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Conductivity of a metal with inverse-power-law correlated impurities

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We calculate the cooperon for a system of noninteracting electrons in the presence of random potentials with correlations $W(\mathbf{r})=W_0\delta(\mathbf{r})+W_1 r^{-(d+\sigma)}$ in d dimensions, for arbitrary values σ and d, to first order in W_1. Our detailed results confirm the exactitude of the universal form for the cooperon proposed earlier by other authors based on general principles. We find that the system will have short-range (long-range) behavior according to σ being positive (negative).

I. INTRODUCTION

The critical behavior of interacting systems depends on general features of the interactions that may be classified in universality classes.[1] One important class is that of short-range (SR) interactions with the Fourier transform expressible as a series in q^2, while a different class constitutes the long-range (LR) inverse power-law interactions decaying like $r^{-(d+\sigma)}$, where d is the space dimensionality and σ the range parameter, with a Fourier transform in q^σ. Crossover from LR to SR critical behavior occurs when $\sigma>\sigma_c$, where the exact value of σ_c, either 2 or $2-\eta_{\rm SR}$, depends on the renormalization procedure.[1,2]

Interesting phenomena occur in systems with random interactions $V(\mathbf{r})$. Within a renormalization-group calculation, the only relevant cumulants[3] are the mean and variance

$$M(\mathbf{r})=\langle V(\mathbf{r})\rangle\ ,$$
$$W(|\mathbf{r}-\mathbf{r}'|)=\langle V(\mathbf{r})V(\mathbf{r}')\rangle-\langle V(\mathbf{r})\rangle\langle V(\mathbf{r}')\rangle\ , \tag{1}$$

which can be of either the SR or the LR type. The case of LR correlations in random spin systems, $W(r)=r^{-(d+\sigma)}$ in Eq. (1), was studied in Ref. 4. The critical value of σ that separates SR and LR behavior is in this case $\sigma_c=0$.

In a previous work[5] we investigated the Anderson localization transition in the presence of random potentials with inverse power-law correlations. We followed Ref. 4 to perform a renormalization-group calculation in a double expansion in $\epsilon=4-d$ and σ. We found a fixed point for $\sigma<0$ and $2\sigma<\epsilon<\sigma$ that we interpreted to indicate a LR-induced localization transition at $d>4$, with the corresponding scaling law for the conductivity.

A microscopic calculation of the conductivity was performed in Ref. 6 for the case of scattering by Yukawa-like potentials in two dimensions. The main result in this work was that, in spite of the mathematical intricacies of the problem, the cooperon in Fig. 1 could be cast into a geometric series with the familiar result for SR correlations

$$\Gamma_c(\mathbf{Q};\omega)=\frac{1}{2\pi\rho_F\tau^2}\frac{1}{-i\omega+Q^2D_{\rm tr}} \tag{2}$$

for small values of the cooperon momentum $\mathbf{Q}$, where $D_{\rm tr}=k_F^2\tau_{\rm tr}/d$ for $d=2$ and ρ_F is the density of states at the Fermi energy. All the details of the interaction were absorbed into the elastic and transport lifetimes τ and $\tau_{\rm tr}$, respectively. In a subsequent publication[7] it was shown that, provided the cooperon exhibits a diffusion pole as in Eq. (2), the conductivity takes the universal scaling form of SR interactions for arbitrary dimensionality d.

