamics and chaos [2–6].

The conservative version of Langmuir turbulence is described by the Zakharov equations that couple the slowly varying amplitude of a high-frequency electric field, the Langmuir field, to slow density fluctuations, the ion-acoustic field. Decay processes deposit energy into Langmuir fluctuations with long wavelengths and if the energy thus accumulated exceeds the threshold for modulational instability, solitons can be formed.

In addition to solitons a certain amount of ion-acoustic radiation is also generated, a fact that creates the possibility of nonlinear wave interaction involving these two types of structures: solitons of the Langmuir field and long wavelength ion-acoustic radiation. In more specific terms, what happens is that as solitons are formed their shapes exhibit temporal oscillations [7]; if ion-acoustic fluctuations are also present, the possibility exists of interaction between the oscillatory degrees of freedom of solitons and the oscillating ion-acoustic waves. One has two length scales in the region of long scales. One of them is the soliton length scale, we shall call it L_s, and the other is the length scale of the ion-acoustic fluctuations, L_i. Both quantities shall be better defined later on, but we can already identify L_s with the length of the spatial region occupied by a single soliton, and L_i with the typical wavelength of an ion-acoustic wave. It has been shown that depending on the general conditions of the system, the mentioned interaction may lead to intense energy transfer from the spectral region of long wavelengths to the region with much shorter wavelengths, we call it L_{sh} with $L_{\mathrm{sh}} \ll L_s, L_i$. As energy moves into modes with small wavelengths, dissipation becomes progressively more important. However, as we are interested only in nonlinear transfer processes, we discard dissipation in a first approximation. It has been suggested that energy transfer occurs when the interaction is of chaotic nature. Presumably the process underlying the transfer is related to the diffusive processes induced by the presence of a stochastic drive in the system; the stochastic drive would be formed by the chaotic degrees of freedom [8].

Now we come to our point. In various earlier simulations [6,7,9] a modulationally perturbed plane wave is launched into the system. If the system is unstable a number of solitons and additional ion-acoustic radiation are formed. Solitons interact with each other and with the radiation, and transfer of energy towards small spatial scales L_{sh} may take place if nonintegrable features are prominent. The problem here is that this kind of simulations does not examine properly the interaction of individual solitons and the radiation, since soliton-soliton collisional processes cannot be disregarded under such conditions. It is not even clear which type of interaction, if soliton-soliton or soliton-radiation, is the dominant one responsible for the transfer. In addition, several systems display a small soliton density so that collisions are unlikely—in these systems one should focus attention on the individual interaction involving one single oscillating soliton and ion-acoustic waves. This is the purpose of the present paper. We shall examine the system evolving from an initial condition where only one single oscillating soliton and some radiation are present. In addition to simulations we develop a model where we perform averages over fast variables in order to make estimates with regard to the behavior of the collective variables of the system.

As will become clear, energy transfer starts to take place when the collective variables become chaotic. In general we shall see that while for moderate and small amplitudes of the perturbing ion-acoustic radiation solitons can be at least seen as metastable structures in the system, for large amplitudes transfer is fast and initial solitary structures are rapidly destroyed. It has been argued that solitons are robust enough to describe final states of this type of system [10–13]. But what we see here is that even if some solitons are present in asymptotic states of large amplitude regimes, those solitons are not the same as those present in earlier times—after the initial solitons are destroyed there are long stretches of times over which no organized structures are seen.

We finally mention that a number of works have already analyzed the interaction of localized structures and perturba-

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tions with longer wavelengths. In some of them only low dimensional models were investigated [14], and in others where full simulations were performed, chaotic dynamics was not the issue, although some nonintegrable features like soliton fusion have been reported [9].

We organize the paper as follows: in Sec. II we introduce the basic model and the numerical techniques to be used here; in Sec. III we discuss our initial conditions and perform the appropriate averages to single out the relevant collective variables; in Sec. IV we compare the low dimensional model with full simulations, and in Sec. V we summarize the work.

