A Review on the Relativistic Effective Field Theory with Parameterized Couplings for Nuclear Matter and Neutron Stars

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Abstract. Nuclear science has developed many excellent theoretical models for many-body systems in the domain of the baryon-meson strong interaction for the nucleus and nuclear matter at low, medium and high densities. However, a full microscopic understanding of nuclear systems in the extreme density domain of compact stars is still lacking. The aim of this contribution is to shed some light on open questions facing the nuclear many-body problem at the very high density domain. Here we focus our attention on the conceptual issue of naturalness and its role in shaping the baryon-meson phase space dynamics in the description of the equation of state (EoS) of nuclear matter and neutrons stars. In particular, in order to stimulate possible new directions of research, we discuss relevant aspects of a recently developed relativistic effective theory for nuclear matter within Quantum Hadrodynamics (QHD) with genuine many-body forces and derivative *natural* parametric couplings. Among other topics we discuss in this work the connection of this theory with other known effective QHD models of the literature and its potentiality in describing a new physics for dense matter. The model with parameterized couplings exhausts the whole fundamental baryon octet (n, p, Σ^-, Σ^0, Σ^+, Λ, Ξ^-, Ξ^0) and simulates n-order corrections to the minimal Yukawa baryon couplings by considering nonlinear self-couplings of meson fields and meson-meson interaction terms coupled to the baryon fields involving scalar-isoscalar (σ, σ^*), vector-isoscalar (ω, ϕ), vector-isovector (ϱ) and scalar-isovector (δ) virtual sectors. Following recent experimental results, we consider in our calculations the extreme case where the Σ^- experiences such a strong repulsion that its influence in the nuclear structure of a neutron star is excluded at all. A few examples of calculations of properties of neutron stars are shown and prospects for the future are discussed.

INTRODUCTION

The nuclear matter equation of state (EoS) at the low, medium and high domains of densities, plays an important role in both nuclear physics and astrophysics. However, the knowledge of the EoS at extremely high densities, such as those found in neutron stars and pulsars, together with an adequate description of the properties of hadrons in dense matter, is still an open problem in physics. Neutron stars represents in particular an unique laboratory for testing the EoS of nuclear matter at high densities. Their properties such as masses, radii and moment of inertia, can be calculated by solving the Tolman-Oppenheimer-Volkov equations[1] of general relativity combined with the EoS of nuclear matter.

The derivation of the EoS of nuclear systems is constrained by the dependence of the energy per particle on the particle number density. The energy per particle (or Boltzman free energy for a system at finite temperature) is described in field theory by an effective energy functional and equilibrium states of the system may be found at each density level by minimizing ist expectation value. Additionally, the other related quantities, for instance the pressure, the incompressibility or the entropy may be determined as derivatives of the energy per particle at equilibrium. Subsequently, in order to build energy functionals one needs nuclear and particle physics model descriptions.

In the description of *infinite* nuclear matter, within the framework of effective meson and baryon degrees of freedom, the formal complexity of Quantum Chromodynamics (QCD) has motivated the development of Quantum

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Hadrodynamics (QHD), a very efficient and economical parametrization: in the QHD-I Walecka model [2], keeping only Hartree self-energy terms in the Lagrangian formulation, the classical attractive σ and repulsive ω meson fields completely exhaust the overwhelming part of the effective NN interaction in the nuclear medium at ordinary nuclear matter density ($\rho_0 \sim 0.15 fm^{-3}$). However, at more high densities, still keeping the Hartree approximation, the theory yields the same result as the mean-field theory if one takes into account additional vacuum fluctuation corrections.

When exploring hadron matter at the extremely high density regime of neutron stars and pulsars, the QHD-I model has to be extended. Alternative formulations have been proposed, following however the same philosophy of the original model. Among these we mention the nonlinear model of Boguta-Bodmer[3], the Zimanyi and Moszkowski[4] model, the formulations of N. K. Glendenning[5], and the relativistic effective theory for nuclear matter with *natural* parametric couplings and genuine many-body forces[6] -[11].

In the following we review the concept of naturalness and its role in shaping the baryon-meson phase space dynamics in the description of the EoS of nuclear matter and neutrons stars. In particular, in order to stimulate possible new directions of research, we review relevant aspects of the relativistic effective theory for nuclear matter with *natural* parametric couplings and genuine many-body forces and we analyze the physical implications of recent results.

NATURALNESS

In the formulation of a relativistic effective quantum field theory of nuclear systems at high densities, two conceptual issues are predominant. The first one is the degree of formal consistency of the theory, since this formulation should embody fundamental symmetries and conservation laws in the description of physical properties of many-body nuclear systems, such as Lorentz covariance, microscopic causality, naturalness, analyticity, among others. The second conceptual issue refers to a type of standard technical procedure, commonly adopted in formal treatments in field theory, which are based on the classification of the dynamical terms of the effective action, taking into account the fundamental scales of QCD, which enables a perturbation expansion in a controlled manner of the interaction Lagrangian density.

In the description of global static properties of nuclear systems, the relevant physical phenomena described by the theory are generally dominated by the presence of long-range components while the short range dynamics, which in turn corresponds to the more massive degrees of freedom of meson fields, is explicitly ignored, and their effects implicitly absorbed in the coupling parameters of the theory.