The question still remains, however, whether the valid-

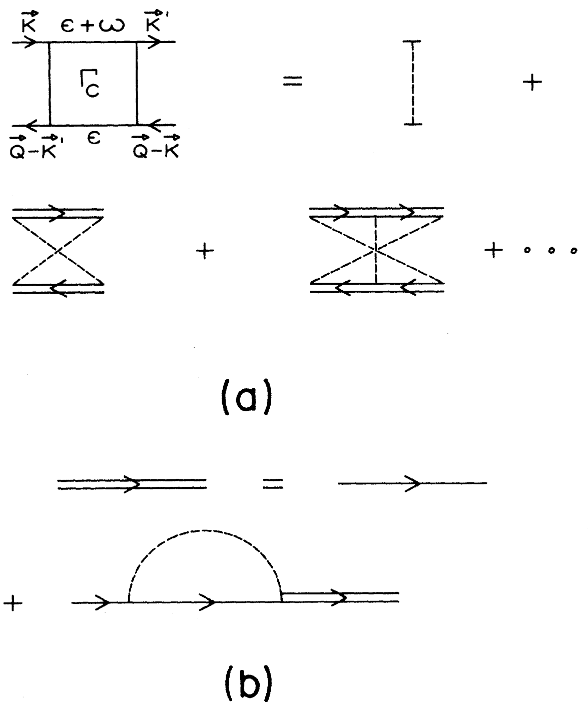

FIG. 1. (a) Series of maximally crossed diagrams for the cooperon $\Gamma_c[\mathbf{k},\mathbf{k}',\mathbf{Q};\omega]$. A full line of momentum $\mathbf{k}$ stands for the Green function $G(\mathbf{k})$ while a broken line stands for the correlation $W(\mathbf{k})$. (b) Green function and self-energy.

ity of Eq. (2) is not restricted to potentials belonging to the SR class,[1] such as the Yukawa potential. Following the formal theory in Ref. 8, it can be established that the relation between the retarded $K^R(\mathbf{k};\omega+i\delta)$ and causal $K^C(\mathbf{k};\epsilon+\omega+i\delta;\epsilon-i\delta)$ density-density correlation functions is given by

$$K^R(k;\omega+i\delta) \approx \frac{1}{2\pi i}\int_{-\infty}^{+\infty} d\epsilon \frac{\partial n_F(\epsilon)}{\partial\epsilon}\{\omega K^C(\mathbf{k};\epsilon+\omega+i\delta;\epsilon-i\delta) -2\pi i\rho(\epsilon)\} , \quad (3)$$

where $n_F(\epsilon)$ is the Fermi function and $\rho(\epsilon)$ the density of states. In the limit $\mathbf{k}=0$ we obtain

$$K^R(\mathbf{k}=0;t_1-t_2) =\theta(t_1-t_2)\langle[\rho_{\mathbf{k}=0}(t_1),\rho_{\mathbf{k}=0}(t_2)]\rangle=0 \quad (4)$$

because the density operator is given by

$$\rho_{\mathbf{k}=0}=\int d\mathbf{p}\, n(\mathbf{p})=N , \quad (5)$$

where $n(\mathbf{p})$ is the particle number operator for momentum $\mathbf{p}$ and N is the conserved total number of particles. From Eqs. (4) and Eq. (5), the *exact* relation[8] follows:

$$K^C(\mathbf{k}=0;\epsilon+\omega+i\delta;\epsilon-i\delta)=\frac{2\pi i\rho(\epsilon)}{\omega} . \quad (6)$$

The causal density-density correlation function corresponds to the vertex part shown in Fig. 1. Then Eq. (6) demonstrates that $\Gamma(\mathbf{Q}=0;\omega)$ in Eq. (2) is *exact* but the theory does not provide rigorously for the first correction in Q^2 in the case of an arbitrary potential. The derivation of Eq. (2) is straightforward for a contact[8] potential, but in the Yukawa case[6] the calculations were far from trivial. Hence we consider interesting the calculation of the cooperon in Fig. 1 for random impurities when the correlations $W(\mathbf{r})$ in Eq. (1) are a combination of a contact potential plus a LR part, with the Fourier transform

$$W(\mathbf{q})=W_0+W_1 q^{\sigma} . \quad (7)$$

This particular form is dictated by a renormalization-group procedure[5] that automatically generates W_0 starting from W_1.

In Sec. II we present results that are obtained to first-order perturbation theory in W_1 by following the method of Ref. 9 and that confirm the exactitude of Eq. (2) also for the LR correlations in Eq. (7). We discuss the results for the localization part of the conductivity[7] and we obtain, according to the range parameter σ being positive or negative, that the system will exhibit SR or LR behavior, as predicted in Ref. 5.

II. CALCULATION OF THE COOPERON AND CONCLUSIONS

We consider a gas of noninteracting electrons in d dimensions, in the presence of random potentials $V(\mathbf{r})$ with mean and variance defined in Eq. (1). Here we consider $M=0$ and $W(\mathbf{r})$ is defined by its Fourier transform in Eq. (7). We work in units $\hbar=m=c=1$.

