Correlation decay and partial coherence in nonlinear wave interactions

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In the present analysis we study the broad-band triplet interaction in regimes of large amplitudes. Linear response theories associated with nonlinear arguments are used to show that even though coherence of high-frequency modes is lost as one first encounters chaotic regimes, it can be restored as field amplitudes grow further. We discuss implications of the feature for fixed-phase interactions.

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I. INTRODUCTION

Nonlinear wave dynamics can be frequently modeled by the interaction of a few monochromatic modes [1]. In many cases the dynamics involves three higher-amplitude modes, in which case one refers to the model as the triplet wave interaction. Triplet interaction is one of the most significant forms of the wave interaction, presenting the most prominent nonlinear features in wave systems. It comes in several versions, and the one we are about to examine here is the conservative regime, where sources and dissipation are absent. A large variety of situations can be described by the conservative triplet interaction, for instance decay instabilities in laser-plasma interactions, three-mode interaction in optics, and others.

The classic pure triplet interaction has been shown to be integrable by explicit quadrature methods, and this happens so because one defines the classic model in terms of resonant modes that can be well described with first order time derivatives only. The modes just keep exchanging energy among themselves in a very regular and periodic fashion. A question analyzed by a number of researchers concerns the behavior of the system when each of the three single modes of the classic picture is replaced with a narrow comb of many modes [1–3]. One relevant result is that the dynamics can still preserve its simplicity if the mode amplitudes are large enough, such that they can create a strong mean field trapping all modes through a phase-locking mechanism [3]. In this case the various modes in each comb behave in a coherent way, i.e., deviating only slightly from their ensemble average. On the other hand, if the field amplitudes are small, phase locking is absent and coherence is lost. In this case the interaction is best described with the aid of random phase approximations.

More recent papers analyze the problem from a yet different perspective [4,5]. The issue addressed was the usage of first order time derivatives to model the interaction. When each comb has its dominant carrier frequency ω, eventual second order time derivatives acting on the appropriate fields can be approximately replaced according to the known slow modulational rule $\partial_t^2 \sim \pm 2i\omega\partial_t$, $i^2=-1$. This means a much slower process than the one scaled by the carrier frequency ω and it establishes, after all, what we understand by resonant interaction. Now since the typical frequencies of amplitude modulations tend to increase with field intensity, it is to be expected that the slow modulational approach outlined above ceases to be valid at some point. In the vicinity of the point where the modulational approach breaks down chaos is likely to appear due to resonance overlaps involving linear and nonlinear frequencies, and when chaos is present, the coherence tends to diminish. This is what we see as we increase the field intensities of the model analyzed in Ref. [5]. But if one increases the field intensities further, the coherence is partially recovered. This result was barely noticed in Ref. [5] and we explore the feature more deeply here given the current interest in the generation of coherent radiation [6]. With use of linear response methods conveniently associated with nonlinear techniques we show that partially coherent states emerge from the interaction with some waves preserving coherence and some not. The general rule is that coherence of higher-frequency waves is harder to breakdown, so one can imagine a situation where a laser packet traverses a nonlinear medium: even if the medium is heavily affected by the chaotic interaction, the laser may still be described in terms of fixed-phase modes, which is a desirable feature. All these points can be observed in the context of the Zakharov equations, for instance. These equations are known as one of the best models to describe the interaction of high- and low-frequency modes in a variety of nonlinear environment and they clearly show the transition from modulational to chaotic regimes [7–9]. Even in more complicated settings, knowledge of the coherent/incoherent behavior of Zakharov triplets allows one to determine whether or not modulational instabilities are present; it is known, for instance, that incoherence inhibits modulational instabilities [10,11].

The paper is organized as follows. In Sec. II we present the model and handle the governing equations to extract information on the transition from coherent to partially incoherent states; in Sec. III simulations are performed to confirm the analytical estimates; and in Sec. IV we draw our final conclusions.

