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Citation: Physics of Plasmas **14**, 022104 (2007); doi: 10.1063/1.2435704
View online: http://dx.doi.org/10.1063/1.2435704
View Table of Contents: http://scitation.aip.org/content/aip/journal/pop/14/2?ver=pdfcov
Published by the AIP Publishing

Mode-coupling of low-frequency electromagnetic waves in dusty plasmas with temperature anisotropy

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(Received 23 August 2006; accepted 18 December 2006; published online 8 February 2007)

This paper studies the effects of the presence of dust particles with variable charge, in fully ionized, homogeneous, magnetized plasma of electrons and ions, with the electrons and ions described by bi-Maxwellian distributions in the equilibrium. The dispersion relation and the absorption rate are obtained for low frequency waves, with frequencies much lower than the ion cyclotron frequency. Two branches are obtained, identified as the whistler branch and the branch of circularly polarized waves, featuring damping due to the Landau damping process and to the collisional charging of the dust particles. The effects of the anisotropy of temperature on the damping rate of low frequency waves, and on the mode coupling which was demonstrated to occur in the isotropic situation, are numerically investigated. The results obtained show that in the anisotropic case the point of mode coupling is displaced to different values of dust density, and that a new point of mode coupling may appear from the effect of the temperature anisotropy. © *2007 American Institute of Physics*.
[DOI: 10.1063/1.2435704]

I. INTRODUCTION

Observations made by different space probes have shown the occurrence of dust grains in the interplanetary medium.[1] In addition to these well known situations in space plasmas, it is not difficult to find other situations where dust particles coexist with plasmas. The dust particles immersed in the plasma inevitably acquire electrical charge due to different charging mechanisms,[2,3] therefore forming what is known as a *dusty plasma*. Particularly, electrically charged dust particles will be present in the solar wind which permeates all the interplanetary medium in the solar system.

The WIND spacecraft, part of a mission initiated in 1994, has collected a large amount of data about the different populations of particles in the solar wind. The analysis of these data have indicated that electrons and ions in the solar wind may feature temperature anisotropy.[4–6]

On the other hand, it is known that low-frequency waves are expected to be the most affected by the presence of charged dust particles. Among the low-frequency waves, Alfvén waves assume particular importance, due to their pervasive presence in the space plasma environment. Alfvén waves have been detected in the solar wind near the Earth orbit by the Mariner V probe, identified by a large correlation coefficient between velocity fluctuations and magnetic field fluctuations.[7] Following the evidence of the presence of Alfvén waves in the solar wind environment, theoretical analysis has incorporated these waves as a possible accelerating mechanism of the solar wind, in the attempts to explain mass ejections from the Sun.[8–10]

The modifications in the behavior of Alfvén waves and its instabilities, due to the presence of a dust component, can be associated with several features characteristic of the dust particles. For instance, these modifications can depend on the size distribution of the dust particles, on the charge imbalance between ions and electrons which is a consequence of the electrical charges added to the dust particles, or on the fluctuations of the charge on the dust particles. Several examples of studies which take into account the presence of a dust component in the plasma can be found in the literature. Some of them focused on the ultralow-frequency waves, with frequencies much below the dust cyclotron frequency, a range in which the waves are affected by the dust grain dynamics.[11–14] Other authors considered the circularly polarized electromagnetic waves propagating parallel to the magnetic field in a plasma with static dust grains, focusing particularly on the case of frequencies much below the ion cyclotron frequency.[2,15,16]

Nonlinear phenomena as parametric instabilities and modulational and decay instabilities of Alfvén waves have been studied using fluid theory, either considering mobile[17,18] or immobile dust particles.[19,20] In Ref. 17 Hertzberg *et al.* have investigated the propagation of Alfvén waves and magnetoacoustic waves modified by the presence of a secondary ion or dust species, allowed to be fully mobile, with the excitation provided by parametrically pumping the magnetic

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field. The modifications suffered by known instabilities due to the presence of an additional ion species were discussed in Ref. 18. It has been shown that in the low-temperature regime the effect of a second ion species is to reduce the usual modulational instability, and to produce a new decay instability. In the high-temperature regime, two new decay instabilities were found, both narrow in wave number. Interaction between the right handed transverse mode and the slow acoustic mode has been shown to occur. In a previous publication, Hertzberg *et al.*[19] had analyzed the excitation of low frequency modes in a magnetized dusty plasma with parametric pumping of the magnetic field, using a two-fluid MHD model where the dust particles were assumed to be immobile. According to the analysis of Ref. 19, the coupling between different modes does not occur, contrary to what had been found in the case in which the particle dynamics was taken into account.[18] More recently, mode-coupling of low frequency waves has been obtained for immobile dust particles in a linear analysis which employs kinetic theory taking into account the collisional charging of the dust particles,[21] indicating a coupling mechanism that is different from the effective one in the case of Ref. 18.

Compressional Alfvén waves, on the interface between dusty plasmas with different densities, or on the interface between a dusty plasma and the vacuum, have also been investigated using fluid theory.[22–24] The case of constant dust charge, with stationary dust grains, has been treated in Ref. 24, and with mobile dust grains in Ref. 23. The resonant damping of the surface wave has been evaluated for an interface of nonvanishing width, and it was shown that for a range of frequencies above the dust cyclotron frequency the surface wave can propagate without resonant damping, in contrast to what occurs for a dustless plasma.[23]

The modification of the Alfvén resonance absorption mechanism due to negative charge residing on the grains in a dusty plasma was investigated in Ref. 25, using a fluid theory. It was shown that the Alfvén resonance process can be strongly modified by the presence of dust, due to the charge imbalance between electrons and ions, even if the amount of charge on the dust particles is quite small (typically the proportion is $\approx 10^{-4}$ in interstellar clouds and in cometary plasmas).[11] The dispersion relation obtained had a form similar to that describing the coupling of Alfvén waves and collisionless ion-sound waves in nonisothermal plasmas with $T_e \gg T_i$.[25]

The effects of the dust particles on small amplitude magnetohydrodynamic waves in interstellar clouds were investigated by Cramer and Vladimirov in Ref. 26, using a fluid formulation, considering obliquely propagating waves in a plasma with a stationary secondary heavy species.

The presence of charged dust particles can also influence the excitation of hydromagnetic waves by instabilities caused by pickup ions in cometary plasmas.[27] The analysis made using a collisionless cold plasma theory, with the effect of the charged dust particles introduced basically by the charge imbalance between electrons and ions, has found that the dust has its greatest effect on the firehose instability at very long wavelength. Effects due to dust charge fluctuation and due to Landau damping were not taken into account. *In passant*, we mention that we have incorporated these effects in an investigation of low-frequency instabilities which has recently been sent for publication.

The studies on linear waves in dusty plasmas also include attempts which discuss the effect of a distribution of size of the dust particles.[28] The analysis in Ref. 28 has been made considering a magnetized, homogeneous, and cold plasma, with a distribution of size of the dust particles representative of that to be found in interstellar molecular clouds. The analysis has discussed the dust-cyclotron damping of the waves. It has been shown that the effect of the size distribution is to introduce a large frequency interval of resonant absorption due to the cyclotron damping.

It is possible to find in the literature some references which apply kinetic formulations to studies of the solar wind.[29,30] However, kinetic approaches to the study of the solar wind plasmas which include the charged dust particles as one of the plasma components are not easily found. That was one of the motivations of our recent publications on the subject of dusty plasmas, particularly of Ref. 31, in which we have used a kinetic description to analyze the propagation of electromagnetic waves in dusty plasmas, taking into account the fluctuation of the dust charges due to inelastic collisions with electrons and ions.

More specifically, in the formulation utilized in Ref. 31, the components of the dielectric tensor depend on the frequency of inelastic collisions between ions and electrons and the dust particles. Although the collision frequency is momentum dependent, we have simplified the evaluation of momentum integrals which appear in the dispersion relation, by assuming as a first approach that the collision frequency could be replaced by an average over the electron or ion distribution function. The formulation has been applied to the particular case of low-frequency waves propagating along the ambient magnetic field, incorporating many details which have appeared in previous publications.[32,33]

The results which we have obtained show that, as in the case of dustless plasmas, the dispersion relation describes two different modes, identified for higher frequencies as the whistler waves and as the circularly polarized waves. In the absence of dust these two modes collapse together, for frequencies well below ion cyclotron frequency, forming the well-known branch of the Alfvén waves. In the presence of dust particles with variable charge, however, our results show that these two modes become separated. Another effect of the presence of the dust particles with variable charge is an additional damping of the Alfvén waves, which may completely override conventional Landau damping for large wavelengths.[31] Reference 31 also has shown the occurrence of mode coupling due to the presence of dust particles, between waves in the branch of circularly polarized waves, propagating in opposite directions.

More recently we have performed a parametric analysis of the dispersion relation of low-frequency waves propagating parallel to the ambient magnetic field, in a dusty plasma.[21] The analysis of Ref. 21 has shown the possibility of occurrence of coupling between waves in the whistler branch and waves in the branch of circularly polarized waves, due to the presence of the dust particles, a phenomenon not yet reported in the literature at the time of the publication of Ref. 21, to the best of our knowledge. The modifications in the dispersion relation of Alfvén waves reported in Ref. 21 appear when the collisional damping on the charged dust particles is taken into account in a kinetic theory, and therefore are not the same as those modifications reported in the literature in which the effect can be attributed to unequal electron and ion Hall currents and to the charge imbalance between electrons and ions which occur in the presence of dust (e.g., Refs. 18 and 23).