The assumption of naturalness in the strong interaction physics means that, unless a more detailed explanation exists, all conceivable dynamical terms, that preserve the required fundamental symmetries and conservation laws, should appear in the effective action of a theory with natural coupling coefficients[12, 13]. Thus the naturalness condition, when applied to an effective field theory of the strong interaction, establishes that once the appropriate dimensional scales have been extracted using the *naive dimensional analysis* proposed by Georgi and Manohar[13], the remaining dimensionless coefficients appearing in the effective action should all remain of order unity. In other words, naturalness is equivalent, at this level, to extract hidden physics from the coupling parameters of the theory. If the naturalness assumption is valid, then the effective strong interaction Lagrangian density can be truncated, with an acceptable confidence, within the phenomenological physical domain of the theory.

A *natural* way to classify strong interaction contributions is to expand the corresponding Lagrangian density in terms of the characteristic scales of QCD, for which different expansion schemes are possible. The fundamental scales are the renormalization invariant parameter $\Lambda_{QCD} \sim 200 MeV$ or the numbers of colors of quarks, N_c, reminiscent of the $SU(3)$ group structure of QCD. However, when we focus our attention on mesons and baryons as effective low-energy degrees of freedom, — equivalently realized in the large-N_c limit as a result of chiral symmetry breaking —, the appropriate scales are the low energy chiral parameters of QCD, *i.e.*, the weak pion decay constant, $f_\pi = 93 MeV$, and the chiral parameter, $\Lambda_\chi \sim 1 GeV$.

To accomplish that goal, the lagrangian density in QHD-I is defined as

$$\mathcal{L} = \left(\frac{\partial \, or \, m_\pi}{M}\right)\left(\frac{\bar{\psi}\Gamma\psi}{f_\pi^2 M}\right)^{\ell} f_\pi^2 \Lambda^2 \sum_{i,k} \frac{\tilde{c}_{i,k}}{i!k!}\left(\frac{g_\sigma \sigma}{M}\right)^{i}\left(\frac{g_\omega \omega}{M}\right)^{k}; \tag{1}$$

in this expression, ψ represents Dirac solutions for nucleon fields, M denotes the nucleon mass, m_π is the pion mass and σ and ω represent respectively the fundamental scalar-isoscalar and vector-isoscalar meson fields.

At least two schemes allow a compact summation of the Lagrangian density (1):

$$\mathcal{L} = \left(\frac{\partial \, or \, m_\pi}{M}\right)\left(\frac{\bar{\psi}\Gamma\psi}{f_\pi^2 M}\right)^{\ell} \exp\left(\frac{\sigma}{M} + \frac{\omega}{M}\right) f_\pi^2 \Lambda_\chi^2 \quad (\text{for } c_{i,\kappa} = 1) \text{ and} \tag{2}$$

$$\mathcal{L} = \left(\frac{\partial \, or \, m_\pi}{M}\right)\left(\frac{\bar{\psi}\Gamma\psi}{f_\pi^2 M}\right)^{\ell} \left(\frac{1}{1+\frac{\sigma}{M}}\right)\left(\frac{1}{1+\frac{\omega}{M}}\right) f_\pi^2 \Lambda_\chi^2 \quad (\text{for } c_{i,\kappa} = i!\kappa!)\,. \tag{3}$$

The first scheme, $c_{i,\kappa} = 1$, corresponds to the *natural* limit. The second one, $c_{i,\kappa} = i!\kappa!$), corresponds to a kind of *derivative coupling*[4] scheme. The *naive dimensional analysis* when applied in the formulation of an effective Lagrangian density involving nucleons and strongly interacting meson fields may be synthesized as follows: a) the amplitude of each strongly interacting field in the lagrangian, i.e. the meson fields, becomes dimensionless when divided by the pion decay weak constant; b) to obtain the correct dimension ((energy)^4) for the Lagrangian density, an overall normalization scale $f_\pi^2\Lambda^2 \simeq f_\pi^2 M^2$, with M denoting the nucleon mass, has to be included; c) for identical meson fields self-interacting terms of power n, a symmetrization factor $n!$, for proper counting, should be included into the formalism. The overall dimensionless coefficients, after the dimensional factors and appropriate counting factors are extracted, are of order $O(1)$ if naturalness holds. Of course, there is no general proof of the naturalness property, since no one knows how to derive the effective strong interaction Lagrangian density from Quantum Chromodynamics (QCD). Nevertheless, the validity of naturalness and *naive power counting rules* is supported by phenomenological studies[12, 13, 14, 15].

LAGRANGIAN DENSITY WITH PARAMETERIZED COUPLINGS

The model discussed here has a philosophy quite similar to the original versions of the models with parameterized couplings[4, 6]. However, while in the most general approach of ref.[6] parameterizations of the coupling constants are introduced in *had hoc* way, we discuss here a method for the derivation of the parametric dependence on the coupling constants following the original formulation of the ZM-model[4] that allows: a) a consistent formal justification for its adoption; b) the extension of the range of possibilities of parameterizations in effective models with derivative couplings in a coherent way. Additionally, this approach exhibits consistency with the concept of naturalness allowing this way, with acceptable confidence, the utilization of perturbation expansions of the Lagrangian density which describes the strong interaction contributions, within the phenomenological physical domain of the theory.

