[11]S. Hayakawa, H. Okuda, Y. Tanaka, and Y. Yamamoto, Progr. Theoret. Phys. (Kyoto) Suppl. No. 30, 153 (1964).

[12]Compton loss saturates at some energy, depending on the photon energy, but for the 3°K radiation this saturation energy is very much higher than a few hundred GeV and the Compton loss due to starlight is anyway a small fraction of the total loss if the electrons spend most of their life outside the galactic disk.

[13]J. E. Felton, Astrophys. J. 145, 589 (1966).

[14]N. Durgaprasad, in Proceedings of the International Conference on Cosmic Rays, Jaipur, India, 1963, edited by R. R. Daniel et al. (Commercial Printing Press, Ltd., Bombay, India, 1964-1965) Vol. 3, p. 17.

[15]R. R. Daniel and N. Durgaprasad, Progr. Theoret. Phys. (Kyoto) 35, 36 (1966).

[16]The large uncertainty in the cross sections which are generally used is exemplified by the following: (i) A direct measurement of the cross sections for reaction $p + C^{12} \rightarrow Li^{6,7}$, which has become available recently, gives a value of ~ 18 mb in the range 50 to 500 MeV of proton energy [R. Bernas, M. Epherre, E. Gradsztajn, R. Klapisch, and F. Yiou, Phys. Letters 15, 197 (1965)] whereas G. D. Badhwar, R. R. Daniel, and B. Vijayalakshmi [Progr. Theoret. Phys. (Kyoto) 28, 607 (1962)] had used for this a value of 40 mb. (ii) Badhwar, Daniel, and Vijayalakshmi (loc. cit.) use a value of 48 mb for $p + C^{12} \rightarrow B^{10,11}$, whereas the experiment from which they have derived most of the information of C^{12} fragmentation indicates a value of 118 ± 13 mb; they argue that this value is too high to be correct.

[17]W. R. Webber, J. F. Ormes, and T. Von Rosenvinge, Proc. Int. Conf. Cosmic Rays (London) 1, 407 (1965).

[18]Badhwar, Daniel, and Vijayalakshmi, Ref. 16.

[19]The implications of the arguments presented in this paper for various models of cosmic-ray production and storage will be discussed by Daniel and Stephens in a forthcoming detailed paper (to be published).

SPONTANEOUSLY BROKEN SYMMETRIES AND CURRENT CONSERVATION*

Th. A. J. Maris and Gerhard Jacob

Instituto de Física and Faculdade de Filosofia,
Universidade Federal do Rio Grande do Sul, Pôrto Alegre, Brasil

(Received 28 November 1966)

It is often argued that spontaneously broken symmetry (SBS)[1-3] theories cannot explain the approximate symmetries of hadron[4,5] and high-energy lepton[6,7] physics because the expected conserved currents (and their Goldstone bosons or long-range interactions) are not observed. The purpose of this Letter is to show that this argument is probably incorrect because it appears that a SBS current is in general not conserved.

We consider the quantum electrodynamics of "electrons" and "muons" without bare masses.[8-11] In a two-component notation for the electron-muon field, this theory is invariant under the group $SU(2) \otimes SU(2) \otimes U(1)$, containing the $\vec{\tau}$, $\gamma^5\vec{\tau}$, and γ^5 gauge transformations.[10] In the Landau gauge, the lowest order Dyson-Schwinger equation for the fermion propagator has finite solutions for which nearly the whole group, the τ_3 symmetry excepted, is spontaneously broken (in agreement with the experimental situation) and for which the mass renormalizations are finite.[10-13] As in the present approximation all renormalizations are finite,[12] we will use the unrenormalized coupling constant and propagators.

Because the field operators are unbounded, one has to define the currents by a limiting procedure.[14,15] We smear out the lepton field operators in the equal-time plane:

$$\psi^{(\epsilon)}(x) = \int f(\vec{x} - \vec{x}')\psi(\vec{x}', x_0)d^3x' \qquad (1)$$

with

$$\int f(\vec{y})d^3y = 1,$$

and take

$$f(\vec{y}) = \epsilon^{-3}\pi^{-\frac{3}{2}}e^{-\vec{y}^2/\epsilon^2},$$

$$\tilde{f}(\vec{p}) = \int e^{i\vec{p}\cdot\vec{y}}f(\vec{y})d^3y = e^{-\vec{p}^2\epsilon^2/4}. \qquad (2)$$

The details of the smearing function are unimportant, as long as it is spherically symmetric and sufficiently localized. Only relations which remain finite and become independent of ϵ for sufficiently small ϵ will have a physical significance.