In the present paper we return to the subject, motivated by the fact that dust particles can be found in the solar wind and in stellar winds in general, and by the fact that anisotropic distributions of electrons and ions have been observed in the solar wind. As an approximation to the study of velocity distributions for electrons and ions which depart from the Maxwellian state and feature temperature anisotropy, bi-Maxwellian distributions are a logical and convenient choice. Therefore, proceeding with our studies on low-frequency waves in dusty plasmas, particularly with the study of the mode-coupling phenomena which has been shown to occur, we introduce in the present work the possibility of temperature anisotropy for electrons and ions, considering bi-Maxwellian distribution for these particles. For this analysis, we take into account in the numerical evaluation of the dispersion relation the momentum dependence of the frequency of inelastic collisions between electrons and ions and the dust particles, which has been neglected in previous analysis by the use of an average collision frequency. A comparison between the two approaches shows that the results obtained with the more exact evaluation are quite similar to the results obtained with the approximated evaluation, for the parameters considered.

The structure of the paper is the following: In Sec. II we briefly outline the model used to describe the dusty plasma. In Sec. III we present essential features of the dielectric tensor to be used in the discussion of wave propagation exactly parallel to the external magnetic field, derived assuming bi-Maxwellian distributions for the electrons and ions in the equilibrium, and the ensuing dispersion relation. In Sec. IV the numerical results obtained from the dispersion relation modified by the dust are presented and discussed. The conclusions are presented in Sec. V.

II. THE DUSTY PLASMA MODEL

We consider a plasma in a homogeneous external magnetic field $\mathbf{B}_0 = B_0\,\mathbf{e_z}$. In this magnetized plasma we take into account the presence of spherical dust grains with constant radius a and variable charge q_d; this charge originates from inelastic collisions between the dust particles and particles of species β (electrons and ions), with charge q_β and mass m_β. For simplicity, we will consider simply charged ions.

The charging model for the dust particles must in principle take into account the presence of an external magnetic field. This field must influence the characteristics of charging of the dust particles, because the path described by electrons and ions is modified; in this case we have cyclotron motion of electrons and ions around the magnetic field lines. However, it has been shown by Chang and Spariosu, through numerical calculation, that for $a \ll \rho_G$, where $\rho_G = (\pi/2)^{1/2} r_{Le}$ and r_{Le} is the electron Larmor radius, the effect of the magnetic field on the charging of the dust particles can be neglected.[34] For the values of parameters used in the present work the relation $a \ll \rho_G$ is always satisfied.

We will consider the dust grain charging process to occur by the capture of plasma electrons and ions during inelastic collisions between these particles and the dust particles. Since the electron thermal speed is much larger than the ion thermal speed, the dust charge will be preferentially negative. As a cross section for the charging process of the dust particles, we use expressions derived from the orbital motion limited (OML) theory.[35,36]

In the present work we focus our attention on low-frequency waves in a weakly coupled dusty magnetoplasma, where the electrostatic energy of the dust particles is much smaller than their kinetic energy. This condition allows for a wide variety of natural and laboratory plasmas, with the exception of the so-called colloidal plasmas.[37,38] Dust particles are assumed to be immobile, because of their mass which is much larger than the masses of ions and electrons, and consequently the validity of the proposed model will be restricted to waves with frequency much higher than the characteristic dust frequencies, excluding the modes that can arise from the dust dynamics. More particularly, we will consider the regime in which $|\Omega_d| \ll \omega_{pd} < \omega \leq \Omega_i \ll |\Omega_e|$, where Ω_d and Ω_β are the cyclotron frequencies of the dust particles and of electrons and ions, respectively, and ω_{pd} is the plasma frequency of the dust particles. The regime of frequencies $\omega \leq \Omega_i$ deserves special attention because it covers the range of the Alfvén waves, although nothing in the formalism prevents the analysis of waves with $\omega > \Omega_i$.

In this range of frequencies the presence of the dust particles modify the dispersion relation, through modifications of the quasineutrality condition and through effects due to dust charge fluctuation. It is known that in the case of Maxwellian distributions the dust charge fluctuations provide an additional damping mechanism for the Alfvén waves, beyond the well-known Landau damping mechanism.[31] In the present paper we investigate the influence of this mechanism in the case of distributions with anisotropy of temperature.

The dielectric tensor for a magnetized dusty plasma, homogeneous, fully ionized, with identical immobile dust particles and charge variable in time, can be written in the following way:

$$\epsilon_{ij} = \epsilon_{ij}^C + \epsilon_{ij}^N, \tag{1}$$

where the explicit expressions for ϵ_{ij}^C and ϵ_{ij}^N are given in Refs. 32 and 33 and also in the Appendix of Ref. 31.

The term ϵ_{ij}^C is formally identical, except for the $i3$ components, to the dielectric tensor of a magnetized homogeneous conventional plasma of electrons and ions, with the resonant denominator modified by the addition of a purely imaginary term which contains the collision frequency of electrons and ions with the dust particles. For the $i3$ components of the dielectric tensor, in addition to the term obtained with the prescription above, there is a term which is proportional to the inelastic collision frequency of electrons and ions with the dust particles. This additional term vanishes for propagation parallel to the magnetic field.

The term ϵ_{ij}^N is entirely new and only exists in the presence of dust particles with variable charge. Its form is strongly dependent on the model used to describe the charging process of the dust particles. However, a general feature can be obtained for the case of propagation exactly parallel to the external magnetic field. The term ϵ_{ij}^N appearing in Eq. (1), only occurs for $i=j=3$, regardless of the detailed form of the distribution function $f_{\beta 0}$.

III. PROPAGATION PARALLEL TO B_0 AND BI-MAXWELLIAN DISTRIBUTION FUNCTION

In the case of propagation parallel to the external magnetic field, the dielectric tensor assumes the form

$$\overleftrightarrow{\epsilon} = \begin{pmatrix} \epsilon_{11}^C & \epsilon_{12}^C & 0 \\ -\epsilon_{12}^C & \epsilon_{11}^C & 0 \\ 0 & 0 & \epsilon_{33}^C + \epsilon_{33}^N \end{pmatrix}, \tag{2}$$

where

$$\epsilon_{11}^C = 1 + \frac{1}{4}\sum_\beta X_\beta [I_\beta^+ + I_\beta^-],$$

$$\epsilon_{12}^C = -\frac{i}{4}\sum_\beta X_\beta [I_\beta^+ - I_\beta^-],$$

$$\epsilon_{33}^C = 1 + \sum_\beta X_\beta I_\beta^0,$$

and where

$$I_\beta^s \equiv \frac{1}{n_{\beta 0}} \int d^3p \frac{p_\perp \mathcal{L}(f_{\beta 0})}{1 - \frac{k_\parallel p_\parallel}{m_\beta \omega} + s\frac{\Omega_\beta}{\omega} + i\frac{\nu_{\beta d}^0(p)}{\omega}},$$

$$I_\beta^0 \equiv \frac{1}{n_{\beta 0}} \int d^3p \frac{p_\parallel (\partial f_{\beta 0}/\partial p_\parallel)}{1 - \frac{k_\parallel p_\parallel}{m_\beta \omega} + i\frac{\nu_{\beta d}^0(p)}{\omega}},$$

with

$$\mathcal{L}(f_{\beta 0}) = \left(1 - \frac{k_\parallel p_\parallel}{m_\beta \omega}\right)\frac{\partial f_{\beta 0}}{\partial p_\perp} + \frac{k_\parallel p_\perp}{m_\beta \omega}\frac{\partial f_{\beta 0}}{\partial p_\parallel},$$

$$\nu_{\beta d}^0(p) = \frac{\pi a^2 n_{d0}}{m_\beta}\frac{(p^2 + C_\beta)}{p} H(p^2 + C_\beta),$$

$$X_\beta = \frac{\omega_{p\beta}^2}{\omega^2}, \quad \omega_{p\beta}^2 = \frac{4\pi n_{\beta 0} q_\beta^2}{m_\beta}, \quad \Omega_\beta = \frac{q_\beta B_0}{m_\beta c},$$

$$C_\beta = -\frac{2 q_\beta m_\beta q_{d0}}{a},$$

and $s=\pm 1$. The subscript $\beta = e, i$ identifies electrons and ions, respectively, $q_{d0} = \varepsilon_d e Z_d$ is the equilibrium charge of the dust particle (positive, $\varepsilon_d = +1$, or negative, $\varepsilon_d = -1$) and H denotes the Heaviside function, or *step function*. The quantity Z_d is the number of charges in each dust particle, and n_{e0}, n_{i0}, and n_{d0} are the equilibrium electron, ion, and dust densities, respectively. The explicit form of the quantity ϵ_{33}^N is not necessary for the present investigation and therefore will not be reproduced here.[31–33]

The general dispersion relation for $\mathbf{k} = k_\parallel \mathbf{e_z}$ therefore follows from the determinant

$$\det \begin{pmatrix} \epsilon_{11}^C - N_\parallel^2 & \epsilon_{12}^C & 0 \\ -\epsilon_{12}^C & \epsilon_{11}^C - N_\parallel^2 & 0 \\ 0 & 0 & \epsilon_{33}^C + \epsilon_{33}^N \end{pmatrix} = 0. \tag{3}$$

In this expression, $N_\parallel = k_\parallel c/\omega$ is the refractive index in the direction parallel to the external magnetic field. The dispersion relation for Alfvén waves is obtained retaining only the components in the upper left 2×2 determinant in Eq. (3), that is, by imposing $E_z = 0$. As a result, we obtain

$$[N_\parallel^2]_s = 1 + \frac{1}{2}\sum_\beta X_\beta I_\beta^s, \tag{4}$$

where we have used the expressions for ϵ_{11}^C and ϵ_{12}^C, from Eq. (2).

The effect of the dust particles on the dispersion relation given by Eq. (4) occurs via the quasineutrality condition ($n_{i0} \neq n_{e0}$), and also via the mechanism of collisional charging of dust particles due to the incorporation of electrons and ions, described by the terms which contain the inelastic collision frequency $\nu_{\beta d}^0(p)$.