The sum of maximally crossed diagrams for the particle-hole vertex part of cooperon shown in Fig. 1 is obtained by solving the Bethe-Salpeter equation[9]

$$\Gamma_c(\mathbf{k},\mathbf{k}',\mathbf{Q};\omega) =W(\mathbf{k}-\mathbf{k}')+\int\frac{d\mathbf{p}}{(2\pi)^d}W(\mathbf{k}-\mathbf{p})G_+(\mathbf{p})G_-(\mathbf{Q}-\mathbf{p}) \times\Gamma_c(\mathbf{p},\mathbf{k}',\mathbf{Q};\omega) , \quad (8)$$

where the electron Green function is given by

$$G_{\pm}(\mathbf{p})=[\tfrac{1}{2}(p^2-k_F^2)-\epsilon_{\pm}]^{-1} \quad (9)$$

and ϵ_0 and ϵ_+ stand for $-i/2\tau$ and $\omega+i/2\tau$, respectively, while we indicate by k_F the Fermi momentum. The pole in the Green function ensures that all relevant momenta will be at the Fermi surface. Then, in Eq. (8) we have, from Eq. (7),

$$W(\mathbf{k}-\mathbf{k}')=W_0+W_1[2k_F\sin(\tfrac{1}{2}\theta_{kk'})]^{\sigma} , \quad (10)$$

where $\theta_{kk'}$ is the angle between the vectors $\mathbf{k}$ and $\mathbf{k}'$. Also we perform the integrals within the standard approximation

$$\int\frac{d\mathbf{p}}{(2\pi)^d}=k_F^{d-2}\int_{-\infty}^{+\infty}d\xi\, d\Omega_p , \quad (11)$$

where $\xi=(p^2-k_F^2)/2$, $d\Omega_p$ is the angular differential on the unit sphere divided by $(2\pi)^d$, and $\int d\Omega_p =\Omega_d=2^{1-d}/\pi^{d/2}\Gamma(d/2)$. The inverse lifetime in Eq. (9) is calculated in the Born approximation from the self-energy in Fig. 1, with the result

$$\frac{1}{\tau}=\frac{1}{\tau_0}\left[1+\frac{W_1}{W_0}f_\tau\right], \tag{12}$$

where

$$\frac{1}{\tau_0}=2\pi\rho_F W_0 , \tag{13}$$

$$f_\tau=2^{\sigma+d-2}\frac{\Gamma^{(d/2)}\Gamma\left[\frac{d-1+\sigma}{2}\right]}{\sqrt{\pi}\Gamma\left[d-1+\frac{\sigma}{2}\right]}k_F^\sigma , \tag{14}$$

and $\rho_F=k_F^{d-2}\Omega_d$ is the density of states at the Fermi energy.

The solution of Eq. (8) to first order in W_1 is obtained by following the method of Béal-Monod and Forgacs.[9] We obtain

$$\Gamma_c(\mathbf{k},\mathbf{k}',\mathbf{Q};\omega)=\frac{W_0}{1-W_0B}+W_1\left[|\mathbf{k}-\mathbf{k}'|^\sigma+2\frac{W_0C}{1-W_0B}+\frac{W_0^2E}{(1-W_0B)^2}\right], \tag{15}$$

where

$$\begin{aligned}B&=\int\frac{d\mathbf{p}}{(2\pi)^d}G_+(\mathbf{p})G_-(\mathbf{Q}-\mathbf{p}) ,\\ C&=\int\frac{d\mathbf{p}}{(2\pi)^d}G_+(\mathbf{p})G_-(\mathbf{p})[2k_F\sin(\tfrac{1}{2}\theta_{pk})]^\sigma ,\\ E&=\int\frac{d\mathbf{p}}{(2\pi)^d}\frac{d\mathbf{q}}{(2\pi)^d}G_+(\mathbf{p})G_-(\mathbf{Q}-\mathbf{p})\\ &\quad\times[2k_F\sin(\tfrac{1}{2}\theta_{pq})]^\sigma G_+(\mathbf{q})G_-(\mathbf{Q}-\mathbf{q}) .\end{aligned} \tag{16}$$

