On the temperature dependence of the magnetoresistance of ferromagnetic alloys

L. Berger, P. P. Freitas, J. D. Warner, and J. E. Schmidt

On the temperature dependence of the magnetoresistance of ferromagnetic alloys

L. Berger, P. P. Freitas, J. D. Warner, and J. E. Schmidt

Physics Department, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213

The ferromagnetic anisotropy of resistivity $\Delta\rho/\rho_0 = (\rho_\| - \rho_\perp)/\rho_0$ depends strongly on the nature of the dominant electron scattering centers, and is usually larger for impurity scattering than for phonon scattering. This explains why $\Delta\rho/\rho_0$ decreases with increasing temperature T in crystalline transition-metal alloys. Parker has proposed an expression for $\Delta\rho/\rho_0$ as a function of T, based on Matthiessen's rule. It contains the parameters $(\Delta\rho/\rho_0)_{im}$ and $(\Delta\rho/\rho_0)_{ph}$, representing the $\Delta\rho/\rho_0$ values for impurity scattering and phonon scattering, respectively. In an alloy series with weak electron scattering, such as Fe-Co, $(\Delta\rho/\rho_0)_{ph}$ is found experimentally to be positive and equal to a fixed fraction $\simeq \frac{1}{4}$ of $(\Delta\rho/\rho_0)_{im}$. On the other hand, in crystalline alloy series with strong, resonant electron scattering, such as Ni-Fe, Fe-Cr, Fe-V, $(\Delta\rho/\rho_0)_{ph}$ is found to be negative and has a minimum at the composition where $(\Delta\rho/\rho_0)_{im}$ is maximum.

INTRODUCTION

The ferromagnetic anisotropy of resistivity is defined as $\Delta\rho/\rho_0 = (\rho_\| - \rho_\perp)/(\rho_\|/3 + 2\rho_\perp/3)$, where $\rho_\|$ and $\rho_\perp$ are the resistivity values when the magnetization is parallel and perpendicular to the current, respectively. It arises from the combination of spin-orbit interaction and *s-d* electron scattering. In most crystalline transition-metal alloys, $\Delta\rho_0/\rho_0$ is positive, and a decreasing function of the temperature T. The present paper concerns such crystalline alloys, but not amorphous materials.

Fert, Campbell, and Jaoul proposed[1] the following relation in the case of nickel alloys at $T \simeq 0$:

$$\Delta\rho/\rho_0 = \gamma(\rho_\perp/\rho_\uparrow - 1), \tag{1}$$

where $\rho_\perp$ and $\rho_\uparrow$ are the resistivities for the spin-down and spin-up bands, respectively, and $\gamma \simeq 0.01$. The condition $\rho_\perp/\rho_\uparrow \gg 1$ is assumed. A similar relation, where $\rho_\perp/\rho_\uparrow$ is replaced by $\rho_\uparrow/\rho_\perp$, holds for Fe-based alloys in the limit $\rho_\uparrow/\rho_\perp \gg 1$.

At $T > 0$, these authors assumed[1] a "spin-mixing" resistivity $\rho_{\uparrow\perp}(T)$ to exist, which couples the spin-up and spin-down currents so that Eq. (1) has to be modified. The introduction of $\rho_{\uparrow\perp}$ is equivalent to having $\rho_\perp/\rho_\uparrow$ approach the value of 1, leading to a reduction of $\Delta\rho/\rho_0$. The gradual increase of $\rho_{\uparrow\perp}$ with increasing T explains why $\Delta\rho/\rho_0$ decreases at increasing temperatures. It was also suggested that $\rho_{\uparrow\perp}$ arises from electron-magnon or electron-electron scattering. Unfortunately, these two scattering mechanisms are usually very weak at 300–400 K, as compared to electron-phonon scattering, and not likely to be significant.