Let us assume that electrons and ions are described by bi-Maxwellian distributions, with anisotropy of the temperature given by $\Delta_\beta = T_\perp^\beta / T_\parallel^\beta$,

$$f_{\beta 0} = A_\beta e^{-p_\perp^2/2m_\beta T_\perp^\beta} e^{-p_\parallel^2/2m_\beta T_\parallel^\beta}, \tag{5}$$

where A_β is the normalization constant,

$$A_\beta = \frac{n_{\beta 0}}{(2\pi m_\beta)^{3/2} T_\perp^\beta (T_\parallel^\beta)^{1/2}}.$$

It is therefore easy to obtain

$$\mathcal{L}(f_{\beta 0}) = -\left(1 - \frac{k_\parallel p_\parallel}{m_\beta \omega}\right)\frac{p_\perp}{m_\beta T_\perp^\beta} f_{\beta 0} - \frac{k_\parallel p_\perp}{m_\beta \omega}\frac{p_\parallel}{m_\beta T_\parallel^\beta} f_{\beta 0},$$

and write the I_β^s integral as follows:

$$I_\beta^s = -\frac{4}{(\pi)^{1/2}\Delta_\beta^2}\int_0^\infty dt t^4 e^{-t^2/\Delta_\beta}\int_{-1}^1 du e^{-t^2 u^2(\Delta_\beta - 1)/\Delta_\beta} \times [\zeta_\beta^0 - tu(1-\Delta_\beta)]\frac{(1-u^2)[(\hat{\zeta}_\beta^s)^* - tu]}{[(\hat{\zeta}_{\beta r}^s) - tu]^2 + (\hat{\zeta}_{\beta i}^s)^2} + i(2\pi)\frac{2}{\sqrt{\pi}}[\hat{\zeta}_\beta^0 - \hat{\zeta}_\beta^s(1-\Delta_\beta)]e^{-(\hat{\zeta}_\beta^s)^2} H[-\hat{\zeta}_{\beta i}^s], \tag{6}$$

where

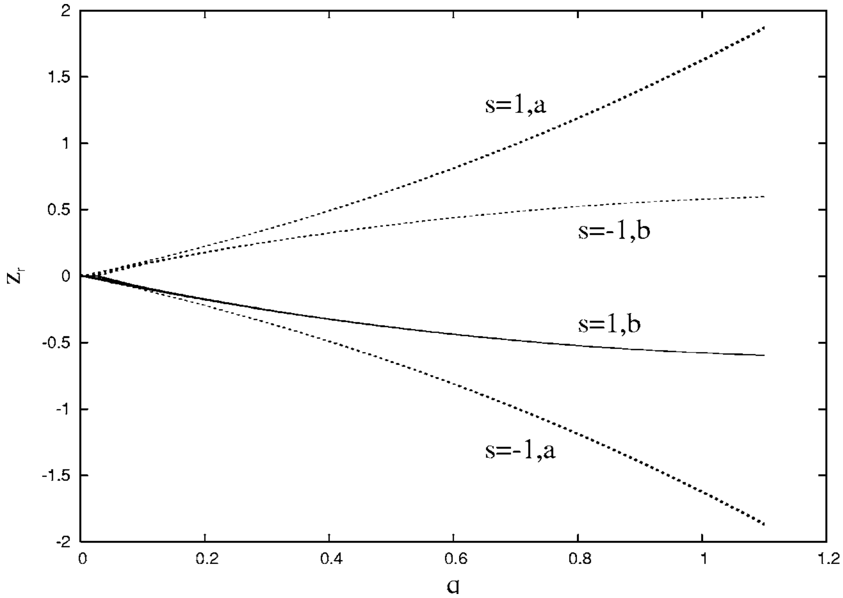

FIG. 1. Real part of the normalized frequency (z_r) for the two roots obtained using $s=1$ and for the two roots obtained using $s=-1$, as a function of q, for five values of ϵ. $B_0=1.0\times10^{-4}$ T, $T_i=1.0\times10^4$ K, $n_{i0}=1.0\times10^9$ cm^{-3}, $Z_i=1.0$, $m_i=m_p$, and $T_e=T_i$. Radius of dust particles, $a=1.0\times10^{-4}$ cm. The dust density n_{d0} is obtained from $\epsilon=n_{d0}/n_{i0}$, and the values of ϵ utilized are $\epsilon=0.0$, 1.25×10^{-6}, 2.50×10^{-6}, 3.75×10^{-6}, and 5.0×10^{-6}. For this range of variation ϵ, the quantity z_r is almost independent of the dust density, so that the five curves appear superposed (Refs. 21 and 31).

$$\hat{\zeta}^0_\beta=\frac{z}{\sqrt{2}qu_{\beta\parallel}},\quad \hat{\zeta}^s_\beta=\frac{z+sr_\beta+i\tilde{\nu}_\beta}{\sqrt{2}qu_{\beta\parallel}},$$

$$z=\frac{\omega}{\Omega_i},\quad q=\frac{k_\parallel v_A}{\Omega_i},\quad u_{\beta\parallel}=\frac{v^\beta_{T_\parallel}}{v_A},$$

$$r_\beta=\frac{\Omega_\beta}{\Omega_i},\quad \tilde{\nu}_\beta=\frac{\nu^0_{\beta d}(p)}{\Omega_i},$$

and where $v^\beta_{T_\parallel}$ and v_A are, respectively, the parallel thermal velocity of particles of species β and the Alfvén velocity,

$$v^\beta_{T_\parallel}=\sqrt{\frac{T^\beta_\parallel}{m_\beta}},\quad v_A^2=\frac{B_0^2}{4\pi n_{i0}m_i}.$$

We have introduced in these expressions the variables $t=p/\sqrt{2m_\beta T^\beta_\parallel}$ and $u=\cos\theta$. The term in the last line of Eq. (6) represents an approximated evaluation of the contribution of the pole to the analytical continuation of the integral. Moreover, $\hat{\zeta}^s_{\beta r}$ and $\hat{\zeta}^s_{\beta i}$ denote, respectively, the real and imaginary parts of $\hat{\zeta}^s_\beta$.

On the other hand, it is possible to follow the same procedure used in Refs. 21 and 31, for evaluation of the integrals I^s_β, replacing the functions $\nu^0_{\beta d}(p)$ by their average values in momentum space,

$$\nu_\beta\equiv\frac{1}{n_{\beta0}}\int d^3p\,\nu^0_{\beta d}(p)f_{\beta0}. \tag{7}$$

In the bi-Maxwellian case the average collision frequency between charged particles and dust particles can be written as follows, in dimensionless form:

$$\tilde{\nu}_\beta=\nu_\beta/\Omega_i=\sqrt{2\pi}\epsilon\gamma\tilde{a}^2\Delta_\beta^{1/2}u_{\beta\perp}I^\beta_\nu, \tag{8}$$

where

$$\epsilon=\frac{n_{d0}}{n_{i0}},\quad u_{\beta\perp}=\frac{v^\beta_{T_\perp}}{v_A},\quad v^\beta_{T_\perp}=\sqrt{\frac{T^\beta_\perp}{m_\beta}},$$

$$\gamma=\frac{\lambda^2 n_{i0}v_A}{\Omega_i},\quad \tilde{a}=\frac{a}{\lambda},\quad \lambda=\frac{e^2}{T^i_\parallel},$$

where $v^\beta_{T_\perp}$ is the perpendicular thermal velocity of particles of species β. Moreover,

$$I^i_\nu=\left[\frac{1}{\Delta_i}+\frac{1+2\chi_{\perp i}}{\sqrt{\Delta_i-1}}\tan^{-1}\sqrt{\Delta_i-1}\right],\quad \Delta_i>1,$$

$$I^i_\nu=\left[\frac{1}{\Delta_i}+\frac{1+2\chi_{\perp i}}{2\sqrt{1-\Delta_i}}\ln\frac{|1+\sqrt{1-\Delta_i}|}{|1-\sqrt{1-\Delta_i}|}\right],\quad \Delta_i<1,$$

$$I^i_\nu=2(1+\chi_i),\quad \Delta_i=1,$$

$$I^e_\nu=\left[\frac{2}{\sqrt{\Delta_e-1}}\int_0^{\sqrt{\Delta_e-1}}dx\frac{e^{-|\chi_{\perp e}|(1+x^2)}}{(1+x^2)^2}\right],\quad \Delta_e>1,$$

$$I^e_\nu=\left[\frac{2}{\sqrt{1-\Delta_e}}\int_0^{\sqrt{1-\Delta_e}}dx\frac{e^{-|\chi_{\perp e}|(1-x^2)}}{(1-x^2)^2}\right],\quad \Delta_e<1,$$

$$I^e_\nu=2e^{-|\chi_e|},\quad \Delta_e=1,$$

with $\chi_{\perp i}=Z_dZ_ie^2/(aT^i_\perp)$, where Z_i is the ion charge number, and $\chi_{\perp e}=-Z_de^2/(aT^e_\perp)$. Using these average collision frequencies, and performing the calculation of the I^s_β integrals, the dispersion relation given by Eq. (4) assumes the form

$$\frac{q^2c^2}{v_A^2}=z^2+\sum_\beta\eta^2_\beta\{(\Delta_\beta-1)+[\zeta^0_\beta+(\Delta_\beta-1)\hat{\zeta}^s_\beta]Z(\hat{\zeta}^s_\beta)\}, \tag{9}$$