II. BASIC EQUATIONS AND NUMERICAL TECHNIQUES

The one dimensional Zakharov equations governing the Langmuir turbulence can be written in the adimensional form [6]

$$i\partial_t E + \partial_x^2 E = nE, \tag{1}$$

$$\partial_t^2 n - \partial_x^2 n = \partial_x^2 |E|^2, \tag{2}$$

with $\partial_t \equiv \partial/\partial t$, $\partial_x \equiv \partial/\partial x$. $E(x,t)$ is the slowly varying amplitude of the high-frequency Langmuir field and $n(x,t)$ are slow density fluctuations associated with the ion-acoustic field. The nonlinear Schrödinger (NLS) equation

$$i\partial_t E + \partial_x^2 E + |E|^2 E = 0 \tag{3}$$

is obtained from the set (1), (2) if one is allowed to approximate Eq. (2) in order to replace n with $-|E|^2 + \text{const}$. This approximation is called subsonic because it requires very slow time scales that $\partial_t^2 n(x,t) \ll \partial_x^2 n(x,t)$.

Our numerical approach is based on a pseudospectral method. We assume spatial periodicity with basic length L and expand $E(x,t)$ and $n(x,t)$ into Fourier series as

$$E(x,t) = \sum_{m=-N/2}^{+N/2} E_m(t) e^{imkx},$$

$$n(x,t) = \sum_{m=-N/2}^{+N/2} n_m(t) e^{imkx}. \tag{4}$$

The basic wave vector is defined in terms of the system length L as $k = 2\pi/L$, and the integer N represents the number of modes used in the simulations. To represent a continuous system one should take the limit $N \to \infty$. In practice we let $N = 1024$, removing half of the modes to cure aliasing problems associated with the fast Fourier transform (FFT) routines. Comparison with $N = 2048$ indicates numerical convergence in terms of number of modes. Accuracy is further checked by varying the tolerance factor of the numerical integrator and by monitoring the conserved energy [1]. We find that relative fluctuations in energy are about one part in $10^6 - 10^8$ and that variations of tolerance factor do not produce alterations in the outcome of runs.

Solitons of amplitude $\sqrt{2} a_s$ are formed when a homogeneous train of Langmuir radiation of amplitude $|E_0| \sim a_s$ becomes modulationally unstable. The subsonic growth rate Γ for a perturbation with wave vector k superimposed on the homogeneous train can be estimated as

$$\frac{\Gamma}{k} \sim \sqrt{|E_0|^2 - k^2}. \tag{5}$$

From relation (5) one sees that the only unstable modes are those for which $|E_0|^2 > k^2$. Now when $|E_0|^2 - k^2 \ll 1$, $\Gamma \ll k$. If this condition holds for the majority of modes, $\partial_t \ll \partial_x$, ion-acoustic fluctuations are mostly enslaved to the Langmuir field, and approximation (3) can be used. On the other hand when $|E_0|^2$ is not exceedingly small ion-acoustic fluctuations with $k \ll |E_0|$ may not be completely enslaved to the Langmuir field. Those free fluctuations are to be seen as independent degrees of freedom whose presence is capable of destroying the integrability of the system. Given that the maximum growth rate occurs for $k_{\max} \sim |E_0|$ and that the typical length scale of a soliton arising from modulational instability induced by a perturbation with wave vector $k_{\max}$ is given by $L_s \sim 2\pi/k_{\max}$ (this means that the one way to calculate the soliton length is to imagine that one has as many solitons as the number of wavelengths along the space—a more formal way shall be indicated shortly), free ion-acoustic radiation of wave vector $k_i = 2\pi/L_i$ (as a matter of fact this relation should serve as a definition for L_i; $L_i \equiv 1/k_i$) typically appears in the spectral region for which

$$L_i \gg L_s. \tag{6}$$

III. COLLECTIVE VARIABLES AND LOW DIMENSIONAL MODEL

Our system is multidimensional but we would like to see whether a small subgroup of modes is more active than the remaining. If this is the case one could try to describe the basic features of the full dynamics by a low dimensional approximation. As it turns out, such an approximation appears to be possible.

To see how to obtain the low dimensional model, we proceed as follows. We first recall that as initial conditions we are interested in configurations with isolated solitary structures. To represent this sort of states either analytically or in the simulations we shall first determine the stationary one-soliton solution for the full problem. We start by taking $\partial_t = 0$ in Eqs. (1) and (2) from which we first get $n \approx -|E_s|^2 + \text{const}$. Substituting this relation into Eq. (1), after some algebra one obtains

$$|E_s(x)| = \sqrt{2}\xi \,\text{sech}(\xi x), \tag{7}$$

which is the expression we are looking for. ξ is an arbitrary factor that measures either the amplitude or the inverse width of the soliton. Given the soliton shape by Eq. (7), we may now better define the soliton length scale L_s, introduced earlier, as $L_s \equiv 1/\xi$. We point out that due to the nonlinearities and dispersion of the problem a precise balance between amplitude and width is needed. If we call $a_s \equiv \xi$ and $w_s \equiv 1/\xi$, it is indeed seen that the following relation holds:

