Nonlocality of the isobar propagation and the effective Δ-nucleus spin-orbit interaction

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(Received 29 September 1981)

The imaginary central and spin-orbit components of the Δ-nucleus optical potential are inves­tigated in a Δ'-exchange model. It is found that the effective spin-orbit interaction of the isobar reflects large nonlocalities from true pion absorption and from the quasielastic Δ decay. The parameters for the absorption and the spin-orbit strength are in qualitative agreement with recent phenomenological results.

The formulation of the isobar doorway model has provided new insight into the dynamics of nuclear reactions at intermediate energies. In particular it turned out that in view of the selective sensitivity of the Δ-particle N-hole (ΔN) states on the nuclear medium, higher order corrections—beyond that of pion multiple rescattering—are crucial for a quantitative understanding. Unfortunately, a microscopic cal-

\[ V_A(r;\omega) = \left\{ \left[ V_c(\omega) + iW_c(\omega) \right] - \left[ V_h(\omega) + iW_h(\omega) \right] \right\} \sum_{r} \frac{1}{r} \frac{d}{dr} \rho(r)/\rho_0 \]

as a function of the scattering energy ω and the nuclear density ρ(r) (r_0 is a scale parameter, taken to be 1 fm). For an energy-independent spin-orbit term W_h(ω) a best fit of elastic π-scattering data on 12C was obtained with (compare Fig. 3)

\[ W_c(\omega) = -40 \text{ MeV} \]

(2)

together with

\[ W_h(\omega) = -W_c(\omega)/10 \]  

(3)

Though the ansatz in Eq. (1) is fairly successful in actual calculations, its shortcomings are obvious: Incorporating different medium corrections in such a simple parametrization necessarily prevents their detailed and systematic investigation. In addition, an interpretation of the various effective coupling constants, obtained from a fit of Eq. (1) to experimental data, is not unique, as the resulting parameters have to reflect the shortcomings of the parametrization itself [for example, they artificially have to mock-up nonlocal effects in the central part of the isobar-nucleus optical potential, absent in the ansatz in Eq. (1)].

In a microscopic approach to the parametrization from Eq. (1) we concentrate in the following on the absorptive part of \( V_A(r;\omega) \): for this piece a diagrammatic expansion is promising due to the small number of inelastic channels (in addition, the real part of the Δ-nucleus potential receives large contri-

\[ \Delta(\pi') \]

\[ \Delta(\pi) \]

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FIG. 1. Leading diagrammatic contributions to the imaginary isobar self-energy (to first order in the nuclear density) through the coupling to the 2p-1h continuum due to true π absorption (π, 2N) (a) and to the quasielastic channel (b). Above the wiggly lines represent the shell model potential of the nucleon; the double lines indicate Pauli blocking for the nucleon.
2π-exchange diagram in Fig. 1 (a)

\[ V_{\pi\pi}(\vec{p}, \vec{p}'; \omega) = \left[ \frac{f_\pi}{m_\pi} \right]^2 \left[ \frac{f_\pi}{m_\pi} \right]^2 \frac{1}{(2\pi)^3} \int \frac{d\vec{k}N(\vec{k}; \vec{p}, \vec{p}')}{\omega_\pi(\vec{k}+\vec{p})\omega_\pi(\vec{k}+\vec{p}')} \frac{F_\pi^2(\vec{k}+\vec{p}')F_\pi^2(\vec{k}+\vec{p})F_\Delta(k_0)}{2T_N(k)-\omega+i\epsilon} \times \frac{1}{2} \left[ \frac{1}{\omega_\pi(\vec{k}+\vec{p})+T_N(k)} - \frac{1}{\omega_\pi(\vec{k}+\vec{p})+T_N(k)-\omega+i\epsilon} \right] \times \frac{1}{2} \left[ \frac{1}{\omega_\pi(\vec{k}+\vec{p})+T_N(k)} - \frac{1}{\omega_\pi(\vec{k}+\vec{p})+T_N(k)-\omega+i\epsilon} \right] \]

(4)

with the numerator

\[ N(\vec{k}; \vec{p}, \vec{p}') = \overline{s}_1(\vec{k}+\vec{p}')\overline{\sigma}_2(\vec{k}+\vec{p})\overline{s}_1(\vec{k}+\vec{p})\overline{\sigma}_2(\vec{k}+\vec{p})T_1^\tau T_1^\tau T_2 \ . \]

(5)

Above \( f_\pi \) and \( f_\pi^* = 2f_\pi \) (Ref. 9) denote the \( \pi NN \) and \( \pi N\Delta \) coupling constants; \( \omega_\pi(q) = (q^2 + m_\pi^2)^{1/2} \) is the pion energy and \( T_N(k) \) the kinetic energy of a nucleon in the intermediate state (corrections on the level of single particle energies are dropped). The form factors \( F_\pi(\vec{k}+\vec{p}')(q) \) (Ref. 9) in Eq. (4) include off-shell corrections both for the virtual pions (we use the same cutoff mass \( \Lambda_\pi \) at the \( \pi NN \) and the \( \pi N\Delta \) vertices) and for the \( \Delta \)-isobar \( \pi \) isobar \( (k_0 = k_\pi \approx 230 \text{ MeV}/c \) at resonance).