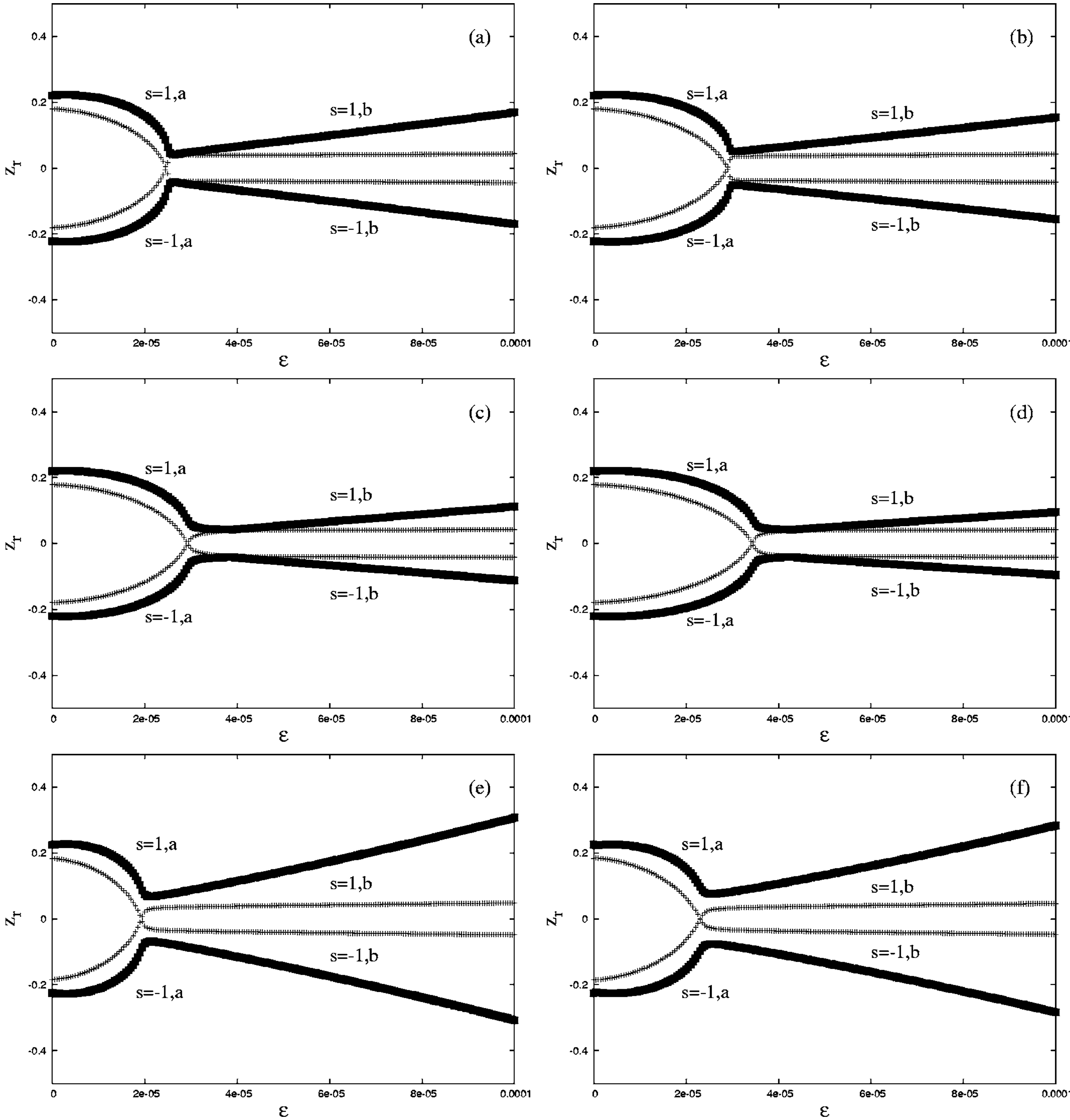

FIG. 2. z_r as a function of ϵ, for $q=0.2$. (a) $\Delta_i=\Delta_e=1.00$, evaluation with the Z function; (b) $\Delta_i=\Delta_e=1.00$, evaluation with the I_β^s function; (c) $\Delta_i=\Delta_e=0.50$, evaluation with the Z function; (d) $\Delta_i=\Delta_e=0.50$, evaluation with the I_β^s function; (e) $\Delta_i=\Delta_e=2.00$, evaluation with the Z function; (f) $\Delta_i=\Delta_e=2.00$, evaluation with the I_β^s function.

where $\eta_\beta=\omega_{p\beta}/\Omega_i$ and where Z is the plasma dispersion function,[39]

$$Z(\zeta)=\frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty}dt\frac{e^{-t^2}}{t-\zeta}.$$

It is interesting to examine Eq. (9) considering some limit situations. For instance, in the isotropic case, $\Delta_\beta\to 1$, Eq. (9) recovers Eq. (19) of Ref. 31. Additionally, if the expansion for large argument of the Z function is utilized, it is easy to obtain the dispersion relation in the following expanded form:

$$\frac{q^2c^2}{v_A^2z^2}=1+\sum_\beta\frac{\eta_\beta^2}{\sqrt{2}qu_\beta z}\left[-\frac{\sqrt{2}qu_\beta}{z+sr_\beta+i\tilde{\nu}_\beta}+i\sqrt{\pi}\exp\left(-\frac{(z+sr_\beta+i\tilde{\nu}_\beta)^2}{2q^2u_\beta^2}\right)\right]. \quad (10)$$

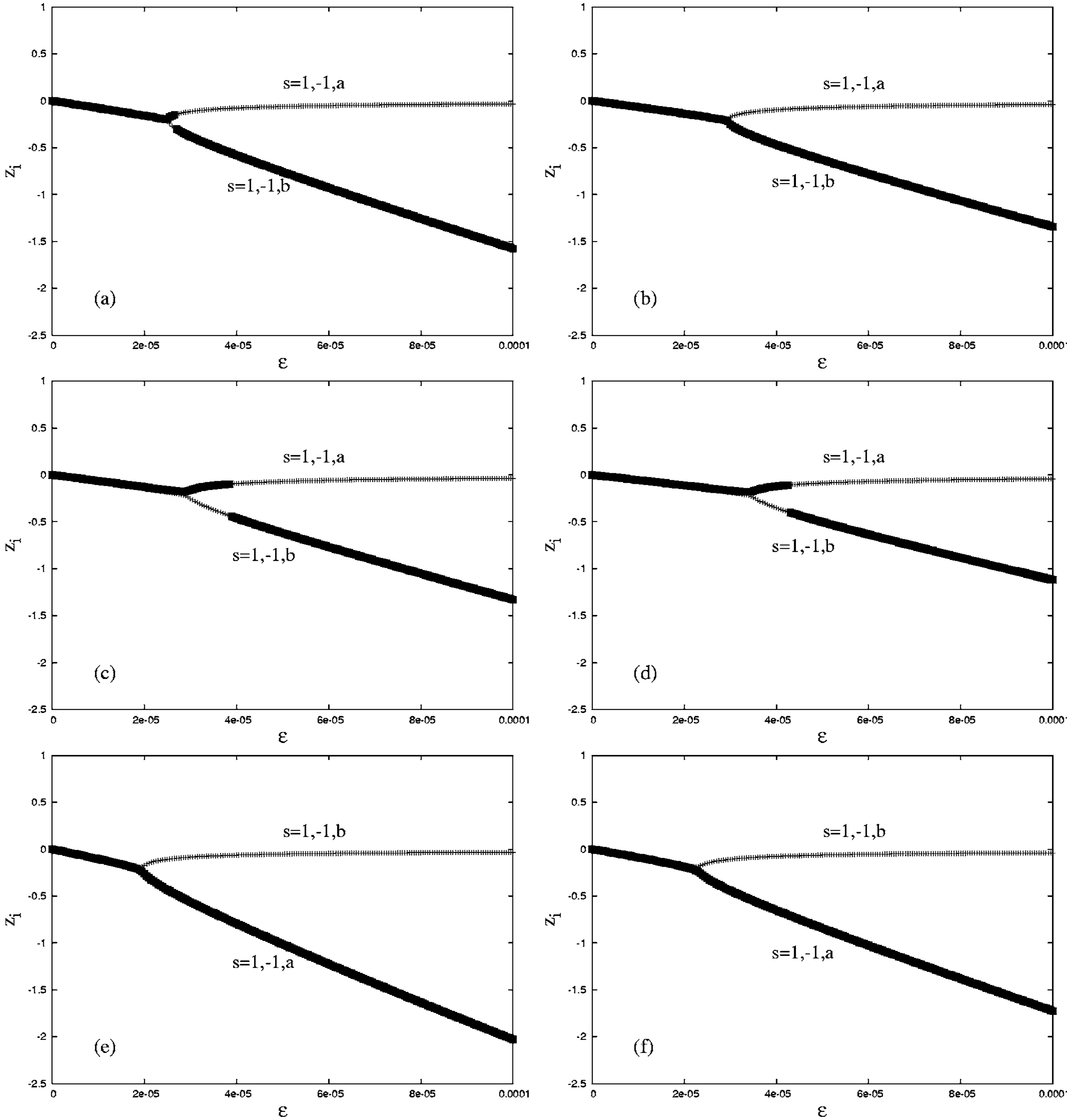

FIG. 3. z_i as a function of ϵ, for $q=0.2$. (a) $\Delta_i=\Delta_e=1.00$, evaluation with the Z function; (b) $\Delta_i=\Delta_e=1.00$, evaluation with the I_β^s function; (c) $\Delta_i=\Delta_e=0.50$, evaluation with the Z function; (d) $\Delta_i=\Delta_e=0.50$, evaluation with the I_β^s function; (e) $\Delta_i=\Delta_e=2.00$, evaluation with the Z function; (f) $\Delta_i=\Delta_e=2.00$, evaluation with the I_β^s function.

Moreover, it is seen that, if the effect of the charge fluctuation is neglected, the quantities $\tilde{\nu}_\beta$ will vanish ($\tilde{\nu}_\beta=0$, for $\beta=i,e$). In that case, the real part of Eq. (10) becomes the same as Eq. (8) of Ref. 15, in the range $\omega_{pd}\ll\omega\ll\Omega_i$, limitation which is necessary because the dynamics of the dust particles has been neglected in the derivation of Eqs. (9) and (10). Equation (10) also reproduces dispersion relations appearing in other well-known works on low-frequency waves, for instance in Refs. 13 and 16.