Both electron-magnon and electron-electron scattering give rise to very inelastic collisions. As a result, the Lorenz number $L(T) = \kappa_0 \rho_0/T$, where κ_0 is the zero-field thermal conductivity and ρ_0 the zero-field resistivity, should be[2] considerably smaller than the value $L_{el} = 2.44 \times 10^{-8}$ W^2 K^{-2} for elastic collisions, if either of these two mechanisms were dominant at 300–400 K. However, existing data[3] for Fe, Ni, and Co show that $L(T)$ rises, and approaches L_{el} above the Debye characteristic temperature $\simeq 400$ K, as expected if

electron-phonon scattering dominates. This confirms our conclusion that no significant spin-mixing resistivity $\rho_{\uparrow\perp}(T)$ exists in nickel, iron, or cobalt at 300–400 K. This conclusion should hold *a fortiori* in alloys where additional scattering by impurities is present.

Instead, we attribute the temperature dependence of $\Delta\rho/\rho_0$ to the well-known fact that $\Delta\rho/\rho_0$ depends strongly on the nature of the dominant electron scattering centers. The characteristic value of $\Delta\rho/\rho_0$ for phonon scattering will be called $(\Delta\rho/\rho_0)_{ph}$, and is usually smaller than the characteristic value $(\Delta\rho/\rho_0)_{im}$ for impurity scattering.

Matthiessen's rule is valid whenever conduction by one specific group of electrons dominates over that of other groups, i.e., when $\rho_\perp/\rho_\uparrow \gg 1$ or $\rho_\uparrow/\rho_\perp \gg 1$. It states that the resistivity ρ_{im} caused by impurity scattering and the resistivity $\rho_{ph}(T)$ caused by electron-phonon scattering are additive, so that $\rho_0(T) = \rho_{im} + \rho_{ph}(T)$. An expression for $\Delta\rho/\rho_0$ at arbitrary temperature T, based on Matthiessen's rule, has been derived by Parker[4] and reformulated by us[5]:

$$\frac{\Delta\rho}{\rho_0}(T) = \left(\frac{\Delta\rho}{\rho_0}\right)_{ph} + \left[\left(\frac{\Delta\rho}{\rho_0}\right)_{im} - \left(\frac{\Delta\rho}{\rho_0}\right)_{ph}\right]\frac{\rho_0(4.2)}{\rho_0(T)}, \tag{2}$$

where $\rho_0(4.2) \simeq \rho_{im}$. Parker's formula also has a factor containing the saturation magnetization, but this is almost constant if T remains sufficiently far below the Curie point.

PARKER PLOTS FOR Fe-Co ALLOYS

We have measured $\Delta\rho/\rho_0$ and ρ_0 on an annealed and furnace-cooled Fe$_{18}$Co$_{82}$ sample[6] between 79 and 295 K, in a variable-temperature cryostat. Also, $\Delta\rho/\rho_0$ and ρ_0 have been measured in liquid baths at 4.2 and 77 K, and in air at 295 K. We plot the measured $\Delta\rho/\rho_0$ vs $\rho_0(4.2)/\rho_0(T)$ in Fig. 1(a). This kind of graph will be called a "Parker plot." If Parker's law [Eq. (2)] is valid, data points should be located on a straight line. We see that this is the case within the accuracy of the data, at 77 K and above.

On the other hand, the data point for 4.2 K is located below the straight line in Fig. 1(a). Similar deviations from
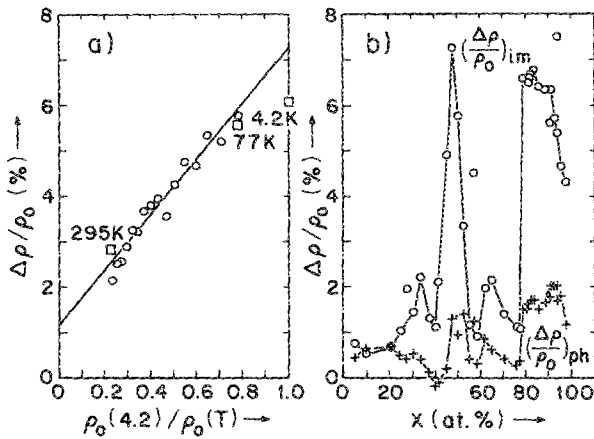
FIG. 1. (a) Parker plot for a $Fe_{18}Co_{82}$ alloy. Circles represent $\Delta\rho/\rho_0$ values obtained in a variable-temperature cryostat, and squares are values obtained in liquid baths or air. (b) Circles represent $(\Delta\rho/\rho_0)_{im}$ values for $Fe_{100-x}Co_x$ as a function of x, while crosses represent $(\Delta\rho/\rho_0)_{ph}$.