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II. THE MODEL

In the case of the triplet interaction one has three combs of fields a_{pq}, $p=1,2,3$ with many modes each, $-(N-1)/2 \leqslant q \leqslant (N-1)/2$. We take N as odd to simplify mode counting. This way we have a central mode $q=0$ surrounded by a symmetric distribution at $q>0$ and $q<0$. Our results are of course independent of this particular choice for N. The interaction can be modeled by the following set [1–3,5]:

$$i\dot{a}_{1q}=\omega_{1q}a_{1q}+\frac{1}{N}\sum_{q',q''}a_{2q'}a_{3q''}, \tag{1}$$

$$i\dot{a}_{2q}=\omega_{2q}a_{2q}+\frac{1}{N}\sum_{q',q''}a_{1q'}a^{*}_{3q''}, \tag{2}$$

$$\ddot{a}_{3q}=-\omega_{3q}a_{3q}-\frac{k_3^2}{N}\sum_{q',q''}a_{1q'}a^{*}_{2q''}. \tag{3}$$

As explained in Ref. [1], Eqs. (1)–(3) are obtained when one writes down the appropriate Fourier analyzed governing equations and averages over large groups of modes in each comb. The assumption is that individual modes in each comb cannot be well resolved spatially, but time is not restricted and the full difference in frequencies is still contemplated by the linear terms of the equations. This is the appropriate procedure if one wishes to study the transition from narrow combs to the strictly pure three-mode interaction. The first two combs describe the amplitude of high-frequency fields like lasers or Langmuir plasma waves, and the latter describe full low-frequency fields.

$$\omega_{pq}\equiv k_{pq}^2, \quad k_{pq}\equiv k_p+q\Delta_k,$$

where Δ_k is the spectral distance of contiguous modes in any comb and $\Delta\equiv(N-1)\Delta_k/2$ is the half width of the combs which we assume to be the same. It is convenient to introduce $\Sigma_q a_{1q}\equiv\rho$. When $N=1$, we retrieve the pure case analyzed in Refs. [4] and [5]. We call the $N=1$ case the synchronized manifold since this situation would represent what happens should all modes oscillate coherently. What has been found in those two cited papers is that if $\rho\gtrsim 1$, the dynamics of the synchronized manifold is chaotic due to the presence of second order time derivatives in the equation controlling a_3. As a matter of fact, we observe here that the Lyapunov exponent σ scales like $\sigma\sim\ln\rho$ in the range $1<\rho<100$. Then, initial studies published in Ref. [5] show that in these chaotic cases coherence is lost if we let $N>1$ and $\Delta\neq 0$ for $\rho\sim 1$—which is expected in view of the presence of chaotic activity in the synchronized manifold—but is at least partially recovered if ρ is further increased to $\rho\gg 1$, a somewhat unexpected behavior. Due to its physical interest, this higher-intensity range is the subject of analysis of the present paper. To see what happens we now introduce our formalism in terms of the concepts of average and fluctuation for an arbitrary quantity g_{pq} within each comb: $\overline{g_{pq}}\equiv\Sigma_q g_{pq}/N$ and $\delta g_{pq}\equiv g_{pq}-\overline{g_{pq}}$. Then if one defines $\overline{a_{pq}}\equiv X_p$ and $\overline{\omega_{pq}}\equiv\overline{\omega_p}$, and recursively $s_{pq,n}\equiv\delta\omega_{pq}\delta s_{pq,n-1}$ with $s_{pq,0}\equiv\delta a_{pq}$ and $s_{pn}\equiv\overline{s_{pq,n}}$, one arrives at the following infinite cumulant hierarchy:

$$i\dot{X}_1=\overline{\omega_1}X_1+s_{1\,1}+N\,X_2\,X_3,$$
$$i\dot{s}_{1\,n}=\overline{\omega_1}\,s_{1\,n}+s_{1\,n+1}+\gamma_1^{(n+1)}X_1, \tag{4}$$

$$i\dot{X}_2=\overline{\omega_2}X_2+s_{2\,1}+N\,X_1\,X_3^{*},$$
$$i\dot{s}_{2\,n}=\overline{\omega_2}\,s_{2\,n}+s_{2\,n+1}+\gamma_2^{(n+1)}X_2, \tag{5}$$

$$\ddot{X}_3+\overline{\omega_3}\,X_3=-s_{3\,1}-N\,X_1\,X_2^{*},$$
$$\ddot{s}_{3\,n}+\overline{\omega_3}\,s_{3\,n}=-s_{3\,n+1}-\gamma_3^{(n+1)}X_3. \tag{6}$$

