Girotti *et al.* Reply: The criticisms raised by Hagen [1] do not apply to the situation studied in our papers [2,3].

We started in Ref. [2] by analyzing the infrared structure of the *fully quantized* (2+1)-dimensional QED (QED_3). It was shown that the only way out of the severe infrared problem was through the resummation of vacuum polarization graphs, in order to dynamically induce a photon mass. Essentially, the massless photon is replaced by a massive vector meson with the mass $|\theta_{in}| = e^2/8\pi$. Then, we calculated the S-matrix element describing the elastic electron-electron scattering when only one vector meson is exchanged. For nonrelativistic electrons, the effective electron-electron potential (V^{QED_3}) was computed as the Fourier transform of the amplitude just mentioned. Afterwards, following a well established procedure [4-6], this potential was used in connection with the Schrödinger equation to prove that electron-electron bound states do exist in QED_3. This is the "crucial junction" between perturbative field theory calculations and the Schrödinger equation to which Hagen refers in his Comment.

We also demonstrated in [2] that the remaining contributions to the electron-electron potential are, up to some power of $\ln(e^2/m)$, of order e^2/m or higher with respect to V^{QED_3}; in this connection, we remind the reader that diagrams involving the exchange of two or more vector mesons contribute to the higher order terms of the Born series [7]. Also, there is no *a priori* reason for altering the systematics for computing the matrix elements of γ_i between initial and final electron states.

The validity of our results is, therefore, restricted to the interval where the parameters e (electron charge) and m (electron mass) verify $e^2/m \ll 1$ or, what amounts to the same thing, that $|\theta_{in}| \ll 2m$. Hence, the particle mediating the interaction cannot decay on shell into an electron-positron pair and it therefore remains in the spectrum. This is exactly the situation opposite to that considered by Hagen in his Comment ($|\theta_{in}| \to \infty$) [1]. Although for computing V^{QED_3} one only needs the elastic electron-electron scattering S-matrix element in the region where the momentum transfer $\mathbf{q}$ is small as compared with the electron mass ($|\mathbf{q}|/m \ll 1$), there are significant contributions to V^{QED_3} coming from values of $\mathbf{q}$ which, as far as the vector meson is concerned, are well in the relativistic region ($|\mathbf{q}| \approx |\theta_{in}|$). In other words, the fact that the incoming electrons are, by assumption, nonrelativistic does not imply that the particle mediating the interaction between them is also nonrelativistic.

Therefore, it does not make sense at all to contrast our results in Ref. [2] with those which may eventually arise in the region $|\theta_{in}| \to \infty$ [1], where the vector meson decays on shell into an electron-positron pair and disappears from the spectrum. Also, it does not make sense to compare the effective electron-electron low energy potential derived by us [2,3] with that obtained when the electromagnetic field is a *classical external field* [8].

As for the arguments about gauge invariance brought by Hagen [1], we start by recalling that the S matrix is a gauge invariant object. Whence, so is the potential V^{QED_3} obtained as the Fourier transform of an S-matrix element. Henceforth, all quantities entering into the Schrödinger equation are assumed to be invariant under gauge transformations. Of course, the scenario changes radically when A^μ is an *external field.* In this last case, one assumes that the wave function acquires a space-time dependent phase under gauge transformations. Then, the quadratic $A^\mu A_\mu$ term becomes essential to secure the gauge invariant character of the entire Schrödinger equation.

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