$$a_s = \frac{1}{w_s}. \tag{8}$$

We had mentioned that our interest is to see what could happen with the soliton when it starts to interact with free ion-acoustic radiation. Based on several results one knows already that the basic soliton solution must be allowed to display temporal oscillations. The problem now is how to describe those oscillations in a compact way. And the answer is known: one first writes an ansatz solution for the soliton field where amplitude is, however, not correlated to the width according to the static relation (8). The ansatz solution is therefore generically written in the form

$$E(x,t)=\sqrt{2}a(t)\mathrm{sech}\left(\frac{x}{w(t)}\right)e^{i\Phi(t)}, \tag{9}$$

where $a(t)$, $w(t)$, and $\Phi(t)$ are all unknown as yet. The phase factor Φ is included to incorporate the complex structure of the solutions of the set (1), (2). As for the ion-acoustic field interacting with the soliton field, one writes

$$n(x,t)=-|E(x,t)|^2+[A(t)e^{ikx}+\text{c.c.}]. \tag{10}$$

Here we write the ion-acoustic field as a sum of the pure adiabatic response to the soliton field, plus some free radiation that will actually interact with the isolated nonlinear structure. $A(t)$ is the amplitude of the radiation field and c.c. stands for complex conjugate. The next step is to derive the appropriate governing equations for the four time dependent parameters, $a(t)$, $w(t)$, $\Phi(t)$, and $A(t)$. This is more easily done with the help of average Lagrangian techniques. The full Lagrangian from which one obtains the original set (1), (2) reads

$$L=\int \mathcal{L}dx \equiv \int \Bigg(\frac{i}{2}(E^*\partial_t E - E\partial_t E^*)-|\partial_x E|^2 - |E|^2\partial_x \nu + \frac{1}{2}[(\partial_t \nu)^2-(\partial_x \nu)^2]\Bigg)dx, \tag{11}$$

where the dynamical variable $\nu(x,t)$ is introduced in the form $n(x,t)\equiv\partial_x \nu(x,t)$. The Euler-Lagrange equation for $E(x,t)$, for instance, is written as

$$\partial_t \frac{\partial \mathcal{L}}{\partial(\partial_t E)}=\frac{\partial \mathcal{L}}{\partial E}-\partial_x \frac{\partial \mathcal{L}}{\partial(\partial_x E)}, \tag{12}$$

with similar expressions holding for the other variables. From expression (12) one obtains the complex conjugate of Eq. (1). In terms of averaged Lagrangians, what has to be done now is to substitute into Eq. (11) the one-soliton solution, Eq. (9), plus the ion-acoustic field, Eq. (10). Doing this and performing the spatial integrations one arrives at

$$L\approx -2\eta\dot{\Phi}+\left[\frac{4W^2}{3w}-\frac{2W}{3w^2}+0.429\frac{W^2\dot{w}^2}{w}-3.290W\dot{w}w\dot{A}\right] + \frac{\pi\dot{A}^2}{k^3}-\frac{\pi A^2}{k}, \tag{13}$$

with $\eta=a(t)^2w(t)$. The various numerical factors appear in Eq. (13) as a result of the integrals involving trigonometric and hyperbolic functions.

The Euler-Lagrange equation with respect to the variable Φ indicates that η is a constant of motion. As a matter of fact this feature has already been used to simplify the form of the Lagrangian (13) by dropping terms proportional to $\dot{\eta}$ up to positive powers. Euler-Lagrange variational equations are then applied to the independent variables $w(t)$ and $A(t)$ to produce a two-degrees-of-freedom conservative dynamical system. If we set $A\to 0$ we have solutions corresponding to free oscillations of the soliton shape. One can construct a convenient phase space to visualize those oscillations. This is done in Fig. 1 where we plot $\dot{w}(t)$ versus $w(t)$. The central fixed point of the figure is simply the static soliton solution analytically represented by Eq. (7), and the curves surrounding the fixed point represent oscillatory modes of the soliton, each mode labeled by a particular constant energy that can be canonically evaluated from Lagrangian (13) with $A=0$. In the absence of ion-acoustic free fluctuations, one can estimate the position of the fixed point,