Parker's law are observed at low temperatures in many alloy series, especially those with low-resistivity ρ_0 and high saturation magnetization M_S. They are caused by the internal field $B = M_S$ which creates[7] a Lorentz force curving the electron trajectories, in turn leading to a negative contribution to $\Delta\rho/\rho_0$ if the latter is measured at ferromagnetic saturation.

The intercept of the straight line on the left-hand vertical axis gives $(\Delta\rho/\rho_0)_{ph}$, here equal to $+1.2\%$. Similarly, the intercept on the right-hand vertical axis gives $(\Delta\rho/\rho_0)_{im} = +7.25\%$.

The values of $\Delta\rho/\rho_0$ and ρ_0 have also been measured[6,8] in a series of $Fe_{100-x}Co_x$ alloys at 4.2, 77, and 295 K. Parker plots similar to Fig. 1(a) have been drawn for all 37 samples. By drawing straight lines through the two data points at 77 and 295 K, $(\Delta\rho/\rho_0)_{ph}$ and $(\Delta\rho/\rho_0)_{im}$ have been determined for all samples as above, and are displayed as a function of cobalt concentration x in Fig. 1(b). Some data points are too close together to be seen in Fig. 1(b). For the reason described above, the data point for 4.2 K often falls below the straight line. We see in Fig. 1(b) that $(\Delta\rho/\rho_0)_{im}$ and $(\Delta\rho/\rho_0)_{ph}$ are usually positive. Also, samples with large $(\Delta\rho/\rho_0)_{im}$ tend to have large $(\Delta\rho/\rho_0)_{ph}$. Except for Fe-rich samples, $(\Delta\rho/\rho_0)_{ph}$ is roughly a fixed fraction $\simeq\frac{1}{4}$ of $(\Delta\rho/\rho_0)_{im}$. The Smit theory[9] of $\Delta\rho/\rho_0$, based on the first Born approximation, predicts $\rho_\downarrow/\rho_\uparrow \propto D_\downarrow/D_\uparrow$, where $D_\downarrow$ and $D_\uparrow$ are the $3d$ densities of states for spin down and spin up. Using Eq. (1) in the limit $\rho_\downarrow/\rho_\uparrow \gg 1$, this implies $\Delta\rho/\rho_0 \propto D_\downarrow/D_\uparrow$. That result should apply to both $(\Delta\rho/\rho_0)_{im}$ and $(\Delta\rho/\rho_0)_{ph}$, though the proportionality coefficients will be different. In other words, $(\Delta\rho/\rho_0)_{im}$ and $(\Delta\rho/\rho_0)_{ph}$ are predicted to vary in similar fashion through the Fe-Co series, reflecting variations of $D_\downarrow$ and $D_\uparrow$ with composition. This agrees with the data of Fig. 1(b). For example, the sharp peaks of $(\Delta\rho/\rho_0)_{im}$ and $(\Delta\rho/\rho_0)_{ph}$ in Fig. 1(b) correspond[6,8] to minima of $D_\uparrow$ caused by atomic ordering.

PARKER PLOTS FOR Ni-Fe, Fe-Cr, AND Fe-V ALLOYS

Measurements of $\Delta\rho/\rho_0$ have been done on $Ni_{100-x}Fe_x$ alloys at $\simeq 290$ and 4.2 K (or 20 K) by several authors,[1,10]

similarly for ρ_0 measurements.[11] Unfortunately, data at 77 K are not available. We have drawn smooth curves representing average values of $\Delta\rho/\rho_0$ and ρ_0 as a function of x. Parker plots have been constructed for $x = 5, 10, 15,..., 50$ at. % Fe. Values of $(\Delta\rho/\rho_0)_{ph}$ and $(\Delta\rho/\rho_0)_{im}$ have been determined for all such compositions by drawing straight lines through the 4.2 and 290 K points, and are shown in Fig. 2(a). In addition, we show the $\Delta\rho/\rho_0$ values measured on pure nickel[12] at $\simeq 290$ K. These should be identified with $(\Delta\rho/\rho_0)_{ph}$ since impurity scattering is absent.