In the limit $|z_i|\ll|z_r|$, and dustless plasma, we obtain from Eq. (9) the approximated solution for anisotropic plasmas[40]

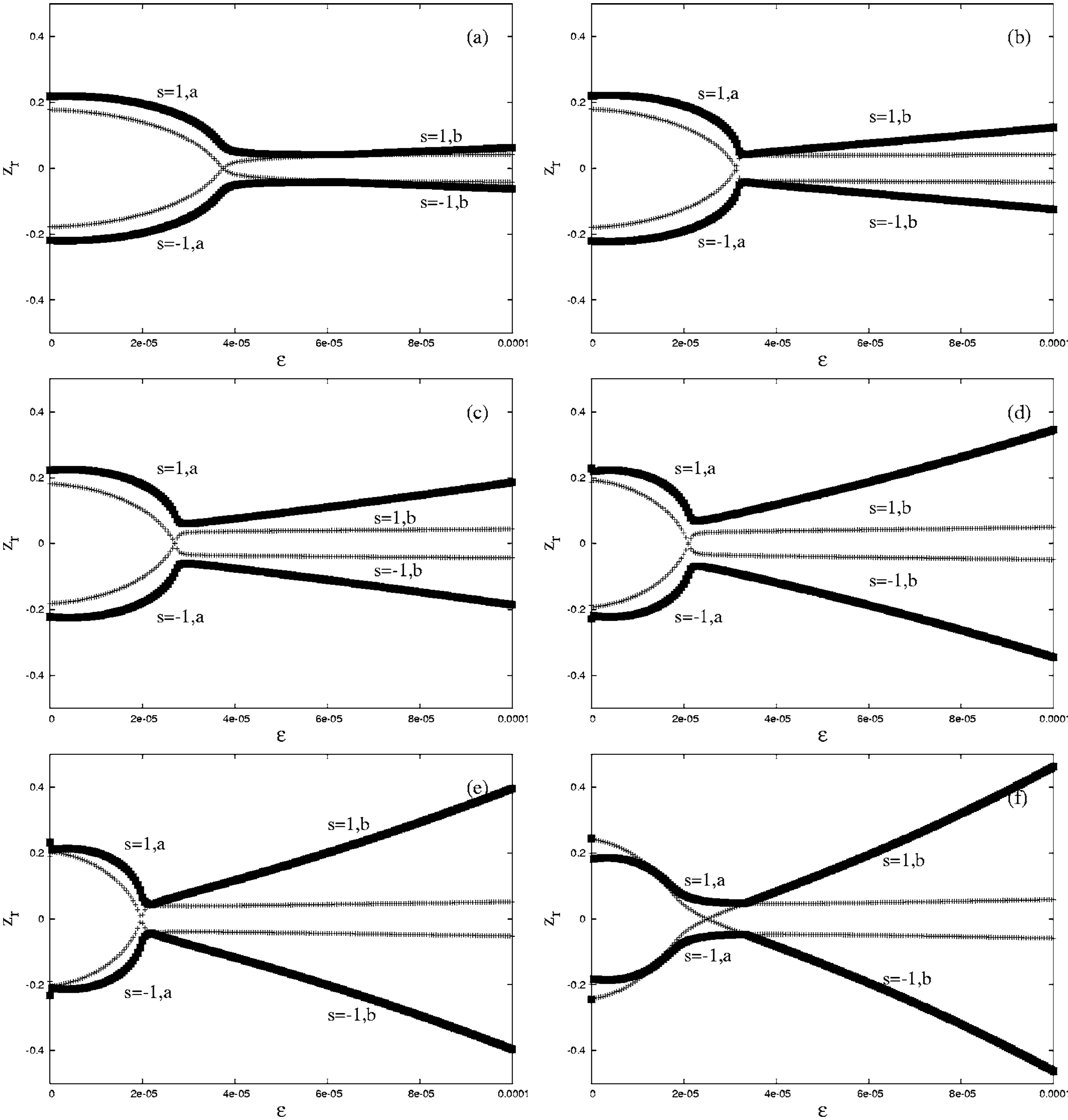

FIG. 4. z_r as a function of ϵ, for $q=0.2$, and eight values of $\Delta_i=\Delta_e$. (a) 0.25; (b) 0.75; (c) 1.25; (d) 2.50; (e) 3.00; (f) 4.00.

$$z_r = \frac{q}{\sqrt{1+\eta_i^2}}\left\{\frac{c^2}{v_A^2} + \eta_i^2 u_{i\parallel}^2[(\Delta_i - 1) + \tau_e(\Delta_e - 1)]\right\}^{1/2},$$

$$z_i = -\sqrt{\frac{\pi}{8}}\frac{1}{z_r q(1+\eta_i^2)}\sum_\beta \frac{\eta_\beta^2}{u_{\beta\parallel}}[\Delta_\beta z_r - s r_\beta(1-\Delta_\beta)] \times \exp[-(z_r + s r_\beta)^2/(2q^2 u_{\beta\parallel}^2)], \tag{11}$$

where $\tau_e = T_\parallel^e / T_\parallel^i$.

In the absence of dust and for isotropic temperatures, the approximated dispersion relation and the damping rate in the range of small q and for $\omega \ll |\Omega_\beta|$ are given analytically as follows, corresponding to the well-known textbook expressions for Alfvén waves:

$$z_r = \frac{c}{v_A}\frac{q}{\sqrt{1+\eta_i^2}},$$

$$z_i = -\sqrt{\frac{\pi}{8}}\frac{\eta_i^2}{q(1+\eta_i^2)u_i}\exp\left(-\frac{1}{2q^2 u_i^2}\right). \tag{12}$$

For the parameters which we utilize in the present paper

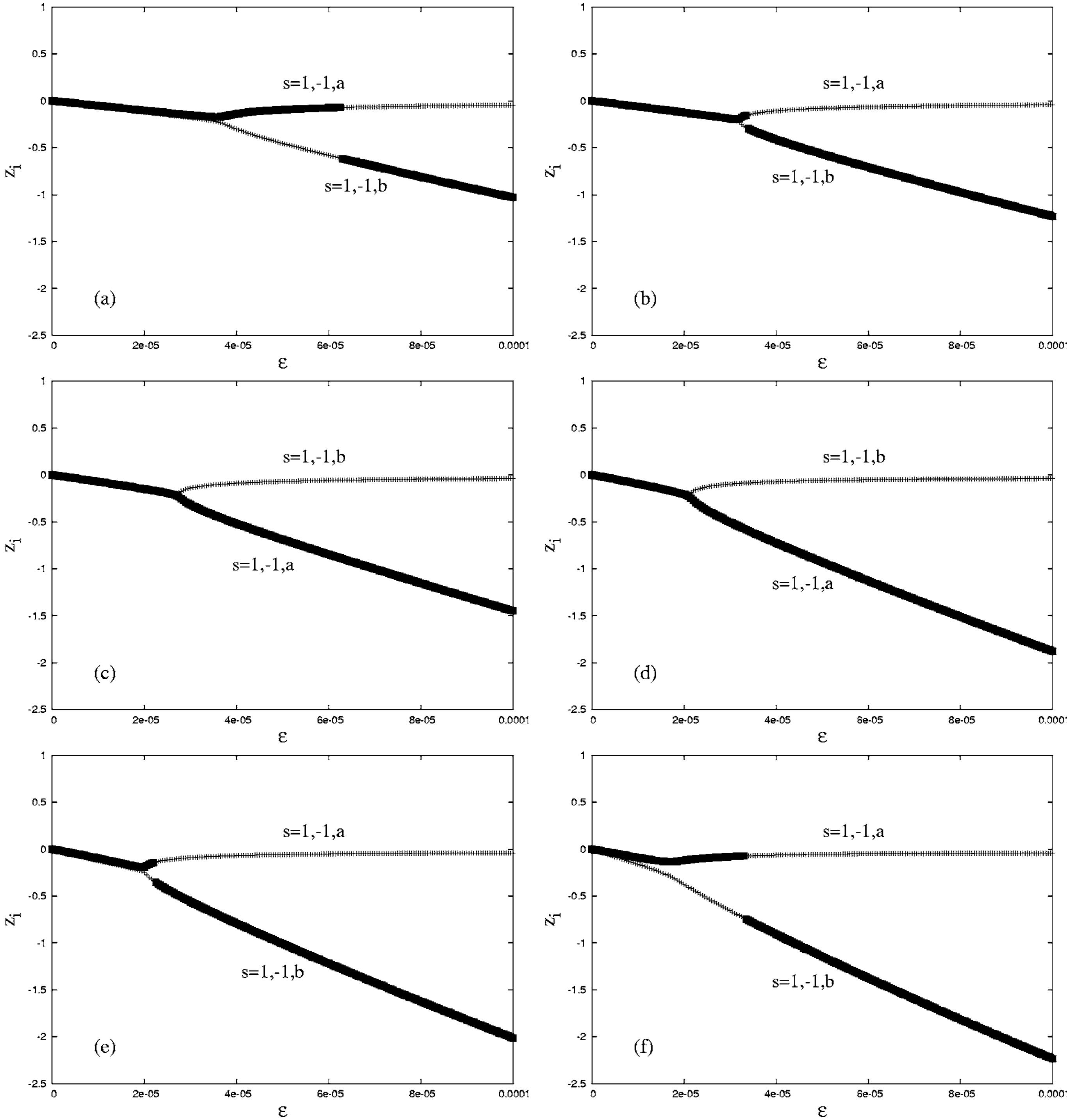

FIG. 5. z_i as a function of ϵ, for $q=0.2$, and eight values of $\Delta_i=\Delta_e$. (a) 0.25; (b) 0.75; (c) 1.25; (d) 2.50; (e) 3.00; (f) 4.00.

(see next section), Landau damping of Alfvén waves in the isotropic case is in practice completely negligible, and the cases with temperature anisotropy do not satisfy the condition for occurrence of instability, which would follow from Eq. (11) when $z_i>0$.