for $n=1,2,3,4,\ldots$, etc. $\gamma_p^{(n+1)}\equiv\overline{[\delta\omega_{pq}\delta:]^n\delta\omega_{pq}}$, where "$\delta$:"calculates the fluctuation of everything on its right side—$\gamma_p^{(2)}\equiv\overline{[\delta\omega_{pq}\delta:]\delta\omega_{pq}}=\overline{(\delta\omega_{pq})^2}$, for instance. This way of displaying the multitude of equations we have to deal with—in terms of averages and fluctuations of the pertinent quantities—is equivalent to the projector method proposed by Martins and Mendonça some time ago [2]. The present alternative is more adequate for the purposes we have in mind, since it introduces all the fluctuations in terms of linear processes governed by differential equations. Let us sketch what can be expected in regimes of relatively high-power waves, for instance $\rho\gtrsim 10$; for further purposes, note that $X_p\sim\rho/N$. Fields of this order of magnitude look big, but in fact result from multiplying factors involving the ion-to-electron mass ratio in the Zakharov equations and can be adequately described with the nonlinearities included in the original model (1)–(3) as explained in Ref. [10]. As a matter of fact our normalized field ρ can be written in the form $\rho^2=(3m_i/4m_e)W$, where W denotes the ratio of electric to thermal energy, $W\equiv\epsilon_0E^2_{physical}/4nk_BT_e$ with n as the particle density, $m_{i,e}$ as the ion (electron) mass, k_B as the Boltzman constant, and T_e as the electron temperature. For $\rho=10$ and $m_i/m_e\sim 10^3$ this yields $W\sim 0.1$, which falls well inside the validity region of the Zakharov equations. Considering typical physical parameters of nonrelativistic beam experiments, $n\sim 10^8$ cm^{-3}, $T_e\sim 1$ eV, $m_i/m_e\gtrsim 10^3$ for argon (hydrogen and helium have been used for these experiments as well), and the electric field of injected Langmuir waves $E_{physical}\sim 10$ V/cm, one has indeed $W\sim 0.1$ or even larger [10,12]. Returning to our arguments, as a first step one assumes initially coherent modes with small values for the various fluctuations $s_{p\,n}$. Then consider the first equations of sets (4) and (5) redefining $X_j=\hat{X}_je^{-i\,\overline{\omega_j}t}$, $j=1,2$, such that one can write

$$i\dot{\hat{X}}_1=N\,\hat{X}_2\,X_3\,e^{i\,\omega_{12}t},\quad i\dot{\hat{X}}_2=N\,\hat{X}_1\,X_3^{*}\,e^{-i\,\omega_{12}t}, \tag{7}$$

$\omega_{12}\equiv\overline{\omega_1}-\overline{\omega_2}$. Equations (7) suggest that variables $\hat{X}_1$ and $\hat{X}_2$ are oscillating on a fast time scale, given the magnitude of the time derivatives when the fields are large. Equations (6) on its turn suggest that X_3 oscillates on a much slower time scale essentially determined by the average frequency

$\sqrt{\overline{\omega}_3}$; even if the driver $X_1X_2^*$ contains high-frequency components, we expect that the main spectral contribution comes from ponderomotive forces at low frequencies of the order of $\sqrt{\overline{\omega}_3}$. The slowness of X_3 allows us to see Eq. (7) as an adiabatic process in which both X_1 and X_2 oscillate with a slowly varying frequency $\Omega_1 \sim \rho$, which is large. Fluctuations $s_{j\,1}$, if now reconsidered, can be treated similarly and shown to scale like $s_{j\,1}/\sqrt{\gamma_j^{(2)}} \sim (\sqrt{\gamma_j^{(2)}}/\Omega_1)X_j$, which is much smaller than X_j simply because the width of the packet is small and Ω_1 is large. Note that we divide s by $\sqrt{\gamma}$ in order to compensate for the $\delta\omega$ factor contained in the definition of s; in general, if $|s/\sqrt{\gamma}| \sim |X|$ fluctuations are significative and when $|s/\sqrt{\gamma}| \ll |X|$ they are not; we shall take the quantity $|s_{p\,1}/(\sqrt{\gamma_p^{(2)}}X_p)|$ as a measure of coherence for each comb. As an alternative view on the meaning of $s/(\sqrt{\gamma}X)$, consider the equation for δa from Eq. (1) neglecting small terms involving $\delta\omega\,\delta a$ above resonance, i.e., neglecting higher-order cumulants (and omitting indexes). Then, $i\dot{\delta a} \sim \overline{\omega}\delta a + \delta\omega X$ which can be written as $(\omega - \overline{\omega})\delta a \sim \delta\omega X$, where we assume harmonic behavior $\delta a \sim e^{-i\omega t}$ to estimate orders of magnitude. One can express $\delta\omega$ in terms of δa or δa in terms of $\delta\omega$ and have $s \equiv \overline{\delta\omega\,\delta a} = [(\omega - \overline{\omega})/X^*]\overline{|\delta a|^2} = [X/(\omega - \overline{\omega})]\gamma$, respectively; this yields $|s|^2 = \gamma\overline{|\delta a|^2}$ or $|s|/(\sqrt{\gamma}|X|) = \sqrt{\overline{|\delta a|^2}}/|X|$, which is equal to the normalized fluctuation in the respective comb.