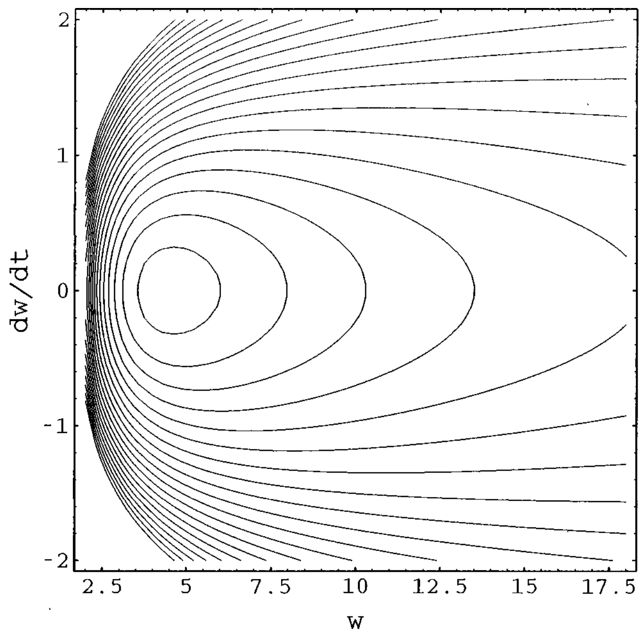

FIG. 1. Contour levels for the unperturbed dynamics $A\to 0$; $\eta=\sqrt{0.1/2}$.

$$a_s=1/w_s=\eta, \tag{14}$$

and the oscillatory frequency around the fixed point,

$$\omega_s=\sqrt{\frac{2\eta^2}{1.29}}\sim a_s. \tag{15}$$

Given that $L_s\equiv w_s=1/a_s$, one has $L_s\sim 1/\omega_s$, and given that $2\pi/\omega_i=L_i\gg L_s$ one obtains a relationship involving the frequencies of soliton and ion-acoustic waves:

$$\omega_i \ll \omega_s. \tag{16}$$

In other words, the components of the ion-acoustic field most weakly enslaved to the Langmuir field are those for which both length and time scales are much longer than the scales corresponding to the solitons. One shall also mention that in addition to trapped orbits around the fixed point, open orbits are also possible. Those would represent decaying solitons for which $w\to\infty$ asymptotically. The fact that one has trapped and untrapped orbits implies that a separatrix does exist in which vicinity some amount of chaotic activity may