IV. NUMERICAL ANALYSIS

In the present section we present a parametric study of the solutions of the dispersion relation, given either by Eqs. (4) and (6), or by Eq. (9). Along the study we consider as basic parameters the ion charge number $Z_i=1.0$ and the ion mass $m_i=m_p$, where m_p is the proton mass. For the classical distance of minimum approach, measured in cm, we use the value $\lambda=1.44\times10^{-7}/T_\parallel^i(\mathrm{eV})$, where $T_\parallel^i(\mathrm{eV})$ means the parallel ion temperature expressed in unit of eV. We choose the ambient magnetic field $B_0=1.0\times10^{-4}$ T, ion density $n_{i0}=1.0\times10^{9}\ \mathrm{cm}^{-3}$, parallel ion temperature $T_\parallel^i=1.0\times10^{4}$ K, and parallel electron temperature $T_\parallel^e=T_\parallel^i$. For the radius of the dust particles, we assume $a=1.0\times10^{-4}$ cm. The electron density and the dust charge number Z_d are obtained from the quasineutrality condition,

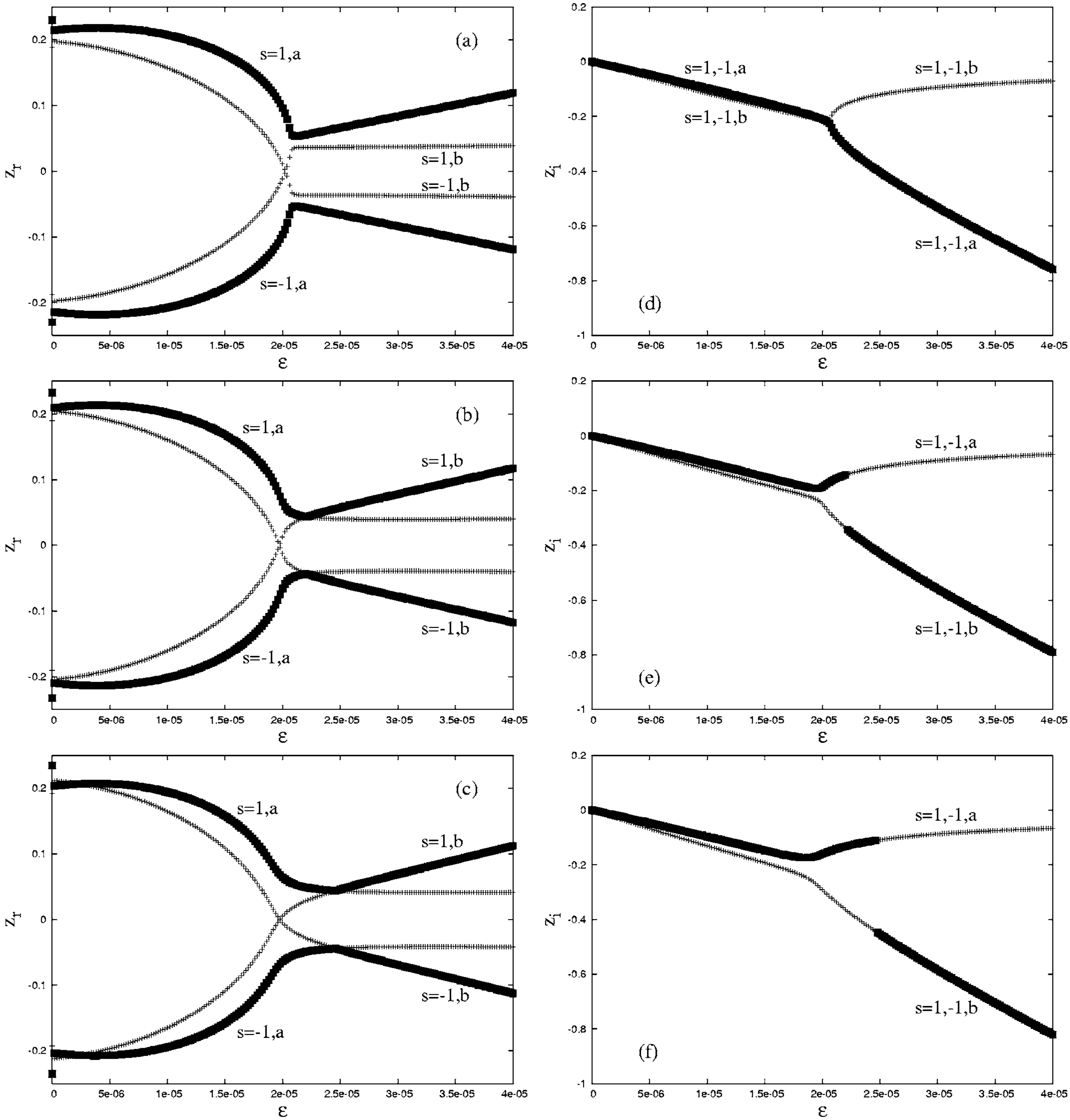

FIG. 6. z_r and z_i as a function of ϵ, for $q=0.2$, and three values of $\Delta_i=\Delta_e$. (a) z_r, 2.75; (b) z_r, 3.00; (c) z_r, 3.25; (d) z_i, 2.75; (e) z_i, 3.00; (f) z_i, 3.25.

$$n_{e0}=n_{i0}(Z_i-\epsilon Z_d), \tag{13}$$

and from the condition for equilibrium of the charging current, which for bi-Maxwellian plasmas becomes the following:

$$Z_i\Delta_i^{1/2}u_{\perp i}I_\nu^i-(Z_i-\epsilon Z_d)\Delta_e^{1/2}u_{\perp e}I_\nu^e=0. \tag{14}$$

The perpendicular temperatures and the dust density enter the analysis via the parameters Δ_β and ϵ, which will be varied for the numerical analysis. The parameters chosen may be representative of plasmas in stellar winds. Except for the possibility of temperature anisotropy, they are the same utilized in Refs. 21 and 31, since we are interested in the investigation of the effect of temperature anisotropy on the mode-coupling phenomena which has been previously demonstrated in Ref. 21 to occur due to the presence of the dust particles.

We start by investigating Eq. (9) in the range of small values of dust density, considering ϵ between 0 and 5.0 $\times 10^{-6}$. As it is known, in the limit $\epsilon=0$, ion and electron densities are equal and $\tilde{\nu}_e=\tilde{\nu}_i=0$, and therefore the dispersion

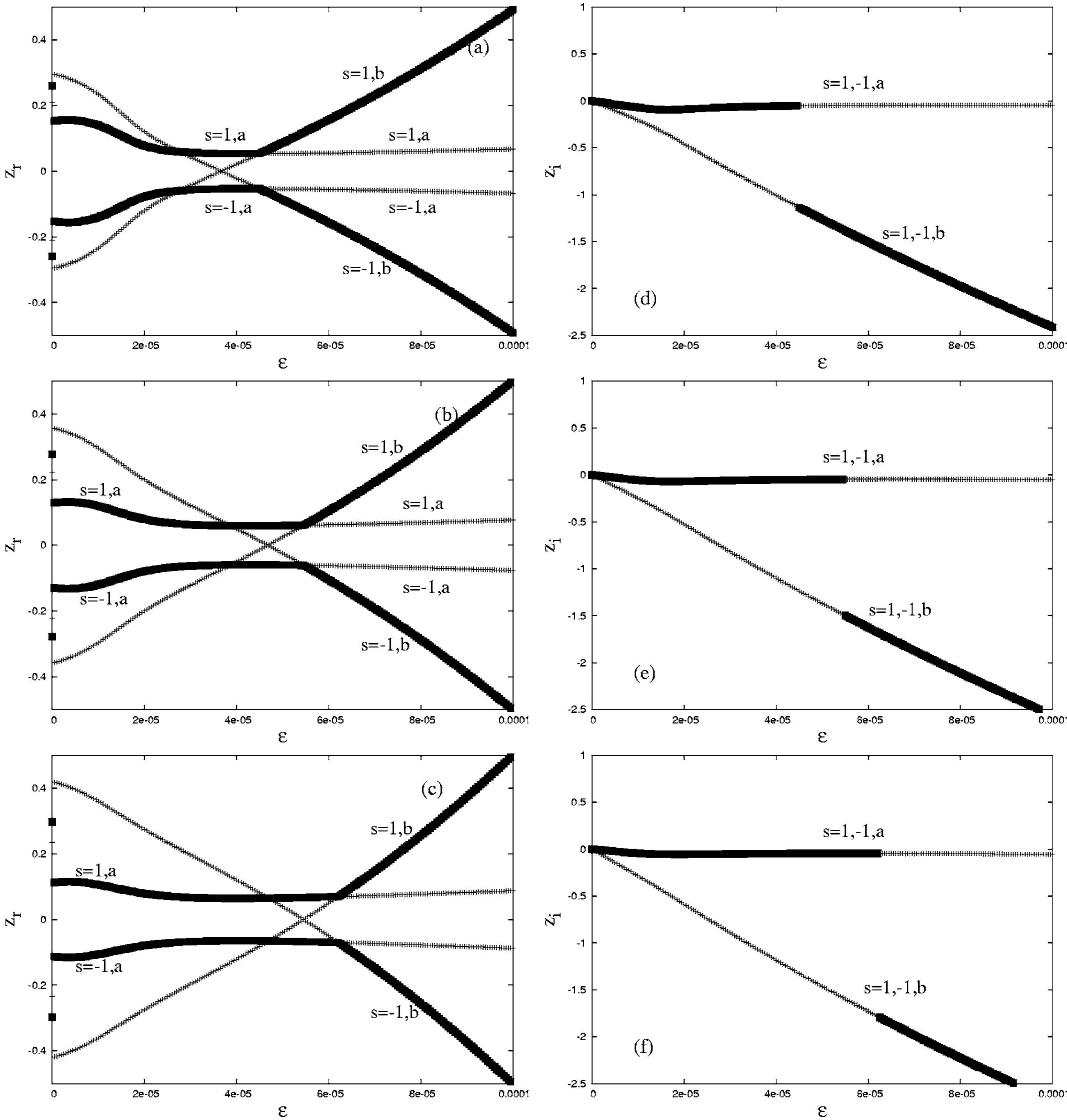

FIG. 7. z_r and z_i as a function of ϵ, for $q=0.2$, and three values of $\Delta_i=\Delta_e$. (a) z_r, 5.00; (b) z_r, 6.00; (c) z_r, 7.00; (d) z_i, 5.00; (e) z_i, 6.00; (f) z_i, 7.00.

relation in the isotropic limit ($\Delta_\beta \rightarrow 1$) becomes the usual dispersion relation for Alfvén waves in the absence of dust.