Returning to our discussion, since the dynamics of the first two combs occurs away from any resonance and with little fluctuation, we do not expect coherence decay there. But when we examine the third comb the reasoning changes. The fact is that the third comb is described by driven harmonic oscillators which in principle cannot be approximated by modulational assumptions given the relatively high field intensities. Even if $s_{3\,n}$ are small initially, we expect that they will be driven resonantly to large values by the $\gamma_3^{(n+1)}X_3$ terms which already contain spectral contributions at the average frequencies $\pm\sqrt{\overline{\omega}_3}$. From this perspective one can use linear response theories to estimate the rate with which the coherent energy dissipates here. One simply represents the solution for X_3 in the causal form

$$X_3(t) = \int_0^{+\infty} g(\tau)[F_{12}(t-\tau) + f(t-\tau)]d\tau \qquad (8)$$

with $F_{12}(t) \equiv NX_1(t)X_2(t)^*$ and $f(t)$ as δ functions implementing the initial conditions on X_3. The propagator $g(t)$ is to be determined from all resonances arising from the linearly coupled set for all $s_{3\,n}$ and X_3, when $F_{12} \to 0$. If all $s_{3\,n}$ are excluded, the resonances are located at $\pm\sqrt{\overline{\omega}_3}$; when $s_{3\,1}$ is included but the remaining $s_{3\,n}$ $n \geqslant 2$ are not, the resonances appear at $\pm\sqrt{\overline{\omega}_3 \pm \sqrt{\gamma_3^{(2)}}}$, and so on. If $\gamma_3^{(2)}$ is small, all poles are thus roughly distributed around the main frequency $\sqrt{\overline{\omega}_3}$ over a region of size $\sqrt{\gamma_3^{(2)}/\overline{\omega}_3}$. The imaginary part $Img(\omega)$ of the Fourier transform of $g(t)$ contains contributions from all poles in the form $Im\,g(\omega) \sim \Sigma_{res}K_{res}\delta(\omega - \omega_{res})$, where K_{res} are constants associated with the resonances, and $g(t)$ can be obtained from the formula [13]

$$g(t) = \frac{2}{\pi}\int_0^{\infty} Img(\omega)\sin(\omega t)d\omega. \qquad (9)$$

The collection of all poles can be replaced with a continuous distribution provided $\int_0^{+\infty}\omega Im\,g(\omega)\,d\omega = \pi/2$, which is a high-frequency summation rule. The problem is thus, how to model the distribution $Img(\omega)$? We adopt the simplest option here and take $Img(\omega) = K$ if $-\sqrt{\gamma_3^{(2)}} \leqslant \omega^2 - \overline{\omega}_3 \leqslant \sqrt{\gamma_3^{(2)}}$ with K as a constant, and $Img(\omega) = 0$ otherwise. This form does not include higher-order corrections $\gamma_3^{(n)}$ $n \geqslant 3$, but is analytically treatable and does observe the crucial fact that one has large number of resonances acting upon the system. If one refines the spectral description with higher-order γ's, the linear spectrum of Eq. (3) is reobtained. We shall see that the approximate model is adequate for our intentions and that it is the numerous resonances that cause decay. From the summation rule $K = 1/\sqrt{\gamma_3^{(2)}}$ and from Eq. (9) one concludes that $g(t)$ decays in time; for earlier times and relatively small γ's, g behaves as $g \sim \sin(\sqrt{\overline{\omega}_3}t)(1 - \gamma_3^{(2)}t^2/6)$, and for later times it goes like $(\gamma_3^{(2)}t^2)^{-1/2}$ times an oscillating function. This is not an exponential decay, but nevertheless shows that the system preserves a past memory for prior times up to the correlation time $\tau_{cor} \sim 1/\sqrt{\gamma_3^{(2)}}$ and that coherent energy decoheres within the same time scale when the drive $F_{12}(t)$ is absent. When the drive is active, it is harder to make precise predictions, but in general terms what we expect to see is: (i) initially, transient chaos while coherence is still present in X_3; (ii) after a time scale comparable to τ_{cor}, decay of X_3 towards an equilibrium or quasiequilibrium state determined by the ponderomotive drive arising from F_{12}. All the while, we expect that coherence be preserved in the first two combs. Note that our results contrast with those obtained by Robinson and Drysdale [3] since even for large fields we do expect to see comb $p=3$ loose coherence due to the resonant behavior introduced by the second derivatives. Reference [3], on the other hand, shows that if all modes are first order in time, large amplitudes imply phase locking and coherence in *all* combs.