The strategy here is to consider a phenomenological and more flexible parametrization of the QHD Lagrangian density which combines the two previous limits and an extension of the interaction phase space of baryon and meson fields. Properties of the fields considered in our formulation are presented in table (1).

The interaction Lagrangian density of the model is defined as:

$$\mathcal{L}_{int} = \prod_{\lambda=\xi,\kappa,\eta}\left(1 + \frac{g_{\sigma B}\sigma + g_{\sigma^* B}\sigma^* + \frac{1}{2}g_{\delta B}\boldsymbol{\tau}\cdot\boldsymbol{\delta}}{\lambda M_B}\right)^{\lambda} \bar{\psi}_B\, i\gamma_\mu\partial^\mu\,\psi_B - \bar{\psi}_B\,\Gamma_{\kappa\eta\xi\zeta B}\,\psi_B\,, \tag{4}$$

where the operators $\boldsymbol{\tau} = (\tau_1, \tau_2, \tau_3)$ represent the Pauli isospin matrices. In this expression, the Lorentz scalar Γ is defined as

$$\begin{aligned}\Gamma_{\kappa\eta\xi\zeta B} = {} & g_{\omega_B}\prod_{\lambda=\kappa,\eta}\left(1 + \frac{g_{\sigma B}\sigma + g_{\sigma^* B}\sigma^* + \frac{1}{2}g_{\delta B}\boldsymbol{\tau}\cdot\boldsymbol{\delta}}{\lambda M_B}\right)^{\lambda}\gamma_\mu\omega^\mu + \frac{1}{2}g_{\varrho B}\prod_{\lambda=\xi,\eta}\left(1 + \frac{g_{\sigma B}\sigma + g_{\sigma^* B}\sigma^* + \frac{1}{2}g_{\delta B}\boldsymbol{\tau}\cdot\boldsymbol{\delta}}{\lambda M_B}\right)^{\lambda}\gamma_\mu\boldsymbol{\tau}\cdot\boldsymbol{\varrho}^\mu \\ & + g_{\phi_B}\prod_{\lambda=\kappa,\xi}\left(1 + \frac{g_{\sigma B}\sigma + g_{\sigma^* B}\sigma^* + \frac{1}{2}g_{\delta B}\boldsymbol{\tau}\cdot\boldsymbol{\delta}}{\lambda M_B}\right)^{\lambda}\gamma_\mu\phi^\mu + M_B\Pi_{\lambda=\kappa,\eta,\xi,\zeta}\left(1 + \frac{g_{\sigma B}\sigma + g_{\sigma^* B}\sigma^* + \frac{1}{2}g_{\delta_B}\boldsymbol{\tau}\cdot\boldsymbol{\delta}}{\lambda M_B}\right)^{\lambda}.\end{aligned} \tag{5}$$

The corresponding expression of the Lagrangian density of the model is

$$\mathcal{L} = \sum_B \bar{\psi}_B\left[\prod_{\lambda=\xi,\kappa,\eta}\left(1 + \frac{g_{\sigma B}\sigma + g_{\sigma^* B}\sigma^* + \frac{1}{2}g_{\delta B}\boldsymbol{\tau}\cdot\boldsymbol{\delta}}{\lambda M_B}\right)^{\lambda} i\gamma_\mu\partial^\mu - \Gamma_{\kappa\eta\xi\zeta B}\right]\psi_B$$

$$
\begin{aligned}
&+ \quad \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma{}^2\sigma^2) + \frac{1}{2}(\partial_\mu\sigma^*\partial^\mu\sigma^* - m_{\sigma^*}{}^2\sigma^{*2}) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega{}^2\omega_\mu\omega^\mu \\
&- \quad \frac{1}{4}\phi_{\mu\nu}\phi^{\mu\nu} + \frac{1}{2}m_\phi{}^2\phi_\mu\phi^\mu - \frac{1}{4}\boldsymbol{\varrho}_{\mu\nu}\cdot\boldsymbol{\varrho}^{\mu\nu} + \frac{1}{2}m_\varrho^2\boldsymbol{\varrho}_\mu\cdot\boldsymbol{\varrho}^\mu + \frac{1}{2}(\partial_\mu\boldsymbol{\delta}\cdot\partial^\mu\boldsymbol{\delta} - m_\delta{}^2\boldsymbol{\delta}^2) \\
&+ \quad \sum_l \bar{\psi}_l\,(i\gamma_\mu\partial^\mu - m_l)\,\psi_l\,,
\end{aligned}
\tag{6}
$$

where the subscripts B and l label respectively the different baryon and lepton (electrons and free muons) species. Its is important to remember that isoscalar meson fields are related to the algebra of the group theory $U(1)$, while isovector meson fields are related to the non-commutative algebra of the group theory $SU(2)$, and that this aspect is responsible for the presence of additional self-coupling terms involving the ϱ meson fields in the above expression of the Lagrangian density (6); those terms characterize many-body interaction contributions. We Introduce in the following a change of scale of the baryon fields in the form

$$
\psi_B \rightarrow \left(\prod_{\lambda=\xi,\kappa,\eta} \left(1 + \frac{g_{\sigma B}\sigma + g_{\sigma^* B}\sigma^* + \frac{1}{2}g_{\delta B}\boldsymbol{\tau}\cdot\boldsymbol{\delta}}{\lambda M_B} \right)^{-\lambda} \right)^{1/2} \psi_B\,. \tag{7}
$$