As we are looking for the singular contributions to Γ_c when $Q\to 0$, we must expand B and E in Eq. (16) to $O(Q^2)$. Also the expressions for C and E should be evaluated for $W_1=0$. The results for B and C are straightforward[10] and read

$$B=2\pi\rho_F\tau\left[1+i\omega\tau-Q^2\frac{k_F^2\tau^2}{d}\right], \tag{17}$$

$$C=2\pi\rho_F\tau_0 f_\tau . \tag{18}$$

The expression for E is more complicated

$$E=(2\pi\rho_F\tau_0)^2\{f_\tau[1-2Q^2\tau_0D_0]-Q^2k_F^2\tau_0^2 2^\sigma k_F^\sigma J\} , \tag{19}$$

where $D_0=k_F^2\tau_0/d$ is the diffusion coefficient and

$$J=\frac{1}{(\Omega_d)^2}\int d\Omega_p\int d\Omega_q\cos\theta_{pQ}\cos\theta_{qQ}[\sin(\tfrac{1}{2}\theta_{pq})]^\sigma . \tag{20}$$

We introduce d-dimensional polar coordinates in momentum space

$$\begin{aligned}q_i&=q\cos\theta_{i-1}\prod_{j=i}^{d-1}\sin\theta_j,\quad i=1,\dots,d-1\\ q_d&=q\cos\theta_{d-1},\quad \theta_0=0 ,\end{aligned} \tag{21}$$

with Jacobian $q^{d-1}\prod_{j=1}^{d-1}(\sin\theta_j)^{j-1}$. Then the angle θ_{qQ} between two vectors $\mathbf{q}(\theta_1,\dots,\theta_{d-1})$ and $\mathbf{Q}(\varphi_1,\dots,\varphi_{d-1})$ is given by

$$\cos\theta_{qQ}=\frac{\mathbf{q}\cdot\mathbf{Q}}{qQ}=\sum_{i=1}^{d-1}\cos\theta_i\cos\varphi_i\prod_{j=i}^{d-1}\sin\theta_j\sin\varphi_j . \tag{22}$$

To perform the integral over $d\Omega_q$ in Eq. (19), we set the d axis along $\mathbf{p}$; then $\theta_{pq}=\theta_{d-1}$ and $\theta_{pQ}=\varphi_{d-1}$ and we obtain[10]

$$J=\frac{1}{(\Omega_d)^2}\int d\Omega_p\cos\theta_{pQ}K(\theta_{pQ}) , \tag{23}$$

where, from Eq. (22),

$$\begin{aligned}K(\theta_{pQ})&=\frac{1}{(2\pi)^d}\int\prod_{j=1}^{d-1}[d\theta_j(\sin\theta_j)^{j-1}]\times\cos\theta_{qQ}\left[\sin\left[\frac{\theta_{d-1}}{2}\right]\right]^\sigma\\ &=-\frac{\Omega_{d-1}}{2\pi}2^{d-3}\sigma\frac{\Gamma\left[\frac{d+\sigma-1}{2}\right]\Gamma\left[\frac{d-1}{2}\right]}{\Gamma(d+\frac{\sigma}{2})}\\ &\quad\times\cos\varphi_{d-1}\end{aligned} \tag{24}$$

because only the term with $i=d-1$ in Eq. (22) gives a nonzero contribution to the integral in Eq. (24). It follows by introducing Eq. (24) into Eq. (23), where now $d\Omega_p=1/(2\pi)^d\prod_{j=1}^{d-1}(\sin\varphi_j)^{j-1}d\varphi_j$, that

$$J=-\frac{2^{d-3}}{d}\sigma\frac{\Gamma\left[\frac{d+\sigma-1}{2}\right]\Gamma\left[\frac{d}{2}\right]}{\sqrt{\pi}\Gamma\left[d+\frac{\sigma}{2}\right]} \tag{25}$$

and, from Eqs. (25) and (19),

$$E=(2\pi\rho_F\tau_0)^2f_\tau\left[1-Q^2\tau_0D_0\frac{4(d-1)+\sigma}{2(d-1)+\sigma}\right] . \tag{26}$$

Introducing Eqs. (17), (18), and (26) into Eq. (15) and expanding the terms originating in $(1-W_0B)^{-1}$ to first order in W_1, we obtain the singular part

$$[\Gamma_c(\mathbf{Q})]_{\omega=0}=\frac{1}{2\pi\rho_F}\frac{1}{\tau_0^2D_0Q^2}\times\left[1+\frac{W_1}{W_0}f_\tau 2\frac{3(d-1)+2\sigma}{2(d-1)+\sigma}\right],\quad Q\to 0 . \tag{27}$$