III. SIMULATIONS

We are now in position to investigate all those issues numerically and see if the predictions are correct. The following conditions are given at $t=0$, which are equilibrium conditions for the pure triplet when modulational approximations are employed, $a_{1q} = a_{3q} = \rho_0/\sqrt{3}N$, $a_{2q} = \sqrt{2}\rho_0/\sqrt{3}N$, and $\dot{a}_{3q} = 0$ for any q, where ρ_0 is a control parameter measuring initial field amplitudes. The central modes of each comb are taken in the form $k_1 = k_3 = 1$ and $k_2 = 0$, the triplet resonance $\omega_1 = \omega_2 + \omega_3$ is satisfied this way. In addition, $\rho_0 = 10$, $N = 401$, and $\Delta = 0.1$, which implies $\gamma_1^{(2)} = \gamma_3^{(2)} \equiv \gamma^{(2)} \approx 0.013$. We first look at X_3 in Fig. 1 and compare our analytical model (8) and (9) with full simu-

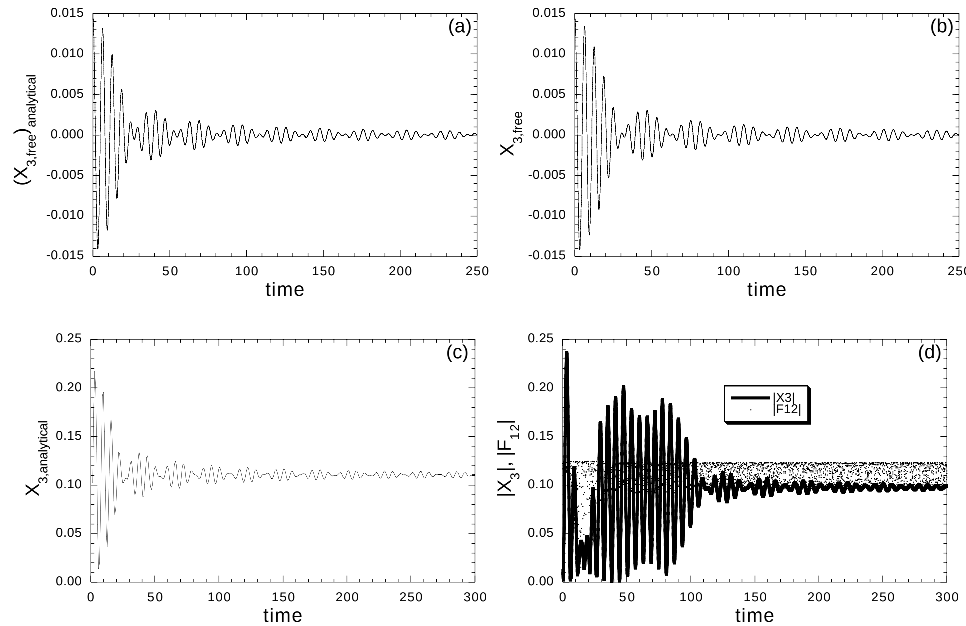

FIG. 1. Comparing the analytical model with simulations. Analytical model and simulations with $F_{12}=0$ in panels (a) and (b), respectively; analytical model and simulations with F_{12} active in panels (c) and (d), respectively.