The kinematics of the two-nucleon process allows a rather natural separation of the nonlocal potential \( V_{\pi\pi}(\vec{p}, \vec{p}'; \omega) \) of Eq. (4) into a local piece and the leading nonlocal correction: At resonance the momentum of the nucleons emitted satisfy

\[ k_N = \sqrt{M\omega} \approx 500 \text{ MeV}/c >> p, p' \]  

for typical momenta \( p \) and \( p' \) of the isobar. By dropping then all the \( \vec{p} \) and \( \vec{p}' \) dependence relative to \( k_N \) except for the invariant \( \overline{s}_1\overline{\sigma}_2\overline{s}_1\overline{\sigma}_2 \) the only dependence of \( V_{\pi\pi}(\vec{p}, \vec{p}'; \omega) \) on the isobar momentum is contained in the spin-orbit term of the numerator, which then reads, after summing over the intermediate \( NN \) state, as

\[ N(\vec{k}; \vec{p}, \vec{p}') = \frac{k^2}{3} \left[ k^2 + \frac{1}{2} (\vec{p}' \times \vec{p}) \right] \]  

(7)

where the transition matrix \( \overline{\Sigma} \) is defined by

\[ \langle \overline{\Sigma} \parallel \overline{\Sigma} \rangle = 12 \sqrt{2} \frac{\Lambda_\pi}{\Lambda_N} \]

In going over to coordinate space we introduce the spin-orbit operator for the isobar by

\[ i \overline{\Sigma}(\vec{p}' \times \vec{p}) \rightarrow \overline{\Sigma} \frac{1}{r} \frac{d}{dr} \]

(8)

As the remaining part of the Box diagram is independent of \( \vec{p} \) and \( \vec{p}' \) its Fourier transform yields schematically

\[ V_{\pi\pi}(\vec{r} - \vec{r}_N; \omega) \sim \delta(\vec{r} - \vec{r}_N) \]  

(9)

By folding in the nuclear density we recover the form of Eq. (1) for the \( \Delta \)-nucleus potential with the imaginary central and spin-orbit part given, respectively, as

\[ W_\ell(\omega) = -\frac{1}{48\pi} \left[ \frac{f_\pi}{m_\pi} \right]^2 \left[ \frac{f_\pi}{m_\pi} \right]^2 \frac{Mk_N^5}{\omega_\pi^2(k_N)^2} F_\pi^4(k_N)F_\Delta(k_0) \rho_0 \frac{1}{\omega_\pi(k_N) + E_N(k_N) - E_\Delta} + \frac{1}{\omega_\pi(k_N) + E_N(k_N) - E_N} \right]^2 \]

(10)

\[ \text{spin-orbit interaction of the nucleon} \]

Pauli blocking reduces the total spin-orbit strength in \( ^{12}\text{C} \) by a factor of

\[ P(k_N) \equiv 1 - \frac{\sqrt{2}}{18} \frac{7 + 4a^2(k_N - \sqrt{2}/a)^2}{ak_N} \times \exp \left[ -a^2 \left( k_N - \frac{\sqrt{2}}{a} \right)^2 \right] \]

(13)

for a \( \Delta \) isobar with an average momentum \( (\vec{p}^2)^{1/2} = (\sqrt{2}/a)(\langle p^2 \rangle^{1/2} = 1.69 \text{ fm} \) (Ref. 7)). The effect of both corrections is very small due to their mutual cancellation.
Similarly, we obtain for the spin-orbit contribution from Fig. 1(b) [the notation is the same as for Fig. 1(a)]

\[ W_b^2(r; \omega) = \frac{\hbar^2}{2} \frac{d}{dr} \frac{\rho(r)}{\rho_0} \left( \frac{f_\pi^2}{m_\pi} \right) \left( \frac{1}{2\pi^2} \right) \int \frac{d^3k}{(2\pi)^3} \frac{\hat{S}^i k \sigma \hat{T} \hat{S}^i k}{2\omega_{e}(k)} F_{N}(k_0) \frac{k^2}{M + \omega} P(k_0) . \]  

(14)

Evaluating the integral with standard techniques we obtain with

\[ \hat{S}^i k \sigma \hat{T} \hat{S}^i k = \frac{k^2}{3} \]  

(15)

the spin-orbit coefficient

\[ W_b^1(\omega) = \frac{\Gamma_\Delta(\omega)}{2} \left( \frac{\omega}{k_0^2} - \frac{1}{3M + \omega} \right) \frac{f_\pi^2}{m_\pi} P(k_0) , \]  

(16)

where \( \Gamma_\Delta(\omega) \) is just the width of the \( \Delta \) isobar at the scattering energy \( \omega \),\(^{14} \) while \( P(k_0) \) again accounts for Pauli blocking of the intermediate nucleon.