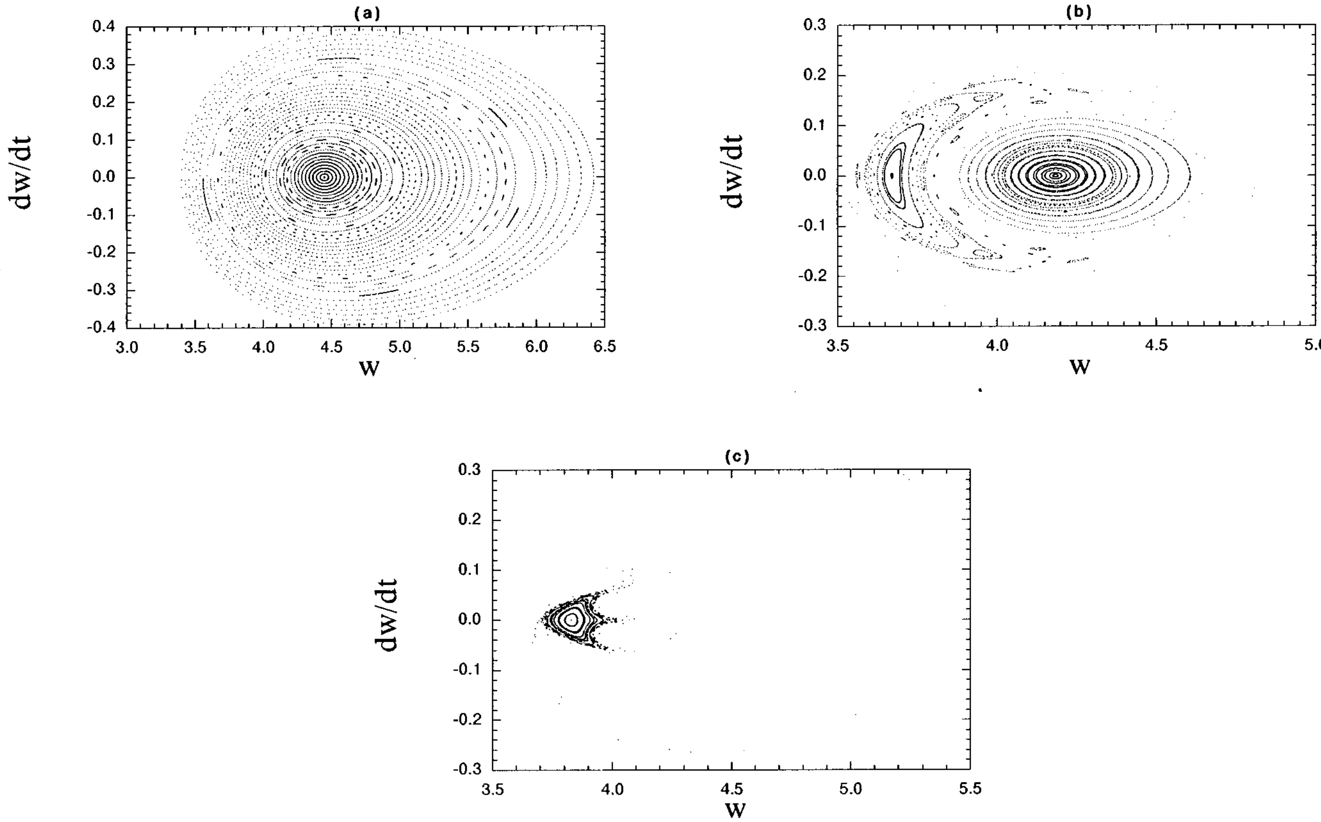

FIG. 2. Poincaré plots $(\dot{w},w)$ of the low dimensional model with $k_i=0.0257$ and $\eta=\sqrt{0.1/2}$. $A_0=0$ in (a), 0.14 in (b), and 0.16 in (c).

be displayed if the system is in fact nonintegrable. The role of chaos, if chaos is indeed present, shall be better explored in the next section.

IV. FULL SIMULATIONS VERSUS THE LOW DIMENSIONAL MODEL

At this point we make use of the numerical techniques discussed in Sec. II to compare results of full one dimensional simulations with the low dimensional model developed in the preceding section. Our full simulations give an account of the behavior of a stationary soliton submitted to the action of long wavelength ion-acoustic perturbations. Our purpose is to test the robustness of the soliton solution and see what happens when it loses stability due to the ion-acoustic radiation. Before embarking on the simulations it is perhaps convenient to preview the basic system behavior, based on possible results obtained with the low dimensional model. If the parameters of the low dimensional model are such that the corresponding nested orbits on the phase plane $\dot{w},w$ are mostly regular, one can expect a negligible influence exerted by the ion-acoustic field on the solitary structure in the full simulations, whether this structure is oscillatory or not. On the other hand it may well happen that the low dimensional system is nonintegrable. Should this be the case, and if low dimensional chaos is indeed well developed, the influence of ion-acoustic waves may be strong enough to destroy the solitary structure. In this case our low dimensional description may be expected to cease furnishing reasonable results. What is likely to happen then is that the chaotic low dimensional degrees of freedom start to act like a random drive, continuously delivering energy in a diffusive way to all the other dynamical modes [8]. Then one may anticipate the soliton to decrease in intensity as its energy flows away. In addition, short wavelength modes are expected to grow and appear in the spectrum. We shall investigate some details of the transfer next.

A. Low dimensional analysis

Let us first explore the integrability properties of the low dimensional approximation. To do that we examine the surface of section obtained when we record the pair of variables $w,\dot{w}$ each time $A=0$ with $\dot{A}>0$. In the context of the low dimensional analysis we examine the system as an ion-acoustic wave of initial amplitude A_0 is added to the central fixed point of Fig. 1—the remaining initial amplitudes corresponding to other orbits are obtained with help of the condition of constant energy. This constant energy is to be obtained from Lagrangian (13). Parameters are specified in the legend of Fig. 2. In both low dimensional and full simulation we work with a soliton of $a_s=\sqrt{0.1/2}$ and with a perturbing ion-acoustic wave vector $k_i=0.0257$. In the simulations the basic wave vector k is chosen as $k=k_i/2$; smaller values and even $k=k_i$ generate the same results. For those parameters,