With this change of scale we obtain the following expression for the Lagrangian density:

$$
\begin{aligned}
\mathcal{L} \quad = \quad & \sum_B \bar{\psi}_B \left[i\gamma_\mu\partial^\mu - g^*_{\omega B\xi}\gamma_\mu\omega^\mu - \frac{1}{2}g^*_{\varrho B\kappa}\gamma_\mu\boldsymbol{\tau}\cdot\boldsymbol{\varrho}^\mu - g^*_{\phi B\eta}\gamma_\mu\phi^\mu - M^*_{B\zeta} \right]\psi_B \\
+ \quad & \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma{}^2\sigma^2) + \frac{1}{2}(\partial_\mu\sigma^*\partial^\mu\sigma^* - m_{\sigma^*}{}^2\sigma^{*2}) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega{}^2\omega_\mu\omega^\mu \\
- \quad & \frac{1}{4}\phi_{\mu\nu}\phi^{\mu\nu} + \frac{1}{2}m_\phi{}^2\phi_\mu\phi^\mu - \frac{1}{4}\boldsymbol{\varrho}_{\mu\nu}\cdot\boldsymbol{\varrho}^{\mu\nu} + \frac{1}{2}m_\varrho^2\boldsymbol{\varrho}_\mu\cdot\boldsymbol{\varrho}^\mu + \frac{1}{2}(\partial_\mu\boldsymbol{\delta}\cdot\partial^\mu\boldsymbol{\delta} - m_\delta{}^2\boldsymbol{\delta}^2) \\
+ \quad & \sum_l \bar{\psi}_l\,(i\gamma_\mu\partial^\mu - m_l)\,\psi_l\,,
\end{aligned}
\tag{8}
$$

where the parameterized coupling constants are:

$$
g^*_{\omega B\xi} \equiv m^*_{B\xi}g_{\omega B}\ ;\ \ g^*_{\varrho B\kappa} \equiv m^*_{B\kappa}g_{\varrho B}\ ;\ \ g^*_{\phi B\eta} \equiv m^*_{B\eta}g_{\phi B}\,, \tag{9}
$$

with $i = \xi, \kappa, \eta$, and

$$
m^*_{Bi} \equiv \left(1 + \frac{g_{\sigma_B}\sigma + g_{\sigma^*_B}\sigma^* + \frac{1}{2}\,g_{\delta_B}\boldsymbol{\tau}\cdot\boldsymbol{\delta}}{iM_B} \right)^{-i}\,. \tag{10}
$$

TABLE 1. Properties of the fields considered in our formulation. In what follows, we use the abbreviations: ISS: isoscalar-scalar; IVS: isovector-scalar; ISV: isoscalar-vector; IVV: isovector-vector.

Fields	Classification	Particles	Coupling Constants	Mass (MeV)
ψ_B	Baryons	N, Λ, Σ, Ξ	N/A	939, 1116, 1193, 1318
ψ_l	Leptons	e^-, μ^-	N/A	0,5, 106
σ	ISS-meson	σ	$g^*_{\sigma_B}$	550
$\boldsymbol{\delta}$	IVS-meson	a_0	$g^*_{\delta_B}$	980
ω_μ	ISV-meson	ω	$g^*_{\omega_B}$	782
$\boldsymbol{\varrho}_\mu$	IVV-meson	ρ	$g^*_{\varrho_B}$	770
σ^*	ISS-meson	f_0	$g^*_{\sigma^* B}$	975
ϕ_μ	ISV-meson	ϕ	$g^*_{\phi_B}$	1020

The resulting expression for the Lagrangian density of the parameterized coupling model allows numerous possibilities of parameterizations. Here we focus our attention to a few examples of parameterizations and their formal relation with known QHD models of the literature. For instance, we may consider variations of the ζ parameter keeping $\xi = \kappa = \eta = 0$ (scalar or S-model). This parametrization reduces to the QHD-I model of Serot and Walecka[2] if $\zeta = 0$ and $g_{\varrho B} = g_{\phi B} = 0$. In case $g_{\varrho B} \neq 0$, this parametrization reduces to the QHD-II model[2]. Assuming $\zeta = 1$, performing a binomial expansion of $m_{B\zeta}$, truncating the perturbative series to cubic and quartic self-interactions terms involving the scalar-isoscalar σ meson (making $g_{\sigma^* B} = g_{\delta B} = 0$), this parametrization reduces to the model of Boguta and Bodmer[3]. The second choice contemplates variations of the parameters ζ and ξ while keeping $\kappa = \eta = 0$ (scalar-isoscalar-vector or SIV-model). In the third choice we may consider variations of ζ, ξ, and κ, and fixing $\eta = 0$ (scalar-isoscalar-vector-isovector-vector or SIIV-I-model), as already discussed in ref.[8]. Finally, in the fourth choice we may consider variations of the four parameters of the theory, ζ, ξ, κ, and ζ (scalar-isoscalar-vector-isovector-vector or SIIV-II-model). These examples of parameterizations of our approach are shown in Table (2).