Our main findings for \( ^{12}\text{C} \) are presented in Figs. 2 and 3. In Fig. 2 the energy dependence of \( W_b^1(\omega) \) and \( W_b^2(\omega) \) is shown for representative values of \( \Lambda_\pi \) and \( \Lambda_\Delta \). Characteristically, the two coefficients show an opposite trend with increasing \( \omega \) as expected from their gross structure

\[ W_b^1(\omega) \propto 1/\omega; \quad W_b^2(\omega) \propto \omega^{3/2} . \]  

(17)

For quantitative details we consider for \( \Lambda_\pi = 800 \) MeV (Ref. 8) two extreme choices with \( \Lambda_\Delta = \infty \) (no cutoff) and \( \Lambda_\Delta = 200 \) MeV (Ref. 11) for the off shell continuation of the \( \Delta \) isobar. The influence of \( \Lambda_\Delta \) is significant; presently, its uncertainty just reflects current problems in defining the \( \Delta \) isobar microscopically, especially off the resonance (for different philosophies compare, for example, Refs. 15–17).

In Fig. 3 we compare our results with the findings by Horikawa et al. For \( \Lambda_\pi = 800 \) MeV without an additional \( \Delta \) cutoff we qualitatively reproduce the \( W_c(\omega) \) from Ref. 7. As the same model fits the total \( \pi \)-absorption cross section \( \pi d \rightarrow NN \) (Ref. 8) we conclude from our qualitative agreement that the phenomenological quantity \( W_c(\omega) \) in Ref. 7 receives its dominant contribution from the true \( \pi \)-absorption process (\( \pi,NN \)). For \( \Lambda_\Delta = 200 \) MeV the agreement is much worse as the resulting energy dependence of \( W_c(\omega) \) is too steep. The same situation persists for

![FIG. 2. Energy dependence of the coefficients \( W_b^1(\omega) \) and \( W_b^2(\omega) \) (without Pauli corrections) for \( \Lambda_\pi = 800 \) MeV and two different cutoff masses \( \Lambda_\Delta = \infty \) (full lines) and \( \Lambda_\Delta = 200 \) MeV (dashed lines), respectively.](image1)

![FIG. 3. Energy dependence of true \( \pi \) absorption \( W_c(\omega) \) [Eq. (11)] and the summed strength \( W_b(\omega) = W_b^1(\omega) + W_b^2(\omega) \) of the spin-orbit interaction of the \( \Delta \) isobar. Shown are the results for \( \Lambda_\pi = 800 \) and 900 MeV without \( \Delta \) cutoff (full and dashed-dotted lines) and for \( \Lambda_\pi = 800 \) MeV together with \( \Lambda_\Delta = 200 \) MeV (dashed line), as well as the corresponding results from Horikawa et al. (Ref. 7) (full dots).](image2)
the total spin-orbit strength $W_\gamma(\omega) = W_\gamma^1(\omega) + W_\gamma^2(\omega)$: only without a $\Delta$ cutoff we find $W_\gamma(\omega)$ approximately constant as in the fit from Ref. 7. For the discrepancy in the absolute magnitude by a factor 1.5--2 there are two obvious interpretations: on the one side we presumably overestimate $W_\gamma(\omega)$, as our model incorporates neither p-exchange nor short range correlations [both mechanisms should cut down $W_\gamma(\omega)$]; on the other side, $W_\gamma(\omega)$ from Ref. 7 might indeed underestimate the complex spin-orbit interaction of a $\Delta$ isobar, as $\pi$ scattering in the isobar-hole model is not yet understood on such a quantitative level.

Summarizing, we can account for some aspects of the parametrization of Ref. 7 in our simple microscopic model. We find that for a local parametrization of the central part of the $\Delta$-nucleus optical potential highly nonlocal medium corrections—dominated by true pion absorption and reflection—have to be mocked-up by a large spin-orbit interaction of the isobar. This explains why isobar doorway calculations, which keep the nonlocalities in the higher order corrections, are similarly successful without a strong spin-orbit term.\(^{5,18}\) Furthermore, the result indicates that only within a more detailed microscopic framework a comparison between the phenomenological spin-orbit interaction of the isobar from Ref. 7 and with the quark model,\(^{19}\) for example, is meaningful.

It is clear that on a quantitative level the diagrammatic approach has its own problems. The sensitivity of the result on the cutoff masses is an unpleasant feature (though the same parameters already enter into a calculation of the $\Delta N$ interaction in first order); more serious are the difficulties in developing on the same basis a quantitative picture for the real parts of the $\Delta$-nucleus potential, as it is not fully clear how well a diagrammatic expansion converges. For a conclusive answer further investigations have to be awaited.

One of us (M.D.) would like to thank the Instituto de Física da Universidade Federal do Rio Grande do Sul for its kind hospitality. The work was partially supported by Conselho National de Desenvolvimento Científico e Tecnológico (Brasil), Financiadora de Estudos e Projetos (Brasil), and by the Bilateral Cooperation Brasil-West Germany (Conselho Nacional de Desenvolvimento Científico e Tecnológico--Kernforschungsanlage).

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