TECHNICAL NOTE

Generalized Abelès relations for an anisotropic thin film with an arbitrary dielectric tensor: comments

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The Cojocaru generalization of the 2×2 extended Jones matrix method, placed in a wider context of previous approaches to anisotropic optical thin films, is analyzed from a complementary perspective. This, contrary to initial belief, allows for a simple proof that one may include multiple reflections by taking into account total fields into the anisotropic film, and this therefore provides support for a more widespread use of the method. © 1998 Optical Society of America

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For more than a century, researchers have known about thin-film optical anisotropy relative to dichroism in metal coatings,[1] which later found application as metal polarizers in the near infrared.[2] In dielectric films, oblique deposition at high angles was needed to demonstrate significant phase-retarder operation under normal incidence of light.[3] This structure-related form birefringence[4] in films is described by the columnar model[5–7] and can be attributed to the self-shadowing effect during thermal deposition.[8] Other sources of optical anisotropy, such as crystalline structure or mechanical stress, may be present in a wide variety of materials, and therefore also influence performance of film devices.

Early theories for light propagation in anisotropic thin films were formulated for particular orientations, either of the principal axes of symmetry[9–11] or of light incidence.[12] They were followed by a general treatment for layered media,[13] although still without the convenience of associating each layer to a distinct entity in the multilayer calculation. This was achieved by 4×4 matrix formulations in the context of magnetic media,[14] liquid crystals,[15] and solid-state physics.[16] For optical multilayer films in which each layer is associated with an arbitrary dielectric tensor, the approach discussed in Ref. 14 was extended to produce a generalized transfer matrix.[5] In the isotropic limit, this 4×4 matrix breaks down into two 2×2 submatrices, which are exactly the well-known characteristic Abelès matrices[17] for the two polarization eigenstates.

More recently, from a 2×2 extended Jones matrix method,[18] Cojocaru reported a generalization to anisotropic optical thin films,[19] which, as discussed in Ref. 5, leads to Abelès relations in the isotropic limit.[17] Since the Abelès approach takes into account total fields and each field corresponds to all waves propagating in the same direction inside a film layer, in Ref. 19 it was inferred, without further argumentation, that this should be also the case for the 2×2 extended Jones matrix method. This is contrary to the statement that this method neglects multiple reflections in the initial extension by Yeh[18] and in subsequent developments.[20,21] Note that, unlike the isotropic case, it is not straightforward in anisotropic media that boundary conditions and propagation relations may imply any pattern regularity under multiple reflections and refractions, corresponding to a zigzag construction that has been treated with Fresnel relations.[22] To clarify the controversy, we present a complementary point of view, which allows for simple physical arguments.

Consider Fig. 1 in which all conventions from Ref. 19 are followed and extended with the subscript n, which refers to the number of reflections that occurred from the last departed interface. If we apply Stoke's reversibility principle[23] at the $x_3 = d$ interface, reversed $\mathbf{k}_\alpha^-$ would reflect onto reversed $\mathbf{k}_\alpha^+$ (and reversed $\mathbf{k}_\beta^-$ onto reversed $\mathbf{k}_\beta^+$). Whatever the film anisotropic structure, this is analogous to a second reflection at $x_3 = 0$ (i.e., once $\mathbf{k}_\alpha^-$ is reflected, resulting $\mathbf{k}_{2\alpha}^+$ from second reflection sees the same microstructure as $\mathbf{k}_\alpha^+$, and is thus parallel to it; the

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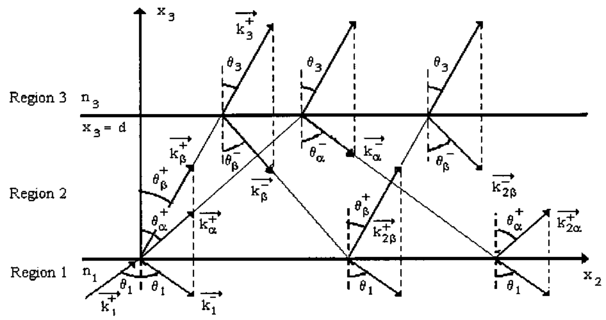

Fig. 1. Multiple reflection and refraction in anisotropic film of thickness d (Region 2), surrounded by isotropic incidence (Region 1) and substrate (Region 3) media. As in Ref. 19, wave vectors $\mathbf{k}_1{}^+$ and $\mathbf{k}_1{}^-$ correspond to the incident and the reflected waves at interface 1–2; $\mathbf{k}_\alpha{}^+$ and $\mathbf{k}_\beta{}^+$ correspond to forward-moving α and β waves after double refraction at interface 1–2; $\mathbf{k}_\alpha{}^-$ and $\mathbf{k}_\beta{}^-$ correspond to backward-moving α and β waves after single reflection at interface 2–3; $\mathbf{k}_3{}^+$ corresponds to the transmitted waves after refraction at interface 2–3. In addition, $\mathbf{k}_{2\alpha}{}^+$ and $\mathbf{k}_{2\beta}{}^+$ correspond to forward-moving α and β waves after second reflection at interface 1–2; $\mathbf{k}_{2\alpha}{}^-$ (not shown) and $\mathbf{k}_{2\beta}{}^-$ correspond to backward-moving α and β waves after second reflection at interface 2–3 (and so on). Note that all $\mathbf{k}_{n\alpha}{}^+$, $\mathbf{k}_{n\beta}{}^+$, $\mathbf{k}_{n\alpha}{}^-$, and $\mathbf{k}_{n\beta}{}^-$, where n is the number of multiple reflections from the last departed interface, are parallel to $\mathbf{k}_\alpha{}^+$, $\mathbf{k}_\beta{}^+$, $\mathbf{k}_\alpha{}^-$, and $\mathbf{k}_\beta{}^-$.

same applies with subscript β). By induction, this also occurs in the third reflection at $x_3 = 0$ and so on, and the same procedure applies for higher-order reflections at $x_3 = d$. With the same reasoning, by refraction, all the $\mathbf{k}_{n\alpha}{}^+$, $\mathbf{k}_{n\beta}{}^+$, $\mathbf{k}_{n\alpha}{}^-$, and $\mathbf{k}_{n\beta}{}^-$ results in reflected wave vectors parallel to $\mathbf{k}_1{}^-$ or transmitted wave vectors parallel to $\mathbf{k}_3{}^+$.

Therefore in this situation the multiple reflections consist of two sets of periodic zigzags in the anisotropic film, which can be represented by the first couple of zigzag periods for which the total fields are taken. In other words, the 2 × 2 extended Jones matrix calculation remains the same as originally performed, and it becomes just a matter of looking at the fields under consideration for single reflection as total fields. As a consequence, the method becomes even more comprehensive than it seemed at first, and the reduction to the total-field Abelès relations in the isotropic limit inevitably follows.

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