$$L_i \sim 40 L_s, \tag{17}$$

and

$$\omega_s = 10.8\omega_i . \tag{18}$$

One thus has $\omega_s \gg \omega_i$ and $L_s \ll L_i$ as required by the assumptions on time and length scales. If one recalls the scalings used to derive the normalized form of the Zakharov equations, Eqs. (1) and (2) [1], one finds that the normalized distance unit approximately corresponds to 65 Debye lengths, where the Debye length λ_D is written as $\lambda_D \equiv \sqrt{\kappa T_e/4\pi n_0 e^2}$, with κ as the Boltzmann constant, T_e and e^2 as the temperature and squared charge of the electrons, respectively, and with n_0 as the equilibrium density of the system. Therefore, going momentarily over dimensional quantities, one has, in the present case, $L_s \sim a_s^{-1} 65\lambda_D \sim 290\lambda_D$ and $L_i \sim 40 L_s \sim 11\,600\lambda_D$, where $\lambda_D \sim 10^{-3}$ cm for a fusion plasma, for instance.

One should also notice that relation (18) says that if the system is indeed nonintegrable, a period one island is likely to appear close to the central fixed point in the $w,\dot{w}$ phase space.

Examining the phase plots of Fig. 2 one sees that for small amplitudes the phase space is mostly regular. However, for larger values two features become noticeable: (i) the dynamics is indeed nonintegrable, and (ii) for large enough values of the amplitude, chaotic dynamics is dominant. In addition, for sufficiently large amplitudes (in the present case $A_0 \sim 0.145$) the central fixed point undergoes an inverse tangent bifurcation and disappears along with the unstable fixed point of the period one island seen in Fig. 2(b). All these features strongly suggest that the stochastic drive mechanism may be operative causing energy transfer into small spatial scales for moderately large values of the perturbation. This type of behavior is found for other choices of the ratios ω_i/ω_s and L_i/L_s as long as relations (6) and (16) are respected.

B. Full simulations

The results of full simulations can be found in Fig. 3 where we make plots of the space-time history of the field $|E(x,t)|^2$, and of the average number of active modes versus time. The average number of modes is an auxiliary tool that can help to study details of energy transfer that are not particularly apparent in the space-time plots. The average number of modes is denoted by $\sqrt{\langle N_{L,i}^2 \rangle}$ for Langmuir and ion-acoustic fields, respectively, and defined according to the following [15]:

$$\langle N_L^2 \rangle \equiv \frac{\sum_m m^2 |E_m|^2}{\sum_m |E_m|^2}, \tag{19}$$

$$\langle N_i^2 \rangle \equiv \frac{\sum_m{}' \, m^2 |n_m|^2}{\sum_m{}' \, |n_m|^2}. \tag{20}$$

The primes in definition (20) mean that the ion modes into which energy is initially placed are to be excluded from the summation. We do this simply to obtain clearer results. The problem is that since all the initial ion-acoustic energy goes into one single mode, the statistics becomes poor if we do not make the exclusion. No problems of that sort occur with the Langmuir field, as solitons already involve a statistically good number of modes.

We launch a solitary structure of shape given by Eq. (9), with a slight mismatch between $a(t=0)$ and $1/w(t=0)$ such that the soliton can oscillate initially: we choose $a(t=0) = \sqrt{0.1/2}$ and $w(t=0) = 1.2/a(t=0)$. For small enough values of the ion-acoustic perturbation Fig. 3(a) shows that the solitary structure maintains its original amplitude without noticeable damping. It is seen from Fig. 3(b) that for this perturbing amplitude the number of modes involved in the dynamics does not change significantly as time evolves. It should be noticed that the present modes are those used to construct the solitary structure.

For larger values of the ion-acoustic amplitude, as in Fig. 3(c), the soliton gradually damps away as time advances. Fig. 3(d) shows that energy diffusion is now present, and that in the ion-acoustic field it is considerably much faster than in the Langmuir field. Diffusion in the Langmuir field becomes in fact almost imperceptible for even smaller perturbing amplitudes as we shall see a little later. Note that plateaus in the plots are formed when all the modes used in the simulation become involved in the dynamics—at this stage energy would be dissipated if we had added dissipation terms for large values of wave vectors. In the present case represented in Figs. 3(c) and 3(d), the central fixed point is still present in low dimensional phase plots, as indicated by Fig. 2(b). One can therefore think in terms of stochastic drive models to describe this type of regime [8]. Although the orbits are chaotic, the presence of the central fixed point offers some resistance against rapid destruction of the low dimensional chaotic system. This low dimensional system might therefore last long enough to serve as a drive delivering energy to short wavelength modes.