We start the analysis with the display of the real part of the normalized frequency, z_r, as a function of q, for several values of ϵ, in the range of small values of dust density, for the isotropic case, $T_e=T_i$, as shown in Fig. 1. This figure has already appeared in Refs. 21 and 31, and it is similar to figures appearing in well-known textbooks.[41] It is repeated here because it is useful for the identification of the modes predicted by the dispersion relation. In Fig. 1 we observe the real part of the two roots obtained from Eq. (9) for each of the signs $s=1$ and $s=-1$. There are two curves with positive values of z_r, describing waves propagating in the positive direction. The uppermost curve is obtained with $s=1$, and corresponds to the so-called *whistler branch*, while the lower curve in the positive side is obtained with $s=-1$ and corresponds to the branch identified with circularly polarized waves propagating along the ambient magnetic field. For negative values of z_r we have perfectly symmetrical solutions propagating in the negative direction, obtained, respectively, with $s=-1$ (the lower curve) and $s=1$ (the upper curve, closer to the axis). For small values of q, $q \leq 0.2$, the

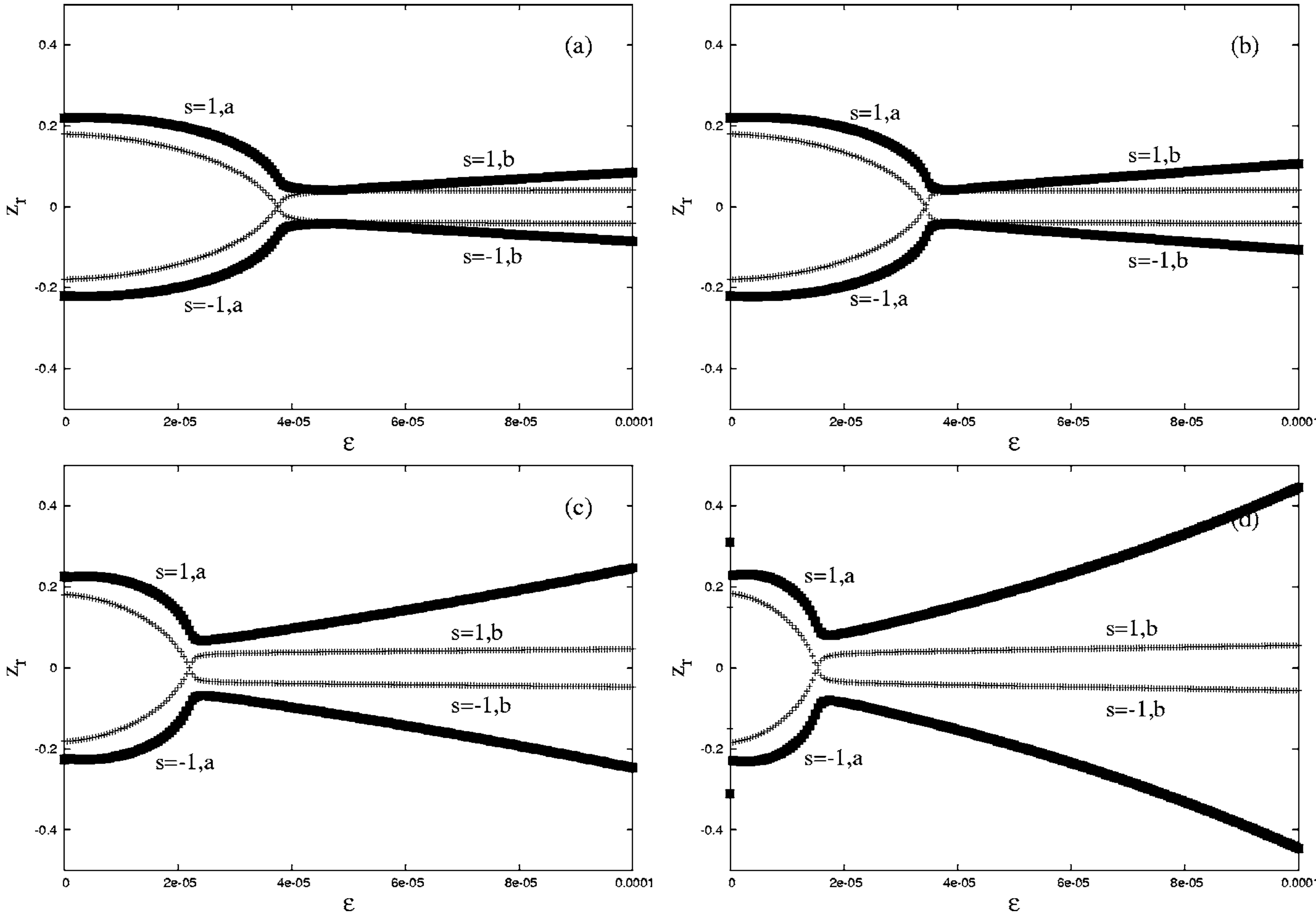

FIG. 8. z_r as a function of ϵ, for $q=0.2$, $\Delta_i=1.00$, and several values of anisotropy in the electron temperature: (a) $\Delta_e=0.25$; (b) $\Delta_e=0.50$; (c) $\Delta_e=2.00$; (d) $\Delta_e=4.00$.

two branches of waves propagating in a given direction collapse together in a single branch known as the branch of the Alfvén waves. Each of the curves appearing in Fig. 1 in fact corresponds to the superposition of five curves, obtained with $\epsilon=0.0$, 1.25×10^{-6}, 2.50×10^{-6}, 3.75×10^{-6}, and 5.0×10^{-6}. These results show that the presence of a small density of dust particles has a negligible effect on the real part of the roots obtained from the dispersion relation of low-frequency waves propagating along the magnetic field, in plasmas with isotropy of temperature.

In Fig. 2 we compare the real part of the roots of the dispersion relation, evaluated using Eqs. (4) and (6), using the momentum dependent collision frequency $\nu^0_{\beta d}(p)$, with the real parts of the roots obtained using Eq. (9), which utilizes the average collision frequency, for $q=0.2$ and three different values of the anisotropy ratios, $\Delta_i=\Delta_e=1.00$, $\Delta_i=\Delta_e=0.50$, and $\Delta_i=\Delta_e=2.00$. The left-hand panels, Figs. 2(a), 2(c), and 2(e), show the values of z_r obtained using Eq. (9), while the right-hand panels, Figs. 2(b), 2(d), and 2(f), show the values of z_r obtained using Eqs. (4) and (6). It is seen that the calculation using the average collision frequency, Eq. (9), provides very close approximation to the results obtained from the more complete dispersion relation featuring the momentum dependent collision frequency between plasma particles and dust particles, at least for the parameters considered in the analysis. The most noticeable effect in all the three cases depicted is a slight displacement of the points of mode coupling toward higher values of dust density ϵ, in the results obtained with the more complete dispersion relation, as compared with the results obtained with the dispersion relation using the average collision frequency. This small displacement appears both for $\Delta_\beta>1$and for $\Delta_\beta<1$, and therefore appears to be independent of the temperature anisotropy. Figure 3 displays the corresponding values of the imaginary part z_i. The same comments made about Fig. 2 can be applied to Fig. 3. It can be noticed that the damping rate of waves in the whistler branch, predicted by the approximated dispersion relation, is larger than the damping rate predicted by the less approximated dispersion relation, for a given value of dust density.

In order to investigate the effect of temperature anisotropy in a dusty plasma, we consider the case of $q=0.2$ and show in Fig. 4 the values of z_r as a function of ϵ, for several values of the ratios $T^\beta_\perp/T^\beta_\parallel$. The corresponding values of the imaginary parts z_i appear in Fig. 5. These graphs have been obtained using Eqs. (4) and (6). In Figs. 4 and 5 we consider the case of electrons and ions with the same anisotropy, with $\Delta_i=\Delta_e=0.25$, 0.75, 1.25, 2.50, 3.00, and 4.00. For $q=0.2$ we had obtained for the same parameters the occurrence of mode coupling between waves in the whistler branch and