Consider the lepton-changing current

$$j_\nu^{(\epsilon)+}(x) = i\bar{\psi}_m^{(\epsilon)}(x)\gamma_\nu\psi_e^{(\epsilon)}(x), \qquad (3)$$

m and e denoting muon and electron. The matrix element of the divergence between momentum eigenstates of the muon and electron is

$$\langle m,p\,|\,\partial^{\nu}j_{\nu}^{(\epsilon)+}(x)\,|\,e,q\rangle = e^{i(q-p)x}\langle m,p\,|\,[\partial^{\nu}j_{\nu}^{(\epsilon)+}(x)]_{x=0}\,|\,e,q\rangle, \tag{4}$$

$$i\langle m,p\,|\,[\partial^{\nu}j_{\nu}^{(\epsilon)+}(x)]_{x=0}\,|\,e,q\rangle = \int d^{3}x''\,d^{3}x'\,\tilde{f}(\vec{x}'')f(\vec{x}')\langle m,p\,|\,[A^{\nu}(x'')-A^{\nu}(x')]ie\bar{\psi}_{m}(x'')\gamma_{\nu}\psi_{e}(x')\,|\,e,q\rangle. \tag{5}$$

We will show in a perturbation theory of infinite order that the right-hand side of Eq. (5) for $\epsilon \to 0$ has a nonzero limit. A similar (but exact) conclusion has been obtained for the Thirring model in another connection[15]; the possible relevance of that result for SBS theories was pointed out by Katz and Frishman.[16]

After applying the reduction technique, performing the perturbation expansion of the resulting T product, and summing over infinitely many lepton self-energy terms, one may represent the result of Eq. (5) by the graphs in Fig. 1.

In these graphs the photon propagator and vertices are the free ones, and the Landau gauge is used. The fermion propagator is a non-normal solution[12,13] of the lowest order Dyson-Schwinger equation and has the form

$$S_{\gamma}^{-1}(p) = i\not{p} + m_{\gamma}(p^{2}), \quad r = e \text{ or } m. \tag{6}$$

The neglected graphs are of fourth and higher order. Graph A corresponds to

$$-\frac{i\alpha}{4\pi^{3}}\bar{u}_{m}(p)\int \tilde{f}(\vec{p}-\vec{k})\tilde{f}(\vec{q}-\vec{k})D_{\mu\nu}(k)\gamma^{\mu}S_{e}(q-k)\gamma^{\nu}d^{4}k\,u_{e}(q)$$

$$=-\frac{i\alpha}{4\pi^{3}}\bar{u}_{m}(p)e^{-\epsilon^{2}/8(\vec{p}-\vec{q})^{2}}\int e^{-\epsilon^{2}t^{2}/2}D_{\mu\nu}\left(\frac{p+q}{2}-t\right)\gamma^{\mu}S_{e}\left(t-\frac{p-q}{2}\right)\gamma^{\nu}d^{4}t\,u_{e}(q). \tag{7}$$

In Ref. 12 it is shown that the integral in Eq. (7), but <u>without</u> the convergence factor, is conditionally convergent, i.e., one may not freely translate the integration variable. Therefore, the argument in the convergence factor defines the integral. As a generalization of a result of Ref. 12, one may prove the relation

$$\lim_{\epsilon \to 0}\frac{-i\alpha}{4\pi^{3}}\int e^{-\epsilon^{2}t^{2}/2}D_{\mu\nu}(b-t)\gamma^{\mu}S_{e}(t-a)\gamma^{\nu}d^{4}t = i\frac{3\alpha}{8\pi}\not{d} + m_{e}[(b-a)^{2}]; \tag{8}$$
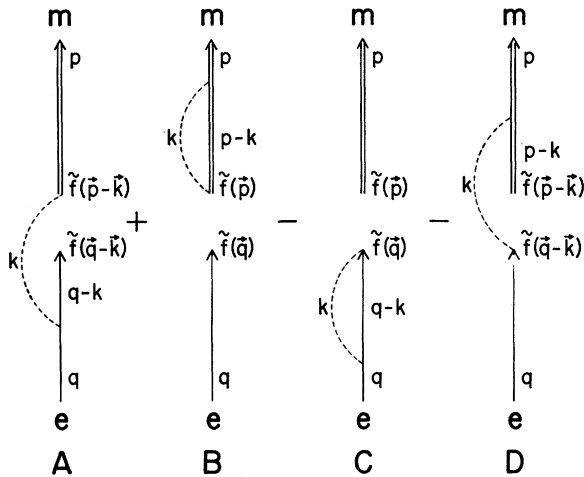

FIG. 1. Feynman graphs corresponding to Eq. (5); the multiplying functions $\tilde{f}$ are defined in Eq. (2).

application to the integral in expression (7) gives the value $i(3\alpha/16\pi)(\not{p}-\not{q})+m_{e}(q^{2})$.