Now, if A is large enough, the localized solitary structure is rapidly destroyed as indicated in Figs. 3(e) and 3(f). Energy is transferred to short wavelengths over short periods of time. We point out that this fast process occurs for ion fields intense enough to destroy the central fixed point of the low dimensional system, as indicated in Fig. 2(c). In addition, diffusive time scales for both Langmuir and ion-acoustic fields become similar in this fast regime. Under such conditions the stochastic drive may not be a very appropriate concept since the lifetime of the solitary structure is too short.

We emphasize, therefore, that three distinct regimes appear to be present.

(i) If the initial perturbation is small, typically $A \ll 0.1$, there is no diffusion whatsoever towards small length scales.

(ii) For larger values of the perturbation, $A \sim 0.1$, diffusion is observed in both Langmuir and ion fields. But diffusion in the ion field is much faster. If one diminishes not too much the perturbing amplitude and reduces the observation time, diffusion in the Langmuir field becomes almost imperceptible although diffusion in the ion field can still be observed. This is what can be seen in Fig. 3(g) where one considers a perturbing amplitude smaller, but of the same order of magnitude, than the one used in Fig. 3(d). In general, within this range of perturbing amplitudes, the central fixed point of the low dimensional phase space is still present. This could explain the persistence of the solitary structure seen in Fig. 3(c). Since solitons are persistent and