We see that $(\Delta\rho/\rho_0)_{im}$ is positive at all compositions, and has a maximum at $x \simeq 15\%$ Fe. On the other hand, $(\Delta\rho/\rho_0)_{ph}$ becomes negative as the iron concentration increases, and has a minimum at a value of x somewhat larger than the one where $(\Delta\rho/\rho_0)_{im}$ is maximum. This behavior of $(\Delta\rho/\rho_0)_{ph}$ is very different from the one observed in Fe-Co [Fig. 1(b)], and cannot be explained merely in terms of $D_\downarrow$ and $D_\uparrow$ variations.

We have measured $\Delta\rho/\rho_0$ and ρ_0 at 299, 77, and 4.2 K on $Fe_{100-x}Cr_x$ alloys with $x = 2.3, 5.1, 12.7, 25.1, 32.2,$ and 41.1 at. % Cr. The first four samples were annealed at 840 °C, and then cooled slowly. The last two were annealed at 1250 °C, and quenched in oil to avoid precipitation. Parker plots were constructed for all samples. Values of $(\Delta\rho/\rho_0)_{ph}$ and $(\Delta\rho/\rho_0)_{im}$ were found from 299 and 77 K data as before [Fig. 2(b)]. We also show the $\Delta\rho/\rho_0 = (\Delta\rho/\rho_0)_{ph}$ value[9] for pure Fe at room temperature. We see that $(\Delta\rho/\rho_0)_{im}$ may have a maximum at $x \simeq 5$ at. % Cr. Also, $(\Delta\rho/\rho_0)_{ph}$ becomes negative as x increases, and has a minimum at an x value equal to or slightly larger than the one where $(\Delta\rho/\rho_0)_{im}$ is maximum. This behavior is very similar to the one for Ni-Fe, and again cannot be explained by $D_\downarrow$ and $D_\uparrow$ variations.

Finally, we analyze existing data[13] of $\Delta\rho/\rho_0$ and ρ_0 for $Fe_{100-x}V_x$ alloys at 273, 77, and 4.2 K in the same way [Fig. 2(c)]. Again, like in Ni-Fe and Fe-Cr, $(\Delta\rho/\rho_0)_{ph}$ becomes negative at increasing x, and looks like an inverted mirror image of $(\Delta\rho/\rho_0)_{im}$.

SPLIT-BAND MODEL AND SATURATION

The existence of the $(\Delta\rho/\rho_0)_{im}$ maximum in Ni-Fe has been attributed[14] to a scattering resonance located in the spin-down $3d$ band, at the boundary T between the Ni and Fe subbands in the "split-band" model. A similar resonance exists[14] in the spin-up $3d$ band of Fe-Cr, Fe-V, etc. In Ni-Fe alloys, point T is predicted[15] to pass through the Fermi level when $x = 18$ at. % Fe. This interpretation is confirmed by an empirical data analysis[16] of $\Delta\rho/\rho_0$ data at 4.2 K performed by Szentirmay and Kedves, which is based on Eq. (1) and shows that $\rho_\downarrow$ has a large maximum $\simeq 70 \times 10^{-8}$ Ω m at $\simeq 15$ at. % Fe while $\rho_\uparrow$ follows a smooth Nordheim behavior in that range.

Note that $(\Delta\rho/\rho_0)_{im}$ is relatively large in all these alloys, because[14] of the resonance, and because they all follow the Slater–Pauling curve for magnetization.