In graph C the f functions do not contain the integration variable. One obtains the usual self-energy integral, symmetrically integrated over the fermion momentum, as is necessary for the consistency of the theory[12] and as follows also from a definition of the electric current in the field equations by a similar limiting procedure as the one of Eq. (3). This same limit, defining the theory, occurs in all graphs and has to be performed <u>before</u> the limiting procedure in the other current operators (representing measurements which can only be made on the system after it has been defined). As we have to keep ϵ finite up to the end of the calculation, the $\tilde{f}$'s determine the cutoff whenever they depend on the integration variable.

With $a = 0$, $b = q$, one finds for graph C the self-energy $m_e(q^2)$ which cancels the diagonal part of graph A. Treating graphs B and D in the same way, one finally obtains

$$\lim_{\epsilon \to 0} \langle m, p | \partial^\nu j_\nu^{(\epsilon)+}(x) | e, q \rangle = \frac{3\alpha}{8\pi} e^{i(q-p)x} \bar{u}_m(p)(\not{p}-\not{q})u_e(q) = i\frac{3\alpha}{8\pi} e^{i(q-p)x}(M_e - M_m)\bar{u}_m(p)u_e(q). \tag{9}$$

The matrix elements of the other SBS currents may be calculated in a similar way; for example,

$$\lim_{\epsilon \to 0} \langle e, p | \partial^\nu j_\nu^{(\epsilon)5}(x) | e, q \rangle = i\frac{3\alpha}{8\pi} e^{i(q-p)x} 2M_e \bar{u}_e(p)\gamma^5 u_e(q), \tag{10}$$

$$\lim_{\epsilon \to 0} \langle m, p | \partial^\nu j_\nu^{(\epsilon)5+}(x) | e, q \rangle = i\frac{3\alpha}{8\pi} e^{i(q-p)x}(M_e + M_m)\bar{u}_m(p)\gamma^5 u_e(q). \tag{11}$$

One may note that if the mass relations are the ones of a certain unbroken symmetry, the one-particle matrix elements of the associated current are conserved and vice versa. Except for a renormalization factor, the calculated divergences equal the ones of the corresponding currents of free, but massive, lepton fields.

The considered mechanism of current nonconservation has some similarity with the one of an extended SBS model of the Heisenberg ferromagnet,[17] where current loss to infinity in space is caused by a static long-range interaction [with the momentum singularity $\delta(p)$ at $p_0 = 0$]; in the present theory the current leaks to the region of infinite momenta through the singularity $\delta(x)$ at $x_0 = 0$ of the commutation relations.

[1]W. Heisenberg, Rev. Mod. Phys. 29, 269 (1957).

[2]Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961).

[3]R. Haag, Nuovo Cimento 25, 287 (1962).

[4]M. Gell-Mann, Phys. Rev. 125, 1067 (1962).

[5]Y. Ne'eman, Nucl. Phys. 26, 222 (1961).

[6]G. Feinberg and F. Gürsey, Phys. Rev. 128, 378 (1962).

[7]H. A. Kastrup, Phys. Letters 3, 78 (1962).

[8]M. Baker and S. L. Glashow, Phys. Rev. 128, 2462 (1962).

[9]R. Haag and Th. A. J. Maris, Phys. Rev. 132, 2325 (1963).

[10]Th. A. J. Maris, V. E. Herscovitz, and G. Jacob, Nuovo Cimento 34, 946 (1964); 40, 214 (1965).

[11]R. Arnowitt and S. Deser, Phys. Rev. 138, B712 (1965).

[12]K. Johnson, M. Baker, and R. Willey, Phys. Rev. 136, B1111 (1964).

[13]Th. A. J. Maris, V. E. Herscovitz, and G. Jacob, Phys. Rev. Letters 12, 313 (1964).

[14]J. Schwinger, Phys. Rev. Letters 3, 296 (1959).

[15]K. Johnson, Phys. Letters 5, 253 (1963).

[16]A. Katz and Y. Frishman, to be published.

[17]Th. A. J. Maris, Nuovo Cimento 45A, 223 (1966).