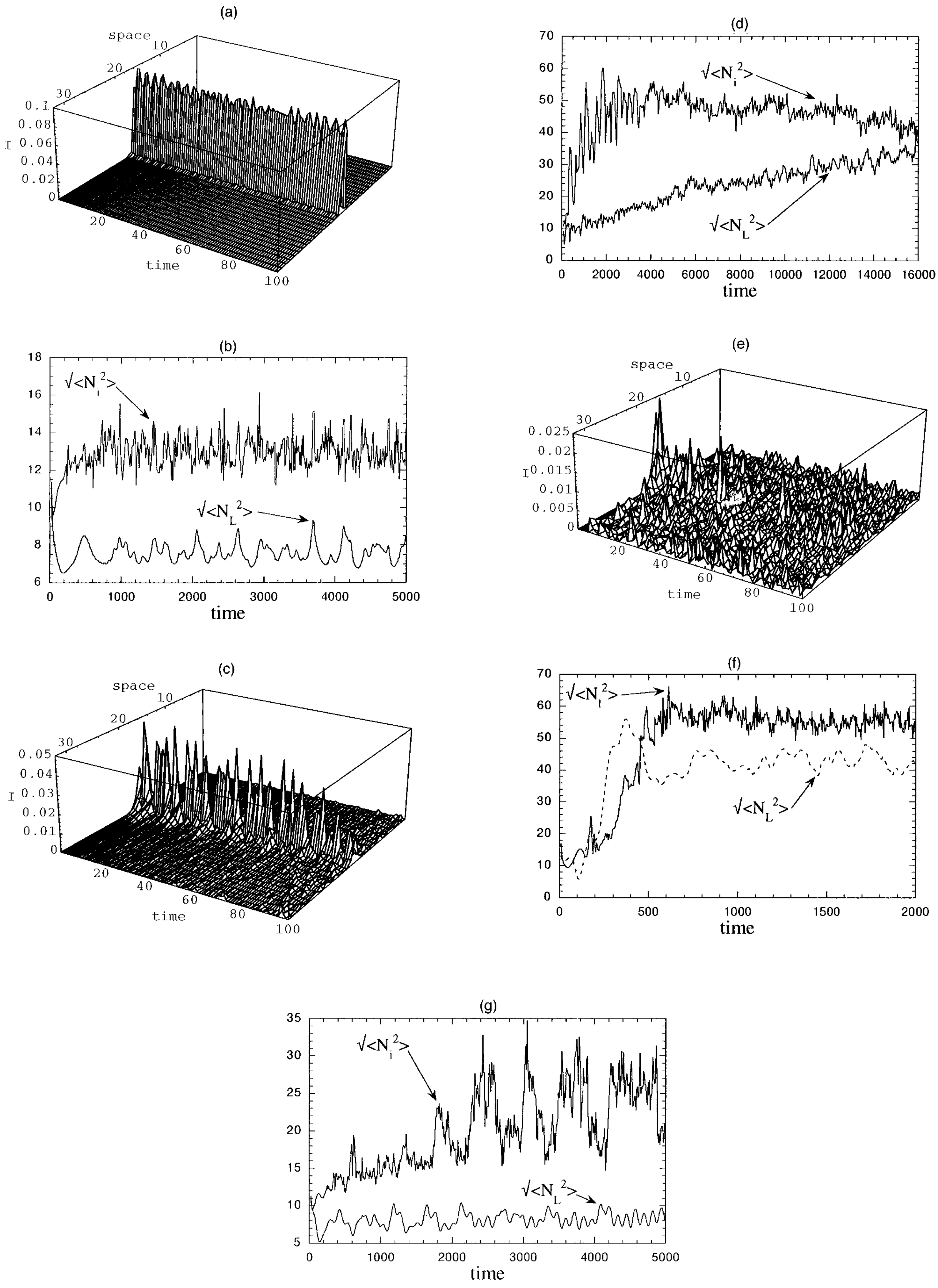

FIG. 3. $I[\equiv|E(x,t)|^2]$ [in (a), (c), (e)] and $\sqrt{\langle N^2\rangle}$ [in (b), (d), (f), (g)] from full simulations with $k_i=0.0257$, $k=k_i/2$, $a(t=0)=\sqrt{0.1/2}$, and $w(t=0)=1.2/a(t=0)$. $A=0.005$ in (a) and (b), 0.1 in (c) and (d), 0.2 in (e) and (f), and 0.05 in (g). Time has been normalized by a factor of $5000/100=50$ and space by a factor of $64\pi/k=32L$.

typically chaotic here, this regime is perhaps the most appropriately described by the stochastic drive. The oscillating low dimensional subsystem formed by the soliton and the ion-acoustic wave would excite the remaining modes of the system. As mentioned, diffusion is very asymmetric, being much faster in the ion-acoustic field. But on examining Eq. (2), it is not unreasonable to say that the Langmuir field term, appearing in the form $\partial_x^2|E(x,t)|^2$ on the right-hand side, can act similarly to a source delivering energy to the ion field on the left-hand side. The sourcelike behavior would enhance diffusion in the ion-acoustic field.

(iii) Finally, when the amplitude attains sufficiently large values, $A>0.1$, fast diffusion takes place in both fields. In contrast to the preceding case, here the time scales for diffusion in both fields are similar. We point out that the central fixed point of the phase plots no longer exists for this range of relatively large perturbing amplitudes. Again, this could explain the short life of the solitary structures, as seen in Fig. 3(e).

V. CONCLUDING REMARKS

In this paper we examined the interaction of an ion-acoustic harmonic mode with a solitary wave of the Zakharov equations. Here the interest is to see how far a solitary wave can resist before it is destroyed by long wavelength radiation and how this destruction takes place. Although some recent works show that solitons can be stable structures even in nonintegrable environments [10–13], what we see here is that if chaos is strong enough in the low dimensional approximations, solitons are in fact destroyed and energy transfer towards small spatial scales takes place.

We have observed that the dynamics can be divided into three categories as a function of the amplitude of the initial ion-acoustic wave. Considering $\sqrt{2}a_s=\sqrt{0.1}$, for small amplitudes, $A\ll 0.1$, there is energy transfer neither in Langmuir nor ion-acoustic fields. For moderately large amplitudes, $A\sim 0.1$, diffusion is observed mostly in the ion-acoustic field, and for sufficiently large amplitudes, $A>0.1$, diffusion is fast and equally present in both fields. In the intermediary regime one can think in terms of a stochastic drive delivering energy to modes with short wavelengths. The drive would be formed as a result of the chaotic, but persistent, low dimensional dynamics. Persistence follows because for not too large amplitudes the central fixed point of the low dimensional phase plots is still unaffected by the interaction, which means that solitons last long enough to serve as stochastic drives. While soliton turbulence may well describe the regime of intermediary amplitudes, it may not be quite appropriate to describe the regime of large perturbing amplitudes since solitons readily damp away there. The characteristics of the stochastic drive are not easy to obtain because the dynamics on the chaotic space $w(t),\dot{w}(t)$ is not pendulumlike. Therefore some known results on pendulumlike settings [8] cannot be directly used here. Details and comparisons with the full simulations are currently under study.

Recalling our initial question in this paper, the conclusion is that the interaction of isolated solitons and ion-acoustic radiation alone is capable of driving energy transfer.

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