Scattering resonances and split-band effects are one manifestation of the breakdown of the first Born approximation, happening[14] because the valence difference $|Z|$ between solvent and solute in Ni-Fe, Fe-Cr, and Fe-V reaches or ex-
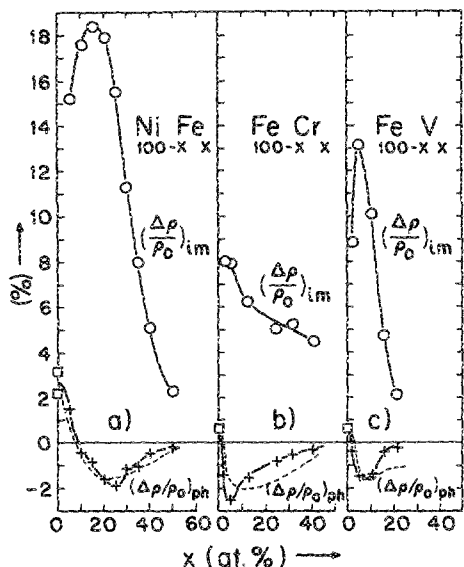
FIG. 2. (a) Circles represent $(\Delta\rho/\rho_0)_{im}$ value for $Ni_{100-x}Fe_x$ as a function of x, while crosses represent $(\Delta\rho/\rho_0)_{ph}$. The squares are $\Delta\rho/\rho_0$ values for pure Ni at $\simeq 290$ K. The dashed line is the prediction of Eq. (4) for $(\Delta\rho/\rho_0)_{ph}$. (b) Same for $Fe_{100-x}Cr_x$. The square is the $\Delta\rho/\rho_0$ value for pure Fe at room temperature. (c) Same for $Fe_{100-x}V_x$.

ceeds 2. On the other hand, the Born approximation holds approximately in Fe-Co because $|Z| = 1$.

It is well known[17] that the temperature coefficient of the resistivity is smaller in alloys where the resistivity ρ_{im} caused by impurity scattering is large. We apply this idea of "resistivity saturation" to $\rho_\downarrow$ in alloys with a scattering resonance, and write

$$\rho_\downarrow(T) \simeq \rho_\downarrow^{im} + \rho_\downarrow^{ph}(T) - (\rho_\downarrow^{im}/\rho_{sat})\rho_\downarrow^{ph}(T) \qquad (3)$$

where $\rho_\downarrow^{im}$ and $\rho_\downarrow^{ph}(T)$ are the impurity and phonon resistivities in the absence of saturation, and $\rho_{sat} > 0$ is an adjustable parameter associated with saturation. By combining Eqs. (1) and (3) in the limit $\rho_\downarrow/\rho_\uparrow \gg 1$, we obtain

$$\left(\frac{\Delta\rho}{\rho_0}\right)_{ph} \simeq \left(\frac{\Delta\rho}{\rho_0}\right)_{ph}^z + \frac{D(x)}{D(0)}\left[1 - \left(\frac{\Delta\rho}{\rho_0}\right)_{im}\frac{\rho_0(4.2)}{\gamma\rho_{sat}}\right]$$

$$\times\left[\left(\frac{\Delta\rho}{\rho_0}\right)_{ph}(x=0) - \left(\frac{\Delta\rho}{\rho_0}\right)_{ph}^z\right], \qquad (4)$$

where $(\Delta\rho/\rho_0)_{ph}(x=0)$ is the value of $(\Delta\rho/\rho_0)_{ph}$ for pure nickel. Also, the $(\Delta\rho/\rho_0)_{ph}^z$ terms are a correction representing the contribution[18] of the L_zS_z part of the spin-orbit interaction to $(\Delta\rho/\rho_0)_{ph}$, assumed independent of x. Finally, $D(x)/D(0)$ is a correction factor taking into account the variation of the electronic density of states D with x. Values of $D(x)$ are derived from electronic specific heat data of the literature. The equation should also hold for Fe-based alloys with a spin-up scattering resonance. The values of