TABLE 2. Examples of parameterizations of our model. S: scalar model; SIV: scalar-isoscalar-vector model; SIIV: scalar-isoscalar-vector-isovector-vector model. Model II differs from I, due to the presence of the ϕ meson.

Model	ζ	ξ	κ	η
S	$\neq 0$	0	0	0
SIV	$\neq 0$	$\neq 0$	0	0
SIIV-I	$\neq 0$	$\neq 0$	$\neq 0$	0
SIIV-II	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$

Of course, there are other possibilities for the parameterizations not included in these examples. The important point to note is the physical interpretation of the parameterizations which represent analytic contributions of the different orderings of many-body density correlations in perturbation theory and moreover, density corrections to the local more conventional Yukawa-type couplings.

We consider, as an example of the effects of the presence of many-body interaction contributions in our formulation, the expansion of the term $g_{\omega B} m^*_{B\xi} \gamma_0 \omega_0$ in expression (8), in the particular case $\xi = 1$. In the framework of a local mean field approximation, expectation values of meson fields correspond to classical numbers[1]. In this context, taking into account that

$$\frac{g_{\sigma B}\sigma_0 + g_{\sigma^* B}\sigma_0^* + \frac{1}{2} g_{\delta B}\tau_3\delta_{30}}{M_B} << 1 \,,$$

we may use the binomial theorem to expand the interaction term $g_{\omega B} m^*_{B\xi} \gamma_0 \omega_0$ for the particular choice $\xi = 1$:

$$\begin{aligned} g_{\omega B} m^*_{B1}\gamma^0\omega_0 \quad &\sim \quad g_{\omega B}\gamma^0\omega_0 - g_{\omega B}\left(\frac{g_{\sigma B}\sigma_0 + g_{\sigma^* B}\sigma_0^* + \frac{1}{2} g_{\delta B}\tau_3\delta_{30}}{M_B}\right)\gamma^0\omega_0 \\ &+ \quad g_{\omega B}\left(\frac{g_{\sigma B}\sigma_0 + g_{\sigma^* B}\sigma_0^* + \frac{1}{2} g_{\delta B}\tau_3\delta_{30}}{M_B}\right)^2 \gamma^0\omega_0 - g_{\omega B}\left(\frac{g_{\sigma B}\sigma_0 + g_{\sigma^* B}\sigma_0^* + \frac{1}{2} g_{\delta B}\tau_3\delta_{30}}{M_B}\right)^3 \gamma^0\omega_0 \ldots \qquad (11) \end{aligned}$$

The ω meson corresponds to the short range repulsive sector of the strong nuclear interaction. However, from this expression we can identify additional many-body attractive and/or repulsive coupling terms associated with the switching of signals generated by the binomial expansion, like for instance $\sigma_0\omega_0$, $\sigma_0^2\omega_0$, $\sigma_0^{*2}\omega_0$, $\delta_{30}\omega_0$, $\delta_{30}^2\omega_0$, $\sigma_0\sigma^*\delta_{30}\omega_0$ and many others. Similarly, the remaining terms of the Lagrangian density also exhibit additional many-body coupling terms. Again, the final selection of the contributions to be considered requires an analysis of the formal coherence of the theory. In this sense, it is important to remember that the theory must embody fundamental symmetries and conservation laws, such as Lorentz covariance, microscopic causality, naturalness, analyticity, uniqueness, among others, as well as physical properties which are relevant for strong interacting relativistic nuclear many-body systems.

[1] It is well known that in the realm of the mean field approximation, expectation values of meson fields may be treated as classical numbers in space-time despite the density dependence of these fields in Fermi space.

The presence of additional positive or negative interaction contributions to the Lagrangian density can be interpreted as density corrections to the local conventional Yukawa-type interaction terms. The parameters of the theory, ξ, κ, η, ζ, control in turn the relevant intervals of values associated to the ordering of the contributions of many-body forces to the dynamics of the system. As we have previously emphasized, these parameters thereby acquire a real physical character, going beyond the usual conceptions of just fitting mathematical parameters.

Expression (8), in the natural limit (with $\wp = i, j, k, m, n, q$ and $c(\wp) \to 1$) may be synthesized as

$$\begin{aligned}\mathcal{L} &= \sum_B \sum_\wp \frac{c_\wp}{\Pi_\wp \wp!}\left(\frac{\partial\, or\, m_\pi}{M_B}\right)\left(\frac{\bar{\psi}_B \Gamma \psi_B}{f_\pi^2 M_B}\right)^\ell f_\pi^2 \Lambda^2 \left(\frac{\sigma}{f_\pi}\right)^i \left(\frac{\sigma^*}{f_\pi}\right)^j \left(\frac{\omega}{f_\pi}\right)^k \left(\frac{\varrho}{f_\pi}\right)^m \left(\frac{\delta}{f_\pi}\right)^n \left(\frac{\phi}{f_\pi}\right)^q \\ &\to \sum_B \left(\frac{\partial\, or\, m_\pi}{M_B}\right)\left(\frac{\bar{\psi}_B \Gamma \psi_B}{f_\pi^2 M_B}\right)^\ell f_\pi^2 \Lambda^2 \exp \sum_\iota \frac{g_\iota \Phi_\iota}{M_B} \,. \end{aligned} \tag{12}$$