lations. In panels (a) and (b) we set $F_{12}\rightarrow 0$, hence $X_{3,free}$, to test the approximations leading to our $g(t)$. Estimate in (a) and simulations in (b) agree well in terms of decay times and the form with which coherence decays. The plots display the diffractive pattern produced by an infinity of modes at least up to long times where discrete effects arise due to our using of a finite number of modes. We have chosen a sufficiently large N so our runs are not affected by discreteness. Then in panels (c) and (d) we compare the analytical model with full simulations when F_{12} is active. In the analytical model, pictured in panel (c), we consider an average $NX_1X_2^*\rightarrow\sqrt{2}\rho_o^2/3N$, which is consistent with the initial conditions. We see from the figures that in both cases X_3 decays. However, decay in the full simulations of panel (d) is slightly postponed due to the fact that here one has to go through initial transient chaos before reaching the steady state. Apart from that, the results are once again equivalent. Note as well that asymptotically X_3 is comparable to the average of $NX_1X_2^*$, as expected from ponderomotive force considerations in our case where $\overline{\omega}_3\sim 1$. It is worth mentioning that the type of decay we estimate and observe here belongs to the same general type of incoherent decay numerically observed in a recent paper [14]. Next we examine the coherence of the $p=1$ and $p=3$ combs, being the analysis of the

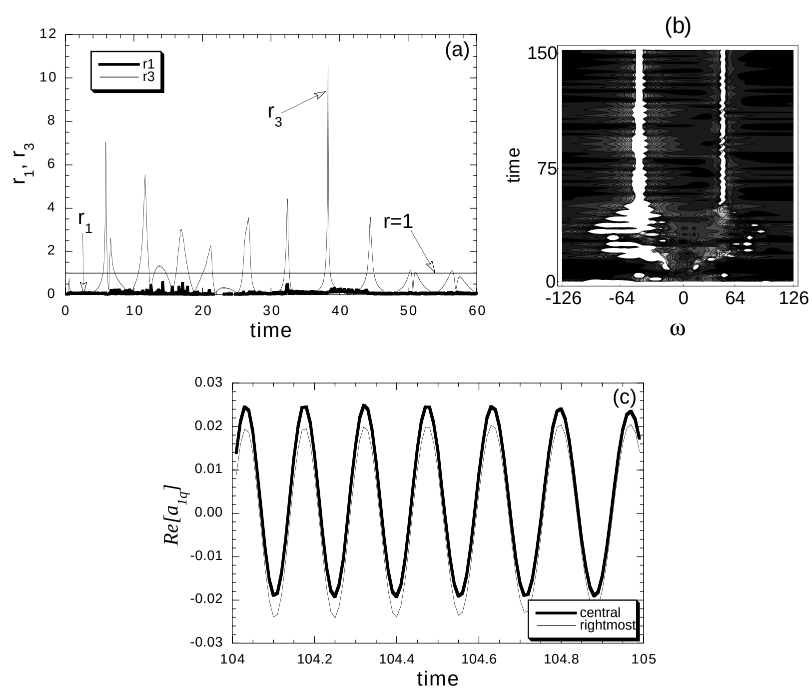

FIG. 2. Coherence analysis. Relative fluctuational levels $r_1(t)$ and $r_3(t)$ in panel (a), contour plots of $|\tilde{F}_{23}(t,\omega)|$ in panel (b), and explicit mode comparison in panel (c).

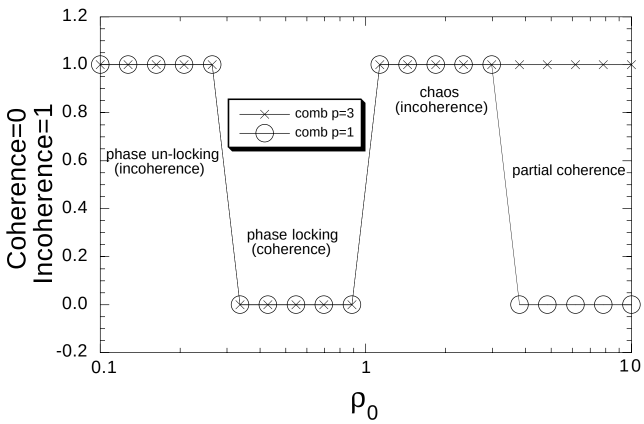

FIG. 3. Binary bifurcation diagram with basis on the control parameter ρ_0. Level '1' denotes incoherence, and level "0" denotes coherence. Symbols indicate where simulations were actually performed.