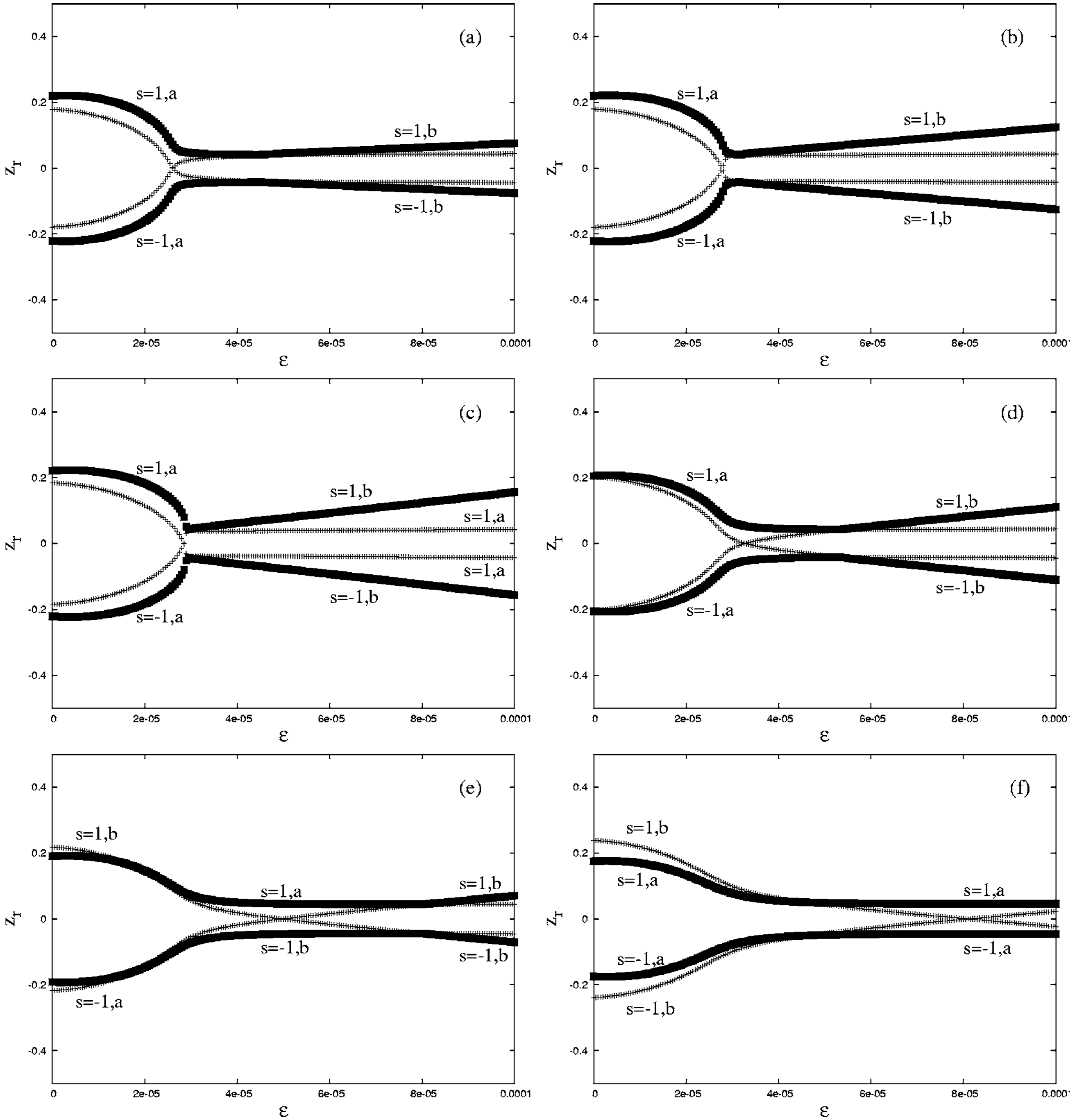

FIG. 9. z_r as a function of ϵ, for $q=0.2$, $\Delta_e=1.00$, and several values of anisotropy in the ion temperature: (a) $\Delta_i=0.25$; (b) $\Delta_i=0.50$; (c) $\Delta_i=2.00$; (d) $\Delta_i=3.00$; (e) $\Delta_i=3.50$; (f) $\Delta_i=4.00$.

waves in the branch of circularly polarized waves, occurring at $\epsilon \simeq 2.7\times10^{-5}$, in the case of isotropic distributions, as seen in Fig. 3(b) of Ref. 21. Starting from the case shown in Fig. 4(a), in which the perpendicular temperature is only one fourth of the parallel temperature, it is seen that the point of coupling between the whistler branch and the branch of circularly polarized waves gradually moves toward smaller dust density, as the temperature anisotropy is decreased. The case of $\Delta_i=\Delta_e=0.75$ shown in Fig. 4(b) is already very similar to the result obtained in the isotropic case, shown in Ref. 21. Figures 4(c) and 4(d) show that for perpendicular temperatures slightly larger than parallel the coupling between these modes do not occur, for the parameters considered. However, the mode coupling reappears for sufficiently large temperature anisotropy, as shown by Fig. 4(e). Moreover, a novel feature emerges. Figures 4(e) and 4(f) show the occurrence of another point of mode coupling, for small values of dust density. For instance, for $\Delta_i=\Delta_e=4.00$, Fig. 4(f) shows the occurrence of coupling between the whistler branch and the branch of circularly polarized waves at $\epsilon \simeq 1.5\times10^{-5}$ and at

$\epsilon \simeq 3.5 \times 10^{-5}$. Panels (a)–(f) of Fig. 4 also show that the coupling between forward and backward propagating circularly polarized waves continues to occur for the whole range considered for the temperature anisotropy.

In Fig. 5 we see the corresponding imaginary parts of the roots of the dispersion relation. For the range of temperature anisotropy considered the curves depicting z_i as a function of ϵ are quite similar to those obtained in the case of isotropic distributions, appearing in Fig. 4 of Ref. 21.

The range of values of anisotropy for which the two points of mode coupling occurs is explored in more details in Fig. 6. Figure 6(a) displays the case of $\Delta_i = \Delta_e = 2.75$, for which the coupling between the two modes do not occur. At $\Delta_i = \Delta_e = 3.00$ the two modes appear coupled for $\epsilon \simeq 0$, and also at $\epsilon \simeq 2.2 \times 10^{-5}$, as shown in Fig. 6(b). For larger anisotropy, the two points of mode coupling are moved toward higher values of dust density, as seen in Fig. 6(c). The corresponding values of the imaginary part z_i are shown in panels (d), (e), and (f), of Fig. 6. The two points of mode coupling continue to appear for considerably larger anisotropy, as seen in Fig. 7, which show the values of z_r and z_i for $\Delta_i = \Delta_e = 5.00$, 6.00, and 7.00.

Situations where only one of the plasma species features temperature anisotropy are shown in Figs. 8 and 9. Figures 8(a)–8(d) show the cases of $\Delta_e = 0.25$, $\Delta_e = 0.50$, $\Delta_e = 2.00$, and $\Delta_e = 4.00$, respectively, with $\Delta_i = 1.00$. It is seen that for isotropic ions the mode-coupling occurs for $T_\perp^e < T_\parallel^e$, and it is known that it occurs in the isotropic case.[21] However, for $T_\perp^e > T_\parallel^e$ the coupling disappears for sufficiently larger anisotropy of the electron temperature, as illustrated by Figs. 8(c) and 8(d).

The effect of anisotropy in the ion temperature appears to be more complex than the effect of the electron anisotropy. Figures 9(a) and 9(b) display the cases of $\Delta_i = 0.25$ and 0.50, respectively, for $\Delta_e = 1.00$. These figures show that the point of coupling between whistler and circularly polarized waves moves toward smaller values of dust density when the ion anisotropy is reduced. On the other hand, it moves again to higher values of dust density ϵ when the anisotropy is increased with $T_\perp^i > T_\parallel^i$, as shown by Figs. 9(c)–9(f). Similarly to what was seen in the case where the two species are anisotropic, in Fig. 4, another coupling point appears for sufficiently large anisotropy, as the ion anisotropy is increased, as shown in Figs. 9(d)–9(f). For the sake of economy of space, the curves for z_i corresponding to the cases appearing in Figs. 8 and 9 are not shown, since they do not introduce new qualitative features.

V. CONCLUSIONS

In the present paper we have used a kinetic description to analyze low frequency waves in dusty plasmas, taking into account the dust charge fluctuations, and taking into account the occurrence of temperature anisotropy in the distributions of electrons and ions. We have considered the case of propagation of waves exactly parallel to the external magnetic field, and bi-Maxwellian distributions for electrons and ions in the equilibrium situation. We have used this kinetic formulation to obtain and solve the dispersion relation, for a set of parameters typical of stellar winds. The emphasis has been on the mode-coupling phenomena which has been previously demonstrated to occur due to the presence of the dust particles,[21] and on the influence of the temperature anisotropy on the mode-coupling. The mode-coupling discussed in the present paper and in Ref. 21 has been obtained by a linear analysis of a kinetic theory, taking into account the collisional charging of the dust particles, for immobile dust particles, and is therefore not of the same nature of the coupling which has been reported in other studies, as in Refs. 18 and 25.

The results obtained show that the coupling between forward and backward propagating circularly polarized waves, which occur at moderately high dust densities, persists in the case of temperature anisotropy, both for the case of perpendicular temperature larger than parallel temperature and in the case of perpendicular temperature smaller than the parallel temperature. The coupling which occurs between waves in the whistler branch and waves in the branch of circularly polarized waves has been shown to remain in the case of perpendicular temperature smaller than the parallel temperature, and disappear in the case of perpendicular temperature moderately larger than parallel temperature, for the parameters considered. However, for larger anisotropy, the coupling reappears, and another coupling point appears for smaller dust density. The ultimate reason for these findings is the dependence of the charging currents on the anisotropy of the temperature of electron and ion distributions, as shown by Eq. (14). Moreover, as shown by Eq. (8), the anisotropy directly affects the evaluation of the inelastic collision frequency which plays a pivotal role in the dispersion relation, through the mechanism of collisional damping incorporated in a kinetic theory.

These features have been obtained using a given set of parameters, which has been sufficient to put in evidence a complex behavior of the dispersion relation. A complete parametric study was not made for the sake of economy of space.

Although it is known that the anisotropy of temperature may be a source of free energy and drive instabilities, we have chosen a set of parameters which do not satisfy the conditions for occurrence of instabilities in the case without dust. Accordingly no instabilities have been observed in the range of parameters investigated, either in the dustless limit or in the presence of dust. A study of instabilities driven by the anisotropy of temperatures, in the presence of dust, will be presented in a forthcoming publication.

ACKNOWLEDGMENTS

M.C.d.J. was supported by the Brazilian Agency Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) (Process Nos. 05/60160-5 and 06/52410-4). L.F.Z. and R.G. acknowledge support from Conselho Nacional de

Desenvolvimento Científico e Tecnológico (CNPq). The work was also supported by Fundação de Amparo à Pesquisa no Estado do Rio Grande do Sul (FAPERGS), Programa/ Convênio PRONEX/CNPq.

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