EFFECTIVE BARYON MASS

The mass term of expression (8), $M_B^* = m_{B\zeta}^* M_B$, corresponds to

$$M_B^* = m_{B\zeta}^* M_B \equiv \left(1 + \frac{g_{\sigma B}\sigma + g_{\sigma^* B}\sigma^* + \frac{1}{2} g_{\delta B} \boldsymbol{\tau} \cdot \boldsymbol{\delta}}{\zeta M_B}\right)^{-\zeta} M_B \,. \tag{13}$$

The resulting Lagrangian density obtained through the substitution of expression (7) in equation (6), is physically equivalent to the original formulation. The introduction of the scaling (7) in equation (6) just results in a reorganization of the original interaction Lagrangian density which allows at one hand, a more direct comparison with well known QHD models and on the other, the use of conventional methods and techniques of field theory when seeking for solutions of the many body nuclear problem.

The effective parameterized baryon mass $M_{B\zeta}^* = M_B \Sigma_{B\zeta}^s = M_B m_{B\zeta}^*$, for

$$\left(\frac{g_{\sigma B}\sigma_0}{\zeta M_B}, \frac{g_{\sigma^* B}\sigma_0^*}{\zeta M_B}, \frac{g_{\delta B}\delta_{03}}{\zeta M_B}\right) \ll 1 \,,$$

in the mean field approximation, becomes

$$M_{B\zeta}^* = M_B - M_B \left\{ \left(\frac{g_{\sigma B}\sigma_0 + g_{\sigma^* B}\sigma_0^* + \frac{1}{2} g_{\delta B} < \tau_3 > \delta_{03}}{M_B}\right) + \binom{\zeta}{2} \left(\frac{g_{\sigma B}\sigma_0 + g_{\sigma^* B}\sigma_0^* + \frac{1}{2} g_{\delta B} < \tau_3 > \delta_{03}}{\zeta M_B}\right)^2 + O(3) \right\}, \tag{14}$$

with $\binom{\zeta}{2}$ representing the generalized binomial coefficients of the expansion. It is important to emphasize the systematic alternating of positive and negative signs in this expression which imply the reduction of the degree of decrease of the effective baryon mass as a function of density due to the presence of many body interaction terms.

PARTICLE POPULATIONS

According to experimental data (see for instance ref.[16]), realistic models for the strong interaction at high densities shall consider the appearance of hyperons at densities $(5-8) \times 10^{14} g/cm^3$. The so called *hyperonization process* however softens the EoS of neutron stars, due to the fact that the Pauli exclusion principle does not manifest between nucleons and hyperons thus causing a decrease in the internal quantum degeneracy pressure in neutron stars. And thereby making it more difficult for nuclear models containing hyperons to describe stars with masses of the order of $2M_\odot$ as recently observed[17, 18].

Thus, it is also important to analyze the role of many-body correlations in the threshold equation for a given species[5]:

$$\mu_n - q_B\,\mu_e \geq g_{\omega B} f_{\sigma\,\sigma^* \delta\xi}\, \omega_0 + g_{\varrho B}\, f_{\sigma\sigma^*\delta\kappa}\, \varrho_{03}\, I_{3B} + g_{\phi B}\, f_{\sigma\sigma^*\delta\eta}\, \phi_0 + f_{\sigma\sigma^*\delta\zeta}\, M_B \,. \tag{15}$$

In this expression, μ_n and μ_e represent respectively the neutron and electron chemical potential, and q_B is the baryon charge. The sign of $g_{\varrho B}\, \varrho_{03}\, I_{3B}$ is determined by the net isospin density of the star. This term determines whether a given baryon configuration is isospin favored or unfavored. Similarly, the term $q_B\, \mu_e$ determines whether a given baryon state is charge favored or unfavored. Moreover, in this expression $f_{\sigma\sigma^*\delta\alpha}$ is defined as

$$f_{\sigma\sigma^*\delta\alpha} = 1 - \left(\frac{g_{\sigma B}\sigma_0 + g_{\sigma^* B}\sigma_0^* + \frac{1}{2}g_{\delta B} <\tau_3> \delta_{03}}{M_B}\right) + \left(\begin{array}{c}\alpha \\ 2\end{array}\right)\left(\frac{g_{\sigma B}\sigma_0 + g_{\sigma^* B}\sigma_0^* + \frac{1}{2}g_{\delta B} <\tau_3> \delta_{03}}{\alpha M_B}\right)^2 + O(3), \qquad (16)$$

with $\alpha = \xi, \kappa, \eta, \zeta$. Theses equations show that the population of hyperons is affected by many-body correlations, which shift the critical density for hyperon saturation to higher densities, contributing this way to the increase of neutron star masses compared to the case where many-body forces are not present.