In Ref. 7, it was proposed, from first-principles arguments based on the Ward identity, that the universal form for the cooperon should be

$$\Gamma_c(\mathbf{Q})=\frac{1}{2\pi\rho_F\tau^2}\frac{1}{DQ^2},\quad Q\to 0 , \tag{28}$$

where

$$D = D_0 \frac{\tau_{tr}}{\tau_0} \tag{29}$$

and τ_{tr} is the transport[7] lifetime, which in our case gives

$$\frac{\tau_0}{\tau_{tr}} = \frac{\tau_0}{\tau} + \frac{W_1}{W_0} f_\tau \frac{\sigma}{2(d-1)+\sigma} , \tag{30}$$

with τ and f_τ defined in Eqs. (12) and (14). The expansion of Eq. (28) to first order in W_1 yields exactly the result obtained in Eq. (27); hence we conclude that our calculation provides a rigorous proof of the validity of Eq. (28) also for inverse power-law correlations.

The calculation of the conductivity now proceeds as indicated in Ref. 7, where it was shown rigorously that *all* the relevant contributions combine into the following expression, which we quote for completeness:

$$\sigma_{dc} = \sigma_0 \frac{\tau_r}{\tau_0} - \frac{\Omega_d}{2\pi} \int_{L^{-1}}^{Q_{max}} Q^{d-3} dQ = \sigma_{Drude} + \sigma_{Loc} , \tag{31}$$

where $L^{-1}=0$ for $d>2, \sigma_0 = e^2 \rho_F D_0$, and σ_{Drude} (σ_{Loc}) indicates the first (second) term in Eq. (31), while

$$Q_{max} = \begin{cases} \dfrac{1}{k_F \tau} & \text{if } \dfrac{1}{\tau} < \dfrac{1}{\tau_{tr}} & \text{(32a)} \\ \dfrac{1}{k_F \sqrt{\tau \tau_{tr}}} & \text{if } \dfrac{1}{\tau} > \dfrac{1}{\tau_{tr}} . & \text{(32b)} \end{cases}$$

We can see, from the last term in Eq. (31), that the localization contribution to the conductivity depends on the potential *only* through Q_{max}: if $\sigma > 0$ then we are in the case (32a) and σ_{Loc} depends solely on the elastic lifetime τ, *just as for a contact potential,* while for $\sigma < 0$ we are in the case (32b) and σ_{Loc} exhibits a long-range behavior through τ_{tr}. These results confirm the renormalization-group prediction in Ref. 5 that the localization transition will have a LR or SR character according to the range parameter σ being negative (LR) or positive (SR). We consider these results to be valid for all values of d and σ that ensure well defined Γ functions by analytic continuation.

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[1]M. E. Fisher, Rev. Mod. Phys. **46**, 597 (1974); M. E. Fisher, S. K. Ma, and B. G. Nickel, Phys. Rev. Lett. **29**, 917 (1972); M. A. Gusmão and W. K. Theumann, Phys. Rev. B **28**, 6545 (1983).

[2]J. Sak, Phys. Rev. B **8**, 281 (1973).

[3]T. C. Lubensky, Phys. Rev. B **11**, 3573 (1975).

[4]A. Weinrib and B. I. Halperin, Phys. Rev. B **27**, 413 (1983).

[5]M. C. Varriale and A. Theumann, J. Phys. A **23**, L719 (1990).

[6]M. T. Béal-Monod, A. Theumann, and G. Forgacs, Phys. Rev. B **46**, 15 726 (1992).

[7]M. T. Béal-Monod, Phys. Rev. B **47**, 3495 (1993).

[8]S. V. Maleev and B. T. Toperverg, Zh. Eksp. Teor. Fiz. **69**, 1440 (1975) [Sov. Phys. JETP **42**, 734 (1976)].

[9]M. T. Béal-Monod and G. Forgacs, Phys. Rev. B **37**, 6646 (1988).

[10]I. S. Gradshteyn and I. M. Ryzhik, in *Table of Integrals, Series and Products,* edited by A. Jeffrey (Academic, New York, 1980).