$(\Delta\rho/\rho_0)_{ph}$ predicted by Eq. (4) are plotted as a dashed line on Fig. 2(a), assuming $\gamma\rho_{sat} = 1.88\times10^{-8}$ Ω m and $(\Delta\rho/\rho_0)_{ph}^z = -4\%$, and using experimental values of $(\Delta\rho/\rho_0)_{im}$, $\rho_0(4.2)$, and $(\Delta\rho/\rho_0)_{ph}$ $(x=0)$. In the case of Figs. 2(b) and 2(c), we use $\gamma\rho_{sat} = 1.25\times10^{-8}$ Ω m, $(\Delta\rho/\rho_0)_{ph}^z = -3\%$. We see that the position and value of the minimum of $(\Delta\rho/\rho_0)_{ph}$ are predicted fairly well by Eq. (4), considering that Eq. (3) is a very rough approximation. This confirms the validity of our explanation of that minimum, in crystalline Ni-Fe, Fe-Cr, and Fe-V, based on the ideas of scattering resonance and resistivity saturation. Also, the $\gamma\rho_{sat}$ values above lead to $\rho_{sat} \simeq 125-188\times10^{-8}$ Ω m, quite consistent with literature values.[17]

[1] I. A. Campbell, A. Fert, and O. Jaoul, J. Phys. C (London) 3, S95 (1970); A. Fert, J. Phys. C (London) 2, 1784 (1969).
[2] C. Herring, Phys. Rev. Lett. 19, 167, (1967); 19, 684 (E) 1967; L. Colquitt, Phys. Rev. A 139, 1857 (1965).
[3] G. K. White, in Proceedings of the 8th Conference on Thermal Conductivity, edited by C. Y. Ho and R. E. Taylor (Plenum, New York, 1969), p. 37 (see Fig. 3); G. K. White and S. B. Woods, Philos. Trans. R. Soc. London Ser. A 251, 273 (1959) (see Table 5); Can. J. Phys. 35, 656 (1957) (see Fig. 4); actually, $L(T)$ seems to exceed L_{el} somewhat at high T, in the case of iron.
[4] R. Parker, Proc. Phys. Soc. London Sect. A 64, 447 (1951); B 64, 930 (1951); B 65, 616 (1952).
[5] L. Berger and D. Rivier, Helv. Phys. Acta 35, 715 (1962) [see Eq. (6) and Fig. 10]; L. Berger and S. A. Friedberg, Phys. Rev. 165, 670 (1968).
[6] P. P. Freitas and L. Berger, J. Magn. Magn. Mater. 54-57, 1515 (1986). Fe-Co sample preparation is described in this paper.
[7] L. Berger and A. R. de Vroomen, J. Appl. Phys. 36, 2777 (1965).
[8] P. P. Freitas and L. Berger, Phys. Rev. B 37, 6079 (1988).
[9] J. Smit, Physica 16, 612 (1951).
[10] H. C. Van Elst, Physica 25, 708 (1959); I. A. Campbell, J. Phys. F (London) 4, L181 (1974); R. M. Bozorth, Phys. Rev. 70, 923 (1946).
[11] Y. Shirakawa, Sci. Repts. Tohoku Imper. Univ. 27, 485 (1939); W. Jellinghaus and M. P. de Andres, Ann. Phys. 5, 187 (1960); T. Farrell and D. Greig, J. Phys. C (London) 1, 1359 (1968); M. C. Cadeville and B. Loegel, J. Phys. F (London) 3, L115 (1973); J. Smit, Physica 21, 877 (1955).
[12] E. Englert, Ann. Phys. 14, 589 (1932); T. R. McGuire, AIP Conf. Proc. 24, 435 (1974).
[13] N. Sueda and H. Fujiwara, J. Sci. Hiroshima Univ. A 35, 59 (1971); N. Sueda, Y. Fujiwara, and H. Fujiwara, ibid. 33, 267 (1969).
[14] L. Berger, AIP Conf. Proc. 34, 355 (1976).
[15] L. Berger, Physica 91B, 31 (1977).
[16] J. Szentirmay and F. J. Kedves, Acta Universitatis Debreciensis, Ser. Phys. Chem., p. 21 (1978); also quoted on Fig. 5 of P. Muth and V. Christoph, J. Phys. F (London) 11, 2119 (1981).
[17] Z. Fisk and G. W. Webb, Phys. Rev. Lett. 36, 1084 (1976).
[18] O. Jaoul, I. A. Campbell, and A. Fert, J. Magn. Magn. Mater. 5, 23 (1977).