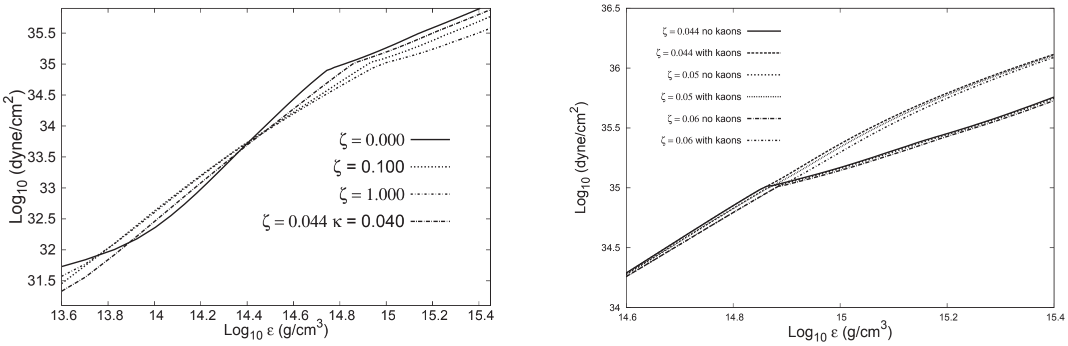

FIGURE 1. (a) On the left: equation of state for a set of parameters. For comparison, the curves labelled with $\zeta = 0.000$ and $\zeta = 1.000$ represent the results of our model corresponding respectively to the results of refs.[2, 4]. The remaining curves labelled with $\zeta = 0.100$ and $\zeta = 0.044, \kappa = 0.040$ represent a particular combination of the parameters of our model. This last result allows, through the calculation of the TOV equations[1], to obtain a maximum mass of neutron stars in good agreement with recent experimental observations[17]. The remaining model parameters, not shown in the curves, are set equal to zero[10]. (b) On the right, for comparison, the curves show the predictions of our model for the equation of state for a particular set of parameters and kaon degrees of freedom. The remaining model parameters, not shown in the curves, are set equal to zero[11].

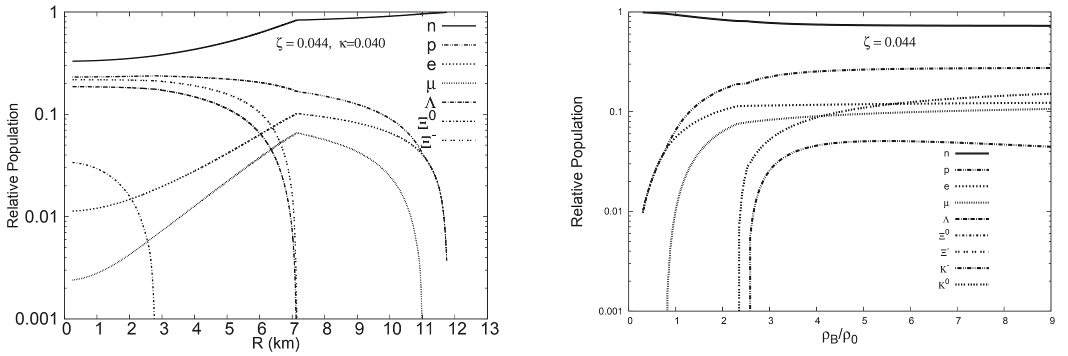

FIGURE 2. (a) On the left, population profile as a function of the stellar radius for the particular set of parameters[10]. (b) On the right, similar results taking into account the presence of kaons[11].

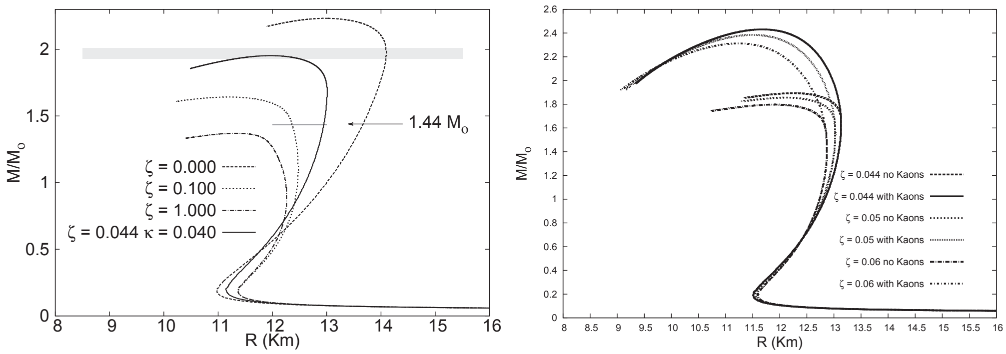

FIGURE 3. (a) On the left, mass-radius relation for different sets of parameters and different versions of the model. The highlighted region determine the observational threshold for the maximum mass $1.97M_{\odot} \pm 0.04$ (see ref.[17]). The thin horizontal line corresponds to a canonical $1.44M_{\odot}$ neutron star with radius 12.5 ± 0.5 km[10]. (b) On the right, similar results taking into account the presence of kaons[11].