$p=2$ comb similar to the analysis of the first. Figure 2(a) compares the relative quantities $r_1 \equiv |s_{1\,1}|/(\sqrt{\gamma^{(2)}}|X_1|)$ and $r_3 \equiv |s_{3\,1}|/(\sqrt{\gamma^{(2)}}|X_3|)$. We see that relative fluctuations in the first comb are always smaller than unity while fluctuations in the third comb can attain much larger values. In other words, coherence is destroyed for $p=3$ but remains solid and steady as far as the first and second combs are considered. To examine the behavior of the high-frequency spectral content of the first comb as a function of the slow time over which the dynamics of the third comb develops, we employ a wavelet transform. Specifically, we wavelet transform the nonlinear driving term $F_{23} \equiv N X_2 X_3$ acting on the first comb and display the resulting time dependent spectral distribution in Fig. 2(b) with higher intensities represented by brighter shades. The spectral distribution indicates that the driver operates mostly at high frequencies with little content in low-frequency bands. This also agrees with the fact that all $\hat{s}_{1\,n} \equiv e^{i\bar{\omega}_1 t} s_{1\,n}$, which have resonant frequencies centered at $\omega=0$ and extending only up to $\sqrt{\gamma^{(2)}}$, are slightly excited. Wavelet analysis is defined in the form

$$\tilde{F}_{23}(t,\omega) \equiv (1/T) \int_{t-T/2}^{t+T/2} [e^{i\bar{\omega}_1 t'} F_{23}(t')] e^{-i\omega t'} dt'$$

with $T=2^4 T_{fast}$, T_{fast} as a quantity with the order of magnitude of the characteristic time scale associated with high-frequency oscillations: $T_{fast} \equiv 1/\rho_0$. Figure 2(c) compares the dynamics of the rightmost and central modes of comb $p=1$, Re$[a_{1\,\Delta/\Delta_k}]$, and Re$[a_{1\,0}]$, respectively. It shows in a more explicit way the high degree of coherence present in the high-frequency modes.

Perhaps, we should end the work giving a brief account on the behavior of the degree of coherence as a function of field strength ρ_0, with ρ_0 varying from very small to the large values employed in this paper. To do that, a binary diagram is made where on the vertical axis we simply record the digit "1" if it happens that $r_p = |s_{p\,1}/(\sqrt{\gamma^{(2)}} X_p)|$ becomes larger than unity during a run of length t_{max}. If r_p always remains smaller than unity, we record digit "0" on the vertical axis; $t_{max}=100$ is taken, but other values yield similar figures with little shifts in the transitions from $1 \rightarrow 0$ and $0 \rightarrow 1$. The result is displayed in our last figure, Fig. 3. Considering combs $p=1$ and $p=3$ as previously (we recall that the second comb is similar to the first), one clearly sees the various regimes detected in the present and earlier works. For small values of ρ_0, one has incoherence and thus large fluctuations $s_{p\,1}$ [3]. As ρ_0 increases one goes through a region where fluctuations are small—this is where phase locking is efficient to maintain coherence [3,5]. Larger values of ρ_0 place the system within chaotic regions where coherence is again lost [5] and yet larger values of ρ_0 finally restore coherence for the first comb, leaving the third comb in its incoherent state; this is the regime of partial coherence analyzed in the present paper.

IV. FINAL CONCLUSIONS

To conclude, we have analyzed the broad-band triplet interaction in regimes of large amplitudes. Linear response theory associated with nonlinear techniques was used to show that partially coherent states emerge from the interaction with some waves preserving coherence and some not. The implications of these results for all matters concerning tuning and coherence of the high-frequency subsystem is of central relevance. If one thinks of a laser interacting with a nonlinear medium, the model suggests that while the medium itself may undergo a number of random alterations as the interaction progresses, the laser modes will remain coherent. Not only that, the excess coherent energy driving the system into chaos is damped out and the final high-frequency modes become regular, as well as coherent. The same kind of comment is true for Langmuir plasma waves interacting with ion waves: the subsystem formed by the ion waves looses coherence, but the plasma waves emerge from the interaction with coherence untouched, and this may be of importance as coherence favors modulational instabilities. The method of analysis seems to be practical, useful, and shall be employed for further analysis of the problem exposed here.

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