COUPLING CONSTANTS

The values of the sets of parameters of the model are chosen to allow the model to reproduce nuclear properties at saturation, like for example the compressibility modulus of nuclear matter smaller than 300 MeV. In general we assume for the saturation density of nuclear matter $\rho_0 = 0.17 fm^{-3}$ and for the binding energy of nuclear matter $\epsilon_B = -16.0 MeV$. The isovector coupling constant g_ϱ is constrained to the symmetry energy coefficient $a_{asym} = 32.5 MeV$[19]. In the study of the equation of state and composition of hypernuclear matter within density-dependent couplings, the parameter space of hyperon-scalar-meson couplings may be explored by allowing for mixing and breaking of SU(6) symmetry, while keeping the nucleon coupling constants fixed. The intensity of the couplings of scalar mesons with hyperons can also be obtained by determining the depth of the hyperon-nucleon interaction potential on saturated nuclear matter and quark counting rules.

RESULTS, DISCUSSION AND PERSPECTIVES

Using the QHD model with parameterized couplings, we may determine the EoS, population profiles and, by solving the Tolman-Oppenheimer-Volkoff (TOV) equations[1], the mass-radius relation for families of neutron stars with hyperon content. Some of our recent results for different values of the set of parameters are illustrated in the figures, where each set generates a sequence of neutrons stars with different equations of state, particle populations, central densities, and maximum masses for neutron stars.

The analysis of these results demands first to remember that a stiffer, or equivalently, more rigid equation of state of nuclear matter is related to higher values of the internal pressure of the system and, accordingly, to higher values of the compressibility modulus $|K_{sym}|$ of nuclear matter. This in turn requires stronger contributions from repulsive components of the nuclear force when compared to the attractive ones. In our general approach, however, many body forces (density correlations) lower the intensities both of attractive and repulsive interaction terms due to *shielding effects*, which result in higher (lower) values of the compressibility modulus $|K_{sym}|$ of nuclear matter in the case of higher (lower) relative reduction of the attractive (repulsive) contributions.

In this sense, when many-body correlations shield the attractive part of the strong interaction, they intensify the corresponding repulsive part, favoring in this way the stiffening of the EoS. On the other hand, the effective masses of baryons increase as the shielding of the attractive part of the strong interaction increases. This favors the growth of the internal pressure of the system and the stiffening of the EoS.

Following recent experimental results[20, 21], we have considered in our calculations the extreme case where the Σ^- experiences such a strong repulsion that it does not appear at all in nuclear matter for densities exceeding those found in neutron stars. The first hyperon species that appears is the Λ: free of isospin-dependent forces, as the density increases, the Λ hyperon continues to accumulate until short-range repulsion forces cause them to saturate. Other hyperon species follow at higher densities.

Our model originates moreover an anti-correlation between the amount of hyperons: for certain values of the parameters, according to equations (15) and (16) an anti-correlation associated with the predominance of the scalar part occurs. This mean that hyperon degrees of freedom become more numerous to the extent that the attractive sector is favored in comparison with the repulsive part, thus favoring smaller neutron star masses. However, the absence of the Σ^- hyperon reduces this effect. For other values of the parameters of the theory, the repulsive part of the strong interaction is enhanced thereby contributing to the stiffening of the EoS and consequently to the increase of the mass of the neutron star.

When considering simultaneously both shielding effects involving the attractive and repulsive contributions of the strong interaction, one would expect that, — since the repulsive part of the strong interaction is more effective in nuclear matter at high densities, on the average, than the attractive sector —, that the shielding of the strong interaction would favor the attractive part, contributing this way to the reduction of the mass of the neutron star. However, our results indicate that the combination of these effects with the others previously reported, favors the stiffening of the EoS. In other words, the shielding of the attractive part of the strong interaction combined with the increase of the effective mass of baryons and the absence of the Σ^- hyperon, are dominant when compared with those effects favoring the softening of the EoS, i.e., the shielding of the repulsive part of the strong interaction and the increase on the population of the remaining hyperons beyond the Σ^-.

Our results also indicate that, to compensate the absence of the Σ^- to bring about charge neutrality and chemical equilibrium, as well as the requirements of the Pauli principle and the rearrangement of Fermi populations to minimize energy, the Λ and Ξ^- thresholds have been reduced. This last reduction is charge favored, replacing a neutron and electron at the top of their Fermi seas, although both Σ^- and Ξ^- are isospin unfavored.

It is our understanding that the naturalness condition is equivalent to exhaust the phase space of the fundamental interactions. This depletion of the phase space can be accomplished by including in nuclear matter the largest possible number of *information*, ie. baryon and meson degrees of freedom and additionally considering many body forces between baryon and meson fields, as well as self-coupling terms involving meson fields. When the condition of naturalness is achieved, it is unnecessary to get rid of heavier degrees of freedom by integrating then out. In this case, the effects of heavier degrees of freedom are not anymore implicitly contained in coupling parameters of the effective field theory. An interesting aspect of our study is that apparently, many body forces occupy a larger role than originally thought in the description of properties of nuclear system at high density. Moreover, the condition of naturalness is achieved with only a small part of the parameter set. This result was expected and can be explained by the saturation property of nuclear matter: for parameter values less than 3, the model with parameterized couplings completely exhausts the phase space of many-body interactions.

The model with parameterized couplings shows promising results. However, the model needs to broaden its scope by the adoption of new parameterizations, expanding the set of parameters of the theory. Interesting issues for future studies will also be the role of finite temperature, neutrino trapping and strong magnetic effects in neutron stars. Work along these lines is in progress.

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